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Classical and modified rescaled range analysis: Sampling properties under heavy tails

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Classical and modified rescaled range analysis: Sampling properties under heavy tails

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Abstract:
Mostly used estimators of Hurst exponent for detection of long-range dependence are biased by presence of short-range dependence in the underlying time series. We present confidence intervals estimates for rescaled range and modified rescaled range. We show that the difference in expected values and confidence intervals enables us to use both methods together to clearly distinguish between the two types of processes. Moreover, both methods are robust against the presence of heavy tails in the underlying process.

Keywords: rescaled range, modified rescaled range, Hurst exponent, long-range dependence, confidence intervals

JEL: G1, G10, G14, G15

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1 Introduction

Long-range dependence in the financial time series has been discussed in number of research papers (e.g. Peters, 1994; Di Matteo, Aste & Dacorogna, 2005; Di Matteo, 2007; Czarnecki, Grech & Pamula, 2008; Grech & Mazur, 2004; Carbone, Castelli & Stanley, 2004; Matos *et al.*, 2008; Vandewalle, Ausloos & Boveroux, 1997; and Alvarez-Ramirez *et al.*, 2008). However, the majority of the papers interpret the results on the basis of simple comparison of estimated self-similarity parameter – Hurst exponent H – with its asymptotic limit of 0.5. Hurst exponent of 0.5 indicates two possible processes – either independent (Beran, 1994) or short-range dependent process (Lillo & Farmer, 2004). Autocovariances of the independent process are insignificant for all non-zero lags whereas for the short-range dependent process (e.g. ARIMA processes, for details, see Box & Jenkins, 1970), autocovariances are significant at low lags and insignificant at high lags and decay exponentially. If $H > 0.5$, the process has significantly positive correlations at all lags and is said to be long-range dependent with positive correlations (Embrechts & Maejima, 2002) or persistent (Mandelbrot & van Ness, 1968). On the other hand, if $H < 0.5$, it has similar properties to the previous case as it has significantly negative correlations at all lags and the process is said to be long-range dependent with negative correlations (Embrechts & Maejima, 2002) or anti-persistent (Barkoulas, Baum & Travlos, 2000). Autocovariances of long-range dependent processes are hyperbolically decaying and are either non-summable for persistent or summable for anti-persistent processes (Beran, 1994).

The efficient distinguishing between the short and the long-range dependence is crucial for portfolio selection as well as option pricing as the long-range dependence is connected to infinite or non-existent variance (Peters, 1994). As the majority of Hurst exponent estimation methods is biased by presence of the short-range dependence in the underlying process (Morariu *et al.*, 2007; and Lo, 1991), we test two methods which are strongly related and one of them is robust against the short-range dependence. These two methods are the rescaled range analysis of Hurst (1951), which is also called the “classical”, and the modified rescaled range analysis of Lo (1991). To be able to distinguish between the two types of dependence, we construct the confidence intervals for both methods as the estimates of Hurst exponent can strongly deviate from the asymptotic limit of 0.5 even for an independent process (Weron, 2002; Grech & Mazur, 2005; and Peters, 1994). However, most of the authors base their simulations on an independent process which is normally distributed. As the markets are known to be non-normal and to have heavy tails (Cont, 2001), we present the results for processes based on both the standard normal distribution and the Cauchy distribution, which possesses the heaviest tails of all stable distributions. It can be said that we test the estimates on two extreme cases – normality and very heavy tails.

The paper is structured as follows. In Section 2, we present and describe both techniques in detail. In Section 3, we present the important aspects of Monte Carlo simulations methodology. In Section 4, we show results of Monte Carlo simulations for time series lengths from 512 to 16 384 observations for both tested methods and both types of distributions of the underlying process. Both methods are robust to heavy tails in the underlying process and even have thinner confidence intervals than for the process based on standard distribution.

2 Classical and modified rescaled range analysis

2.1. Procedure

The classical rescaled range analysis (R/S) was developed by Edwin Hurst while working as an engineer in Egypt (Hurst, 1951) and was later applied to financial time series by Mandelbrot (1970). For the time series of length *T*, Hurst exponent *H* can be estimated from the behavior of rescaled range of the series which is described in the following theorem (for details, see Hurst, 1951; and Taqqu, Teverovsky & Willinger, 1995).

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Theorem. *Let us have time series* $\{V_t\}_{t=-\infty}^{\infty}$ *and* $T = N * \upsilon$ *where* $N \in \mathrm{N}, \upsilon \in \mathrm{N}$. *Denote* $I_n = \{r_{k,n} = V_{(n-1)\upsilon+k}: k = 1,2,\ldots,\upsilon\}$, $n = 1,2,\ldots,N$ *and put arithmetic mean as* $\bar{r}_n = \sum_{k=1}^{\upsilon} r_{k,n} / \upsilon$. *Denote a profile of the series as* $X_{k,n} = \sum_{i=1}^{k} (r_{i,n} - \bar{r}_n)$. *Let's further put a range of the series as* $R_n = \max_{1\leq k\leq\upsilon} X_{k,n} - \min_{1\leq k\leq\upsilon} X_{k,n}$ *and a standard deviation as* $S_n = \left(\sum_{k=1}^{\nu} (X_{k,n} - \bar{X}_n)^2 / (\upsilon - 1)\right)^{0.5}$. *Moreover, put*

$$\left(\frac{R}{S}\right)_{I_n} = \frac{R_n}{S_n} \quad \text{and} \quad \left(\frac{R}{S}\right)_{\upsilon} = \frac{1}{N}\sum_{n=1}^{N}\frac{R_n}{S_n}.$$

Finally, for $c \in R$, *rescaled range scales by the means of Hurst exponent* H *as*

$$\left(\frac{R}{S}\right)_{\upsilon} \approx c * \upsilon^H$$

where "$\approx$*" means asymptotic equality.*

The linear relationship in double-logarithmic scale indicates a power scaling (Weron, 2002). To uncover the scaling law, we use a simple ordinary least squares regression on logarithms of each side of the previous equation. Thus, we get

$$\log(R/S)_{\upsilon} \approx \log c + H \log \upsilon,$$

where *H* is Hurst exponent.

R/S was shown to be sensitive to a heteroskedasticity and the short-range dependence in the underlying process (DiMatteo, 2007; Lo & MacKinlay, 1999; and Alfi *et al.*, 2008). To deal with the problem, Lo (1991) proposed a modified rescaled range analysis (M-R/S). The procedure differs in the definition of the standard deviation which deals with both the heteroskedasticity and the short-range dependence. The modified standard deviation is defined with a use of auto-covariance γ of the selected sub-interval I_n up to the integer lag ξ as

$$S_{I_n}^M = \sqrt{S_{I_n}^2 + 2\sum_{j=1}^{\xi}\gamma_j\left(1-\frac{j}{\xi+1}\right)}.$$

Thus, R/S turns into a special case of M-R/S with $\xi = 0$ (Dülger & Ozdemir, 2005). The most problematic and also the crucial issue of the new standard deviation measure is the number of lags which are used for its estimation (Wang *et al.*, 2006). On one hand, if the chosen lag is too low, it omits lags which may be significant and therefore the estimates of rescaled ranges and Hurst exponent are still biased. On the other hand, if the used lag is too high, the estimates of rescaled range differ significantly from the true values (Teverovsky, Taqqu & Willinger, 1999).

One can either estimate rescaled ranges and Hurst exponents for arbitrary values of ξ (Zhuang, Gree & Maggioni, 2000; and Alptekin, 2006) or use an automatic estimator of ξ for each sub-interval of the length υ. There are two estimators of the optimal lag suggested in the literature. Lo (1991) sets the optimal lag ξ^* based on the first-order autocorrelation coefficient $\hat{\rho}(1)$ of the sub-interval I_n as (where [] is the nearest lower integer operator)

$$\xi^* = \left[\left(\frac{3\upsilon}{2}\right)^{\frac{1}{3}}\left(\frac{2\hat{\rho}(1)}{1-(\hat{\rho}(1))^2}\right)^{\frac{2}{3}}\right].$$

Chin (2008) bases the optimal lag choice on the length of the sub-interval solely as

$$\xi^* = \left[4\left(\frac{\upsilon}{100}\right)^{\frac{2}{9}}\right].$$

Teverovsky, Taqqu & Willinger (1999) showed that M-R/S is biased towards rejecting long-range dependence in the process when high number of lags is used. It implies that in the case of the significant short-range dependence in the process, the method of Lo would lead to biased estimates of H. Moreover, if the short-term memory is not significant or low, the method of Lo does not significantly differ from the method of Chin. For values of the autocorrelation $\hat{\rho}(1)$ lower than 0.5, both methods do not differ significantly.

2.2. Brief literature review

Finite sample properties of the rescaled range analysis and the modified rescaled range analysis were discussed in several papers. Most importantly, the condition for the process to be characterized as independent, which is $H = 0.5$, was shown to hold only asymptotically. For finite samples, the estimated Hurst exponent can differ significantly. To deal with the problem, Anis & Lloyd (1976) showed that for the finite samples, expected value of the rescaled range behaves as

$$E(R/S)_\upsilon = \frac{\Gamma\left(\frac{\upsilon-1}{2}\right)}{\sqrt{\pi}\Gamma\left(\frac{\upsilon}{2}\right)} \sum_{i=1}^{\upsilon-1} \sqrt{\frac{\upsilon-i}{i}} \,.$$

Further, Peters (1994) proposed to adjust the expected value of the rescaled range, which is better for low scales $\upsilon < 50$, to

$$E(R/S)_\upsilon = \frac{\upsilon-\frac{1}{2}}{\upsilon} \sqrt{\frac{2}{\upsilon\pi}} \sum_{i=1}^{\upsilon-1} \sqrt{\frac{\upsilon-i}{i}} \,.$$

However, Couillard & Davison (2005) and Grech & Mazur (2005) showed that the expected values based on the Monte Carlo simulations are closer to the method of Anis & Lloyd (1974). Importantly, they also showed that the standard deviation of the estimates does not converge according to the central limit theorem, which is as a square root of the time series length. Such finding is essential for the confidence interval construction, since as the standard deviation converges slower, the confidence intervals are much wider.

3 Monte Carlo simulations methodology

The choice of parameters is crucial for final results of the simulations and estimates. The thorough description is needed for possible comparison with other results. In the following section, we base the estimates so that we use the sub-length υ equal to the power of a set integer value (Weron, 2002). Thus, we set a basis b, a minimum power $pmin$ and a maximum power $pmax$ so that we get $\upsilon = b^{pmin}, \ldots, b^{pmax}$ where $b^{pmin} \geq b$ is a minimum scale and $b^{pmax} \leq T$ is a maximum scale. The minimum scale is set to 32 observations and the maximum as the whole time series length. Such choice avoids biases of the low scales where the standard deviations can be estimated inefficiently (Peters, 1994; Grech & Mazur, 2004; and Matos *et al.*, 2008). Moreover, the used intervals are contingent and non-overlapping (see Ellis, 2007 for discussion).

The majority of research papers dealing with the finite sample properties of the rescaled range base their estimates on simulations of the standardized normal distribution $N(0,1)$. However, the returns on financial markets are known to be non-normal and fat-tailed (Cont, 2001). To show whether R/S and M-R/S are robust to fat tails in the underlying process, we show the estimates for both the standardized normal distribution and the Cauchy distribution, which is a stable distribution with the fattest tails.

In the simulations, we simulate 1 000 time series with lengths from 2^9 to 2^{14} for the standard normal distribution and estimate Hurst exponent based on R/S and M-R/S for each series. We apply the same procedure for the Cauchy distribution. Both processes are independent as each element of the time series is drawn randomly so that the estimates are expected to be 0.5. Note that we use the optimal lag choice based on Chin (2008) as first-order autocorrelation of independent process is equal to zero so that the method of Lo (1991) would turn into classical rescaled range.

4 Results

Table 1 shows the descriptive statistics of all Monte Carlo simulations as well as the Jarque-Bera test for normality (Jarque & Bera, 1981) and 2.5% and 97.5% percentiles. Chart 1 and Chart 2 present histograms of the estimates of Hurst exponents for R/S and M-R/S, respectively. Chart 3 shows confidence intervals based on different percentiles for both methods.

We can see that the expected value for R/S for standard normal process is the highest for the shortest time series length of 512 observations and is equal to 0.5316 and quite steadily decreases to 0.5211 for 16 384 observations. Interestingly, the expected values of R/S estimates for the Cauchy process are lower than the ones for the normal distribution up to time series length 4 096. For the two highest lengths, the estimates are similar. The expected values of Hurst exponent for Cauchy-based process range from 0.5281 to 0.5248 and are thus quite stable. Nevertheless, R/S is robust against presence of heavy tails and its estimates are not affected when compared to the estimates for independent process based on normal distribution. Moreover, the standard deviations are lower for the Cauchy-based estimates (Chart 4).

The results are similar for M-R/S. The expected values range from 0.5192 for 512 observations to 0.5124 for the length 16 384 for standard normal process. Again, expected values for the Cauchy-based estimates of Hurst exponent are lower than the ones of normal-based ones up till length 4 096. However, these expected values are increasing with time series length. Still, the expected values remain very close for both types of underlying processes so that M-R/S is robust against the presence of heavy tails in the underlying process as well. The standard deviations are lower for process based on the Cauchy distribution and almost equal to the ones of R/S for the same type of the process. The results for standard deviations for the simulated Cauchy-based series show that the maximum scale equal to the time series length can be used as even very heavy tails of the Cauchy distribution do not bias the estimates of Hurst exponent for both R/S and M-R/S.

To clearly distinguish between the short and the long-range dependence in the process, we need to estimate Hurst exponent by both methods and compare them with the confidence intervals (Chart 4). This way, we can distinguish between three possibilities. If the estimates of Hurst exponent based on both R/S and M-R/S are inside the corresponding confidence intervals, the process can be either independent or slightly short-range dependent. If the estimates of H are outside the confidence intervals for R/S but inside the ones for M-R/S, the process is short-range dependent. Finally, if the Hurst exponent estimates are outside of confidence intervals for both methods, the process is long-range dependent.

5 Conclusion

In the paper, we researched on behavior of the rescaled range analysis and the modified rescaled analysis under both the normal distribution assumption as well as under the heavy tails represented by the Cauchy distribution. As most of the studies focus on the normal distribution only, we tried to fill this gap as returns on financial markets are known to be non-normal and with heavy tails. Moreover, the short and the long-range dependence have different implications for portfolio selection and options pricing so that clear distinguishing between both processes can be crucial.

Monte Carlo simulations showed that both methods are robust to the presence of the heavy tails in the underlying process. Interestingly, the standard deviations of the estimates for the process with the Cauchy distribution are lower than for the normal case for both methods. We also presented the confidence intervals which enable us to clearly distinguish between the short and the long-range dependence in the process.

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Table 1 Results of Monte Carlo simulations

		512	*1024*	*2048*	*4096*	*8192*	*16384*
mean	*R/S (normal)*	0,5316	0,5345	0,5305	0,5292	0,5222	0,5211
	R/S (Cauchy)	0,5281	0,5250	0,5258	0,5236	0,5244	0,5248
	M-R/S (normal)	0,5192	0,5186	0,5157	0,5155	0,5151	0,5124
	M-R/S (Cauchy)	0,5012	0,5091	0,5122	0,5133	0,5151	0,5167
SD	*R/S (normal)*	0,0853	0,0608	0,0473	0,0379	0,0328	0,0268
	R/S (Cauchy)	0,0688	0,0528	0,0407	0,0344	0,0292	0,0254
	M-R/S (normal)	0,0743	0,0568	0,0496	0,0392	0,0315	0,0279
	M-R/S (Cauchy)	0,0678	0,0551	0,0421	0,0351	0,0285	0,0251
skewness	*R/S (normal)*	-2,2315	-0,0812	-0,0881	0,0350	0,0661	-0,0444
	R/S (Cauchy)	0,0928	0,1192	0,0792	0,0254	0,0458	0,0814
	M-R/S (normal)	-0,1319	-0,0525	-0,0010	-0,0669	-0,1155	-0,1057
	M-R/S (Cauchy)	-0,0448	-0,2013	0,0445	0,0000	-0,1094	-0,1583
kurtosis	*R/S (normal)*	27,9929	-0,1830	-0,0685	-0,0372	-0,0968	-0,2790
	R/S (Cauchy)	0,0540	0,1476	-0,0207	-0,1679	0,1114	0,1629
	M-R/S (normal)	-0,0602	-0,3193	-0,2894	0,0055	-0,0266	0,0083
	M-R/S (Cauchy)	0,3288	1,3961	0,2073	-0,2088	0,1467	0,1341
Jarque-Bera	*R/S (normal)*	33480,1323	2,4950	1,4902	0,2613	1,1189	3,5705
	R/S (Cauchy)	1,5553	3,2763	1,0630	1,2824	0,8667	2,2100
	M-R/S (normal)	3,0493	4,7071	3,4907	0,7474	2,2539	1,8662
	M-R/S (Cauchy)	4,8401	87,9678	2,1199	1,8164	2,8930	4,9264
p-value	*R/S (normal)*	0,0000	0,2872	0,4747	0,8775	0,5715	0,1678
	R/S (Cauchy)	0,4595	0,1943	0,5877	0,5267	0,6483	0,3312
	M-R/S (normal)	0,2177	0,0950	0,1746	0,6882	0,3240	0,3933
	M-R/S (Cauchy)	0,0889	0,0000	0,3465	0,4032	0,2354	0,0852
$P_{97.5}$	*R/S (normal)*	0,6779	0,6505	0,6237	0,6037	0,5882	0,5733
	R/S (Cauchy)	0,6598	0,6324	0,6093	0,5902	0,5825	0,5772
	M-R/S (normal)	0,6602	0,6275	0,6144	0,5898	0,5726	0,5636
	M-R/S (Cauchy)	0,6361	0,6162	0,5993	0,5819	0,5677	0,5623
$P_{2.5}$	*R/S (normal)*	0,3740	0,4119	0,4353	0,4581	0,4627	0,4669
	R/S (Cauchy)	0,3982	0,4244	0,4455	0,4549	0,4677	0,4773
	M-R/S (normal)	0,3685	0,4077	0,4217	0,4376	0,4522	0,4554
	M-R/S (Cauchy)	0,3695	0,3998	0,4305	0,4466	0,4582	0,4678

Chart 1 Distribution of estimated Hurst exponents for R/S

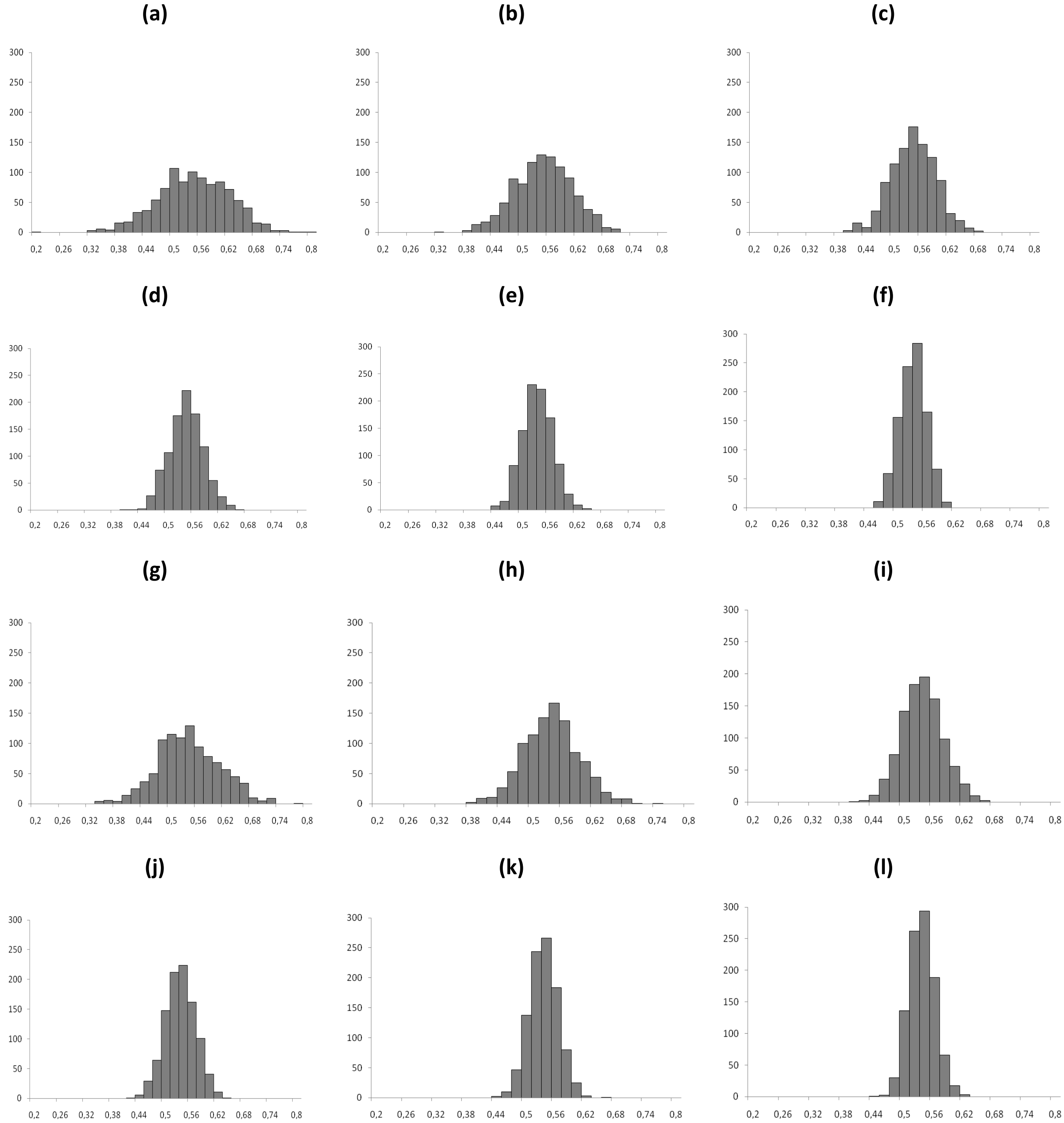


Chart 1: The first two rows show the distributions for Monte Carlo simulations based on standard normal underlying process. The second two rows show the distributions based on Cauchy distribution of the underlying process. Length of the time series starts at 512 observations (**a** and **g**) and ends at 16 384 observations (**f** and **l**).

Chart 2 Distribution of estimated Hurst exponents for M-R/S

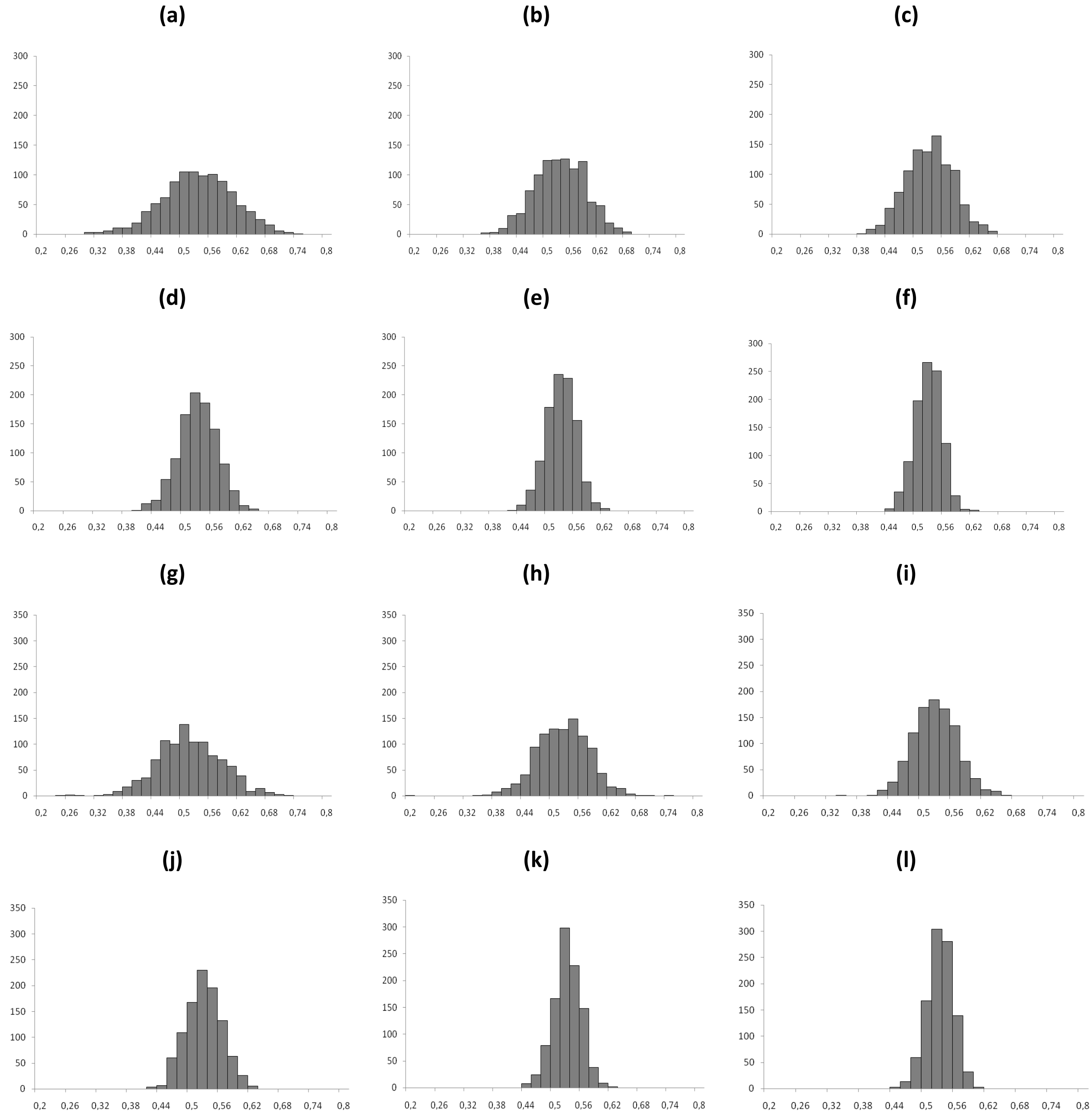
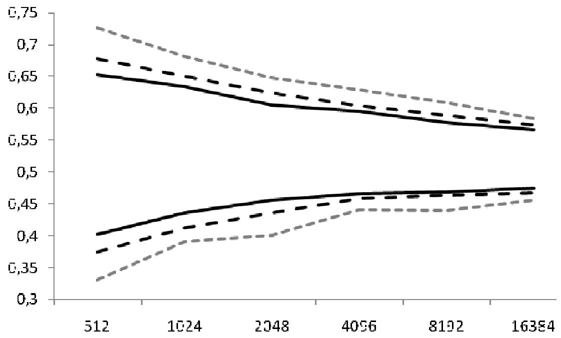


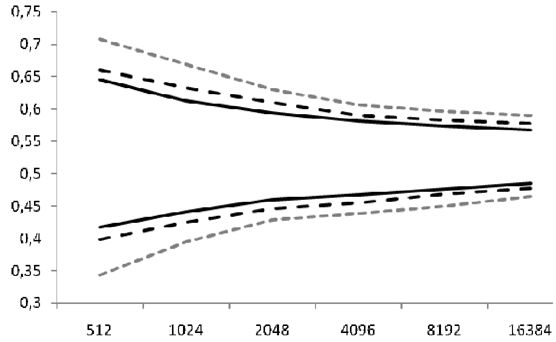
Chart 2: The first two rows show the distributions for Monte Carlo simulations based on standard normal underlying process. The second two rows show the distributions based on Cauchy distribution of the underlying process. Length of the time series starts at 512 observations (**a** and **g**) and ends at 16 384 observations (**f** and **l**).

Chart 3 Confidence intervals for R/S and M-R/S

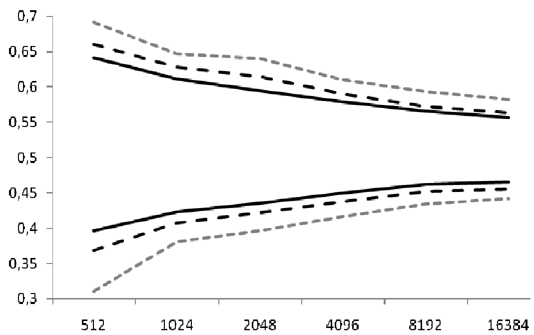
(a)



(b)



(c)



(d)

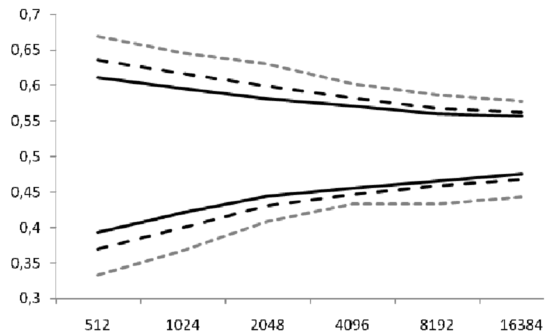
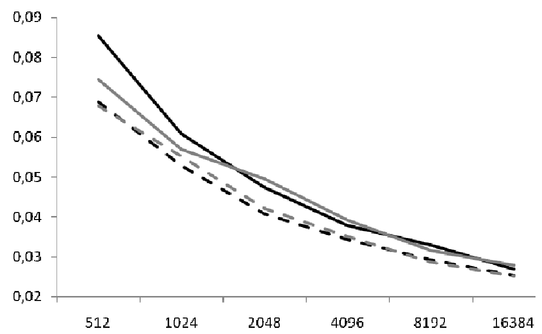


Chart 3: Confidence intervals for **(a)** R/S with standard normal underlying process, **(b)** R/S with Cauchy distributed underlying process, **(c)** M-R/S with standard normal underlying process and **(d)** M-R/S with Cauchy distributed underlying process. Bold black line represents the 5% and 95% confidence intervals. Dashed black line represents 2.5% and 97.5% confidence intervals. Finally, dashed gray line represents 0.5% and 99.5% confidence intervals.

Chart 4 Confidence intervals for R/S and M-R/S

(a)



(b)

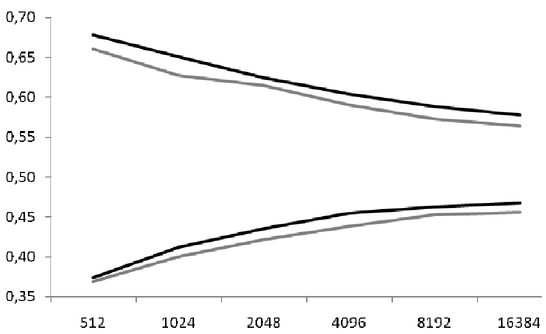


Chart 4: **(a)** Standard deviations of estimated Hurst exponents for R/S (black line) and M-R/S (grey line) with standard normal distribution (full line) and Cauchy distribution (dashed line) of the process. **(b)** Comparison of 2.5% and 97.5% confidence intervals for R/S (black) and M-R/S (grey). For each method, the higher value of upper confidence interval from standard normal and Cauchy distribution of the underlying processes is shown as is the lower value for the lower confidence interval.

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