SENTIMENT PATTERNS IN THE HETEROGENEOUS AGENT MODEL

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Abstract

In this paper we extended the original model of heterogeneous agent model by introducing smart traders and changes in the agents sentiment to the model. The idea of the smart traders is based on endeavor of the market agents to estimate the future price movements. By adding smart traders and sentiment changes we try to improve the original heterogeneous agents model so it will be able to come to closer description of the real markets. The main result of the simulations is that probability distribution functions of the price deviation changes significantly with adding smart traders to the model, and it also changes significantly with introduction of the sentiment changes. We use also Hurst exponent to measure the persistence of the price deviations and we find that the Hurst exponent is significantly increasing with smart traders in simulations. This means that the introduction of smart traders concept into the model results in significantly higher persistence of the simulated price deviations. On the other hand, introduction of changing sentiment in the proposed form does not change the persistence of simulated prices significantly.

Keywords: *heterogeneous agent model, market structure, smart traders, Hurst exponent*

JEL: *C15, D84, G14*

1. Introduction

Modern finance is undergoing an important change in the perception of economic agents, i.e., from a representative rational agent approach towards a behavioral, agent-based approach with markets boundedly rational, where heterogeneous agents apply rule of thumb strategies. The traditional approach, rested on a simple analytically tractable models with a representative, perfectly rational agent and mathematics as the main tool of analysis. The new behavioral approach fits much better with agent-based simulation models and computational and numerical methods have become an important tool of analysis, Hommes (2005). The new behavioral, heterogeneous agents approach challenges the traditional representative rational agent framework. Heterogeneity in expectations can lead to market instability and complicated price dynamics. Prices are driven by endogenous market forces. Typically, in the heterogeneous agents model (HAM), two types of agents are distinguished: fundamentalists and chartists. Fundamentalists base their expectations about future asset prices and their trading strategies on market fundamentals and economic factors, such as dividends, earnings, macroeconomic growth, unemployment rates, etc. Chartists or technical analysts try to extrapolate observed price patterns, such as trends, and exploit these patterns in their investment decisions. One such model was developed by Brock, Hommes (1998). In our previous papers, Vosvrda, Vacha (2002, 2003), we introduced a memory and some learning schemes to the model of Brock and Hommes. We also introduced further extensions, such as stochastic formation of beliefs and parameters including the memory length. Another extension is the application of the Worst Out Algorithm (WOA), which periodically replaces the trading strategies with the lowest performance, Vacha, Vosvrda (2005, 2007). In Vacha, Vosvrda (2005) we showed how memory length distribution in the agents' performance measure affects the persistence of the generated price time series.

In this paper we introduce a new concept of smart predictors. The idea of the smart traders is based on endeavor of the market agents to estimate the future price movements. By adding smart traders we try to improve the original heterogeneous agents model so it will be able to better approximate the real markets. The smart traders are designed to forecast the future trend parameter of the price deviations using the information set consisting of past deviations. They are modeled to assume that the deviations are AR(1) process and they use maximum likelihood estimation method for forecasting. Thus in our model we use 2 groups of traders, smart traders and group of stochastically generated trading strategies which are moreover selected by Worst Out Algorithm. Furthermore we introduce the changes in sentiment which we define as a shift of the beliefs about future trend of newly incoming investor strategy to the market. This allows us to model trend-followers and contrarians. In this paper we use only the form of jumps of sentiment. Our main expectation is that introduction of smart traders and changes in sentiment to the market will change the simulated market prices significantly.

The first part of the paper concerns a heterogeneous agent model, which is an extension of the Brock and Hommes (1998) model. The second part briefly introduces the implementation of the smart traders into the heterogeneous agent model framework. The last part of the paper investigates how the presence of smart predictors and sentiment jumps qualitatively changes the market structure.

2. Model

Capital markets are perceived as systems of interacting agents who immediately process new information. Agents adapt their predictions by choosing from a limited number of beliefs (predictors or trading strategies). Each belief is appreciated by a performance measure. Agents on the capital market use this performance measure to make a rational choice which depends on the heterogeneity in agent information and subsequent decisions of the agent either as a fundamentalist or as a chartist, Chiarella, He (2000). The model presents a form of evolutionary dynamics, called Adaptive Belief System, in a simple present discounted value (PDV) pricing model. The first part of this model was elaborated by Brock and Hommes (1998).

Consider an asset pricing model with one risky asset and one risk free asset. Let p_t denote the price (ex dividend) per share of the risky asset at time t, (random variables at time $t+1$ are denoted in bold), and let $\{\mathbf{y}_t\}$ be a stochastic dividend process of the risky asset. The supply of the risk free asset is perfectly elastic at the gross risk free interest rate R , which is equal to $1+r$, where r is the interest rate. Then the dynamics of the wealth is defined as:

$$\mathbf{W}_{t+1} = RW_t + \left(\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - Rp_t\right) z_t, \tag{2.1.}$$

where z_t denotes the number of shares of the asset purchased at time t . Let us further consider E_t and V_t as the conditional expectation and conditional variance operators based on the set of publicly available information consisting of past prices and dividends, i.e., on the information set $\mathbf{F}_t = \{p_t, p_{t-1}, ?; y_t, y_{t-1}, ?\}$. Let $E_{h,t}$ and $V_{h,t}$ denote beliefs (or forecast) of type h investor about the conditional expectation and conditional variance. Investors are supposed to be a myopic mean-variance maximizers so that the demand $z_{h,t}$ for risky asset is obtained by a solving of the following criterion

$$\max_{z_{h,t}} \left\{ E_{h,t}\left[\mathbf{W}_{t+1}\right] - \frac{a}{2} V_{h,t}\left[\mathbf{W}_{t+1}\right] \right\}, \tag{2.2.}$$

where the risk aversion coefficient, $a > 0$, is assumed to be the same for all traders. Thus the demand $z_{h,t}$ of type h for risky asset has the following form

$$E_{h,t}\left[\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - Rp_t\right] - a\sigma^2 \mathbf{z}_{h,t} = 0, \tag{2.3.}$$

$$z_{h,t} = \frac{E_{h,t}\left[\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - Rp_t\right]}{a\sigma^2}, \tag{2.4.}$$

assuming that the conditional variance of excess returns is a constant for all investor types

$$V_{h,t}\left(\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - Rp_t\right) = \sigma_h^2 = \sigma^2. \tag{2.5.}$$

Let z_t^s be the supply of outside risky shares. Let $n_{h,t}$ be a fraction of type h investor at time t. The equilibrium of the demand and supply is

$$\sum_{h=1}^{H} n_{h,t} \left\{ \frac{E_{h,t}\left[\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - Rp_t\right]}{a\sigma^2} \right\} = z_t^s, \tag{2.6.}$$

where H is the number of different trader types. In the case of zero supply of outside shares , i.e., $z_t^s = 0,$ the market equilibrium is as follows

$$Rp_t = \sum_{h=1}^{H} n_{h,t} \left\{ E_{h,t}\left[\mathbf{p}_{t+1} + \mathbf{y}_{t+1}\right] \right\}. \tag{2.7.}$$

In a market, where all agents have rational expectations, the asset price is determined by economic fundamentals. The price is given by the discounted sum of future dividends

$$p_t^* = \sum_{k=1}^{\infty} \frac{E_t\left[\mathbf{y}_{t+k}\right]}{\left(1+r\right)^k}. \tag{2.8.}$$

The fundamental price p_t^* depends upon the stochastic dividend process y_t . In the special case when the dividend process $\{y_t\}$ is an independent, identically-distributed (IID) process, with constant mean $E_t\{y_t\} = \overline{y}$, then we have the fundamental price given by

$$p^* = \sum_{k=1}^{\infty} \frac{\overline{y}}{\left(1+r\right)^k} = \frac{\overline{y}}{r}. \tag{2.9.}$$

2.1. Heterogeneous Beliefs

This part deals with traders' expectations about future prices. As in Brock, Hommes (1998), we assume beliefs about future dividends to be the same for all trader types and equal to the true conditional expectation, i.e.

$$E_{h,t}\left[\mathbf{y}_{t+1}\right] = E_t\left[\mathbf{y}_{t+1}\right], \quad h = 1,...,H, \tag{2.10.}$$

In the case when the dividend process $\{\mathbf{y}_t\}$ is an independent, identically-distributed (IID) process, $E_t\{\mathbf{y}_{t+1}\} = \overline{y}$, then all traders are able to derive the fundamental price p_t^* that would dominate in a perfectly rational world. Abandoning the idea of rationality and moving to the real world we allow prices to deviate from their fundamental value p_t^*. For our purpose, it is convenient to work with a deviation x_t from the benchmark fundamental price p_t^*, i.e.,

$$x_t = p_t - p_t^*. \tag{2.11.}$$

In general, beliefs about the future price $E_{h,t}\left[\mathbf{p}_{t+1}\right]$ have the following form

$$E_{h,t}\left[\mathbf{p}_{t+1}\right] = E_t\left[p_t^*\right] + f_h\left(x_{t-1},\ldots,x_{t-L}\right), \qquad \text{for all } h,t, \tag{2.12.}$$

where $f_h\left(x_{t-1},\ldots,x_{t-L}\right)$ represents a model of the market. The trader type h believes that the market price will deviate from its fundamental value p_t^*. The heterogeneous agent market equilibrium, defined in Eg. (2.7.), can be reformulated in deviations from the benchmark fundamental as

$$Rx_t = \sum_{h=1}^{H} n_{h,t} E_{h,t}\left[\mathbf{x}_{t+1}\right] \equiv \sum_{h=1}^{H} n_{h,t} f_{h,t}. \tag{2.13.}$$

2.2. Selection of Strategies

Beliefs are updated evolutionary, the selection is controlled by the endogenous market forces, Brock and Hommes (1997). The fractions of trader types on the market $n_{h,t}$, are given by the multinomial logit probabilities of discrete choice

$$n_{h,t} = \exp\left(\beta U_{h,t-1}\right) / Z_t, \tag{2.14.}$$

$$Z_t = \sum_{j=1}^{H} \exp\left(\beta U_{h,t-1}\right), \tag{2.15.}$$

where $U_{h,t-1}$ is the fitness measure of strategy h evaluated at the beginning of period t. As a fitness measure of trading strategies we use moving averages of realized profits, where m_h denotes the length of the moving average filter. In a real market this parameter can be interpreted as a memory length or evaluation horizon for a trading strategy h at time t. Fitness measure $U_{h,t}$ is defined as

$$U_{h,t} = \frac{1}{m_h} \sum_{k=1}^{m_h} \left[\left(x_t - Rx_{t-1}\right) \frac{\left(f_{h,t-1} - Rx_{t-1}\right)}{a\sigma^2} \right]. \tag{2.16.}$$

3. Trading Strategies

In Brock and Hommes (1998) have investigated simple linear rules with one lag with fixed g_h :

$$f_{h,t} = g_h x_{t-1} + b_h, \tag{3.1.}$$

which served as a basic framework for the heterogeneous agent models. In this paper we enrich the model by introducing the smart traders who are able to forecast these linear rules and thus are able to forecast future trend parameter $g_{h,t}$ which is variant in time. In our model we have two groups of trading strategies. The first group consists of smart trading strategies which use simple linear regression predictions of $f_{h,t}$, and the second group comprises trading strategies which are generated stochastically and they are selected with the Worst Out Algorithm (WOA) during simulations. These two groups $f_{h,t}^1$ and $f_{h,t}^2$ will be defined below.

3.1.Smart Traders

The idea of the smart traders is based on endeavor of the market agents to estimate the future price movements. By adding smart traders we try to improve the original heterogeneous agents model so it will be able to come to closer description of the real markets.

The simplest way how to implement this type of market behavior into the Brock and Hommes model is to use the simple linear forecasting techniques. The smart trading strategies thus use the maximum likelihood estimation of AR(1) process to estimate the trend parameter $g_{h,t}$ for the next period. Smart traders thus assume that the deviations x_t follows an AR(1) process, and they base their forecasts of x_{t+1} on the information set $\mathsf{F}_t = \{x_t, x_{t-1}, ?, x_{t-k-1}\}$. Then the trading strategies of the smart traders are defined as follows:

$$f_{h,t+1}^1 = \widehat{f}_{h,t} = \varphi_1 x_{t-1}, \tag{3.2.}$$

where φ_1 is a estimated trend $\widehat{g}_{h,t}$. In the simulations, we use various types of smart traders with different length of the information set k .

3.2. Stochastic Beliefs

Trading strategies of the second group, $f^2_{h,t}$, are generated stochastically. The trend parameter g_h and the bias parameter b_h of the trader type h are realizations from the normal distribution $N(0,\sigma^2)$. In this paper we use $N(0,0.16)$ and $N(0,0.09)$ respectively. The memory parameter m_h of the trading strategy $f^2_{h,t}$ is a realization from the uniform distribution, specifically $U(1,100)$. The memory parameter can be interpreted as an evaluation horizon for the trading strategy h .

Further on, the WOA periodically replaces the trading strategies that have the lowest performance level of the strategies presented on the market by new ones. Without loss of generality, this algorithm is constructed to evaluate and rank the performance of all the strategies from the second group after every 40 iteration in descending order. The four strategies with the lowest performance are then replaced by the newly generated strategies. The use of the WOA in simulations can significantly change price time series parameters and modify the behavior of investors on the simulated market, see Vacha, Vosvrda (2005, 2007).

When $m_h = 1$ for all types h , we get the Brock and Hommes model. If $b_h = 0$ and $g_h > 0$, the investor is called a pure trend chaser. If $b_h = 0$ and $g_h < 0$, the investor is called a contrarian. Moreover, if $g_h = 0$, and the $b_h > 0$ ($b_h < 0$), the investor is said to have an upward (downward) bias in beliefs. In the special case of $g_h = b_h = 0$, the investor is fundamentalist, i.e., the investor believes that the price always return to its fundamental value.

3.3. Stochastic Beliefs with Change of Sentiment

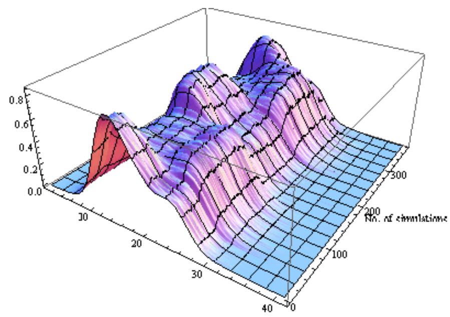
Further on, we investigate the impact of sentiment change of the market on the simulated price. We define the change of the sentiment as a shift of the beliefs about the future trend $g_{h,t}$. In this paper we will focus on the direct jump pattern of the agents' sentiment. This can be seen as jumps of the sentiment from optimism to pessimism and vice versa. We model changes in sentiment by jumps in the trend parameter g_h of newly incoming investor strategy to the market. More precisely, the trend parameter g_h jumps between the realizations from the normal distributions $N(0.4,0.16)$ and $N(-0.4,0.16)$ each 4000 iterations. Changes in sentiment can be observed in Figure 1 in the next section.

4. Simulations Results

This section describes the methodology of our simulations and summarizes the main results. The main purpose of the simulations is to examine influence of the proposed smart traders concept and sentiment changes on the simulated market prices. We compare the initial model without smart traders (0ST) with the model with 5 smart traders (5ST) and the model with 5 smart traders in the first group and sentiment changes in the second group (5STS).

Altogether we consider 40 trading strategies for each simulation. For the model without smart traders, all the strategies are second group strategies $f_{h,t}^2$. For the simulations with 5 smart traders, there are 5 first group strategies $f_{h,t}^1$ and 35 second group strategies $f_{h,t}^2$. For the simulations with 5 smart traders and change of sentiment, there is 5 first group strategies $f_{h,t}^1$ and 35 second group strategies $f_{h,t}^2$ where the trend parameter g_h jumps between the realizations from the normal distributions $N(0.4, 0.16)$ and $N(-0.4, 0.16)$ each 4000 iterations. Figure 1 shows the empirical probability density function of the trend parameter observed at the simulated market. It is the crossection through the iterations, and changes of sentiment can be clearly observed.

Figure 1: Empirical PDF of the trend parameters g_h through iterations



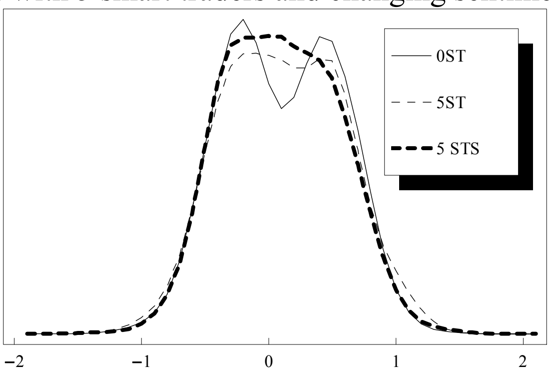
The lengths of the information set, defined by k, used for the trend parameter estimation by various types of smart traders is $\{k_i\}_{i=1}^5 = \{80, 60, 40, 20, 5\}$ for both models.

Other parameters of the simulation are: $\beta = 300$, number of all iterations $N = 15000$, $\alpha\sigma^2 = 1$, $R = 1.1$. Each of the six models has been simulated 36 times to achieve robust results. Table 1 shows the descriptive statistics of the simulated deviations x_t . Further on, Figure 2 shows the kernel estimation[1] of the probability density functions (PDFs).

Table 1: Descripive statistics

Statistics	0 ST	5 ST	5 STS
Mean	0.00419734	0.0131363	-0.00717577
Median	0.00178642	0.00574712	-0.0233304
Variance	0.215544	0.228529	0.213265
St. Dev.	0.46168	0.474366	0.458831
Skewness	0.00228263	0.0421294	0.0957988
Kurtosis	2.54122	3.86883	4.32745
Min.	-2.97239	-6.01944	-7.57153
Max.	4.35099	9.53744	8.46828

Figure 2: Empirical PDF of x_t for simulated models without smart traders, with 5 smart traders and with 5 smart traders and changing sentiment



We begin the results summary with descriptive statistics of x_t . We can see that the means and variances of the x_t are not changing with increasing number of smart traders. The models with smart traders produce the leptokurtic distributions of x_t , while the model without smart traders produces platykurtic distribution. While the values of skewness and kurtosis are the arithmetic means of all the simulations, we use Kruskal-Wallis (1952) test to compare the distributions of simulated x_t . This test does not assume normal distribution of compared sets of data as analogous analysis of variance. The null hypothesis of the test is equal population of medians against alternative of unequal population of medians. The

[1]we use Epanechnikov kernel which is of following form: $K(u) = \frac{3}{4}\left(1 - u^2\right)(|u| \leq 1)$.

Kruskal-Wallis test rejects the null hypothesis of equal medians of the sets on the 10% significance level, thus the skewness and kurtosis of all 3 models are significantly different. We can conclude that adding the smart traders as well as adding change of sentiment to the original model significantly changes the simulated distributions of x_t .

We continue our analysis with estimation of the Hurst (1951) exponent for all the simulated models. Our expectation is that introduction of the smart traders and sentiment change to the simulated market will increase also the Hurst exponent significantly. We again use the Kruskal-Wallis test which rejected the null hypothesis of equal medians of estimated Hurst exponents on the 1% significance level for compared models without smart traders and with smart traders. Thus we conclude that implementing smart traders into the model significantly increases the Hurst exponent and thus increases the persistence of simulated market. On the other hand, Kruskal-Wallis test does not reject the null hypothesis of equal medians when comparing the models 5ST and 5STS. This means that introduction of changing sentiment in the proposed form does not change the persistence of simulated prices significantly.

5. Conclusion

In this paper we extended the original model of heterogeneous agent model by introducing smart traders concept. The idea of the smart traders is based on endeavor of the market agents to estimate the future price movements. By adding smart traders we try to improve the original heterogeneous agents model so it will be able to come to closer description of the real markets.

Smart traders are able to forecast the trend parameter of the price deviations using the information set consisting of the past deviations. They are modeled to assume that the deviations are AR(1) process and they use maximum likelihood estimation method for forecasting. Thus in our model we use 2 groups of traders, smart traders and group of stochastically generated trading strategies which are moreover selected by Worst Out Algorithm. Furthermore, we investigate the impact of sentiment change of the market participants on the simulated price. The change of the sentiment is defined as a shift of the beliefs about the future trend value.

The main result of the simulations is that probability distribution functions of the price deviation changes significantly with increasing number of smart traders in the model, and it also changes significantly with introduction of the sentiment changes. We use also Hurst exponent to measure the persistence of the price deviations and we find that the Hurst exponent is significantly increasing with adding smart traders into the simulations. This means that the introduction of smart traders concept into the model results in significantly higher persistence of the simulated price deviations. On the other hand, introduction of changing sentiment in the proposed form does not change the persistence of simulated prices significantly. These are the preliminary results as more forms of sentiment changes needs to be tested to make more general implications for the real markets.

As this paper introduces the new concept for modeling heterogeneous agents, it also opens large space of possibilities for further research. The most interesting part is to show the impact of the different sentiment changes on the market price.

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