Comment on "Sunspot Cycle: A Driven Nonlinear Oscillator?"

In a recent Letter Paluš and Novotná [1] reported statistical evidence based on surrogate data testing for linearity that a driven nonlinear oscillator is the mechanism underlying the sunspot cycle.

While their result might be true we doubt the correctness of their formal conclusion. Surrogate data testing for linearity [2] tests the null hypothesis that a linear, Gaussian, stationary, stochastic dynamical process underlies the data, including a possible invertible, static nonlinear observation function. To perform the test a feature is chosen that can capture a violation of the null hypothesis. This feature is evaluated for the original time series and for numerous realizations of a process which only exhibits the linear statistical properties of the given data. A significant deviation of the feature evaluated for the original time series from the simulated distribution suggests a rejection of the null hypothesis. The feature is usually chosen according to a specific type of alternative hypothesis on the underlying dynamics. In their Letter Paluš and Novotná [1] chose the amplitude-frequency correlation as a property of nonlinear (driven) oscillators. But the rejection of the null hypothesis based on a certain feature does, in general, not give evidence that the specific type of alternative that has motivated the choice of the feature is present. To provide evidence for a specific alternative one has to show that the chosen feature has high power to detect the violation by which it was motivated but no power to detect other types of violations. Unfortunately, the null hypothesis under consideration is such restrictive that the possible alternatives span a huge class of processes, see e.g. [3,4].

With respect to the amplitude-frequency correlation considered in their Letter by Paluš and Novotná [1], for example, if the frequency of a second order linear stochastic process is modulated with time, the resulting process analytically shows an amplitude-frequency correlation [3]. A physically more plausible alternative hypothesis for the sun spot data arises from solar physics, see [5] for review: The sun spots are an effect of the dynamics of the magnetic field of the sun which exhibits a 22 years cycle. This dynamics, a magnetohydrodynamic dynamo, is described by a nonlinear partial differential equation which is eventually stochastically driven. The sun spot number represent a very specific mapping from the spatio-temporal magnetic field to a scalar time series. Since nonlinear driven partial differential equations include nonlinear driven oscillators as special cases, the latter can not be distinguished from the former based on surrogate data for the sun spots.

Summarizing, a significant amplitude-frequency correlation is a feature of driven nonlinear oscillators, but it is not a specific feature of these type of processes. Thus, the specific alternative of a driven nonlinear oscillator can not be concluded from a rejection of the null hypothesis.

Generally speaking, assuming that (1) no process in nature is indeed a linear, Gaussian, stationary, stochastic dynamical one and (2) that one is using a feature that is capable to detect the actual deviation, without any further information about the process, the only thing one can infer from surrogate data testing is whether there are enough data for the power of the test to be large enough to reject the by assumption untrue null hypothesis.

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