Modeling of Network Dynamics: From Dynamic Nodes to Dynamic Structure

Rudolf Kulhavý

Institute of Information Theory and Automation
Academy of Sciences of the Czech Republic
Pod vodárenskou věží 4, 182 08 Prague, Czech Republic
Tel: +420 26605 2061 Fax: +420 26605 2068
E-mail: kulhavy@utia.cas.cz

ABSTRACT. The paper presents a conceptual framework for modeling of dynamic systems with a *variable structure*. Practical motivation comes from analysis of *value networks* – complex systems of stand-alone business entities that bond together, more or less tightly, through exchange of goods, services, and money. The existence of bonds between certain businesses, and their strength, are determined by the relative performance of individual businesses in terms of a value they add along a network. The collection of effective bonds defines the structure of a network. As the performance of businesses changes over time, so does the network structure. Better performing nodes are more likely to get bonded, and nodes with stronger bonds are better positioned to further improve their performance. Having an operational model of the value network behavior, with the capability to predict changes in the network structure as a result of changes in the individual node performance, is a crucial prerequisite for effective management of the network performance. The proposed model combines probabilistic graphical modeling and stochastic system dynamics to model the network structure and individual node performance, respectively.

KEYWORDS. Networked systems; network dynamics; system dynamics; variable structure; Markov processes; Markov random fields; Bayesian inference; sequential Monte Carlo methods.

Understanding Value Networks

Unbundling of Traditional Enterprise

The market demand for products and services becomes increasingly complex in most industries. The business organizations face extensive variety, nonlinear dynamics and significant uncertainty associated with changing customer preferences. The conventional *open-loop* management strategies, based on forecasting a demand and planning a production, are giving way to *closed-loop* policies, where quick response to change in customer preferences compensates for the lack of reliable prediction into the future. *Make and sell* strategies are being replaced with *sense and respond* policies (Haeckel, 1999). Flexibility, agility and short time to market are accordingly receiving much attention in today's management literature.

In response to the growing complexity of market demand, the enterprise organization has been in the last three decades gradually evolving

- from vertically integrated structures
- to a mix of vertical structures and shared services
- to horizontally cutting processes
- to componentized architecture.

The reasons for breaking the traditional enterprise organization into stand-alone components may be sought in different directions, depending on the perspective of an analyst.

For an *economist*, the primary reason for unbundling is in the decreasing transaction costs. As Coase (1937) noted, a firm tends to expand until the cost of organizing an extra transaction within the company becomes equal to the cost of carrying out the same transaction on the open market. If transaction costs go down, the need to keep all business activities in-house diminishes. As transactions costs decline, largely because of developments in information technology, corporations come to function at lower levels of aggregation.

A *mathematician* recognizes the inherent conflict between efficiency and robustness of optimizing the enterprise performance. The "no free lunch" theorem of combinatorial optimization (Wolpert and Macready, 1997) suggests that the increased robustness comes at the cost of lower efficiency, and vice versa. Intuitively, the globally optimized performance (the maximum efficiency) is difficult to sustain if the environment in which the business operates changes frequently. It is through autonomous business components that an organization can increase robustness of its performance in volatile market conditions.

A *manager* tends to attribute the failure to cope with the dynamic environment to rigid organizational structure. According to Drucker (1990), the traditional factory is like a "battleship" – a large, inflexible structure designed for one task. The postmodern factory is more like a "flotilla," consisting of modules centered either around a stage in the production process or around a number of closely related operations. The flotilla model allows for changes in the production process in order to respond to surges in market demand.

Whatever the perspective, the shift from vertical to virtual integration is real and under way. The traditional enterprise is increasingly replaced with a network of value added components, allowing the virtual enterprise to benefit from a mix of capabilities provided by internal and external specialists. The degree to which the enterprise adds value directly (as opposed to products and services purchased from outside) varies greatly across industries. While in car manufacturing it may be as low as 20%, in financial services it is still about 80% (Sokolovsky and Löschenkohl, 2005). The trend toward enterprise componentization (Pohle *et al.*, 2005) is notwithstanding common to all industries.

Effective Management of Virtual Enterprise

Managing a virtual, vertically unbundled enterprise is clearly a more difficult task than managing the traditional, vertically integrated firm. The value network exhibits a complex dynamic behavior, which results from superposition of two distinct mechanisms, namely

- the formation of a functioning network out of available components and
- the changes in performance of individual components over time.

The link between the two follows from the tendency that components that perform better – by adding higher value or operating at lower cost – are more likely to get selected into the network (and vice versa).

Three levels of value network analysis can be distinguished in this respect:

- *Single component:* The analyst's goal is to optimize the performance of a component as a special capability supplier so as to increase its chances to get selected into an existing or emerging network.
- *Virtual enterprise:* The analyst's goal is to design a mix of internal and external specialists so as to optimize the performance of a virtual enterprise.
- *Extended value network:* The analyst's goal is to gain a better understanding of an extended value network, which may involve multiple virtual enterprises, their supply chains and the market segments served by them.

The objective of the present paper is to propose

- a dynamic simulation model of the value network behavior,
- a method for estimating the network state and parameter values from available data.

The management of a virtual enterprise can seize direct control over only a fraction of business components that compose the entire enteprise. The strategic control is exercised primarily through selection of the core, value-added competencies that the enterprise either has access to or can develop in reasonable time. Optimum selection requires quantitative understanding of how quickly the critical competencies can be developed and how these competencies affect the "bonding potential" of the respective business components within the network (i.e., how attractive these components are to potential suppliers, buyers and partners).

An operational model of the value network behavior that link changes in the network to changes in the individual node performance is thus a crucial prerequisite for effective management of the network performance.

State of the Art in Value Network Modeling

System dynamics and agent-based modeling have established themselves as two major simulation-based approaches to investigation of non-linear social and socio-economic systems. Both aim at better understanding and qualitative prediction of a system's behavior.

The *agent-based modeling* approach comes from the field of complexity science. At its core is the assumption that complexity arises from the interaction of individuals – agents. The behavior of the agents is prescribed by their schemata, mostly modeled as sets of simple generative rules. The schemata can evolve over time, allowing the agents to adapt to their environment (Jennings and Wooldridge, 1998). From a modeling perspective, the adaptation can be achieved by the use of feedback and learning algorithms (Phelan, 2001).

The *system dynamics* centers around the concept of information feedback in social and economic sciences (Forrester, 1961; Richardson, 1991; Sterman, 2000). The model takes the form of a system of nonlinear differential equations. System dynamics can thus be regarded as an application of nonlinear dynamics (Strogatz, 1994; Mosekilde, 1996) or nonlinear state-space modeling (Hangos *et al.*, 2004) to socio-economic systems, with predefined structure and state variables taking on "information, money, orders, materials, personnel, and capital equipment in a company, an industry, or a national economy" (Forrester, 1961).

Neither agent-based modeling, nor system dynamics taken in isolation allows for modeling of value networks with variable structure of dynamic nodes. The two approaches, being complementary in many respects, can be combined, however (Schieritz, 2002; Schieritz and Milling, 2003).

The challenge of such integration is in systematic and consistent treatment of uncertainty that both the network structure and the individual node performance are subject to. This is not only a theoretical challenge, but also a practical problem as many industries operate nowadays under significant uncertainty on the supply or demand side (or both). This uncertainty propagates through the value network in a complex manner, which is difficult to seize the traditional way – by running multiple simulations or stress tests.

Value Network Model Ingredients

The ambition of the present paper is to propose a stochastic model of the dynamic behavior of a value network, which would explain changes in the network structure as an endogenous process resulting from changes in the performance characteristics of available business components.

Our approach combines

- *system dynamics*, in a stochastic setting (Kulhavý, 2007), for the description of continuous changes in the performance characteristics of individual business components,

- *pattern theory* (Grenander and Miller, 2007) for the description of discrete changes in the network structure.

Business Component Performance

Modeling of the individual business component performance is based primarily on the work of Warren (2002, 2007) in competitive strategy dynamics. In the resource-based view of strategy (Penrose, 1995; Wernerfelt, 1984; Dierickx and Cool, 1989), the performance of a business component (typically expressed via financial measures) is supposed to be a function of

- the resources that the component owns or has access to,
- the capabilities associated with particular resources.

From the system dynamics viewpoint, the resources and capabilities represent state (stock) variables, which either accumulate or are depleted over time, but cannot change abruptly.

In contrast to Warren (2002, 2007), the dependence of a business component performance on its resource and capability variables is modeled via stochastic differential equations. These are approximated for the purpose of numerical solution – and for easier interpretation – by stochastic difference equations. Estimation of the component state and parameter values from available observations is formulated as the Bayesian inference problem, a numerical solution of which is assumed using a sequential Monte Carlo algorithm (Kulhavý, 2007).

The stochastic extension of the system dynamics model appears a crucial step if the uncertain behavior of individual business components is to be integrated consistently into the entire value network behavior.

Value Network Structure

The formation of a value network out of available business components can be regarded as a mechanism of combining elementary building blocks (generators) into larger structures (configurations) so that certain externally observable variables (bonds) "fit" together (forming *regular* configurations). This is the initial schema of general pattern theory (Grenander, 1993; Grenander and Miller, 2007; cf. Tarnopolsky, 2003).

In practice, a weaker form of regularity is usually adopted where the degree of regularity is measured by a probability measure of a Gibbs form, which penalizes the differences in bond values for neighboring generators. The construction is closely related to the concept of *Markov random field* (Kindermann and Snell, 1980; Geman and Geman, 1984; Grenander and Miller, 2007).

General pattern theory has been applied successfully in quite diverse areas, including image analysis, computer vision, automated target recognition, computational anatomy, natural language analysis, or cognitive science. Its application to value network modeling requires to

- define the bonding potential between any two components that can potentially "do business together,"
- quantify the probability of possible network configurations resulting from establishing business bonds between particular nodes.

Making and Breaking Bonds

What makes two businesses bond together? What factors determine the likelihood of a particular configuration of market bonds?

Bonding as Value Exchange

Praxeology, the study of human action as a purposeful behavior (von Mises, 1998; Rothbard, 2004), provides a useful starting point for analysis of the bonding mechanism. According to praxeology, humans act whenever they use means to achieve ends that they subjectively *value*. Murphy (2007) summarizes the fundamental traits of every action as follows:

- All action involves an *exchange*, or a *choice*: the actor attempts to achieve a more satisfactory state of affairs than what would have occurred had the actor chosen differently.
- The *benefit* of an action is its psychic revenue, while its *cost* is the value the actor places on the next-best alternative.
- Each actor can arrange various possible ends on a *scale of value*. This is a purely ordinal ranking that can only show which end is first-best, second-best, and so forth.
- Every action involves not only a value judgment concerning different ends, but also a *belief* on the part of the actor that he or she possesses adequate means to achieve his desired end.
- Only *individuals* can act, because only individuals have valuations and can make choices.
- All action takes place in *time*. All individuals possess time preference – preferring satisfaction sooner rather than later.
- Individuals make decisions on the *margin* – taking into account a definite amount of goods or services they may acquire in addition to those they already own or have access to.

- All action involves *uncertainty* of the future.

Since our goal is to construct a practical behavioral model of the value network dynamics, we consciously divert from the above postulates in several directions. In particular, we replace

- acts of individuals with acts of *aggregate bodies* (business organizations) involved;
- individual acts (transactions) with *aggregate flows* of goods, services and money;
- ordinal ranking of possible ends with their *cardinal valuation.*

As we shall see shortly, the first two points allow for system dynamics modeling of the network nodes while the third point enables us to quantify the probability of a network configuration.

System Dynamics with Variable Structure

Every bond within a value network thus represents an exchange of goods, services, and money valued (subjectively) by the involved parties (cf. Fig. 1).

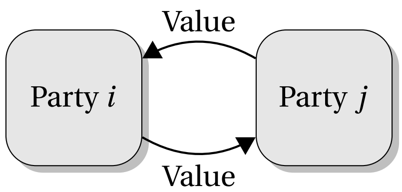


Figure 1: A high-level view of bonding as an exchange of value between two parties.

A typical value network involves multiple bonds, see Fig. 2.

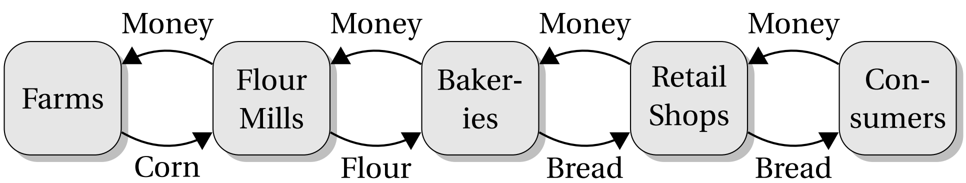
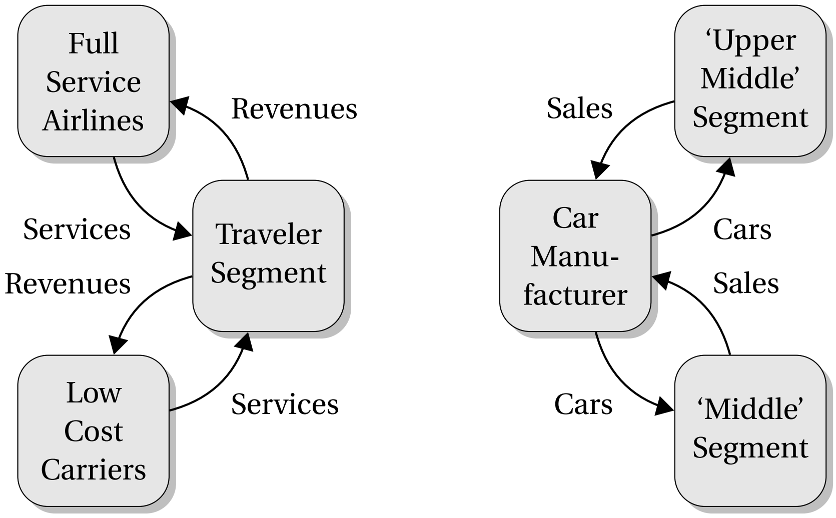


Figure 2: A simple value chain with goods moving in one direction and revenue stream flowing back.

Every node in the network may have multiple suppliers and/or customers, competing for the same resources, cf. Fig. 3.

In terms of system dynamics, the exchange of products, services, and money between two parties can be modeled via *flows* connecting the corresponding *stocks* (product inventory, service capacity, funds, etc.) within system dynamics models of the involved parties' individual behaviors (cf. Fig. 4).



(a) Two types of suppliers. **(b)** Two customer segments.

Figure 3: Simple examples of multiple-supplier and multiple-customer bonds along a value network.

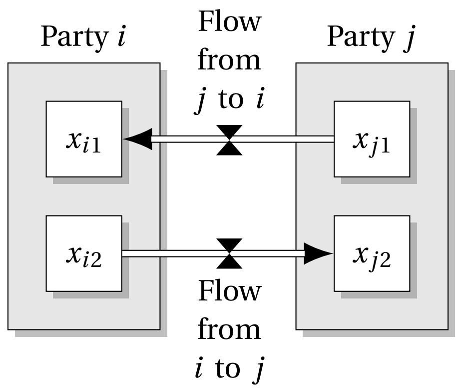


Figure 4: A system dynamics representation of a value exchange. Only stocks affected directly by the exchange are shown here.

In system dynamics modeling, the structure of flows is typically chosen beforehand and fixed during simulation. In order to model value networks, we need to relax this assumption and allow for variable structure.

The variable structure requires adequate definitions and notation. We adopt the following conventions:

- The vector of state variables (stocks) at discrete time k is denoted by x_k.
- The network is composed of n nodes.
- The subvector of state variables composing the i-th node, $i = 1, \ldots, n$, is denoted by $x_{i,k}$.
- There are m potential bonds in the network that consist of value-exchange flows between the network nodes.

- The vector of network flows u_k^s together with the network structure s_k form the *network configuration* $u_k = (s_k, u_k^s)$.
- The network structure is encoded as a binary vector, $s_k \in \{0,1\}^m$, where $s_{j,k} = 1$ if the j-th bond is present and $s_{j,k} = 0$ if the j-th bond is absent.
- The bond configuration for the j-th bond, $j = 1, \ldots, m$, is denoted as $u_{j,k} = (s_{j,k}, u_{j,k}^s)$.

In terms of the above notation, our task is to construct an endogenous model of the network configuration (s_k, u_k^s). Note that the dimension of the vector u_k^s depends on the network structure s_k. As the number of active bonds increases, so does the dimension of the vector of network flows u_k^s. The domain of (s_k, u_k^s) is thus the union – for all possible structures σ – of Cartesian products of a set composed of a particular structure σ and the corresponding domain U_k^σ of u_k^σ,

$$(s_k, u_k^s) \in \bigcup_{\sigma \in \{0,1\}^m} \{\sigma\} \times U_k^\sigma. \tag{1}$$

The set U_k^s is naturally constrained by the existing stocks x_k. For instance, a firm selling the same product to multiple customers cannot deliver in any time interval more than what its product inventory is. Similarly, a customer buying the same product or service from multiple suppliers cannot purchase in any time interval more that its funds allow.[1]

Note that for the case of no bonds, $s_k = (0, \ldots, 0)$, the set U_k^s is empty, $U_k^{(0,\ldots,0)} = \emptyset$.

Modeling of Network Configuration

To sum up, our objective is to model a value network using a system dynamics model composed of n nodes, represented by the state vectors x_i, with m potential bonds between the nodes. The network configuration is defined by the network structure s_k and network flows u_k^s for active bonds. We assume that the dynamic behavior of the nodes themselves is understood well enough for system dynamics modeling to apply. Therefore, in the sequel we do not consider the node models in detail. Instead, we concentrate on modeling of the network configuration u^k.

The Dynamics of Network State and Configuration

Simulation of the network state and configuration over a period of time $(1, \ldots, N)$

$$\begin{aligned} x^{N+1} &= (x_1, x_2, \ldots, x_{N+1}), \\ u^N &= (u_1, u_2, \ldots, u_N) \end{aligned}$$

requires understanding of their dependency structure. Four major cases can be distinguished in this respect.

[1] The constraints can be circumvented by creating additional bonds. A firm can cover the temporary shortage in its inventory by reselling products from other suppliers. A customer temporarily short of funds can buy on credit from a bank.

NO BONDS. With no bonds present between nodes, the nodes are independent of each other and can be treated separately, cf. Fig. 5a. The joint probability density function of the state time series can be factorized using the standard formulae of probability calculus as follows.

$$p(x^{N+1}) = p(x_1) \prod_{k=1}^{N} p(x_{k+1}|x_k)$$

EXOGENOUS BONDS. The presence of bonds interconnects the neighboring nodes. The evolution of state $x_k \to x_{k+1}$ depends now on the network configuration u_k, which is supposed to be a known (exogenously defined) parameter of the model, cf. Fig. 5b.

$$p(x^{N+1}|u^N) = p(x_1) \prod_{k=1}^{N} p(x_{k+1}|x_k, u_k)$$

This is the case of, e.g., a central planner or a single firm's management.

ENDOGENOUS BONDS. The introduction of a feedback loop from the state x_k to the network configuration u_k creates an additional dependency, cf. Fig. 5c.

$$p(x^{N+1}, u^N) = p(x_1) \prod_{k=1}^{N} p(u_k|x_k)\, p(x_{k+1}|x_k, u_k)$$

This case exhibits circular causality where the network configuration u_k is affected by the current network state x_k and, at the same time, affects the future network state x_{k+1}.

ENDOGENOUS BONDS "WITH MEMORY." In contrast to the previous case, the network configuration u_k depends also on the previous configuration u_{k-1}, cf. Fig. 5d.

$$p(x^{N+1}, u^N) = p(x_1) \prod_{k=1}^{N} p(u_k|x_k, u_{k-1})\, p(x_{k+1}|x_k, u_k)$$

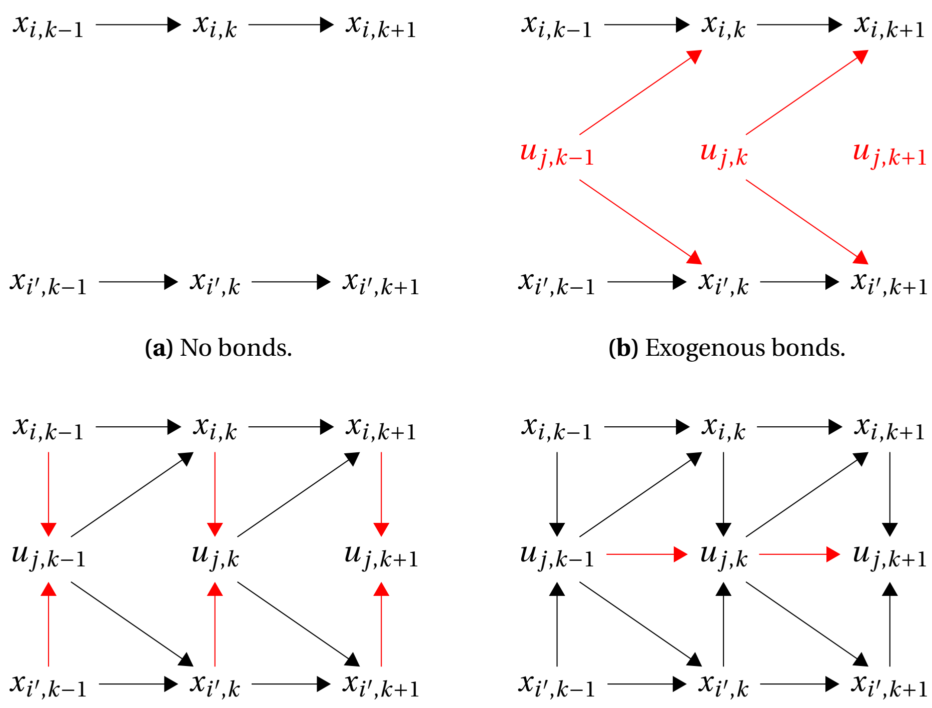
This is the case when bonding exhibits inertia, i.e., when it takes extra energy to create a new bond, break an existing bond or make significant change to the flows between nodes.

The last two cases – of endogenous bonds – are of interest to us in the following.

The Probability of Network Configuration

The modeling of value networks with endogenous bonds combines two terms. The transition probability density function $p(x_{k+1}|x_k, u_k)$ is nothing but a probabilistic representation of conventional system dynamics in a stochastic setting (Kulhavý, 2007). The bond-generating distribution

$$p(u_k|x_k) \quad \text{or} \quad p(u_k|x_k, u_{k-1})$$



(a) No bonds. **(b)** Exogenous bonds. **(c)** Endogenous bonds. **(d)** Bonds "with memory."

Figure 5: Four basic cases of a network configuration for two nodes. The arrows indicate conditional independence – the distribution of any of the variables at k and $k+1$ is determined by its predecessors (variables pointing at the variable in question). The red color is used to stress the new structural artifacts.

is, however, a novel term, which needs to be constructed first.

To simplify notation, we omit below the time index as well as the conditioning variables.

JOINT DISTRIBUTION. This is a key choice in our model – we select the joint distribution of s and u^s in an exponential form with the density[2]

$$p(u) = p(s, u^s) = \frac{1}{Z} e^{-\frac{1}{T}[A(s)+B(u^s)]} \tag{2}$$

where Z denotes a normalizing constant, T stands for a "temperature" parameter and $A(s) + B(u^s)$ quantifies a total *bonding energy*.

More specifically,

- the term $A(s)$ captures the cost of bonding associated with a particular network structure s but independent of the actual network flows u^s;

[2]Given that s and u^s are discrete and continuous random variables, respectively, $p(s, u^s)$ is strictly speaking a Radon-Nikodym derivative of the associated probability measure with respect to a product of the counting and Lebesgue measures.

- the term $B(u^s)$ measures the cost associated with a particular set of flows u^s.

The functions $A(\cdot)$ and $B(\cdot)$ are not constrained to positive values. In fact, it is not difficult to see that modifying the total energy function by an additive constant does not change (because of normalization) the probability of a network configuration.

The temperature T controls the degree of peaking in the density (2). The smaller T is, the more concentrated the samples from (2) are around the mode(s) of (2).

The normalizing constant follows by integration of (2) over the domain (1) of (s, u^s)

$$\begin{aligned} Z &= \sum_s \int_{U^s} e^{-\frac{1}{T}[A(s)+B(u^s)]} \,\mathrm{d}u^s \\ &= \sum_s e^{-\frac{1}{T}A(s)} \int_{U^s} e^{-\frac{1}{T}B(u^s)} \mathrm{d}u^s \end{aligned}$$

Introducing the partition function

$$Z(s) = \int_{U^s} e^{-\frac{1}{T}B(u^s)} \,\mathrm{d}u^s$$

we can rewrite the normalizing constant as follows

$$\begin{aligned} Z &= \sum_s e^{-\frac{1}{T}A(s)} \, Z(s) \\ &= \sum_s e^{-\frac{1}{T}A(s)+\log Z(s)} \end{aligned}$$

Altogether, the joint density takes the form

$$p(s, u^s) = \frac{e^{-\frac{1}{T}[A(s)+B(u^s)]}}{\sum_s e^{-\frac{1}{T}A(s)+\log Z(s)}} \tag{3}$$

MARGINAL DISTRIBUTION. The marginal probability mass function of the network structure s follows by integrating (3) over u^s

$$\begin{aligned} p(s) &= \int_{U^s} \frac{e^{-\frac{1}{T}[A(s)+B(u^s)]}}{\sum_s e^{-\frac{1}{T}A(s)+\log Z(s)}} \,\mathrm{d}u^s \\ &= \frac{e^{-\frac{1}{T}A(s)} \int_{U^s} e^{-\frac{1}{T}B(u^s)} \,\mathrm{d}u^s}{\sum_s e^{-\frac{1}{T}A(s)+\log Z(s)}} \\ &= \frac{e^{-\frac{1}{T}A(s)} Z(s)}{\sum_s e^{-\frac{1}{T}A(s)+\log Z(s)}} \end{aligned}$$

which results in

$$p(s) = \frac{e^{-\frac{1}{T}A(s)+\log Z(s)}}{\sum_s e^{-\frac{1}{T}A(s)+\log Z(s)}} \tag{4}$$

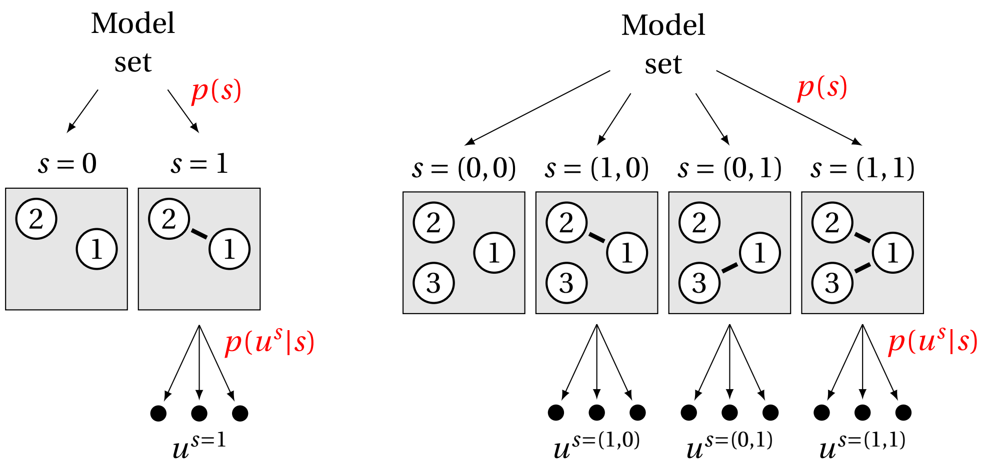
CONDITIONAL DISTRIBUTION. The chain rule formula

$$p(s, u^s) = p(u^s|s)\, p(s)$$

applied to (3) and (4) yields the conditional probability density function of the network flows u^s

$$p(u^s|s) = \frac{e^{-\frac{1}{T}B(u^s)}}{\int_{U^s} e^{-\frac{1}{T}B(u^s)}\, \mathrm{d}u^s} \tag{5}$$

Figure 6 illustrates the connection between the marginal probability mass function of s and the conditional probability density function of u^s.



(a) Single bond. **(b)** Two bonds.

Figure 6: A schematic view of a network configuration and its probability.

Single-Bond Energy

We define the total bonding energy $B(u^s)$ in (5) as the sum of local energies $B(u^s_j)$ over all active bonds

$$B(u^s) = \sum_{\substack{j\in\{1,\ldots,m\} \\ \text{such that} \\ s_j=1}} B(u^s_j) \tag{6}$$

For the sake of illustration, we define the local energy via the formula

$$B(u^s_j) = -\left(g_{i,j} + g_{i',j}\right) + a\left|g_{i,j} - g_{i',j}\right| \tag{7}$$

where

$$\begin{aligned} g_{i,j} &= g(x_i, u^s_j) \\ g_{i',j} &= g(x_{i'}, u^s_j) \end{aligned}$$

stand for the gains of exchange that the nodes i and i' do experience through the bond j given their current states x_i and $x_{i'}$, respectively. The form (7) of the local energy associated with a single bond is motivated by praxeological considerations again.

First, the gains experienced by both parties correspond to valuation of the *acquired* goods, services, or money relative to the value of goods, services, or money *provided* in exchange, cf. Fig. 7. The valuation often counts on the future benefits of the acquired goods, services, or money – and may thus work with concepts like *net present value*.

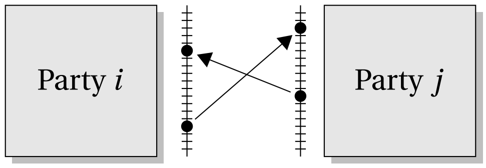


Figure 7: The gains of exchange are calculated by comparison of the acquired value to the value provided in exchange.

Second, the local energy (7) decreases with the increasing gains on both sides of an exchange. On the other hand, the increasing disproportion between the gains makes the local energy grow. The latter effect is controlled by the parameter a, cf. Fig. 8. An exchange that is highly profitable and at the same time fair to both parties is thus assigned a low energy.

In the case that the probability of a network configuration depends on the previous network configuration through $p(u_k|x_k, u_{k-1})$, both the individual bond energy $B(u_j^s)$ and the overall configuration energy $A(s)$ can be easily expanded with additional terms penalizing large changes in the network flows and network structure, respectively.

The Benefits and Costs of Bonding

The probability of a network configuration depends critically on definition of the *gains of exchange*, which enter the energy $B(u^s)$, and the *cost of bonding*, which enter the energy $A(s)$. When looking for a proper definition of these terms, it is useful to consult the theory of the firm as well as the new institutional economics (Mahoney, 2004; Furubotn and Richter, 2005). According to them, the behavior of business organizations in a market environment is crucially affected by two phenomena, namely

- transaction costs,
- cognitive limitations of human agents.

TRANSACTION COSTS. Coase (1960) defined the transaction costs as follows: "In order to carry out a market transaction it is necessary to discover who it is that one wishes to deal with, to inform people that one wishes to deal with and to what terms, to conduct negotiations leading up to a bargain, to draw up the contract, to undertake the inspection needed to make sure that the terms of the contract are being observed, and so on."

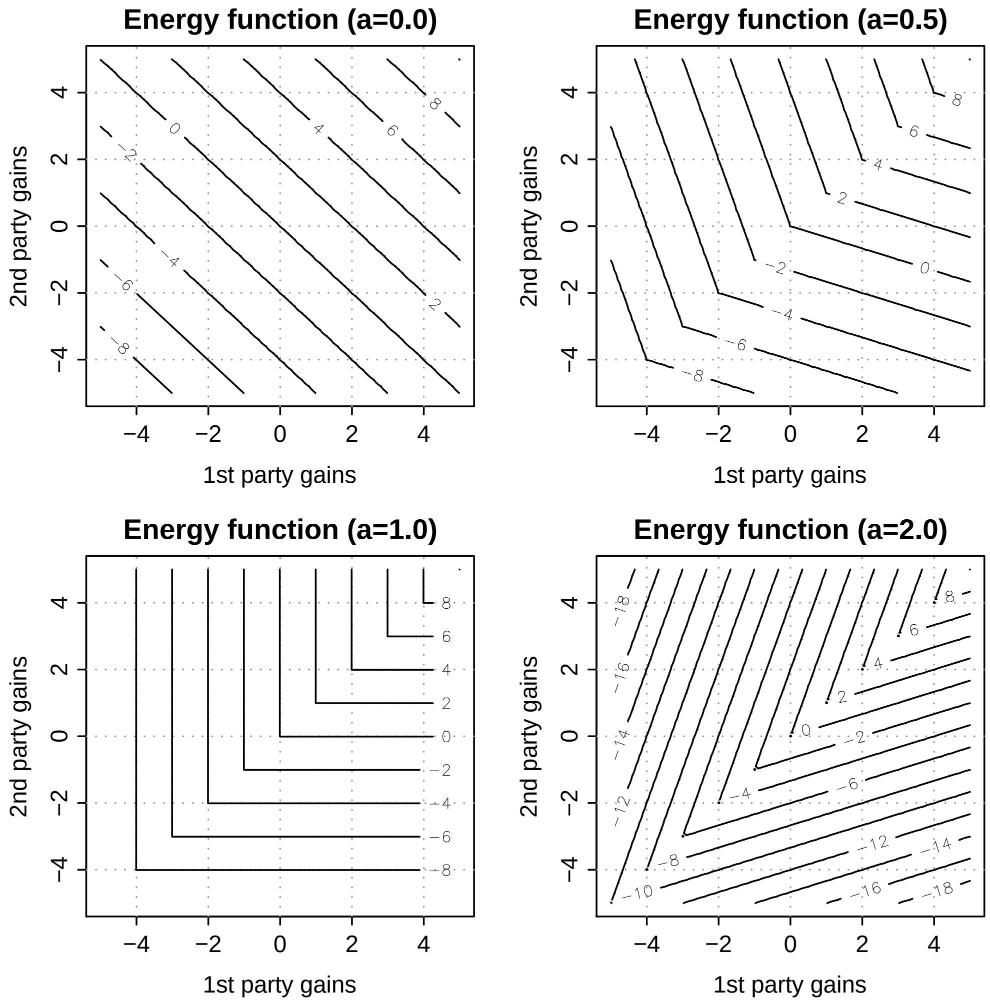


Figure 8: The effect of the parameter a on the local energy function $B(u_j^s)$.

Transaction costs are encountered universally. In our model, they enter directly into the energy component $A(s)$. The treatment of transaction costs in the model can vary from applying universal transaction costs, applicable to every single bond across a market, to highly specific transaction costs, accounting for familiarity of the involved parties with each other, for search and information costs, for bargaining and decision making costs, or for the costs of monitoring and enforcing the contractual obligations.

BOUNDED RATIONALITY The concept of bounded rationality was first introduced by Simon (1957). The bounded rationality admits that a decision maker cannot acquire and process information costlessly and instantly. Individuals face a variety of constraints that prevent them from acting like rational utility maximizers. All intendedly rational behavior is behavior within constraints. As a result, optimizing is replaced by satisficing, a behavior which attempts to achieve at least some minimum level of a particular variable, but which does not necessarily maximize its value.

The bounded rationality affects the capability of individual actors to

- make a perfectly rational calculation of the expected gains of an exchange,
- identify and close bonds that maximize the expected gains of an exchange.

In our model, the bounded rationality can be accounted for through

- calculation of the expected gains $g_{i,j}$ from an exchange (ranging from comprehensive analyses to quick estimations),
- adjustment of the temperature T (low temperature pushes the model toward a strictly maximizing behavior, high temperature moves it closer to a random search).

Network Simulation

The calculation of the marginal density of the network state x_k and network configuration u_k can be organized recursively according to the formula

$$p(x_{k+1}, u_{k+1}) = \int\int p(u_{k+1}|x_{k+1}, u_k)\, p(x_{k+1}|x_k, u_k)\, p(x_k, u_k)\, \mathrm{d}x_k\, \mathrm{d}u_k$$

for $k = 1, \ldots, N$.

A sequential Monte Carlo method (Doucet *et al.*, 2001) can be used for effective numerical integration.

1. *Initialization:* Draw M samples from the prior distribution

$$\begin{aligned} x_1^{(l)} &\sim p(x_1),\ l = 1, \ldots, M \\ u_1^{(l)} &\sim p(u_1|x_1^{(l)}),\ l = 1, \ldots, M \end{aligned}$$

   and set $k := 1$.

2. *Simulation Step:* Draw M samples from mixture approximations to the predictive distributions

$$x_{k+1}^{(l)} \sim \frac{1}{M}\sum_{l'=1}^{M} p(x_{k+1}|x_k^{(l')}, u_k^{(l')}),\ l = 1,\dots,M$$
$$u_{k+1}^{(l)} \sim \frac{1}{M}\sum_{l'=1}^{M} p(u_{k+1}|x_{k+1}^{(l')}, u_k^{(l')}),\ l = 1,\dots,M$$

3. *Iteration:* Increment $k := k+1$ and go back to the simulation step.

Learning from Data

Let us suppose that we are capable to observe on a value network a vector y_k that is a known function of the network state x_k and network configuration u_k (cf. Fig. 9). The past observations $y^k = (y_1,\dots,y_k)$ can then be used for estimation of both x_k and u_k. Following the Bayesian paradigm (Peterka, 1981), we quantify the uncertainty of (x_k, u_k) via the densities $p(x_k, u_k|y^{k-1})$ and $p(x_k, u_k|y^k)$ conditional on the past observations y^{k-1} and y^k, respectively.

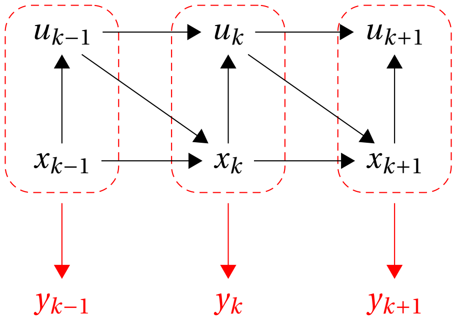


Figure 9: Observations y_k on a network are functions of the network state x_k and network configuration u_k.

Recursive Bayesian Estimation

The Bayesian estimation can be organized recursively – by sequentially updating the conditional densities

$$p(x_k, u_k|y^{k-1}) \overset{\substack{\text{data}\\ \text{update}}}{\longrightarrow} p(x_k, u_k|y^k) \overset{\substack{\text{time}\\ \text{update}}}{\longrightarrow} p(x_{k+1}, u_{k+1}|y^k) \tag{8}$$

for $k = 1,2,\dots$ The recursion starts at $k = 1$ from the prior density $p(x_1, u_1|y^0) = p(x_1, u_1)$. Every iteration combines data update (conditioning on the latest observation) and time update (prediction into the next time step).

DATA UPDATE. The posterior density $p(x_k, u_k|y^k)$ results from the prior density $p(x_k, u_k|y^{k-1})$ through the Bayes rule

$$p(x_k, u_k|y^k) \propto p(y_k|x_k, u_k)\, p(x_k, u_k|y^{k-1}) \tag{9}$$

where the symbol $\propto$ stands for equality up to a normalizing factor.

TIME UPDATE. The predictive density $p(x_{k+1}, u_{k+1}|y^k)$ follows from the posterior density $p(x_k, u_k|y^k)$ by elementary operations of probability calculus

$$p(x_{k+1}, u_{k+1}|y^k) = \int\!\!\int p(u_{k+1}|x_{k+1}, u_k)\, p(x_{k+1}|x_k, u_k)\, p(x_k, u_k|y^k)\, \mathrm{d}x_k\, \mathrm{d}u_k \tag{10}$$

A sequential Monte Carlo method (Doucet *et al.*, 2001) is used again for effective numerical approximation of the ideal Bayesian recursion.

Particle Filter Algorithm

1. *Initialization:* Draw M samples from the prior distribution

$$\begin{aligned} x_{1|0}^{(l)} &\sim p(x_1),\ l = 1, \dots, M \\ u_{1|0}^{(l)} &\sim p(u_1|x_{1|0}^{(l)}),\ l = 1, \dots, M \end{aligned}$$

   and set $k := 1$.

2. *Data Update:* Collect the observation y_k and evaluate the importance weights

$$\pi_l = \frac{p(y_k|x_{k|k-1}^{(l)}, u_{k|k-1}^{(l)})}{\sum_{l'=1}^{M} p(y_k|x_{k|k-1}^{(l')}, u_{k|k-1}^{(l')})},\ l = 1, \dots, M$$

   and draw M samples from a kernel approximation to the posterior distribution

$$(x_{k|k}^{(l)}, u_{k|k}^{(l)}) \sim \sum_{l'=1}^{M} \pi_{l'}\, K(x_k - x_{k|k-1}^{(l')}, u_k - u_{k|k-1}^{(l')}),\ l = 1, \dots, M$$

3. *Time Update:* Draw M samples from mixture approximations to the predictive distributions

$$\begin{aligned} x_{k+1|k}^{(l)} &\sim \frac{1}{M} \sum_{l'=1}^{M} p(x_{k+1}|x_{k|k}^{(l')}, u_{k|k}^{('l)}),\ l = 1, \dots, M \\ u_{k+1|k}^{(l)} &\sim \frac{1}{M} \sum_{l'=1}^{M} p(u_{k+1}|x_{k+1|k}^{(l')}, u_{k|k}^{(l')}),\ l = 1, \dots, M \end{aligned}$$

4. *Iteration:* Increment $k := k + 1$ and iterate from data update.

The function $K(x)$ stands for a suitable kernel function (Silverman, 1986; Hastie *et al.*, 2001), normalized so as to integrate to one. The use of a kernel density estimate in the particle filter prevents the degeneracy of a sample set.

Illustrative Example

To help the reader develop a better feeling for the abstract concepts introduced above, we use a simple example.

Crop Lien System

Bowles (2006, chap. 9) analyzes a counterintuitive trend in farming that developed in the U.S. South after the Civil War when the production of cotton relative to corn increased in a quarter of a century by 50 percent in spite of the decreasing price, lower yields, and higher labor intensiveness of cotton. Bowles (2006) explains the growing dominance of cotton by the structure of *local credit markets*. To finance the crop cycle, most farmers purchased food and other goods on credit during the growing season. The loans were repaid after the crop was sold at the end of the season. Since most farmers were too poor to post collateral, the lenders secured their loans by means of a claim (a lien) on the farmer's future crop in case of default.

Ransom and Sutch (1977) shows that, in the view of the lender, cotton provided greater security than food crops; cotton was easy to store and easy to sell in a well-organized market. According to Ransom and Sutch (1977), the cotton farmer purchasing corn on credit could have increased his income by 29 percent by shifting resources from cotton to corn. But for such a shift, the farmer would have needed sufficient wealth to cover his costs. Since this was not typically the case, the farmer was dependent on credit, which was conditioned on planting cotton. Consequently, the farmer was locked in a lower-income option.

From the present paper's perspective, the crop lien system produces a simple value network that, besides farmers, includes lenders and merchants as well as markets for the cotton and corn crops, cf. Fig. 10.

This historical case study has parallels with, e.g., the contemporary mortgage markets. Residential tenancy incurs similar inefficiencies (typical for principal-agent relationships), yet a significant portion of families rent rather than own their home.

Dynamic Model

A thorough analysis of the crop lien system is beyond the scope of this paper. In the sequel, we discuss the modeling phase only.

NETWORK. Let us consider a value network composed of the nodes Farmer, Lender, Merchant, Corn Market, and Cotton Market where each of the nodes account for the *aggregate* behavior of individual farmers, lenders, merchants, and corn and cotton distributors in a *definite region* of the country (the last constrains the market in question). The network operates in natural production cycles – growing seasons, from planting to harvest. The cycles are labeled with time index $k = 1, 2, \ldots$

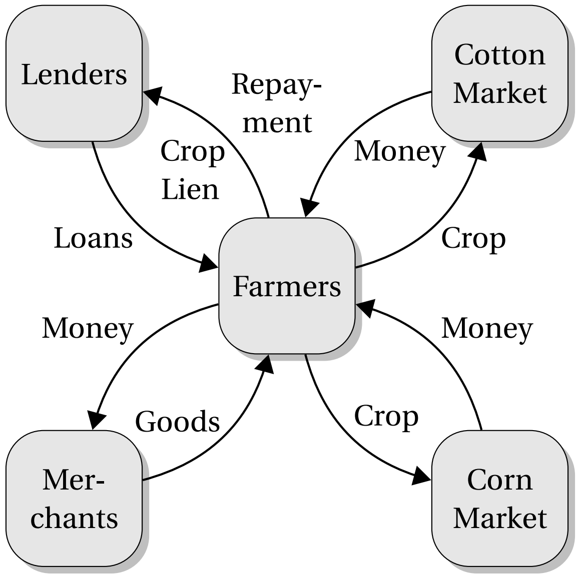


Figure 10: A value network around the crop lien system.

We adopt several simplifying assumptions. First, we consider only aggregate performance for the entire cycle; no attempt is made to model what happens during the cycle. Second, we describe in detail only the Farmer node. Information about other nodes is considered only to the extent that allows to define the network configuration for the Farmer node. Third, we consider a single product only. Generalization to multiple products is straightforward, though possibly tedious.

NODE STATE. The simplest model of the Farmer node has a three-dimensional state

$$x_k = \begin{bmatrix} x_{1,k} \\ x_{2,k} \\ x_{3,k} \end{bmatrix} = \begin{bmatrix} \text{cash balance at } k \\ \text{outstanding debt at } k \\ \text{total crop at } k \end{bmatrix}$$

The state x_k refers to the end of the k-th production cycle – the values of x_k are taken after harvest but before selling the crop.

NETWORK CONFIGURATION. For any of the products grown, the Farmer node bonds potentially with the Lender, Merchant, and Distributor nodes. The network flows include the following

$$u_k = \begin{bmatrix} u_{1,k} \\ u_{2,k} \\ u_{3,k} \\ u_{4,k} \\ u_{5,k} \\ u_{6,k} \end{bmatrix} = \begin{bmatrix} \text{amount of crop sold at } k \\ \text{crop revenue at } k \\ \text{new loan extended at } k \\ \text{repayment of loan taken at } k \\ \text{amount of factors of production at } k \\ \text{cost of factors of production at } k \end{bmatrix}$$

By "factors of production," the whole set of complementary factors is meant, including food, clothing, tools, seed, etc. A natural unit for such aggregate could be, e.g., "all factors needed to equip fully one cropper for the entire season." Note that the factors of production purchased during the production period $k+1$ are aggregated and discounted back to the end of the previous period. This is because the decisions about $u_{5,k}$ and $u_{6,k}$ need to be made before the period $k+1$ starts.

STATE DYNAMICS. Given the Farmer state x_k and network configuration u_k and introducing the cash balance

$$b_k = x_{1,k} + u_{2,k} - x_{2,k}$$

we can calculate the next state according to the equations

$$\begin{aligned}
x_{1,k+1} &= \max\left\{\max\{b_k, 0\} + u_{3,k} - u_{5,k}, 0\right\} \\
x_{2,k+1} &= -(1+i_1)\min\{b_k, 0\} - (1+i_2)\min\left\{\max\{b_k, 0\} + u_{3,k} - u_{5,k}, 0\right\} + u_{4,k} \\
x_{3,k+1} &= f(u_{5,k})\, e^{n_{5,k}},\ n_{5,k} \sim N(0, \sigma_5^2)
\end{aligned}$$

The notation may look more complex than what it actually does. The cash balance $x_{1,k+1}$ at the end of the production period $k+1$ is – after collecting revenue from the last crop $u_{2,k}$, repaying the past debt $x_{2,k}$, taking a new loan $u_{3,k}$, and paying for the factors of production $u_{5,k}$ – either positive or zero. The negative cash balance adds to the outstanding debt $x_{2,k+1}$. Note that there are two interest rates in the model: i_1 applies to the outstanding debt, $i_2 > i_1$ applies to factors of production purchased during the production period on credit. The crop $x_{3,k+1}$ results from a production function $f(\cdot)$ for a particular amount of factors of production $u_{5,k}$. The volatility of the future crop is modeled by multiplicative, log-normally distributed noise. The crop is considered perishable – any part unsold by the beginning of the next growing season is lost.

PROBABILITY OF NETWORK CONFIGURATION. In the model (7), the probability of a particular set of network flows, for a particular network structure, is determined by the gains of exchange for all active bonds. To calculate the gains for any of the bonds, one must evaluate the utilities and opportunity costs for both the buyer and seller in the exchange. Murphy (2007) gives succinct definitions of the four critical values:

- The *utility from selling* a good for money is the value of the most highly ranked use to which the additional money can be devoted.
- The *cost of selling* a good (the *seller's opportunity cost*) is the value of the most highly ranked alternative end to which the good could have been devoted, had it not been sold.
- The *utility from buying* a good with money is the value of the most highly ranked end to which the good can be devoted.
- The *cost of buying* a good with money (the *buyer's opportunity cost*) is the value of the most highly ranked alternative use that the units of money can no longer satisfy.

Compare the schematic view of a value exchange in Fig. 11.

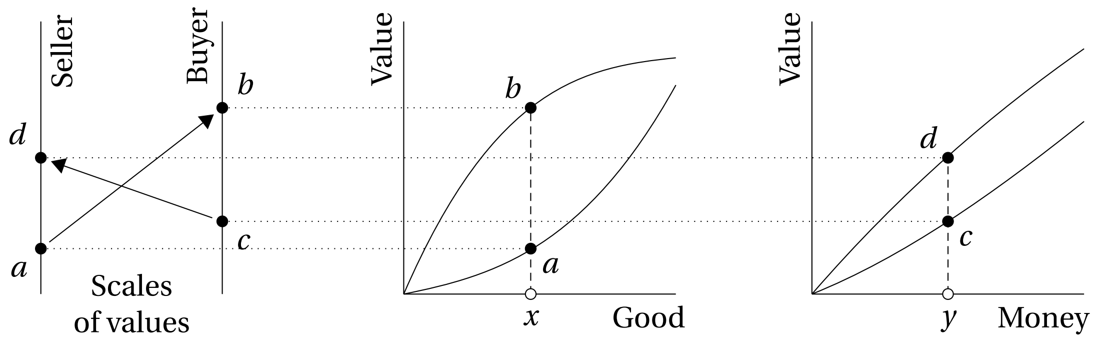


Figure 11: The gains of exchange of x units of a good for y units of money are evaluated against the seller's and buyer's scales of values. The points a, b, c, and d stand for the cost of selling, utility from buying, cost of buying, and utility from selling, respectively.

For the sake of illustration, let us assume that the Farmer assesses the cost of factors of production, the production capability and the expected revenue from crop through functions shown in Fig. 12.

Assume further that the returns of the Lender and Merchant are 50% and 30%, respectively, and that both the Lender's and Merchant's utilities are bounded by 300 (resulting in diminishing marginal utility). Assume that the prevailing interest rate (used for discounting future values back to the current value) is 10%. Using all these assumptions, we can construct the buyer's and seller's utilities and opportunity costs in all three exchanges (namely of the Farmer with the Lender, Merchant and Distributor) as shown in Fig. 13.

The Farmer's utility is imputed back from the crop revenue (after being discounted accordingly). The Lender's and Merchant's utilities result from the assumed returns on the loan and factors of production. The Distributor's utility results from selling the crop to the Distributor's customers, cf. the lower left plot in Fig. 12.

Assuming that the value network is largely isolated so that there are limited alternative uses to the goods and money outside the network, the opportunity costs are mostly represented by the utility of putting money in a bank (for the prevailing interest rate). The Merchant's opportunity cost follows by inversion of the function in the upper left plot in Fig. 12. We assume here that the Merchant buys factors of production only after they are actually ordered by the Farmer. If the factors of production were in stock already, their cost to the Merchant would be sunk and thus the Merchant's opportunity cost would be either the revenue from selling the factors outside the value network or, if no such opportunity existed, it would be zero.

We consider these two options explicitly in selling the crop to the Distributor: the Farmer's opportunity cost is either given by selling the crop outside the value network, at an assumed price 0.2 per unit of crop, or it is zero (cf. solid and dashed lines in the lower left plot in Fig. 13).

After the utilities and opportunity costs for all of the Farmer's bonds are settled, the rest of the algorithm is relatively straightforward. Following the schema in Fig. 11, we

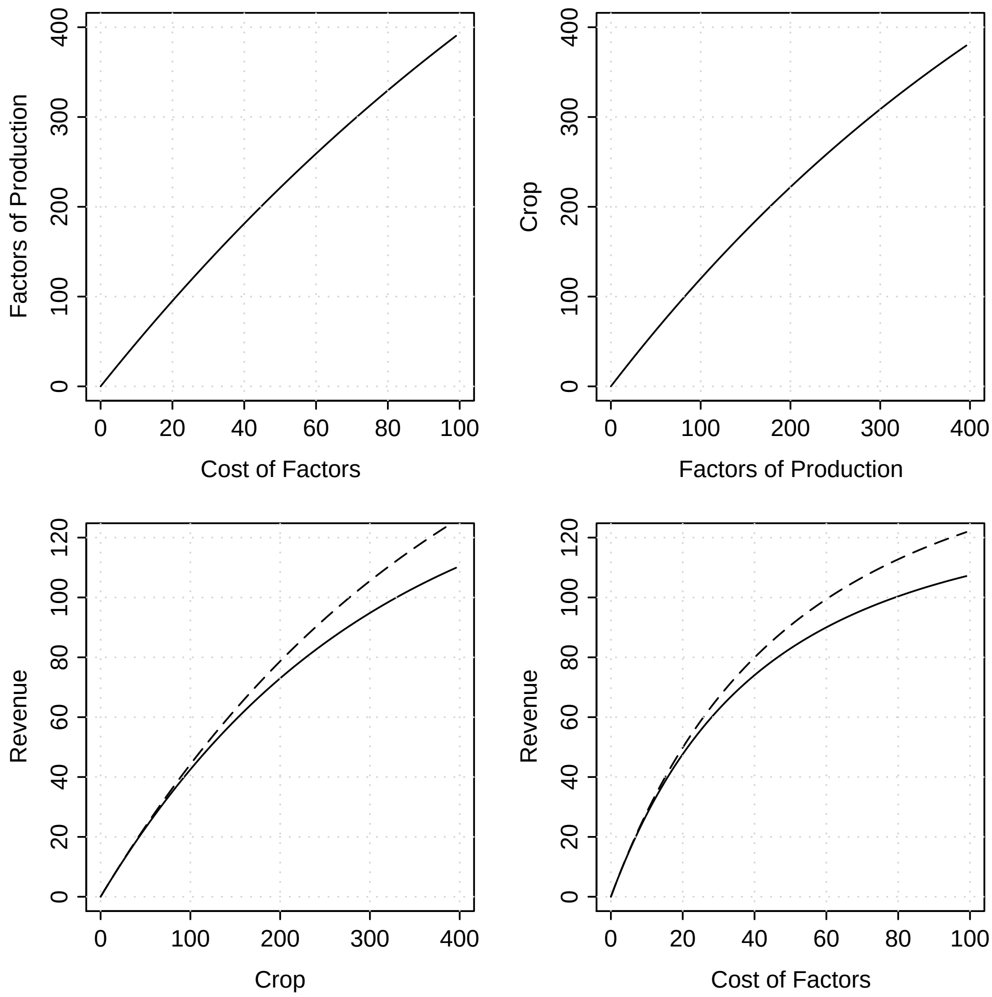


Figure 12: The upper left, upper right and lower left plots capture the Farmer's perception as to the factors of production that money can buy, the crop that given factors of production can yield and the revenue that a given crop can be sold for, respectively. The lower right plot is a composition of the first three functions, showing the revenue as function of the cost of factors of production. The dashed lines in the lower left and lower right plots show the expected Distributor's revenue along with the Farmer's one.

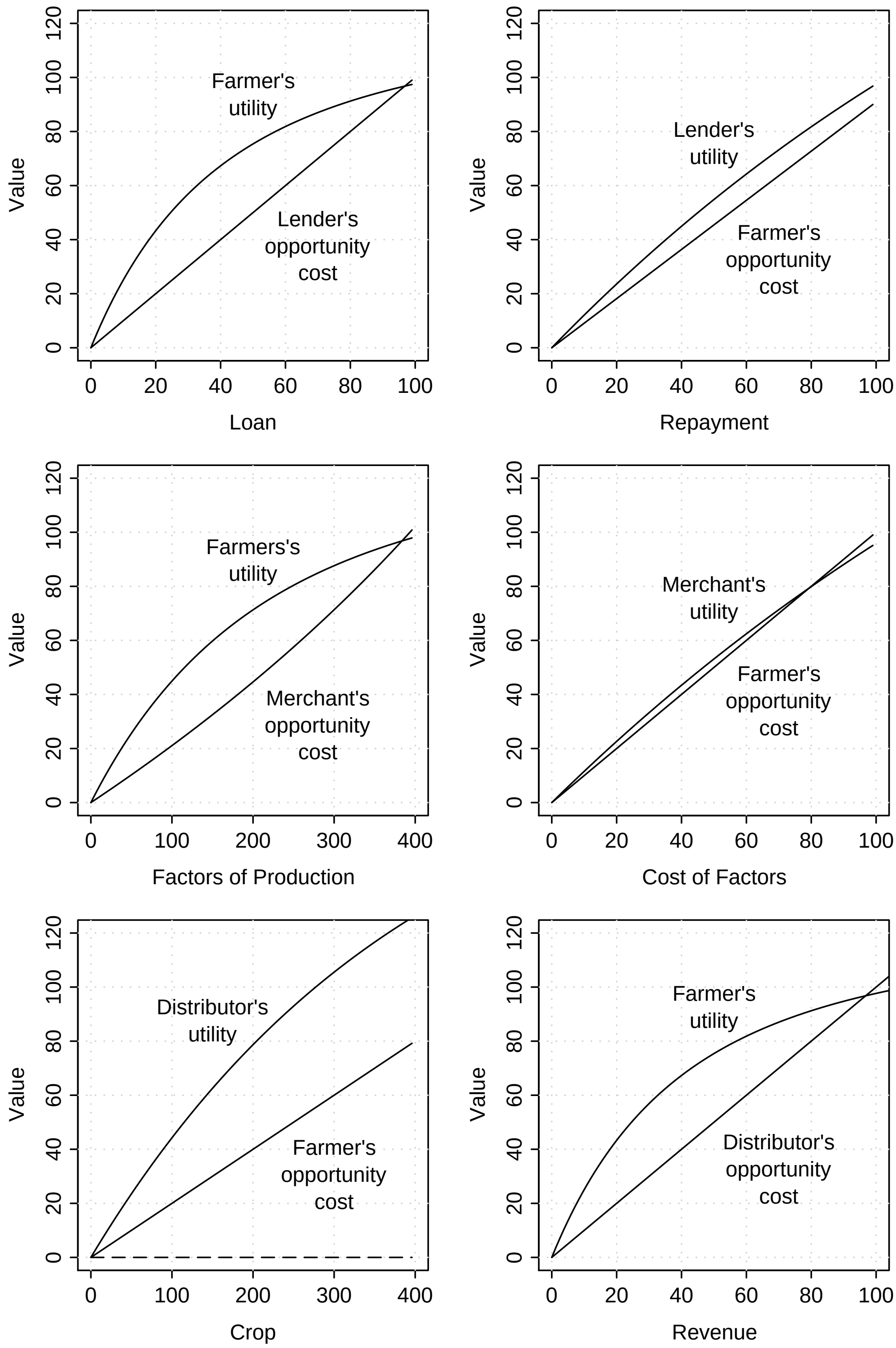


Figure 13: The upper, middle and lower pairs of plots show the utilities and costs of buying and selling for the Farmer-Lender, Farmer-Merchant and Farmer-Distributor bonds.

calculate the gains of exchange $g_{i,j}$ and substitute them in formulae (7), (6) and (5).

Figure 14 shows the local energies (7) associated with the Farmer-Lender, Farmer-Merchant and Farmer-Distributor bonds (for the parameter $a = 1$). Note that all local energies exhibit well-defined minima, except for the lower right plot that corresponds to the case that the Farmer's cost of selling crop is zero and the Farmer is thus motivated to sell all of the crop available.

It is important to stress that the gains of exchange depend dynamically on the performance of network nodes. Thus, e.g., the Farmer's utilities and opportunity costs depend on both the value added in production (crop growing) and on the experience gathered from past exchanges concerning the cost of credit, cost of factors of production and the crop revenue. This information – namely parameters specifying the production function and the cost and revenue curves shown in Fig. 12 – belongs in a full-fledged model to the Farmer's state.

The Power of Bonding

There is a general perception that the world around us becomes less hierarchical and more networked and "flat." While the shift toward a networked and decentralized business environment generally creates more freedom to act, it does not increase automatically the chances of success. Understanding the dynamics of networked systems – in particular the interplay between the performance of an individual node and of the entire network, and the importance of effective bonding for the well-being of an organization – becomes a critical skill.

The proposed approach to modeling of networked systems combines

- stochastic system dynamics modeling of individual nodes within a network,
- probabilistic graphical modeling of a network configuration.

The latter is closely related to theoretical constructs such as the Ising model in statistical mechanics (Kindermann and Snell, 1980) or Markov random fields in image analysis (Winkler, 2006). Modeling of business networks turns out to be even more complex because of the random configuration of a network.

Preliminary results indicate that the proposed approach can

- extend the resource-based view of the firm beyond the firm itself (by linking the bonding potential of a business entity to its internal resources),
- account for the importance of transactions costs,
- model bounded rationality in behavioral modeling,
- estimate value scales of individual parties.

In the present paper, we have paid attention exclusively to business networks. However, since the concept of value exchange is applicable in principle to any human ac-

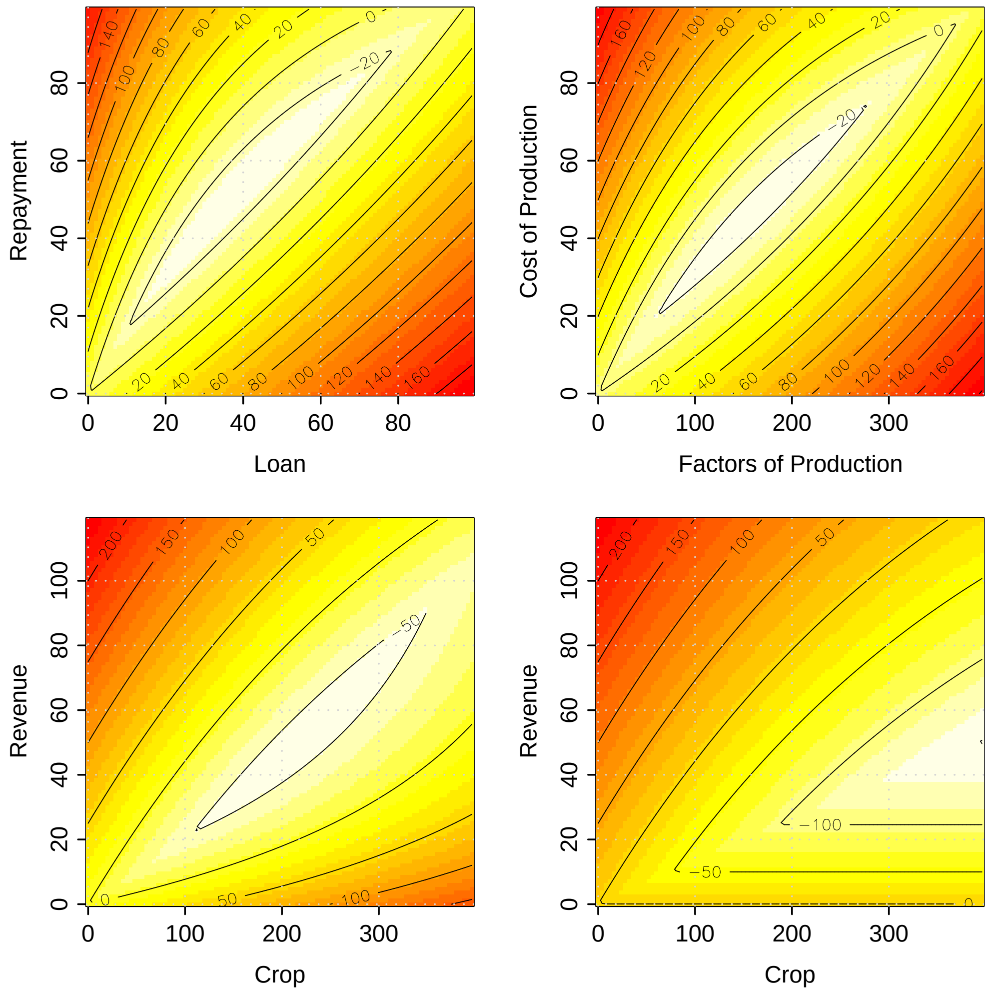


Figure 14: The upper left, upper right and lower (left and right) plots show local energies associated with the Farmer-Lender, Farmer-Merchant and Farmer-Distributor bonds, respectively. Two options are considered for the last bond, namely that the Farmer *can* and *cannot* sell crop outside the value network – the results are shown in the lower left and lower right plots, respectively.

tion (von Mises, 1998), the approach can be of relevance to modeling of social networks as well (Degenne and Forse, 1999).

Acknowledgments

The author gratefully acknowledges research support from the Grant Agency of the Academy of Sciences of the Czech Republic through Grant No. IAA700750701.

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