Homing, Calibration and Model-Based Predictive Control for Planar Parallel Robots

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Abstract: Parallel robots represent way to considerably improve accuracy and speed of industrial machine tools and their centres. This paper deals with the preparatory operations: homing and calibration, which precede the start-up of the robot work, i.e. real control process. Their procedures are discussed with respect to planar parallel robots and their control. In this paper, as a suitable control strategy, the model-based predictive control is considered. The predictive control offers operator to continuously influence the control process. The control issues relating to planar parallel robots are discussed here.

1. INTRODUCTION

Parallel robots (Merlet, 2006; Tsai, 1999), i.e. robots based on parallel kinematic structures (Fig. 1) can be simply characterized as movable truss constructions or as movable platforms supported by several links, where platforms serve as a place for a tool or for a gripper. These structures represent closed-loop constructions, flexibility (dynamics) of which allowing high productivity follows from possible small number of moving masses. Therefore, in comparison with serial types (Sciavicco and Siciliano, 1996), they may offer higher stiffness and dexterity. This feature is given by the fixing almost all drives directly on the basic frame without loading of movable parts of the robot construction. It contributes to the decrease of inertial forces. However, a larger number of the links and passive joints often require more sophisticated procedures in a robot adjustment, start-up and real control process.

All procedures in general should ensure a cooperation of each drive participating in robot motion. The first two procedures are limited by unknown initial robot state, which is detected during them. Once the robot state is determined at least roughly, then the drive cooperation can be ensured. Since adjustment and start-up procedures (calibration and homing) have no so strict time limitation, they may be performed in such a way not to come to undesired states. The time conditions have to be considered in the control procedure, which follows from requirements of real technological process.

Conventional local control strategies (e.g. cascade drive control) have no energy optimization. It is a limiting factor of a robot capacity. In case of parallel robots, it may produce undesired antagonistic drive behaviour i.e. behaviour without necessary drive cooperation. The antagonism, i.e. drive mutual fighting, is caused by presence of interrelations among individual drives through parallel links and appropriate movable platform. The conventional control cannot consider these interrelations and therefore it does not represent economic and safe solution. To avoid mentioned undesirable states, some model describing energy decoupling in the robot structure is needful. However, when some model is available, then global model-based control strategies are more effective. They can provide design of control actions, which are optimized for appropriate robot structure.

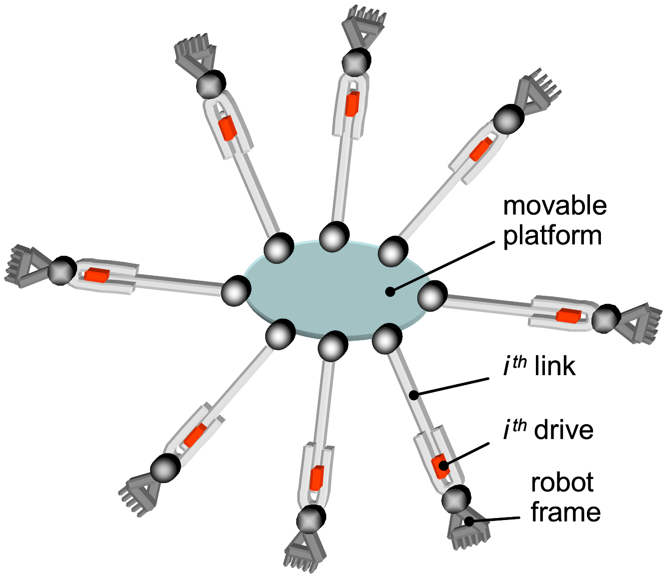

Fig. 1. Principle of parallel kinematical structures

This paper is organized as follows. Firstly, homing and calibration procedures intended for planar parallel robots are investigated. Then the paper deals with control issues relating to parallel structures. The drawbacks of conventional control are discussed. Thereafter, as suitable promising model-based strategy, predictive control is introduced. All procedures being described here are documented in several representative examples arising from their implementation on one real laboratory model of parallel robot.

2. HOMING

Homing procedure represents set of actions automatically leading the all robot elements from initial generally unknown position (i.e. arbitrary position from a robot workspace) to predefined known deterministic position, so-called home position. This position may be an initial position for a sensor calibration, for real work operations or for a work-tool replacement etc. The real homing procedure is determined by used robot drives:

- linear drives
- rotational drives

more precisely, by character of the drive sensors (possible other additional sensors):

- positional (absolute) sensors
- incremental sensors

denoting the pieces of information on the robot position, generally on the robot state. Furthermore, the procedure depends on type of the robot structure and configuration, i.e. if the structure has serial, parallel or hybrid character.

2.1 Homing Procedure

To start the procedure, it is necessary to know rough robot position, appropriate drive positions, respectively, e.g. known quadrant in the case of rotational drives or interval in the case of linear drives in relation home position. For this purpose, specific mechanical or electromechanical elements (marks) are suitable to be considered (Fig. 2 and Fig. 3).

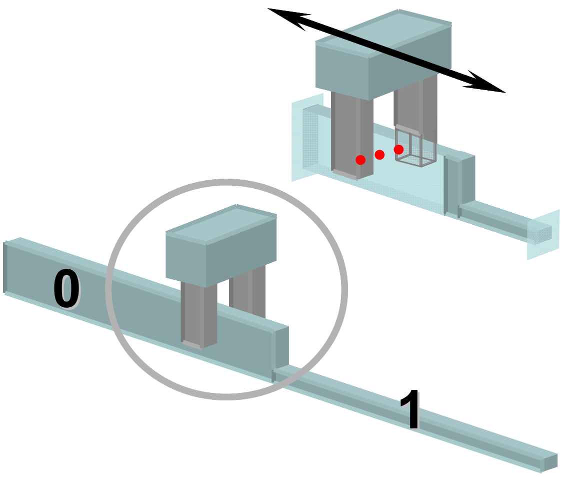

Fig. 2. Example of homing mark - case of linear drive

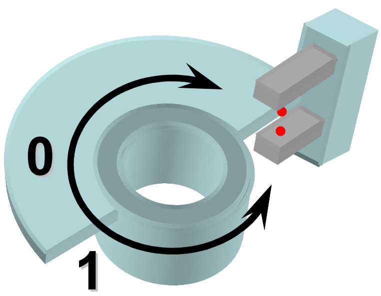

Fig. 3. Example of homing mark - case of rotational drive

These elements stand for positional (absolute) homing marks, one part of which representing reference (e.g. robot frame, previous link) is fixed, and the second part is mounted on appropriate place moving relative to part one (e.g. slide rest, drive spindle). In case of positional (absolute) sensors, mainly at linear drives, such orientation element may be directly sensor itself, but it need not be a rule.

In Fig. 2 and Fig. 3, the elements – homing marks represent two possible states (e.g. values 0 or 1), if the mark shade is in the view (value 0) or not (value 1). According to mark value, representing certain drive position related to the position of movable platform, the sense of rotation or linear motion of robot drives at homing procedure is chosen. It represents limited information on the robot position. Therefore, the homing procedure can be realized as a feed-forward or rather weak feed-back decentralized control, where drives are fed with safe energy keeping selected motion sense up to changing mark values.

The real homing procedure can be generally done as follows:

① The robot is switched on and the control unit tests the values of the marks.

② In a correspondence to these values, the control unit set the input value and its sign for each drive. The robot starts to move slightly towards home position.

③ During the process, the positional drive sensors should register their values, although the values are not directly connected to physical meaning.

④ Once some of drivers achieves the change of marks (change $0 \rightarrow 1$ or $1 \rightarrow 0$), the value of position from drive sensor is stored in memory and the appropriate drive is stabilized in this position. The stabilization can be provided by a simple PI controller.

⑤ When all drives achieve the mark change, the robot reaches the home position and it is stabilized in this position.

2.2 Specification of Homing for Parallel Robots

In case of the parallel robots, which can be moreover redundantly actuated, the homing is limited by kinematical relations through chained robot links. In case of serial robots, i.e. open-loop kinematical structures, the individual drives can be homed independently; there are no direct kinematical relations. Conversely, the homing of parallel robots has to be proceeded for all drives simultaneously. The individual homing of each drive is not possible. It can cause collisions of the robot links and unsafe robot motion, which can lead to the damage of the robot and its neighbourhood.

However, by the simultaneous homing, the individual drives reach the home position in different time. When one drive reaches mark change, it is stabilized in mark neighbourhood. Nevertheless, its stabilization has to be soft not to restrict other drives to reaching the mark changes. After the change of all marks, it is necessary to watch over the all stabilizing actions, especially in case of redundant robot actuation, where mutual drive fighting may appear.

The problem of the mutual drive fighting with suggestion of suitable solution will be discussed in the section dealing with control design.

3. CALIBRATION

The calibration is a procedure of identifying the real geometrical parameters in given kinematical structure of the robot relative to the position and orientation of links and joints in the robot (Andreff, et al. 2004). The calibration is important for the guaranty of the repeatability and for meeting the tolerance requirements.

There are distinguished two types of the calibration:

- mechanical calibration
- software calibration

either off-line (one-off or regularly repeated) prior to real control process or on-line during the process.

3.1 Calibration Procedure

The calibration procedure starts already during the installation of a new robot in a place of the use (in the robot workplace). This phase represents one-off mechanical calibration, where relative positions of individual robot elements are necessary to be adjusted.

Except structural robot elements, the adjustment of the homing elements – marks, described in previous section, belongs to this phase. These elements have to be calibrated together with appropriate drives; i.e. their mutual position has to be fixed and to be corresponding to robot homing position.

The elements – marks can be used in second phase – software calibration, where the accurate drive sensors are calibrated to robot coordinates. Limited mechanical calibration including length measurement of robot links and the like may be pursued also on-line. The result of this phase is set of dimensions served for control design.

The second phase, usually executed regularly e.g. upon robot switching on or possibly upon tool replacement, is software calibration. It can be characterized in simple meaning as mapping accurate drive sensors (incremental counters, measuring rules) and others to real physical coordinates (rotational angles, distances). In other words, during this phase, all offsets and compensatory constants are set up in the program, where the data from sensors are processed.

3.2 Specification of Calibration for Parallel Robots

The first phase called mechanical calibration is more or less equivalent for both serial and parallel robots. The second phase – software, actually local sensor calibration differs already again. In comparison with the homing procedure of parallel robots in previous section, that calibration phase proceeds individually for each sensor and appropriate drive by reason of accuracy.

Contrary to serial robots, where the movement during the calibration is not limited except for occurrence of link collisions, the movement of each drive is strictly limited to small vicinity of selected representative positions (e.g. home position), since larger movement causes undefined movement of a movable platform, links and other robot elements. Thus, the second phase is discontinuous.

After each sensor calibration, the homing procedure like has to be executed in order to provide the same initial conditions for other sensors waiting for calibration.

Single calibration procedure inclusive initial homing procedure for planar parallel robots e.g. with n drives and their appropriate sensors can be formulated by the following four steps:

Step 1. Let the homing procedure from section 2. is done.

Step 2. Let the all drives slowly change the marks and stop immediately after that change on the predefined mark side (i.e. in the unique mark value, the same for all cases).

Step 3. Let the i^{th} drive is moved back and when the new drive sensor hit is appeared, let the sensor value is reset to its new origin, which values correspond to real physical coordinates for given position.

Step 4. Let the procedure goes to Step 1. and all four steps are repeated for other drive and sensor pairs up to state when all sensors are reset to real values.

The correctness of the procedure can be verified by backward controlled motion with the reference values recomputed from record of sensor data and inversed in their own direction. The robot, which starts in this backward motion in known home position, should reach accurately the initial unknown position. This proof can be done due to the known starting home position. Moreover, the robot can be already controlled in both coordinates systems: drive coordinate system or operational system. Due to the known relation between the sensor values and real robot position, the direct and inverse kinematical transformations are computable even for redundantly actuated parallel structures (Böhm, et al., 2001). The mentioned kinematical transformations enable user to use the both coordinate systems.

Another possibility, in case of redundantly actuated robots, is on-line calibration (Valášek, et al., 2002). Since all drives (adequate and redundant) are equipped with drive sensor, then during each motion of the robot, there is a redundant number of measurements. The assembled equations of constraints for certain number of robot positions can be solved for both robot dimensions and initial positions. This approach enables the redundant parallel robots to be calibrated on-line during their operations without using any external equipment or breaking their work.

4. CONTROL OF PARALLEL ROBOTS

The general issue of the robot control is tracking the desired trajectory – path control. To design suitable control, the following fact has to be considered. The robot structures represent nonlinear multivariate systems, dynamics of which is relatively fast in comparison with computation time for control actions. For that reason, the discrete methods are considered. They can ensure the finite computation time of the control.

The control can be solved either on local level, by design of independent control of the drives as servos, or by global control design for the whole robot. This approach can be especially considered, because sufficiently accurate model of the robot can be composed.

4.1 Model of the robot

In general, the parallel robots represent multi-body systems, which can be straightforwardly described in physical coordinates by Lagrange's equations of mixed type (Stejskal and Valášek, 1996). These equations lead to the system of differential algebraic equations – DAE (1)

$$\begin{aligned} \mathbf{M}\,\ddot{\mathbf{s}} - \mathbf{\Phi}_s^T \boldsymbol{\lambda} &= \mathbf{g} + \mathbf{T}\,\mathbf{u} \\ \mathbf{f}(\mathbf{s}) &= \mathbf{0} \end{aligned} \tag{1}$$

where $\mathbf{M}$ is a mass matrix, $\mathbf{s}$ is a vector of physical coordinates (their number is usually greater than number of degrees of freedom – DOF), $\mathbf{\Phi}_s$ is a Jacobian, $\boldsymbol{\lambda}$ is a vector of Lagrange's multipliers, $\mathbf{g}$ is a vector of other internal relations, matrix $\mathbf{T}$ connects inputs $\mathbf{u}$ to appropriate differential equations and algebraic equations $\mathbf{f(s)} = \mathbf{0}$ represent geometrical constraints.

As mentioned, the model (1) is a DAE system and moreover nonlinear. It is not suitable for control design. However, it can be transformed to different form (Stejskal and Valášek, 1996), to the system of ordinary differential equations based on independent coordinates $\mathbf{y}$, which correspond to DOF:

$$\mathbf{R}^T\mathbf{M}\mathbf{R}\ddot{\mathbf{y}} + \mathbf{R}^T\mathbf{M}\dot{\mathbf{R}}\dot{\mathbf{y}} = \mathbf{R}^T\mathbf{g} + \mathbf{R}^T\mathbf{T}\mathbf{u} \tag{2}$$

where matrix $\mathbf{R}$ is the basis of the null space of the overall Jacobian $\mathbf{\Phi}_s$. The equation (2) can be rewritten in condensed notation as follows

$$\ddot{\mathbf{y}} = \mathbf{f}(\mathbf{y}, \dot{\mathbf{y}}) + \mathbf{g}(\mathbf{y})\mathbf{u} \tag{3}$$

where $\mathbf{f}(\mathbf{y}, \dot{\mathbf{y}})$ represents robot dynamics and $\mathbf{g}(\mathbf{y})$ is input matrix.

The model (3) can be transformed to the state-space formula:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}) + \mathbf{G}(\mathbf{x})\,\mathbf{u} \\ \mathbf{y} &= \mathbf{H}\,\mathbf{x} \end{aligned} \quad , \quad \mathbf{x} = \begin{bmatrix} \mathbf{y} \\ \dot{\mathbf{y}} \end{bmatrix} \tag{4}$$

which is simpler and more transparent for handling with multi-input multi-output systems as robots usually are. Due to discrete realization of the control, the model (4) is discretized. To use standard discretization via expansion of exponential functions, the nonlinear vector $\mathbf{f}(\mathbf{x})$ in (4) has to be linearized. It can be provided by decomposition according to Valášek and Steinbauer (1999), which leads to the linear form:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{F}(\mathbf{x})\,\mathbf{x} + \mathbf{G}(\mathbf{x})\,\mathbf{u} \\ \mathbf{y} &= \mathbf{H}\;\;\mathbf{x} \end{aligned} \quad , \quad \mathbf{x} = \begin{bmatrix} \mathbf{y} \\ \dot{\mathbf{y}} \end{bmatrix} \tag{5}$$

The obtained form (5) represents the robot dynamics identically as model (3) or (4) ; the individual elements of state and input matrices $\mathbf{F}(\mathbf{x})$ and $\mathbf{G}(\mathbf{x})$ has to be recomputed on-line for appropriate topical robot state $\mathbf{x}$. Output matrix $\mathbf{H}$ is rectangular identity matrix (it is equal output matrix $\mathbf{C}$ in (6)). The real use of the decomposition is shown in (Belda, et al., 2003).

Then, after discretization of (5), the obtained model has following form:

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}(k)\,\mathbf{x}(k) + \mathbf{B}(k)\,\mathbf{u}(k) \\ \mathbf{y}(k) &= \;\mathbf{C}\;\;\mathbf{x}(k) \end{aligned} \quad , \quad \mathbf{x}(k) = \begin{bmatrix} \mathbf{y}(k) \\ \dot{\mathbf{y}}(k) \end{bmatrix} \tag{6}$$

That form is convenient for control design.

4.2 Conventional Control

The conventional control strategy – PID/PSD feedback control considers the robots – manipulators as a set of single-input / single-output subsystems, which represent individual drives. The mutual interactions are taken into account as disturbance entering each drive subsystem.

However, in the case of the parallel robots, mainly redundantly actuated, the unproductive part of Integral/Sum channels must be solved. It does not occur at serial open-loop structures.

Undesirable unproductive part of I/S channels is caused by inaccuracies in mechanism. It means that the drive coordinates designed according to inverse kinematical transformation from independent coordinates $\mathbf{y}$ in some cases cannot be attainable.

This causes unpredictable increase of I/S channels, which does not contribute to motion and moreover leads to instability of the whole robot system. To damp this undesired property, the specific reduction projection is used.

From mathematical point of view the following minimization problem is solved:

$$\begin{aligned} \mathbf{A}\;\;\mathbf{u} &= \mathbf{b} \\ \min|\,\mathbf{u}\,| & \end{aligned} \tag{7}$$

where matrix $\mathbf{A}$ is $\mathbf{R}^T\mathbf{T}$, generally horizontal rectangular redistribution matrix, which transforms drive torques to the generalized force effects, and $\mathbf{u}$ are control actions required on drives, and $\mathbf{b}$ is a vector of generalized force effects.

To obtain reducing projection in view of $\mathbf{u}$, the quadratic criterion is used:

$$J = \frac{1}{2}\mathbf{u}^T\mathbf{u} + \boldsymbol{\lambda}^T\left(\mathbf{A}\mathbf{u} - \mathbf{b}\right) \overset{!}{=} \min \tag{8}$$

minimization of which gives the projection formula:

$$\mathbf{u}_{red} = \mathbf{A}^T(\mathbf{A}\mathbf{A}^T)^{-1}\mathbf{A}\mathbf{u} \tag{9}$$

with assumption that $|\,\mathbf{A}^T(\mathbf{A}\mathbf{A}^T)^{-1}\mathbf{A}\,| \le |\,\mathbf{I}\,|$.

4.3 Model-Based Predictive Control

The model-based control in general uses the knowledge of the dynamic model (e.g. (2) or its state-space form (6)) and that way it globally optimises control actions in the view of the whole robot. Due to the model, the undesired properties of conventional control discussed in previous subsection do not occur.

One model-based representative strategy is predictive control. It represents a multi-step control based on equations of predictions and the local repetitive minimization of quadratic criterion (Ordys and Clarke, 1993).

The equations of predictions serve for the expression of feed-forward within horizon of predictions N. On their basis, the dominant part of control actions is determined. Using discrete state-space form (6) the equations have following form:

$$\hat{\mathbf{y}} = \mathbf{f} + \mathbf{G}\,\mathbf{u}\,, \quad \hat{\mathbf{y}} = [\hat{\mathbf{y}}_{k+1} \cdots \hat{\mathbf{y}}_{k+N}]^T \tag{10}$$

where $\mathbf{f} = \begin{bmatrix} \mathbf{CA} \\ \vdots \\ \mathbf{CA}^N \end{bmatrix} \mathbf{x}_{(k)}, \quad \mathbf{G} = \begin{bmatrix} \mathbf{C} & \mathbf{B} \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{CA}^{N-1} & \mathbf{B} \cdots & \mathbf{CB} \end{bmatrix}$

Vector $\mathbf{f}$ represents free responds from instant k, i.e. predicted system outputs for $\mathbf{u} = \mathbf{0}$. The product $\mathbf{G}\,\mathbf{u}$ compensates differences of these outputs from desired values within considered horizon of the prediction N.

The control $\mathbf{u}$ is computed from the quadratic criterion (11)

$$J_k = \sum_{j=1}^{N} \{ \| (\hat{\mathbf{y}}_{k+j} - \mathbf{w}_{k+j})\, \mathbf{Q}_y \|^2 + \| \mathbf{u}_{k+j-1}\, \mathbf{Q}_u \|^2 \} \tag{11}$$

where $\mathbf{Q}_y$ and $\mathbf{Q}_u$ are penalizations; and $\mathbf{w}_{(k+j)}$ is a vector of desired values.

Obtained vector $\mathbf{u}$ represents the control actions for whole horizon N. However, only the first appropriate actions are really applied to the robot. This process is repeated in every time step for appropriately updated model (6).

To provide effective and stable algorithm for the computation of control actions $\mathbf{u}$, the minimization of the criterion can be suitably transformed to the minimization in the specific matrix square-root form (Belda, 2005):

$$\mathbf{J} = \begin{bmatrix} \mathbf{Q}_y^{\frac{1}{2}} (\hat{\mathbf{y}} - \mathbf{w}) \\ \mathbf{Q}_u^{\frac{1}{2}} \mathbf{u} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_y^{\frac{1}{2}} \mathbf{G} \\ \mathbf{Q}_u^{\frac{1}{2}} \end{bmatrix} \mathbf{u} + \begin{bmatrix} \mathbf{Q}_y^{\frac{1}{2}} (\mathbf{f} - \mathbf{w}) \\ \mathbf{0} \end{bmatrix} \stackrel{!}{=} \mathbf{0} \tag{12}$$

The minimization of such form leads to the solution of simple system of algebraic equations relative to unknown vector of control actions $\mathbf{u}$.

The predictive control, due to multi-step character, generates more suitable control actions for drives, which fit the desired robot motion without addition of unproductive parts. It represents economic solution in the view of energy consumption in comparison to conventional approaches.

5. APPLICATION EXAMPLES

In this section, the application of explained theoretical results will be shown on one specific planar parallel robot, which is being developed for top milling machine. Main spindle of the machine, which serves for milling cutters, is located in centre of the movable platform of the robot structure. The control is focused on coordinates in plane. Vertical axis (coordinate) is set independently and is fixed during motion.

5.1 Description of Considered Parallel Robot

For experiments, the robot called 'Moving Slide' was used. It is illustrated in Fig. 4. This robot represents horizontal planar parallel configuration, which has 4 × Rotational + Prismatic + Rotational joints. Moreover, it is redundantly actuated, because there are four drives for only three degreases of freedom here. This feature furthermore improves robot stiffness and dynamics.

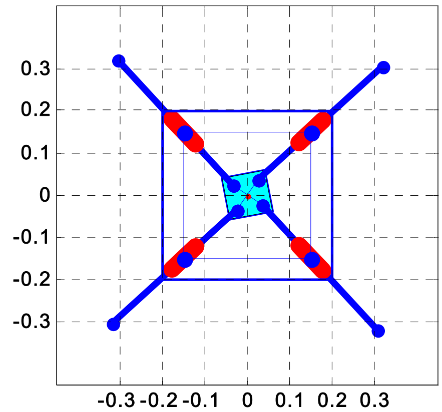

Fig. 4. 4 × RPR parallel structure 'Moving Slide'

5.2 Robot Homing and Calibration

According to sections 2. and 3., the robot is led to home position, which is, for this case, selected in the centre of the robot workspace. The robot has rotational drives (see Fig. 5).

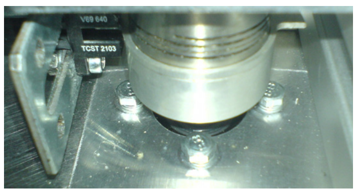

Fig. 5. Homing mark - case of rotational drives

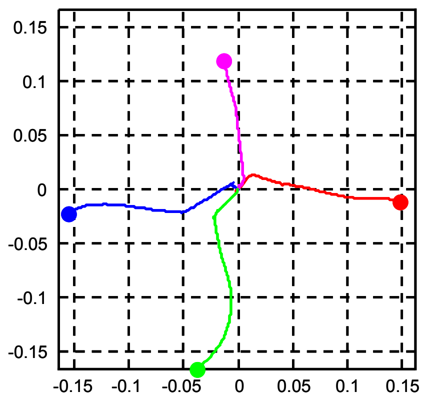

Fig. 6. XY graph of real robot motion during homing

In Fig. 6, the lines represent homing trajectories of four different unknown initial robot positions. As mentioned in section 3., their coordinates were reconstructed by backward robot motion using direct kinematical transformation (Böhm, et al., 2001). It is possible due to known coordinates of the home robot position as initial conditions for the direct transformation.

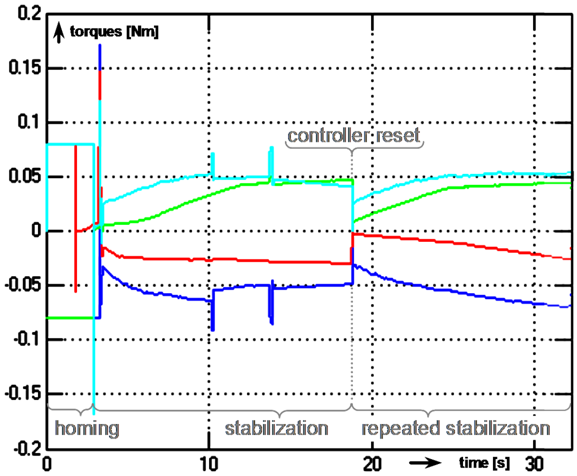

Fig. 7. Time history of real homing and calibration

The Fig. 7 represents time history for one homing trajectory from Fig. 6. After homing of all drives, the robot is stabilized in the home position. It is perceptible undesirable increase of I/S channels, which cannot be still compensated before finishing the calibration.

5.3 *Robot Control*

The example illustrated in Fig. 8 demonstrates real control.

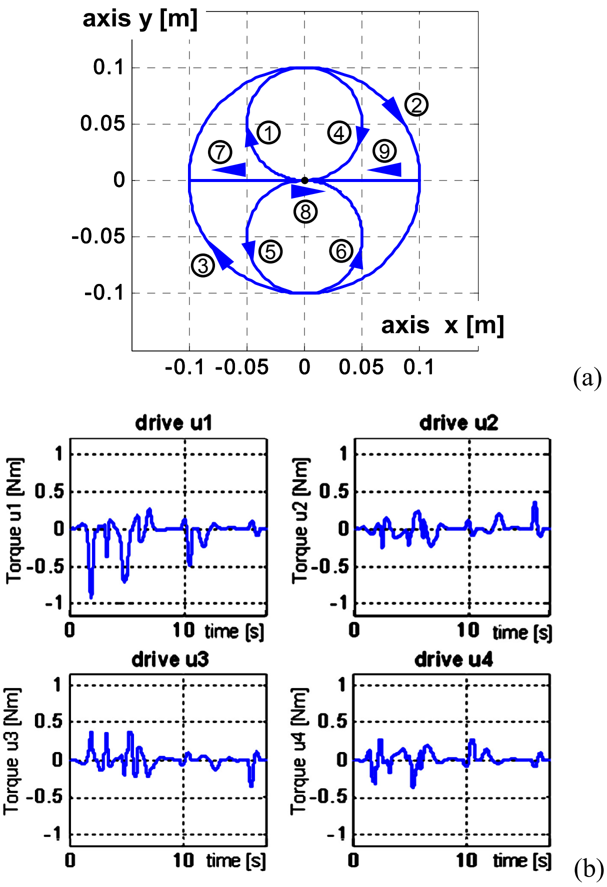

Fig. 8. Trajectory (a) and time history of control action (b)

The structure (Fig. 4) tracks individual trajectory segments in order indicated in Fig. 8 (a) by numbers 1 - 9. Initial and final point is in the workspace centre [x, y] = [0, 0]. The trajectory includes two turn points between segments 7 and 8 [x, y] = [-0.1, 0], and segments 8 and 9 [x, y] = [0.1, 0]. In them, the robot is stopped. During the whole motion, the robot has three acceleration stages (start and two turn points) and three braking stages (two turn and end points). The designed controller using predictive strategy was executed with horizon of prediction $N = 10$ and sampling period $T_s = 0.02\,s$.

6. CONCLUSIONS

This paper focuses on the homing, calibration and control of planar parallel robots. Homing and calibration procedures were discussed initially for general robot structures, then the particularities of the procedures applied to parallel robots were explained. The individual procedures were demonstrated and verified on laboratory model of 4 × RPR parallel structure 'Moving Slide'. In regards to control, predictive strategy proved to be a suitable approach for robot control.

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