

Geometric Structures of Stable Output Feedback Systems

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Abstract: In this paper, we investigate the geometric structures of the stable time-varying and the stable static output feedback systems. Firstly, we give a parametrization of stabilizing time-varying output feedback gains subject to certain constraints, that is, the subset of stabilizing time-varying output feedback gains is diffeomorphic to the Cartesian product of the set of time-varying positive definite matrices and the set of time-varying skew symmetric matrices satisfying certain algebraic conditions. Further, we show how the Cartesian product satisfying certain algebraic conditions is imbedded into the Cartesian product of the set of time-varying positive definite matrices and the set of time-varying skew symmetric matrices. Then, we give some eigenvalue properties of the stable time-varying output feedback systems. Notice that the stable static output feedback system, which does not depend on the temporal parameter t , is just a special case of the stable time-varying output feedback system. Moreover, we use the Riemannian metric, the connections and the curvatures to describe the subset of stabilizing static output feedback gains. At last, we use a static output feedback system to illustrate our conclusions.

Keywords: diffeomorphism; geometric structure; output feedback; immersion;

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