BAYESIAN TRACKING OF THE TOXIC PLUME SPREADING IN THE EARLY STAGE OF RADIATION ACCIDENT

Petr Pecha, Radek Hofman and Václav Šmídl
Institute of Information Theory and Automation of ASCR
Pod Vodarenskou vezi 4
182 08 Prague 8, Czech Republic
E-mail: pecha@utia.cas.cz

KEYWORDS
Pollutant Spreading, Data Assimilation, State-Space models, Particle Filtering, Resampling

ABSTRACT

The article deals with the predictions of time and space evolution of pollution dispersion during the early phase of a hypotetical radiation accident. The goal is to design a proper fast algorithm which could enable more precise online estimation of radioactivity propagation on basis of recursive procedure of Bayesian filtering. Predicted trajectory of the plume of pollutants is refined online according to the values of observations incoming from terrain. The technique should be sufficiently robust to cope an expected lack of information in the same beginning of the event. A certain modification of the particle filter (PF) method is investigated here. Its robustness is illustrated on a real but atypical meteorological situation. Short time meteorological forecast entering the model is for this case in poor correspondence with the real time local meteorological measurements. Radiological measurements are assumed to be coming periodically from the Czech Early Warning Network (EWN). The respective radiological values in the real positions of EWN receptors are generated "artificially" drawing inspiration from the real local meteorological measurements.

INTRODUCTION

Ongoing efforts on improvement of safety requirements cover both implementation of inherent safety features of the new constructed facilities and substantial improvement of emergency preparedness and response. Tracking and predictions of hazardous material spreading through the living environment provide decision-makers fundamental information for effective emergency management. Modelers should be capable to generate relevant information even in the lack of some basic input information. Correct chain of simulated consequences requires as realistic as possible description of the accident evolution from the same beginning of the harmful substances release. Just at the moment the accident scenario is not known completely and large uncertainties are involved. The evolution of emergency situation is usually so far varied and complicated that specific ad hoc solutions have to be introduced.

In this paper we are studying an application of data assimilation (DA) procedure insisting in optimum combination of prior knowledge with real observations incoming from terrain. The observations bring simultaneously an indirect information related to the system state. Advanced statistical assimilation methods account for both model and measurements error covariance structure. The problem of pollution spreading in the atmosphere is described by nonlinear and generally non-Gaussian model. The attention is focused on Bayesian tracking of the toxic plume propagation over the terrain. It was shown (e.g. Doucet et al. 2001, Doucet et al. 2008, Hoteit et al. 2008, Moradkhani 2008) that except simple problems the Bayesian inference in such complex systems is not analytically tractable.

Consequently, the technique implemented here tries to solve a certain particular task of recursive Bayesian filter by Monte Carlo simulations. The objective of tracking is to refine recursively model predictions on basis of incoming measurements. Tracking in Bayesian approach concerns of recursive evaluation of the state posterior probability density function (*pdf*) evolution based on all available information. The article addresses the Bayesian tracking procedure from the same beginning of the complicated toxic plume spreading under (possibly) incomplete scenario description.

PROBLEM FORMULATION

We restrict our attention to the stochastic state-space models

$$\begin{aligned} x_t &= b(x_{t-1}) + w_t \\ y_t &= h(x_t) + v_t \end{aligned} \qquad (1)$$

in discrete time steps $t=1,\dots,T$. Here, $\boldsymbol{x}_t$ is N-dimensional vector unobserved internal quantities describing state of the model at time t, and $\boldsymbol{y}_t$ is M-dimensional vector of measurements obtained during the time step $< t\text{-}1; t >$. Nonlinear vector functions $b()$ and $h()$ describe evolution of the state in time, and mapping of the state to measurements, respectively. Disturbance (noise) vectors w_t and v_t are considered to be independent realizations of random variables with zero mean and known variances, Q_t and R_t, respectively.

Formalization (1) is intuitively appealing for stationary additive disturbances (noises). However, it may be misleading when e.g. variance of the disturbance is state-dependent. Then, we consider a slightly more general version of (1)

$$\begin{aligned} x_t &\sim p(x_t \mid x_{t-1}) \\ y_t &\sim p(y_t \mid x_t) \end{aligned} \qquad (2)$$

Where $p(\boldsymbol{x}_t|\boldsymbol{x}_{t-1})$ denotes probability density function *pdf* of random variable $\boldsymbol{x}_t$ given realization of $\boldsymbol{x}_{t-1}$. Model (1) arises as a special case (2) for choice $p(\boldsymbol{x}_t|\boldsymbol{x}_{t-1})=N(b(\boldsymbol{x}_{t-1}),Q_t)$ and $p(\boldsymbol{y}_t|\boldsymbol{x}_t)=N(h(\boldsymbol{x}_t),R_t)$. The recursion starts at t=0 for $\boldsymbol{x}_0 \sim p(\boldsymbol{x}_0)$ which is known as *prior pdf*.

Model (2) enforces too strong restrictions: (i) realization of state variable $\boldsymbol{x}$ at time t depends only on values of $\boldsymbol{x}_{t-1}$, and (ii) realization of the measurement $\boldsymbol{y}_t$ depends only on current realization of the state $\boldsymbol{x}_t$. These assumptions may seem very restrictive, however, wide range of different models can be converted into the form (2) under appropriate choice of state variable $\boldsymbol{x}_t$. For example, when initial conditions of the process or time-invariant parameters of the *pdfs* are not known, they are considered to be part of the state. In that case, $\boldsymbol{x}_t$ is sometimes called the augmented state, however, we will not make such distinction. In this paper, $\boldsymbol{x}_t$ denotes aggregation of all uncertainty in the model. Specific meaning of different parts of the state will be discussed later.

State-space formulation has been used in DA problem in the later stages of accident in post-emergency phases. Long term evolution of ^{137}Cs deposited on terrain was predicted recursively (Hofman et al. 2008a) using Kalman filter technique, which is an optimal estimator for linear functions $g()$ and $h()$ and Gaussian *pdfs* in (2). But such linear model is insufficient for formulation of more complicated problems arising in the early phase of accident (Rojas-Palma 2005) and more general nonlinear dynamic model (2) is required. Bayesian approach to estimation of unknown quantities $\boldsymbol{x}_t$ is based on recursive evaluation of posterior density $p(\boldsymbol{x}_t|\boldsymbol{y}_{1:t})$ using the Bayes rule:

$$p(x_t|y_{1:t}) \propto p(y_t|x_t)\, p(x_t \mid y_{1:t-1})$$
$$p(x_{t+1} \mid y_{1:t}) = \int p(x_{t+1} \mid x_t) p(x_t \mid y_{1:t}) dx_t. \tag{3}$$

Here, $\boldsymbol{y}_{1:t} = [\boldsymbol{y}_1,\dots,\boldsymbol{y}_t]$ and $\propto$ denotes equality up to multiplicative constant, see (Ducet et al. 2001) for details. Note that since $\boldsymbol{x}_t$ aggregates all uncertainty in the model, posterior density $p(\boldsymbol{x}_t|\boldsymbol{y}_{1:t})$ potentially provides estimates of unknown parameters, unknown initial conditions, or --- under appropriate parameterization --- even unknown variants of the model.

PARTICLE FILTERING

Except for few special cases (such as the Kalman filter), integration (3) is intractable. Therefore, various approximation has been proposed. The particle filter (also known as sequential Monte Carlo) is based on approximation of the posterior density by a weighted empirical approximation

$$p(x_t \mid y_{1:t}) \approx \sum_{i=1}^{n} w_{i,t} \delta(x_t - x_t^{(i)}) \tag{4}$$

where $x_t^{(i)}$, i=1,…,n are samples of the random variable, i.e. the particles, and $w_{i,t}$ >0, $\sum_{i=1}^{n} w_{i,t} = 1$ are particle weights. Under this approximation, integration (3) is reduced to sampling from densities (in our case $p(\boldsymbol{x}_t|\boldsymbol{x}_{t-1})$), and recursive evaluation of particle weights $w_{i,t}$.

$$w_{i,t} \propto p(y_t \mid x_t^{(i)}) w_{i,t-1} \tag{5}$$

Key advantages of this approximation are easy evaluation of an arbitrary moment, $m(x_t)$,

$$m(x_t) = \sum_{i=1}^{n} w_{i,t} m(x_t^{(i)}) \tag{6}$$

ability to handle arbitrary non-linear functions, and guaranteed convergence to the true posterior with growing number of particles n. The main disadvantage of the approach is its excessive computational cost.

Adaptation of particle filtering scheme to the early phase of the plume propagation

Intuitively, the key state variable of the scenario is distribution of the pollutant in the atmosphere over the terrain. We model this distribution via segmented Gaussian plume model (SGPM). This is a discrete model with one-hour time step. Within each hour, given amount of a pollutant is released and evolution of this quantity is simulated taking into account all environmental effects (Pecha et al. 2007).

Real release dynamics is partitioned into a number of fictive one-hour segments with equivalent homogenous averaged release source strength. Synchronization with hourly forecast of meteorological conditions is performed. Hourly segment of release is spread during the first hour as a "Gaussian droplet". In the following hours of spreading according to available hourly meteorological forecast, the droplet is treated as "prolonged puff" and its dispersion and depletion during the movement is simulated numerically by large number of elemental shifts. More detailed description of the procedure is described in (Pecha et al. 2008, Hofman et al. 2008). Each hourly segment g is consecutively modelled in its all hourly meteorological phases f. Output vector $\boldsymbol{s}_T$ of values of interest at time T after the release start are superposed as:

$$s_T = \sum_{(g=1)}^{G} \left\{ \sum_{f=g}^{f=T} s^{g,f} \right\} \tag{7}$$

Each plume segment is uniquely described by the vector variable $\boldsymbol{s}^{g,f}$. Evolution of each such plume segment over the terrain is described by deterministic SGPM model mentioned above. Let rewrite symbolically $\boldsymbol{s}^{g,f}$ to $\boldsymbol{s}(\tau)_t$, where $\tau < t$ denotes time of the release of the plume segment. The SGPM model contains many input and model parameters (Pecha et al. 2005). Most of them are treated as single values that enter the model by their best estimate values. Important random parameters are selected on basis of sensitivity analysis of the SGPM model and constitute random vector $\boldsymbol{\Theta}$. Independent random components of the vector $\boldsymbol{\Theta}$ are labelled as $\Theta_{m,m=1,\dots,M}$. Random samples i from $pdf(\Theta_m)$ are marked as θ_m^i . The components Θ_m selected for our scenario demonstrates Table 1. The aim of investigations calls for inclusion of as large as possible number of random parameters M. So far, because of computational practicability, the Table 1 presents the case with M truncated to 10.

Variable $\boldsymbol{s}(\tau)_t$ is now parameterized by vector of parameters $\boldsymbol{\Theta}^t$. This vector contains both time invariant parameters, such are dispersion and dry deposition characteristics, and time-variant parameters, such are wind direction and wind velocity at time t.

Under probabilistic formalization (2), the original SGPM model is interpreted as conditional density

$$p(s(\tau)_t \mid s(\tau)_{t-1}, \Theta^t) = \delta\left(s(\tau)_t - SGPM(s(\tau)_{t-1}, \Theta^t)\right) \quad (8)$$

Parameters Θ^t were considered to be known in the original formulation. In this text, we consider them to be unknown, hence we consider them to be part of the state. The state is then $x_t = [s(1)_t, s(2)_t, ..., \Theta^t]$ and its evolution model

$$p(x_t \mid x_{t-1}) = \prod_{\tau=1}^{t} p(s(\tau)_t \mid s(\tau)_{t-1}, \Theta^t) p(\Theta^t) \quad (9)$$

Distribution of the parameter vector $p(\Theta^t)$ is composed of independent *pdfs* of components Θ_m^t given in Table 1.

Table 1: Components Θ_m of random parameter vector Θ. Count of components truncated to M=10.

random parameter	*unit*	*implementation in code*	*uncertainty bounds*
Θ_1: radioactiv. release during hour 1 (f=1)	[Bq.h^{-1}]	$Q = c_1 \times Q^b$; Q^b in f=1	LU; $c1 \in$ <0.31;3.1>
Θ_2 : horizont. dispersion	[m]	$\sigma_y = c_2 \times \sigma_y^b$	N_{trunc} ; $c2 \in$ <0.89;1.12>
Θ_3 : dry depo velocity	[m.s^{-1}]	$vg = c_3 \times vg^b$	LU ; $c3 \in$ <0.91;1.10>
Θ_4 : Wind direction f=1	[rad]	$\varphi=\varphi^b + \Delta\varphi$, $\Delta\varphi=c_4 \times 2\pi/80$	U ; $c4 \in$ <-12;+12>
Θ_5 : Wind direction f=2	[rad]	$\varphi=\varphi^b + \Delta\varphi$, $\Delta\varphi=c_5 \times 2\pi/80$	U ; $c5 \in$ <-12;+12>
Θ_6 : Wind direction f=3	[rad]	$\varphi=\varphi^b + \Delta\varphi$, $\Delta\varphi=c_6 \times 2\pi/80$	U ; $c6 \in$ <-12;+12>
Θ_7 : Wind speed f=1	[m.s^{-1}]	$V_{10}= c_7 \times V^b_{10}$; V^b_{10} in f=1	U ; $c7 \in$ <0.5;3.0>
Θ_8 : Wind speed f=2	[m.s^{-1}]	$V_{10}= c_8 \times V^b_{10}$; V^b_{10} in f=2	U ; $c8 \in$ <0.5;3.0>
Θ_9 : Wind speed f=3	[m.s^{-1}]	$V_{10}= c_9 \times V^b_{10}$; V^b_{10} in f=3	U ; $c9 \in$ <0.5;3.0>
Θ_{10}: radioativ. release during hour 2 (f=2)	[Bq.h^{-1}]	$Q = c_{10} \times Q^b$; Q^b in f=2	LU ; $c10 \in$ <0.31;3.1>

Index b stands for "best estimate " values; V_{10} – wind speed at 10 m height; f – phase (hour) after the release start;

Type of distribution: LU-loguniform; N_{trunc} – Normal, truncated; U – Uniform;

The measurements are modelled to have Gaussian distribution:

$$p(y_t \mid x_t) = N\left(\sum_{\tau=1}^{t} SGPM(s(\tau)_t), \Sigma_t\right) \quad (10)$$

The mean value is given by the sum of outputs from each plume segments and is approximated by bilinear approximation of the SGPM model predictions at the points of measurements. For the experiment purposes the covariance matrix Σ_t is constructed as

$$\Sigma_t = \lambda_{\text{model}} I_M + \lambda_{\text{prop}} \text{diag}(y_t) \quad (11)$$

with chosen constants λ_{model} and λ_{prop}. The first term models inaccuracies of the chosen Gaussian plume approximation, the second term models inaccuracies of the measuring devices. This model is almost an arbitrary choice, that is used to show potential of the considered methodology. Model of observation for practical purpose should be designed using exact characteristics of the application specific measurement devices.

Implementation of PF algorithm

The following steps represent computational flow of recursive particle filtering applied here:

1. Generate n realizations of parameter vector Θ^0 from densities listed in Table 1, $\{ [\theta_m^i]_{m=1:M} \}^{i=1:n}$. Substitution of sets of realisations $[\theta_m^i]_{m=1:M}$ for each i into SGPM model (7) yields n corresponding plumes (in the following text interpreted as "particle"). Initially, the same weight $w_{i,0}=1/n$ is assigned to each particle i.
2. For each time t=1...T :
   a. Generate a set $\{ [\theta_m^i]_{m=1:M} \}^{i=1:n}$ of realizations of Θ^t and for each plume (particle) compute one step prediction using the SGPM algorithm. The term "particle prolongation" is introduced.
   b. If measurements are available, recompute the weights $w_{i,t}$ using (5).
   c. Compute posterior values of parameters of interest using (6)

Parameter vector Θ is expressed in Equation (8) as Θ^t. It means that count of the components treated as random within a certain time interval can vary, symbolically:

$$\Theta^{t=1} \approx \boxed{\Theta_1 \;\; \Theta_2 \;\; \Theta_3 \;\; \Theta_4} \;\; \Theta_5 \;\; \Theta_6 \;\; \boxed{\Theta_7} \;\; \Theta_8 \;\; \Theta_9 \;\; \Theta_{10} \ldots.$$

$$\Theta^{t=2} \approx \boxed{\Theta_1 \;\; \Theta_2 \;\; \Theta_3} \;\; \Theta_4 \;\; \boxed{\Theta_5} \;\; \Theta_6 \;\; \Theta_7 \;\; \boxed{\Theta_8} \;\; \Theta_9 \;\; \boxed{\Theta_{10}} \ldots.$$

$$\Theta^{t=3} \approx \Theta_1 \;\; \Theta_2 \;\; \Theta_3 \;\; \Theta_4 \;\; \Theta_5 \;\; \boxed{\Theta_6} \;\; \Theta_7 \;\; \Theta_8 \;\; \boxed{\Theta_9} \;\; \boxed{\Theta_{10}} \ldots.$$

Let assume only the first three hours from the same beginning of an accident. It corresponds to 10 parameters from Table 1. Bounded components stand for the relevant components that enter the sampling procedure in the particular time step. Alternative resampling schemes could be constructed (e.g. locally dependant land use characteristics when corresponding θ_2 and θ_3 could be assumed relevant in all time steps).

Experimental results

The sampling scheme consists of generation of 5000 particles corresponding to n=5000 realisations of random parameter vector Θ with 10 components $\Theta_{m,m=1,...,10}$ according to uncertainty characteristics described in Table 1.

Evaluated values of the particle weights using $\lambda_{\text{model}} = 10^4$ and $\lambda_{\text{prop}} = cov \times \kappa$, with cov =1,…5 , are illustrated in Figure 1. The smallest values of variance (top) sharply selects only a few particles. With increasing variance, cov=2, …,5, uncertainty in the weight grows and more particles become non-negligible. Constant κ=1.0E+6 ensures link to measured magnitudes of radioactivity deposition.

Prior and posterior histograms of distributions of some parameters Θ_m from Table 1 are compared in Figure 2. Note that the posterior is sharply peaked for the three leftmost parameters while it is still widespread for the remaining parameters. But we should distinguish between parameter estimation for the concrete analysed situation and common average conditions. It should not be confused with parameter estimation which could give recommendation on parameter values commonly valid "in average".

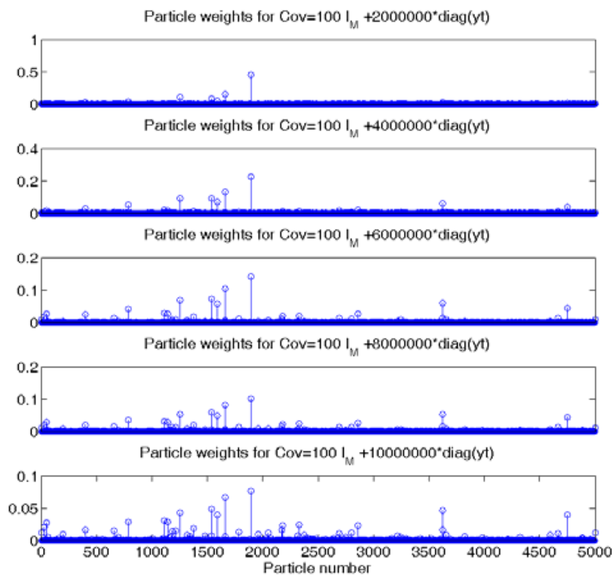

Figure 1: Posterior weights w_t for five choices *cov*, update using measurements incoming just after 2 hours from start.

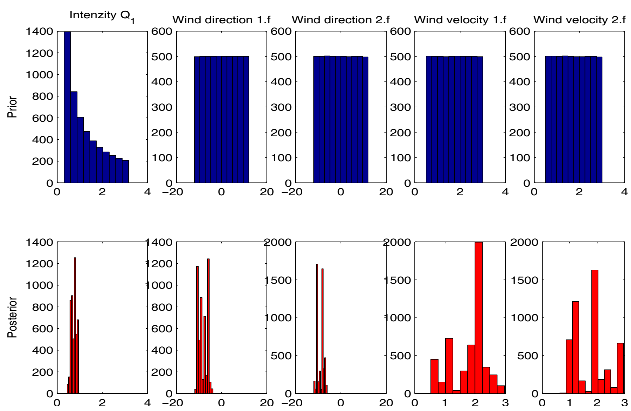

Figure 2: Comparison of prior (top row) and posterior (bottom row) histograms of distribution of selected parameters for *cov*=3.

ILLUSTRATION OF PARTICLE FILTERING APPLIED IN THE EARLY STAGE OF A HYPOTHETICAL ACCIDENT

The robustness of the PF method outlined above is illustrated for case of a certain circumstance when in the same beginning of an accident the decision maker is not provided by fully clear and unambiguous information. Experience from former radiation accidents pointed out the side effects leading to an information shocks with possible temporal paralysis of communication lines. In this sense we have adjusted a hypothetical accident scenario. Real meteorological situation from March 31, 2009 is taken into consideration and the moment of hypothetical radioactivity release is set to 10.00 CET. Available real meteorological observations measured at the point of nuclear power plant (NPP) and short term meteorological forecast are somewhat inconsistent (see next Table 2). Following ex post analysis can give a retrospective view on such the atypical situations (their occurrence rate is surprisingly not negligible). Due to a possible information shock mentioned above we shall assume conservatively a delay of two hours in recovery of radiation monitoring. Thus, the first measurements from terrain are coming just two hours after the release started. A decision maker has a dilemma how to manage the prediction of harmful substances in the early stage.

Available meteorological data

Let release of ^{131}I radioactivity has started at 10.00 CET, March 31, 2009, and lasted for 2 hours (Table 2).

Table 2: Accidental release scenario of ^{131}I , short-term meteorological forecast and real meteorological measurements for "point" of NPP Temelin (49°10'48.53"N × 14°22'30.93"E), time stamp 20090331-1000 CET.

CET hour	10.00	11.00	12.00	13.00
activity release of ^{131}I Bq/hour	5.68 × e+14	7.92 × e+14	0	0
wind direction[1] **METLOC/METOBS**	95.0 / 54.0	101.0 / 69.0	84.0 / 65.0	80.0 / 80.0
wind speed[2] **METLOC/METOBS**	2.0 / 3.8	2.1 / 3.0	1.9 / 3.8	2.2 / 3.8
Pasquill atm. stability	A	A	B	B

1) ... at 10 m height, blowing "from" (degrees measured clockwise from North); 2) ... at 10 m height (m/s)

At moment of accident, the three kinds of meteorological data were directly available:

- Short term meteorological forecast generated twice a day, sequences up to 48 hours):
  - Label METLOC: Simple local forecast for the point of NPP (hourly sequences of wind direction and speed, category of atmospheric stability according to Pasquill and precipitation),
  - Label METGRID: 3-D meteorological forecast in HIRLAM format for vicinity 160 × 160 kilometers around NPP.
- Label METOBS: Observed values (real online meteorological measurements) incoming automatically from the point of NPP.

All the data are provided by the Czech meteorological service and are available online through ORACLE DB server.

Deterministic calculations according to SGPM model with METLOC meteorology for the first two hours of the release are illustrated in Figure 3. Superposition according to Equation (7) was used for quantity of ^{131}I deposition on the ground (first segment g=1, in phases f=1 and 2; second segment g=2, in phase f=2).

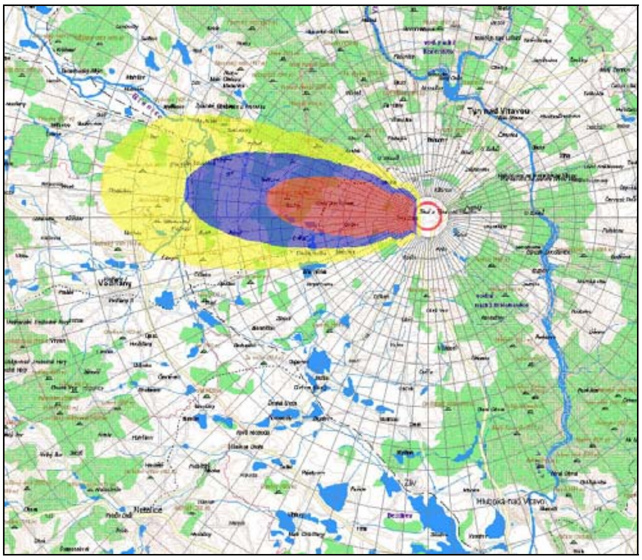

Figure 3: Release scenario with meteodata METLOC - model predictions for “best estimate” values of model parameters, just 2 hours after the release start.
<u>^{131}*I deposition ranges (Bq.m^{-2}):*</u> red: 5.00e+06 ÷ 1.30e+08 ; blue: 1.00e+06 ÷ 5.00e+06 ; yellow: 1.00e+05 ÷ 1.00e+06 ;

Arrangement of the real positions of monitoring sensors

Early Warning Network (EWN) such a component of existing Radiation Monitoring Network (RMN) of the Czech Republic can be exploited for purposes of DA procedures. The main part of EWN is teledosimetric system (TDS) which for the NPP Temelin consists of two circles. The inner circle is positioned on the NPP-fence (see red circles in Figure 4 very close to NPP or in better discrimination in Figure 5) and consists from 24 stations 2,5m above ground. The outer II. circle of measurement positions is drown in Figure 4 by red squares. The dose-rate data are transferred each 4 minutes and stored to the ORACLE DB server for online access. We are assuming all these receptors to be operable. An ability to measure selected magnitudes of deposition is a question of a future monitoring development.

For DA purposes we have 79 sensors located in vicinity of the nuclear facility. In this number we have included 3 mobile stations located randomly in the middle distances.

Artificial simulation of the missing real accidental radiological data

We hope that all considerations remain only in hypothetical level and that the testing accidental radiological data will be always generated artificially. The technique is sometimes known as “twin experiment”.

A degree of belief to the initial near-range estimation using the SGPM model predictions with METLOC meteorological forecast (see Figure 3) will be low if we take into considerations the similar calculations with METOBS real meteorological measurement (see Figure 4). We should respect the fact that if something happens, the shape of the corresponding accidental trajectory close to the source should correspond more likely with the Figure 4.

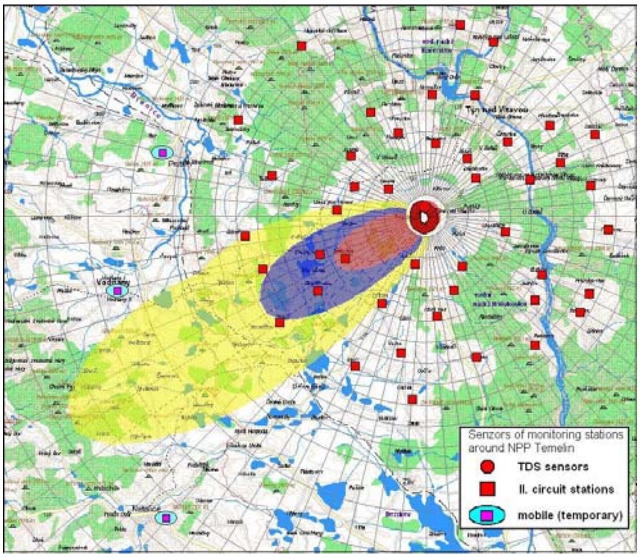

Figure 4: Release scenario with meteodata METOBS - model predictions for “best estimate” values of model parameters, just 2 hours after the release start.
^{131}*I deposition ranges (Bq.m^{-2})*: the same as in Figure 3. This figure also illustrates configuration of the inner part of the Czech EWN around NPP Temelin.

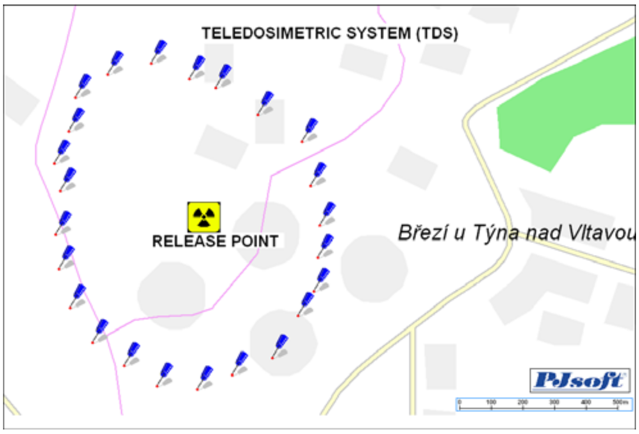

Figure 5: TDS on fence of NPP Temelin – 24 detectors

Without more discussion, we use this subjective assumption and generate the “artificial measurements” on the basis of METOBS real meteorological measurement in Figure 4. Though the model requirements so far exceed possibilities of monitoring in the Czech Republic, the cooperation between modelers and monitoring has growing importance. The assimilation subsystem is developed in cooperation with National Radiation Protection Institute (NRPI) which is administrator of RMN.

THE RESULTS ACHIEVED FOR SEVERAL FIRST TIME STEPS

Finally, the following hypothetical data assimilation scenario defined for the early phase is accomplished:

1. Predictions of 5000 particles (trajectories) for 5000 realisations of the parameter vector **Θ** according to the SGPM model. It covers time interval 2 hours from the same beginning of accident, no measurements from terrain are not yet avail-

able. Gridded meteorological forecast METGRID is always used. Prior probabilistic density function and its moments can be estimated.

2. Let the first set of "artificial measurements" is incoming just two hours after the release start. The values of "measurements" are generated according to Figure 4 and the speculations introduced above.
3. Update step of recursive procedure of PF estimates the posterior density function on basis of weighted empirical approximation given by Equation (4).
4. Recursion continues in the next time interval performing the transition step with resampled particles.

Approximation of posterior *pdf* is generated for 5 choices of covariances *cov*=1,….,5 according to Equation (11) (where $\lambda_{prop} = cov \times \kappa$) and "measurements" from Figure 4. Expected mean values are calculated using common expression according to Equation (6), specifically in the form:

$$I(f_t) := E_{p(x_t|y_{1:t})}\left[f(x_t)\right] = \int f(x_t)\, p(x_t \,|\, y_{1:t})\, dx_t \tag{12}$$

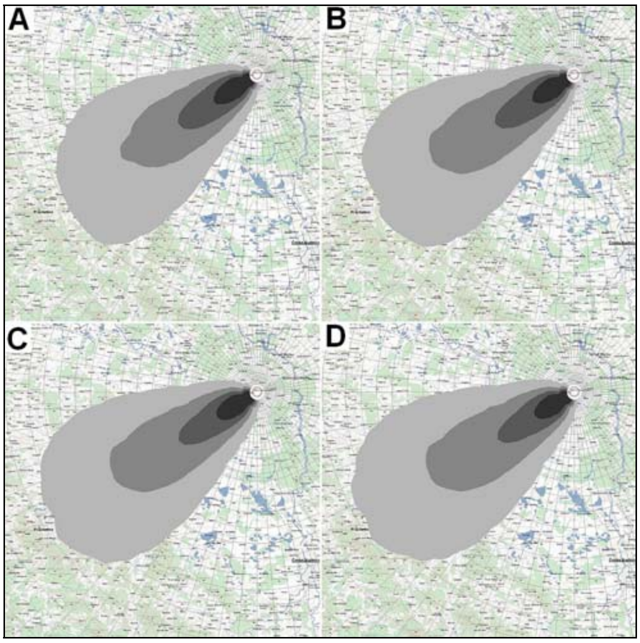

Figure 6: Expectations of posterior *pdf* of the radioactivity deposition in dependency on covariance matrix (according to Equation (11)). A,B,C,D stand for *cov*=1,2,4,5.

An estimation of the expectations on basis of *n* generated particles $x_t^{(i)}$, $i=1:n$ from posterior distribution is given by:

$$I^n(f_t) = \frac{1}{n}\sum_{i=1}^{n} f\left(x_t^{(i)}\right) \tag{13}$$

For $n \to \infty$ is achieved almost sure convergence of $I^n(f)$ to $I(f)$.

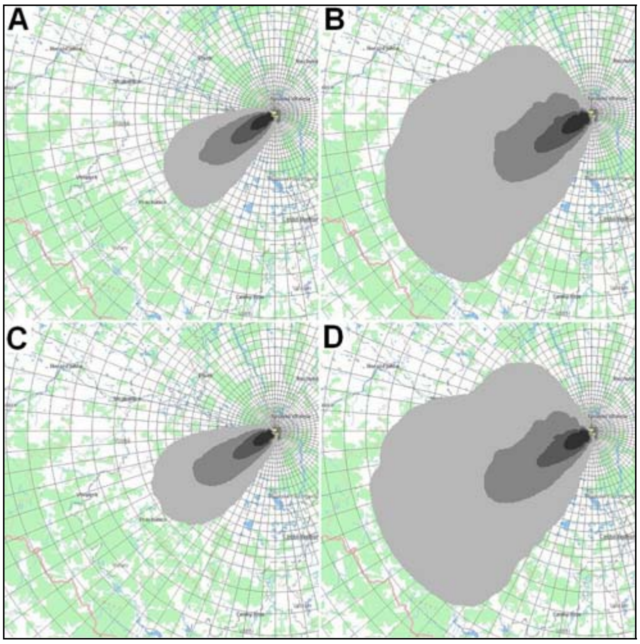

Figure 7: Transition step for the next time interval. Prior *pdf* expectations for transition from hour 2 to hour 3. (case A → B for *cov*=1; case C → D for *cov*=5).

The expectations of the quantity of activity deposition are given in Figure 6 for cases of *cov*=1,2,4,5. The outer contour corresponds to the level of 1.00 E+03 Bq.m^{-2}. The results show tendency of the updated model to approach the measurements with low noises. The values are slightly spreading when inaccuracies of measurements grows (higher *cov*). Covariances of the measurement errors were selected rather low. At present new tests with increased covariance are running and tendency to lean to either model predictions or measurements are mapping.

Figure 7 demonstrates prolongation one time step forward. Case A concerns *cov*=1 (also in Figure 6 A) expectation from the posterior density just after 2 hours after the release start. Numerical approximation of the SGPM model is used for solution of the second part of Equation (3) which stands for transition equation for specific formulation $p(\mathbf{x}_{f=3} \mid \mathbf{x}_{f=2})$. Prediction from analysis (data update) in the second hour (upper left A) to the third hour (upper right B) is done (prediction step). SGPM model prolongs the weighted particles within the hour 2 → 3. The similar shift for *cov*=5 stands for cases C → D.

CONCLUSION

The article extends former investigations in DA methodology (Hofman et al. 2007) where analysis of the input model parameters uncertainty and both model error and observation error covariance structure were examined. DA in early stage of accident requires much more sophisticated access. From all possible techniques is adopted particle filter, which has one significant attribute. In PF the state ensemble trajectories are kept unchanged during the update step as for the forecast step and only their weights are updated. The particles remain unchanged after the correction (update) step and only receive the new weight (according to Equation (5)) reflecting closeness of the particle with respect the new observations.

This evident PF feature has favourable impact on exploitation of nonlinear prediction model SGPM in DA process in the early stage. SGPM model is in principle a trajectory model. The PF does not disrupt the trajectory information and it can be easily recursively forwarded.

The presented approach brings advantage of fast computation even for large number of realisations. One PF step of update and predictions with 5000 realisations is accomplished during about 15 minutes (common PC config.) and promises to support the decision making process in real time.

The adopted procedure seems to be robust and suitable to manage a certain discrepancies and scenario incompletness occurring from the same beginning of an accident. The authors narrow down anxiously the range of some uncertainties. For example the range of horizontal dispersion uncertainty c_2 and dry deposition c_3 should be much higher (in correspondence with expert judgments). Afterwards, the traces (e.g. in Figure 6) would be more dispersed in horizontal and longitudinal directions. Even the calculations have covered only the first time step and demonstrated code ability to predict in the second step, the full recursive PF application seems to be easily feasible.

Still open remains a question of availability of measurements, capability to provide specific quantities and configuration and density of monitoring stations. The first negotiation between modellers and specialists responsible for monitoring was launched (Kuca et al. 2008). The poor information can result from rare measurements. On the other hand, requirements issued from DA experience should be reflected in the future development of radiation monitoring networks.

DA plays substantial role in realistic prediction of evolution of radiation situation during nuclear emergency. Reliable information arriving on time provides decision makers with necessary time on judgement and introduction of efficient urgent countermeasures on population protection.

ACKNOWLEDGEMENTS

This work is part of the grant project GAČR No. 02/07/1596, which is founded by Grant Agency of the Czech Republic. The activities has been also supported by Research center DAR. Thanks to National Radiation Protection Institute in Prague, the online connection of developed software to meteorological forecast and radiological data from Early Warning Network of the Czech Republic could be tested.

REFERENCES

Arulampalam, M. S.; S. Maskell, N. Gordon and T. Clapp. 2002. "A Tutorial on Particle Filters for Online Nonlinear/Non-Gaussian Bayesian Tracking". *IEEE Transactions on Signal Processing, Vol. 50, No. 2.*

Drécourt, J-P. 2004. "Data assimilation in hydrological modeling". Environmental&Resources, Techn. Univ. of Denmark, ISBN 87-89220-84-6.

Doucet, A.; N. De Freitas and N.J. Gordon (eds.). 2001. S*equential Monte Carlo method in practice.* Springer-Verlag, New York.

Doucet, A. and A.M. Johansen. 2009. "A Tutorial on Particle Filtering and Smoothing: Fifteen years later". *Will be published in Handbook of Nonlinear Filtering (2009).*

Moradkhani, H. 2008. "Hydrologic Remote Sensing and Land Surface Data Assimilation". *Sensors, 8, 2986-3004, ISSN 1424-8220.*

Hofman, R. and P. Pecha 2007. "Integration of data assimilation subsystem into environmental model of harmful substances propagation". In *Proc. of the 11^{-th} Int. Conf. on Harmonisation within Atmospheric Dispersion Modelling* (Cambridge, UK, July 2-5, 2007), 111-115.

Hofman, R.; P. Pecha ; and E. Pechová . 2008. "A simplified approach for solution of time update problem during toxic waste spreading in the atmosphere", *Hrvatski Meteorološki Časopis, 12^{-th} Int. Conf. on Harmonisation within Atmospheric Dispersion Modelling* (Cavtat, HR, Oct. 6-10, 2008) .

Hofman, R. and P. Pecha 2008a „Data Assimilation of Model Predictions of Long-Time Evolution of ^{137}Cs Deposition on Terrain" , Proceeding of the IEEE International Geoscience & Remote Sensing Symposium 2008, IEEE International Geoscience & Remote Sensing Symposium 2008, (Boston, US, 06.07.2008-11.07.2008) (2008).

Hoteit, I., D.-T. Pham, G. Triantafyllou and G. Korres. 2008. Particle Kalman Filtering for Data Assimilation in Meteorology and Oceanography. *Mon. Wea. Rev., 136.*

Kalnay, E. 2003. *Atmospheric modeling, data assimilation and predictability*. Cambridge University Press, ISBN 0-521-79179-0.

Kuca, P., R. Hofman and P. Pecha. 2008. "Assimilation techniques in consequence assessment of accidental radioactivity releases – the way for increase of reliability of predictions". In. *Proc. of ECORAD 2008 Int. Conf. On Radioecology & Environ.Radioactivity* (Bergen, Norway, June 15-20, 2008), 138-141, ISBN 978-82-90362-25-1.

Pecha, P., R. Hofman and E. Pechova. 2007. "*Traini*ng simulator for analysis of environmental consequences of accidental radioactivity releases". In *Proc. of the 6^{th} EUROSIM Congress on Modelling and Simulation* (Ljubljana, Slovenia, Sept. 9-13, 2007), 18 pp., CD– ISBN 978-3-901608-32-2.

Pecha, P. and E. Pechova. 2005. "Modeling of random activity concentration fields in air … ". *In Proc. of the 10^{-th} Int. Conf.. on Harmonisation within Atmospheric Dispersion Modelling* (Sissi, Greece,), paper No. H11-069.

Pecha, P. and R. Hofman. 2008. "Fitting of segmented Gaussian plume model predictions on measured data". *In Proc. of the 22th European Simulation and Modelling Conference ESM'2008* (LeHavre, FR, Oct 27-29,2008)

Pecha, P., R. Hofman and V. Šmídl. 2009. "Simulation of random 3-D trajectories of the toxic plume spreading over the terrain". In *Proc. of the 5^{th} WMODA conf on Data Assimilation (Melbourne, Australia, Oct. 5-9, 2009).*

Rojas-Palma, C. 2005, Data assimilation for of site nuclear emergency management. Technical report, SCK-CEN, DAONEM, RODOS(RA5)- final report, RODOS(RA5)-RE(04)-01, 2005.