Non-linear transport and quantum interaction corrections in disordered systems

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Outline

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i) long wire

ii) mesoscopic wire

iii) ultrashort wire

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Conclusions

Introduction

- There are two types of quantum corrections to the Drude formula for σ

  I) Weak localization (WL): a purely one-particle effect due to the interference of time-reversed trajectories

  II) Interaction corrections (IC): due to the interplay of interaction and disorder

- In the following we focus on **how** type II **affect** electrical transport beyond linear regime

- This may be relevant for various experiments

- In general non-linear behavior may probe **dephasing** in type II corrections

Non-linear transport: Drude-Boltzmann theory

Simple example: a wire attached to leads

Diffusive regime: $\lambda_F \ll l \ll L$

The current is given in terms of distribution function

$$\mathbf{I} = \mathrm{e} D N_0 S \int \mathrm{d}\epsilon \partial_x F(x,t,\epsilon)$$

One determines F via

i) Boltzmann eq. (B.E.) $\Rightarrow$ diffusion equation

ii) Boundary conditions at the leads

$$F(x=0,t,\epsilon) = F_{equilibrium}(\epsilon)$$

Effect of interfaces

The current through the interface

$$\mathbf{I} = \frac{G_T}{2\mathsf{e}} \int \mathsf{d}\epsilon \left[F(x = 0^+, t, \epsilon) - F(x = 0^-, t, \epsilon) \right]$$

G_T interface conductance

By matching the currents at the interface $\Rightarrow$ extra boundary conditions to use with B.E.

$\Rightarrow$ Standard result for combining resistive elements

What happens in the presence of quantum interaction corrections?

One expects corrections to

i) distribution function

ii) density of states

iii) diffusion coefficient

To appreciate this use Keldysh (1964) non-equilibrium technique

$$\mathbf{I} = 2(-\mathrm{e}) \int \frac{\mathrm{d}\epsilon}{2\pi} \sum_p \frac{p}{m} G^K(p, \epsilon, x, t)$$

At equilibrium, the *spatial* and *temporal* dependence drops out

$$G^K(p, \epsilon) = F_{equilibrium}(\epsilon) \left[G^R(p, \epsilon) - G^A(p, \epsilon) \right]$$

With interaction corrections $G^K \to G^K + \delta G^K$

$$\delta G^K \sim \delta F + \delta G^R$$

$\delta F \to \delta V, \qquad \delta G^R \to \delta N_0, \delta D$

By a diagrammatic analysis one can prove

$$\delta \mathbf{I} = \delta \mathbf{I}_A + \delta \mathbf{I}_B$$

$\delta \mathbf{I}_A$ associated with F-corrections

$\delta \mathbf{I}_B$ associated with DoS- and D- corrections

Consider the structure: reservoir-interface-wire-interface-reservoir

By current conservation

$$\begin{aligned} \delta \mathbf{I} &= \delta \mathbf{I}_{A,L} + \delta \mathbf{I}_{B,L} \\ &= \delta \mathbf{I}_{A,wire} + \delta \mathbf{I}_{B,wire} \\ &= \delta \mathbf{I}_{A,R} + \delta \mathbf{I}_{B,R} \end{aligned}$$

By requiring that the voltage drop across the system is fixed

$$\delta \mathbf{I} = \frac{R_L \delta \mathbf{I}_{B,L} + R_{wire} \delta \mathbf{I}_{B,wire} + R_R \delta \mathbf{I}_{B,R}}{R_L + R_{wire} + R_R}$$

Diagrammatic analysis provides expressions for $\delta\mathbf{I}_B$

Let us consider first the wire

$$\delta\mathbf{I}_{B,wire} = \delta I^1(x) + \delta I^2(x)$$

$$\frac{\delta I^1(x)}{eDN_0} = 2\text{Im}\int \mathrm{d}\epsilon \mathrm{d}x_1 \frac{\mathrm{d}\omega}{2\pi} F_\epsilon(x) P_\omega(x,x_1) F_{\epsilon-\omega}(x_1) \partial_x \Phi_\omega(x_1,x)$$

$$\frac{\delta I^2(x)}{eDN_0} = \text{Im}\partial_x \int \mathrm{d}\epsilon \mathrm{d}x_1 \frac{\mathrm{d}\omega}{2\pi} F_\epsilon(x) P_\omega(x,x_1) F_{\epsilon-\omega}(x_1) \Phi_\omega(x_1,x)$$

$P_\omega(x,x')$ describes propagation of a diffusive density fluctuation:

$\Phi_\omega(x,x')$ is the effective potential created by a density fluctuation

$\Phi_\omega(x,x') = \int \mathrm{d}x'' V_\omega(x,x'') P_\omega(x'',x')$

$V_\omega(x,x')$ screened Coulomb interaction

A few comments

- The two terms correspond to the diffusive (^{2})and drift (^{1}) term of the phenomenological expression of the current

$$j = -\mathrm{e} D \partial_x n + \sigma E$$

- For a wire attached to ideal leads by ideal interfaces $\delta \mathbf{I}^2 = 0$

- In the presence of interfaces, there is charge accumulation close to the boundary and δI^2 has to be taken into account

- The ingredients of the calculation: $F, P.\Phi$ which have to calculated

  i) F obeys B.E.

  ii) P obeys diffusion equation

  iii) Φ depends on screening and geometry

For the current at an interface

$$\delta \mathbf{I}_{B,L}(x) = -\frac{1}{2\mathrm{e}R_L}\mathrm{Im}\int \mathrm{d}\epsilon \mathrm{d}x_1 \frac{\mathrm{d}\omega}{2\pi}\left(F_\epsilon(0) - F_{L,\epsilon}\right) P_\omega(x, x_1) F_{\epsilon-\omega}(x_1) \Phi_\omega(x_1, x)$$

$\delta \mathbf{I}_{B,L}$ is similar

Note: we have neglected quantum interaction corrections in the leads

First example: long wire $L \gg L_{ph}, L_{in}$

L_{ph} e-phonon relaxation time

L_{in} e-e relaxation time

electrons in the wire scatter inelastically many times

$\Rightarrow$ distribution function has a local equilibrium form with spatial dependent μ and T (Nagaev 1995)

$$\delta I = -\frac{2\mathrm{e}}{h} 2 \int_0^\infty \mathrm{d}r \int_\tau^\infty \mathrm{d}t \left(\frac{T_e}{\sinh(\pi T_e t)} \right)^2 P_t(r) \sin\left(\frac{\mathrm{e}Vrt}{L}\right)$$

τ the elastic scattering time

At low voltages

$$\delta I(V) \approx \frac{2\mathrm{e}^2}{h} \frac{\sqrt{D/T_e}}{\pi L} V \left(-4.92 + 0.21 \frac{D(eV/L)^2}{T_e^3} + \cdots \right)$$

The first term is the AAL correction (PRL 1980)

Second example: mesoscopic wire $L_T \ll L \ll L_{in}$

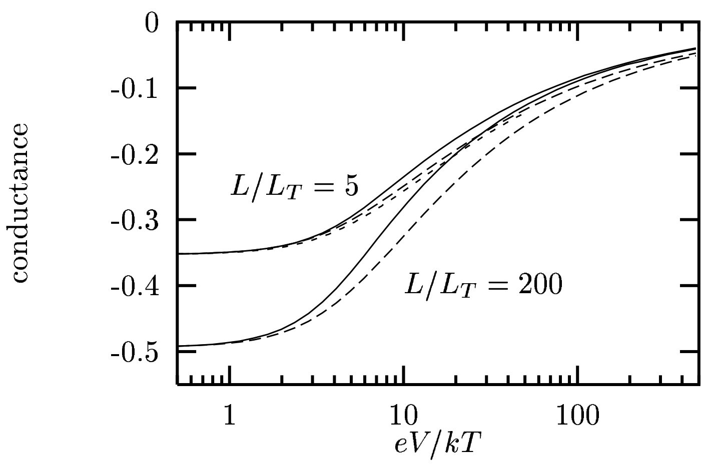
$L_T = \sqrt{D/T}$

- The wire is phase coherent, no inelastic scattering
- The distribution function linearly interpolates between the distribution functions in the leads

$$\delta I = -\frac{2\mathrm{e}}{h} 2 \int_0^\infty \mathrm{d}r \int_\tau^\infty \mathrm{d}t \left(\frac{T}{\sinh(\pi T t)} \right)^2 P_t(r) \sin(eVt) r/L$$

A comment about interplay with heating

- For the local-equilibrium case, non-linear behavior also due to heating
- T_e estimated with energy balance arguments $P_{\text{in}} = P_{\text{out}}$
- Weak heating, for instance, $T_e - T \approx \frac{3}{\pi^2} D (eV/L)^2 \tau_{\text{ph}}/T$
- Following Nagaev (PRB 1995) one calculates $T_e(x)$
- Generally, heating is important when $eVL \approx T$ while for non-heating non-linear $eVL_T \approx T$



- I/V is plotted in units of $(e^2/\hbar)L_T/L$
- Full line corresponds to the non-equilibrium distribution function
- Long dashed line corresponds to the local equilibrium distribution function
- Short dashed line ($L/L_T = 5$) is the non-linear conductivity due to the heating contribution only

A comment on the diffuson

- $P_\omega(x,x')$ obeys a diffusion equation with boundary conditions

- In the case of ideal interfaces (open boundary conditions)

$$P_\omega(x,x')|_{x=0,L} = 0$$

- This condition may be derived by observing that in the leads the diffusion coefficient is much larger than in the wire

$$P_\omega(x,x') = \sum_{n=1}^{\infty} \frac{2}{L} \frac{\sin(k_n x)\sin(k_n x')}{-\mathrm{i}\omega + D k_n^2}, \quad k_n = \frac{n\pi}{L}$$

- For $L \gg L_T$

$$P_\omega(x,x') = \int \frac{\mathrm{d}k}{2\pi} \frac{\exp(\mathrm{i}k(x-x'))}{-\mathrm{i}\omega + Dk^2}$$

Third example: ultrashort wire $L \ll L_T$

One can make a lowest mode approximation for the diffuson

$$\delta I = -\frac{\mathrm{e}}{h} A \int_\tau^\infty \mathrm{d}t e^{-\gamma_0 t} \left(\frac{T}{\sinh(\pi T t)} \right)^2 \sin(eVt)$$

$$A \approx 0.25$$

$\gamma_0 = Dk_1^2 = \pi^2 D/L^2 = \pi^2 E_{Th}$, Thouless energy

The linear conductance

$$G \approx -\frac{2\mathrm{e}^2}{h} \frac{1}{\pi^2} \ln \frac{1}{\tau \max(T, E_{Th})}$$

i.e., G depends logarithmically at $T > E_{Th}$ and then saturates at $T \sim E_{Th}$

Fourth example: short wire attached to leads by non-ideal interfaces

- In this case the voltage drop is concentrated at the interface

- The distribution function is spatially independent and a linear superposition of those in the leads

$$F_{wire}(\epsilon) \approx \frac{R_L^{-1} F_L + R_R^{-1} F_R}{R_L^{-1} + R_R^{-1}}$$

- The diffuson is evaluated in the lowest mode approximation with boundary condition

$$\partial_x P_\omega(x, x')|_{x=0^+} = \frac{R_{wire}}{R_L} P_\omega(x, x')|_{x=0^+}$$

- For $R_{wire} \ll R_L$ this condition reduces to that of an interface with the vacuum or an insulator

The current

$$\delta I = -\frac{\mathsf{e}}{h} A \int_{\tau}^{\infty} \mathsf{d}t e^{-\gamma_0 t} \left(\frac{T}{\sinh(\pi T t)} \right)^2 \sin(eVt)$$

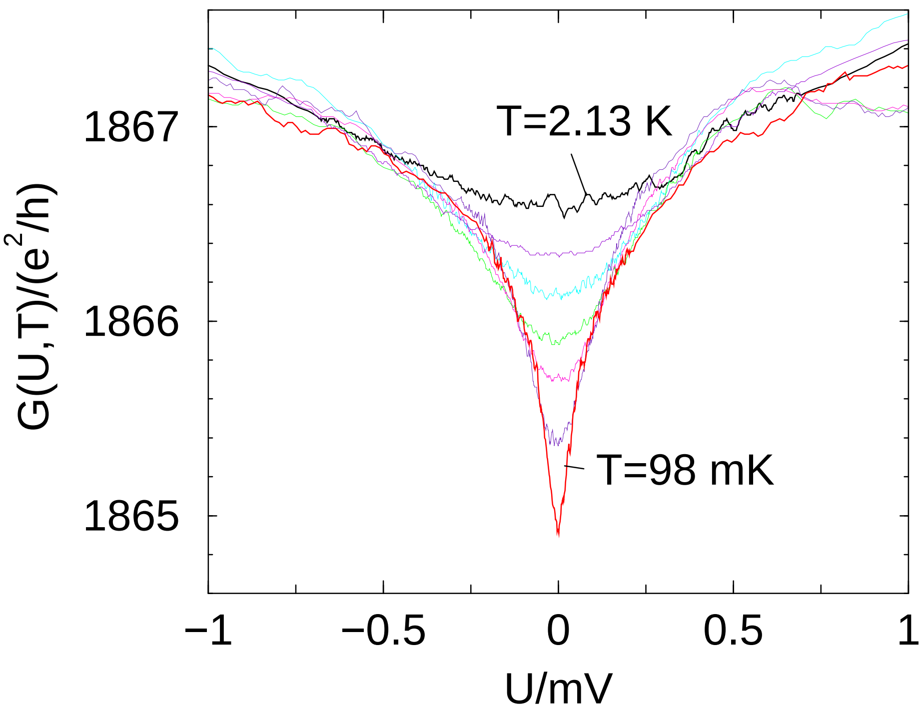
ii) resistive intefaces:

$$A = \frac{2 R_L R_R}{(R_L + R_R)^2} \approx .5$$

for symmetric system

$$\gamma_0 = E_{Th} R_{wire} (R_L + R_R) / R_L R_R \ll E_{Th}$$

Comparison with experiment (Weber et al. PRB **63**, 165426)



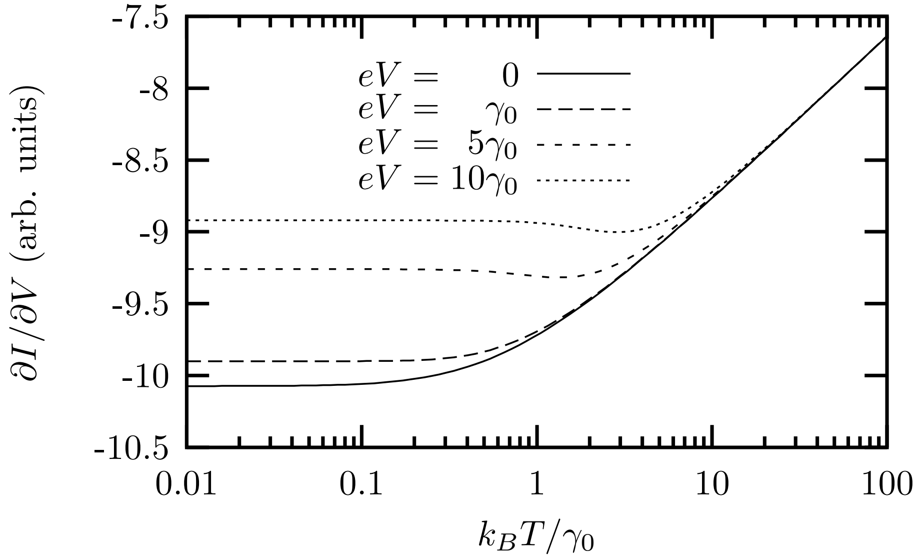
Log-T dependence between $T = 100mK$ and $T = 2K$

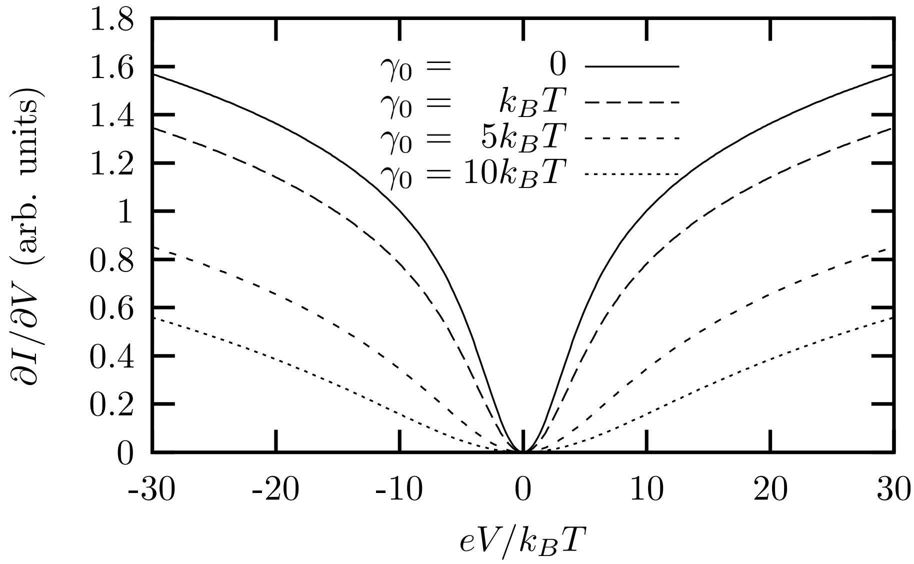
$$G(0,T) = G(0,T_0 = 1K) + A\ln(T/T_0), \quad A = 0.49e^2/h$$

Saturation below $T = 100mK$

Scaling law

$$\frac{G(V,T) - G(0,T)}{A} \equiv f(eV/T)$$





From saturation temperature and prefactor we conclude that main resistive behavior at interfaces

Changing transparency would result in change of saturation temperature and prefactor

The same analysis can be done for a 2D macroscopic film in the presence of a DC electric field $\mathbf{E}$

$$\delta \mathbf{I} = -\mathbf{E}\frac{e^2}{(\pi h)}\int_{\tau}^{\infty}\frac{dt}{t}\left(\frac{\pi T_e}{\sinh \pi t T_e}\right)^2 \frac{\sinh \frac{(tT_E)^3}{2}}{\frac{(tT_E)^3}{2}} e^{-\frac{(tT_E)^3}{2}}$$
$$T_E^3 = De^2E^2$$

Low field expansion (The first term is the AAL logarithmic correction)

$$\frac{\delta \mathbf{I}}{e^2/(\pi h)} = -\mathbf{E}\left[\ln\frac{1}{T\tau} - 1.62\frac{De^2E^2}{\pi^3 T^3}\right]$$

High field limit: T_E replaces T in the log and gives rise to a "dephasing" in the particle-hole channel $\tau_\phi^{ph} \sim E^{-2/3}$

$$\frac{\delta \mathbf{I}}{e^2/(\pi h)} = -\mathbf{E}\ln\frac{1}{T_E\tau}$$

Non-linear effect possibly relevant for 2D SiMOSFET and GaAs heterostructure

Positive magnetoresistance (Simonian et al. 97, Popovic et al. 97, Coleridge et al. 99) implies that the spin-triplet channel contribution is important (Finkelstein 83, Castellani et al.84, Castellani et. al. 98)

Electric field scaling in 2D SiMOSFET (near MIT) (Kravchenko et al. 96, Heemsterk and Klapwijk 98)

Non-linear effects used to probe metallic or insulating behavior in 2D GaAs/AlGaA (Yoon et al. 98)

$T_E \ll T$ limit (γ_2): triplet channel scattering amplitude

$$\delta\sigma_2 = \frac{e^2}{2\pi^2}\left[-f_2^1(\gamma_2)\ln\left(\frac{\mathrm{e}}{2\pi T\tau}\right) + \frac{\pi}{30} f_2^3(\gamma_2)\frac{T_E^3}{T^3}\right]$$

The function $f_1^1(\gamma_2)$ controls the RG flow.

$$f_2^1(\gamma_2) = 1 + 3\left[1 - \frac{1+\gamma_2}{\gamma_2}\ln(1+\gamma_2)\right]$$

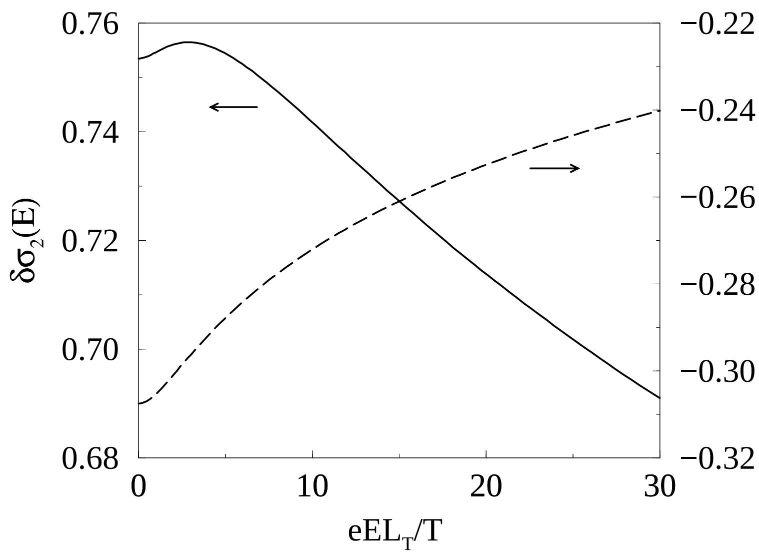
$$f_2^3(\gamma_2) = \frac{1}{2} + \frac{3}{2}\left[\frac{6+5\gamma_2}{\gamma_2^2} - \frac{(6+2\gamma_2)(1+\gamma_2)}{\gamma_2^3}\ln(1+\gamma_2)\right]$$

Non-linear effects also appear in the magnetoconductance from the $M = \pm 1$ triplet contributions (Ω_s Zeeman energy)

$$\Delta\sigma_2 = -\frac{e^2}{2\pi^2}\frac{\Omega_s^2}{T^2}\left[\frac{3\zeta(3)}{2\pi^2} g_2^1(\gamma_2) + \frac{\pi}{42} g_2^3(\gamma_2)\frac{T_E^3}{T^3}\right]$$

Note

- $\gamma_2 = 0$ (dashed line) localizing
- $\gamma_2 = 5$ (solid line) metallic
- At small fields, $f_2^3(\gamma_2) > 0$, non-linear conductivity always positive $\Rightarrow$ we need a careful analysis of experimental data at low fields (compare with Yoon et al. 98)
- At large electric fields $\Rightarrow$ log-behavior with the sign of $f_2^1(\gamma_2)$



In semiconductors devices GaAs and Si MOSFET $E_{DC} \sim 1$V/m, $T \sim 100mK$ one estimates $T_E \sim 10mK$ (Yoon et al. 98) (Kravchenko et al. 96) smaller than what indicated by the experiment

- Need to go beyond lowest order perturbation theory and possible renormalization of the scale T_E $\Rightarrow$ see next
- Relevance of dishomogeneity and nonuniform electric field in the sample (Meir 99,)
- Complicated interplay with heating effects and one has to measure T_{el} independently $\Rightarrow$ see next

Possible consequences for scaling

T_E gives a mechanism for scaling

- Close to QCP (If any (cf. Belitz and Kirkpatrick 94, Sondhi et al. 97)) $T \sim \xi^{-z}$ where ξ is the correlation length and z is the dynamical critical exponent.

- In a diffusive system $T \sim D_{\text{qp}}(\xi)/\xi^2$ with scale-dependent $D_{qp}(\xi)$ diffusion and quasi-particle DOS N_{qp} related by $D_{qp} = D/(N_{qp}/N_0)$ (Finkelstein 83, Castellani and DiCastro 86). $\Rightarrow$ D_{qp} scales near the QCP as $D_{\text{qp}} \sim \xi^{2-z}$.

- From $T_E^3 = D_{\text{qp}} e^2 E^2 \rightarrow E \sim \xi^{-(1+z)}$.

In the experiments

- $z \approx 1$ which corresponds to growing D_{qp} and a vanishing N_{qp} quasi-particle density of states near MIT.

- Then one expects large non-linear effects near the QCP point.

- The small value of $z < 2$ implies $c_v \sim T\xi^{z-2} \sim T^{2/z}$.

Conclusions

- Formulation of non linear transport including quantum interaction corrections in disordered systems

- Analysis of 1D and 2D systems

- Good agreement for 1D metallic systems

- Qualitative agreement with 2D semiconducting systems