On Markov Properties in Evidence Theory

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Abstract

The goal of the paper is to recall a recently introduced concept of conditional independence in evidence theory and to discuss Markov properties based on this independence concept.

1 Introduction

Any application of models of artificial intelligence to practical problems must manage two basic issues: uncertainty and multidimensionality. At present the most widely used models managing it are so-called *probabilistic graphical Markov models*.

In these models, the problem of multidimensionality is solved with the help of the notion of conditional independence, which enables factorization of a multidimensional probability distribution into small parts, usually marginal or conditional low-dimensional distributions (or generally into low-dimensional factors). Such a factorization not only decreases the storage requirements for representation of a multidimensional distribution but it usually also induces efficient computational procedures allowing inference from these models. Many analogies of results concerning conditional independence and Markov properties from probabilistic framework were achieved also in possibility theory [7, 8].

It is easy to realize that if we need efficient methods for representation of probability and possibility distributions (requiring an exponential number of parameters), the greater is the need of an efficient tool for representation of belief functions, which cannot be represented by a distribution (but only by a set function), and therefore the space requirements for its representation are superexponential.

After a thorough study of relations among stochastic independence, possibilistic T-independence, random set independence and strong independence [9, 10] we came to the conclusion that the most proper independence concept in evidence theory is random set independence. Furthermore, in [11] we introduced a new concept of conditional independence and proved in which sense it is superior to the formerly introduced one [1]. Therefore, all results presented in this contribution are based on this new conditional independence concept.

The contribution is organized as follows. After a short overview of necessary terminology and notation (Section 2), in Section 3 we recall the recently introduced concept of conditional independence and its properties [11] and in Section 4 we discuss its Markov properties.

2 Basic Notions

The aim of this section is to introduce as briefly as possible basic notions and notations necessary for the understanding the following text.

For an index set $N = \{1, 2, \ldots, n\}$ let $\{X_i\}_{i \in N}$ be a system of variables, each X_i having its values in a finite set $\mathbf{X}_i$. In this paper we will deal with *multidimensional frame of discernment*

$$\mathbf{X}_N = \mathbf{X}_1 \times \mathbf{X}_2 \times \ldots \times \mathbf{X}_n,$$

and its *subframes* (for $K \subseteq N$)

$$\mathbf{X}_K = \times_{i \in K} \mathbf{X}_i.$$

When dealing with groups of variables on these subframes, X_K will denote a group of variables $\{X_i\}_{i \in K}$ throughout the paper.

A *projection* of $x = (x_1, x_2, \ldots, x_n) \in \mathbf{X}_N$ into $\mathbf{X}_K$ will be denoted $x^{\downarrow K}$, i.e. for $K = \{i_1, i_2, \ldots, i_k\}$

$$x^{\downarrow K} = (x_{i_1}, x_{i_2}, \ldots, x_{i_k}) \in \mathbf{X}_K.$$

Analogously, for $M \subset K \subseteq N$ and $A \subset \mathbf{X}_K$, $A^{\downarrow M}$ will denote a *projection* of A into $\mathbf{X}_M$:[1]

$$A^{\downarrow M} = \{y \in \mathbf{X}_M \mid \exists x \in A : y = x^{\downarrow M}\}.$$

In addition to the projection, in this text we will also need an opposite operation, which will be called an extension. By an *extension* of two sets $A \subseteq \mathbf{X}_K$ and $B \subseteq \mathbf{X}_L$ $(K, L \subseteq N)$ we will understand a set

$$A \otimes B = \{x \in \mathbf{X}_{K \cup L} : x^{\downarrow K} \in A \;\;\&\;\; x^{\downarrow L} \in B\}.$$

Let us note that if K and L are disjoint, then

$$A \otimes B = A \times B.$$

In evidence theory (or Dempster-Shafer theory) two measures are used to model the uncertainty: belief and plausibility measures. Both of them can be

[1] Let us remark that we do not exclude situations when $M = \emptyset$. In this case $A^{\downarrow \emptyset} = \emptyset$.

defined with the help of another set function called a *basic (probability* or *belief) assignment* m on $\mathbf{X}_N$, i.e.

$$m : \mathcal{P}(\mathbf{X}_N) \longrightarrow [0,1]$$

for which

$$\sum_{A \subseteq \mathbf{X}_N} m(A) = 1.$$

Furthermore, we assume that $m(\emptyset) = 0$.

A set $A \in \mathcal{P}(\mathbf{X}_N)$ is a *focal element* if $m(A) > 0$. A pair $(\mathcal{F}, m)$, where $\mathcal{F}$ is the set of all focal elements, is called a *body of evidence*. A basic assignment is called *Bayesian* if all its focal elements are singletons. A body of evidence is called *consonant* if its focal elements are nested.

For a basic assignment m on $\mathbf{X}_K$ and $M \subset K$ a *marginal basic assignment* of m is defined (for each $A \subseteq \mathbf{X}_M$):

$$m^{\downarrow M}(A) = \sum_{B \subseteq \mathbf{X}_K : B^{\downarrow M} = A} m(B).$$

Having two basic assignments m_1 and m_2 on $\mathbf{X}_K$ and $\mathbf{X}_L$, respectively $(K, L \subseteq N)$, we say that these assignments are *projective* if

$$m_1^{\downarrow K \cap L} = m_2^{\downarrow K \cap L},$$

which occurs if and only if there exists a basic assignment m on $\mathbf{X}_{K \cup L}$ such that both m_1 and m_2 are marginal assignments of m.

3 Conditional Independence and Its Properties

Let us start this section by recalling the notion of random sets independence [2].[2]

Definition 1 Let m be a basic assignment on $\mathbf{X}_N$ and $K, L \subset N$ be disjoint. We say that groups of variables X_K and X_L are *independent with respect to basic assignment* m if

$$m^{\downarrow K \cup L}(A) = m^{\downarrow K}(A^{\downarrow K}) \cdot m^{\downarrow L}(A^{\downarrow L})$$

for all $A \subseteq \mathbf{X}_{K \cup L}$ for which $A = A^{\downarrow K} \times A^{\downarrow L}$, and $m(A) = 0$ otherwise.

It has been shown in [9] that application of Definition 1 to two consonant bodies of evidence leads to a body of evidence which is not consonant any more.

It seemed that this problem could be avoided if we took into account the fact that both evidence and possibility theories could be considered as special kinds of imprecise probabilities. Furthermore, we proved that strong independence implies possibilistic independence based on product t-norm. Nevertheless,

[2] Klir [4] uses the notion *noninteractivity*.

in [10] we showed that the application of strong independence to two general bodies of evidence (neither Bayesian nor consonant) leads to models beyond the framework of evidence theory.

Therefore, at present random sets independence seems to be the most appropriate independence concept in the framework of evidence theory. For this reason in [11] we introduced the following generalization of this concept.

Definition 2 Let m be a basic assignment on $\mathbf{X}_N$ and $K, L, M \subset N$ be disjoint, $K \neq \emptyset \neq L$. We say that groups of variables X_K and X_L are *conditionally independent given* X_M *with respect to* m (and denote it by $K \perp\!\!\!\perp L|M\ [m]$), if the equality

$$m^{\downarrow K\cup L\cup M}(A) \cdot m^{\downarrow M}(A^{\downarrow M}) \quad = \quad m^{\downarrow K\cup M}(A^{\downarrow K\cup M}) \cdot m^{\downarrow L\cup M}(A^{\downarrow L\cup M}) \quad (1)$$

holds for any $A \subseteq \mathbf{X}_{K\cup L\cup M}$ such that $A = A^{\downarrow K\cup M} \otimes A^{\downarrow L\cup M}$, and $m(A) = 0$ otherwise.

Let us note that for $M = \emptyset$ the concept coincides with Definition 1, which enables us to use the term conditional independence. Let us also note that (1) resembles, from the formal point of view, the definition of stochastic conditional independence [5].

Theorem 1 taken from [11] expresses the fact that this concept of conditional independence is consistent with marginalization. What that means can be seen from the following definition.

An independence concept is *consistent with marginalization* iff for arbitrary projective basic assignments (probability distributions, possibility distributions, etc.) m_1 on $\mathbf{X}_K$ and m_2 on $\mathbf{X}_L$ there exists a basic assignment (probability distribution, possibility distribution, etc.) m on $\mathbf{X}_{K\cup L}$ satisfying this independence concept and having m_1 and m_2 as its marginals.

Furthermore, the following assertion presents a form expressing the joint basic assignment by means of its marginals.

Theorem 1 *Let* m_1 *and* m_2 *be projective basic assignments on* $\mathbf{X}_K$ *and* $\mathbf{X}_L$*, respectively. Let us define a basic assignment* m *on* $\mathbf{X}_{K\cup L}$ *by formula*

$$m(A) = \frac{m_1(A^{\downarrow K}) \cdot m_2(A^{\downarrow L})}{m_2^{\downarrow K\cap L}(A^{\downarrow K\cap L})}$$

for $A = A^{\downarrow K} \otimes A^{\downarrow L}$ *such that* $m_1^{\downarrow K\cap L}(A^{\downarrow K\cap L}) > 0$ *and* $m(A) = 0$ *otherwise. Then*

$$\begin{aligned} m^{\downarrow K}(B) &= m_1(B), \\ m^{\downarrow L}(C) &= m_2(C) \end{aligned}$$

for any $B \in \mathbf{X}_K$ *and* $C \in \mathbf{X}_L$*, respectively, and* $(K \setminus L) \perp\!\!\!\perp (L \setminus K)|(K \cap L)\ [m]$*. Furthermore,* m *is the only basic assignment possessing these properties.*

This is the main advantage of conditional independence concept in comparison with the widely used notion of conditional noninteractivity based on commonality functions instead of basic assignments [1]. For more details the reader is referred to [11].

Among the properties satisfied by the ternary relation $K \perp\!\!\!\perp L|M\ [m]$, the following are of principal importance:

(A1) $K \perp\!\!\!\perp L|M\ [m] \implies L \perp\!\!\!\perp K|M\ [m]$,

(A2) $K \perp\!\!\!\perp L \cup M|I\ [m] \implies K \perp\!\!\!\perp M|I\ [m]$,

(A3) $K \perp\!\!\!\perp L \cup M|I\ [m] \implies K \perp\!\!\!\perp L|M \cup I\ [m]$,

(A4) $K \perp\!\!\!\perp L|M \cup I\ [m] \ \wedge\ K \perp\!\!\!\perp M|I\ [m] \implies K \perp\!\!\!\perp L \cup M|I\ [m]$,

(A5) $K \perp\!\!\!\perp L|M \cup I\ [m] \ \wedge\ K \perp\!\!\!\perp M|L \cup I\ [m] \implies K \perp\!\!\!\perp L \cup M|I\ [m]$.

Let us remind that stochastic conditional independence satisfies so-called *semigraphoid* properties (A1)–(A4) for any probability distribution, while axiom (A5) is satisfied only for strictly positive probability distributions. Conditional noninteractivity [1], on the other hand, satisfies axioms (A1)–(A5) for general basic assignment m, as proven in [1]. For conditional independence introduced by Definition 2 we get the following assertion proven in [11].

Theorem 2 *Conditional independence $K \perp\!\!\!\perp L|M\ [m]$ satisfies (A1)–(A4).*

Analogous to probabilistic case, conditional independence $K \perp\!\!\!\perp L|M\ [m]$ does not generally satisfy (A5), as can be seen from the following example.

Example 1 Let X_1, X_2 and X_3 be three variables with values in $\mathbf{X}_1, \mathbf{X}_2$ and $\mathbf{X}_3$ respectively, $\mathbf{X}_i = \{a_i, \bar{a}_i\}, i = 1, 2, 3$, and their joint basic assignment is defined as follows:

$$m(\{(x_1, x_2, x_3\}) = 1/16,$$
$$m(\mathbf{X}_1 \times \mathbf{X}_2 \times \mathbf{X}_3) = 1/2,$$

for $x_i = a_i, \bar{a}_i$, values of m on the remaining sets being 0. Its marginal basic assignments on $\mathbf{X}_1 \times \mathbf{X}_2, \mathbf{X}_1 \times \mathbf{X}_3, \mathbf{X}_2 \times \mathbf{X}_3$ and $\mathbf{X}_i, i = 1, 2, 3$ are

$$m^{\downarrow 12}(\{x_1, x_2\}) = 1/8,$$
$$m^{\downarrow 12}(\mathbf{X}_1 \times \mathbf{X}_2) = 1/2,$$

$$m^{\downarrow 13}(\{x_1, x_3\}) = 1/8,$$
$$m^{\downarrow 13}(\mathbf{X}_1 \times \mathbf{X}_3) = 1/2,$$

$$m^{\downarrow 23}(\{x_2, x_3\}) = 1/8,$$
$$m^{\downarrow 23}(\mathbf{X}_2 \times \mathbf{X}_3) = 1/2,$$

and

$$m^{\downarrow i}(x_i) = 1/4,$$
$$m^{\downarrow i}(\mathbf{X}_i) = 1/2,$$

respectively. It is easy (but somewhat time-consuming) to realize that

$$m(A^{\downarrow 13} \otimes A^{\downarrow 23}) \cdot m^{\downarrow 3}(A^{\downarrow 3}) = m^{\downarrow 13}(A^{\downarrow 13}) \cdot m^{\downarrow 23}(A^{\downarrow 23})$$

and

$$m(A^{\downarrow 12} \otimes A^{\downarrow 23}) \cdot m^{\downarrow 2}(A^{\downarrow 2}) =^{\downarrow 12} (A^{\downarrow 12}) \cdot m^{\downarrow 23}(A^{\downarrow 23})$$

for any A such that $A = A^{\downarrow 12} \otimes A^{\downarrow 23}$ and $A = A^{\downarrow 13} \otimes A^{\downarrow 23}$ (the values of remaining sets being zero), while e.g.

$$m(\{(a_1, \bar{a}_2, \bar{a}_3)\}) = \frac{1}{16} \neq \frac{1}{4} \cdot \frac{1}{8} = m^{\downarrow 1}(\{a_1\}) \cdot m^{\downarrow 23}(\{(\bar{a}_2, \bar{a}_3)\}),$$

i.e. $\{1\} \perp\!\!\!\perp \{2\}|\{3\}$ $[m]$ and $\{1\} \perp\!\!\!\perp \{3\}|\{2\}$ $[m]$ hold, but $\{1\} \perp\!\!\!\perp \{2,3\}|\emptyset$ $[m]$ does not. ♢

This fact perfectly corresponds to the properties of stochastic conditional independence. In probability theory (A5) need not be satisfied if the joint probability distribution is not strictly positive. But the counterpart of strict positivity of probability distribution for basic assignments is not straightforward. It is evident, that it does not mean strict positivity on all subsets of the frame of discernment in question — in this case variables are not (conditionally) independent (cf. Definitions 1 and 2). On the other hand, it can be seen from Example 1 that strict positivity on singletons is not sufficient (and, surprisingly, as we can see in Theorem 3, also not necessary).

Theorem 3 *Let m be a basic assignment on X_N such that $m(A) > 0$ if and only if $A = \times_{i \in N} A_i$, where A_i is a focal element on $\mathbf{X}_i$. Then (A5) is satisfied.*

Example 1 suggests that the assumption of positivity of $m(A)$ on any $A = \times_{i \in N} A_i$, where A_i is a focal element on $\mathbf{X}_i$ is substantial. On the other hand, the assumption that $m(A) = 0$ otherwise may not be so substantial and (A5) may hold for more general bodies of evidence than those characterized by the assumption of Theorem 3 (at present we are not able to find a counterexample).

Let us note that for Bayesian basic assignments assumption of Theorem 3 seems to be more general than that of strict positivity of the probability distribution. But the generalization is of no practical consequence — if probability of a marginal value is equal to zero, than this value may be omitted. On the other hand, nothing similar can be done (in general) in evidence theory, because this value may belong to some bigger focal element.

4 Markov Properties

Markov properties of probability distributions belong to the area of mathematics called *graphical modeling*. In this section we will introduce their counterparts in evidence theory, but first let us present a few notions from graph theory that will appear necessary.

A *graph* is a pair $G = (V, E)$, where V is a finite set of *vertices* and the set of *edges* E is a subset of the set $V \times V$ of (unordered) pairs of distinct vertices. A subset of the vertex set $K \subseteq V$ *induces* a subgraph $G_K = (K, E_K)$, where the edge set $E_K = E \cap (K \times K)$ is obtained from G by keeping edges with both endpoints in K.

A graph is *complete* if any pair of vertices is joined by an edge. A subset is complete if it induces a complete subgraph. A maximal (with respect to set inclusion) complete subset is called a *clique*.

If there is an edge between $k \in V$ and $l \in V$, k and l are said to be *adjacent*, otherwise they are *nonadjacent*. The *boundary* $bd(K)$ of a subset K of vertices is the set of vertices in $V \setminus K$ that are adjacent to vertices in K. The *closure* of K is $cl(K) = K \cup bd(K)$.

If there is a *path* from k to l (a sequence $k = k_0, k_1, \ldots, k_n = l$ of distinct vertices such that $(k_{i-1}, k_i) \in E$ for all $i = 1, \ldots, n$) we say that k and l are in the same *connectivity component*. A subset $S \subseteq V$ is called an (k, l)-*separator* if all paths from k to l intersect S. The subset S *separates* $K \subseteq V$ *from* $L \subseteq V$ if it is an (k, l)-*separator* for every $k \in K$ and $l \in L$.

Now, let us consider the conditional independence in a special situation: we have a graph $G = (V, E)$ and a finite collection of variables $\{X_i\}_{i \in V}$. We will identify variables with vertices of the corresponding graph.

We can (analogous to probability theory) associate three different Markov properties with any undirected graph $G = (V, E)$ and a collection of variables $\{X_i\}_{i \in V}$. Basic assignment m is said to obey

(P) *the pairwise Markov property*, relative to G if, for any pair (i,j) of non-adjacent vertices,

$$\{i\} \perp\!\!\!\perp \{j\}|V \setminus \{i,j\}[m];$$

(L) *the local Markov property*, relative to G if, for any vertex $i \in V$,

$$\{i\} \perp\!\!\!\perp V \setminus cl(\{i\})|bd(\{i\})[m];$$

(G) *the global Markov property*, relative to G if, for any triple (K,L,S) of disjoint subsets of V such that S separates K from L in G,

$$K \perp\!\!\!\perp L|S[m].$$

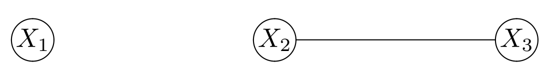
The global Markov property (G) gives a general criterion for deciding whether two groups of variables K and L are conditionally independent, given a third group of variables S. It is the strongest property, as can be seen from Proposition 1 and Examples 2 and 3, inspired by Examples 3.6 and 3.5 from [5], respectively.

The following two assertions (Proposition 1 and Theorem 4) are presented without proofs. Their proofs completely depend on semigraphoid and graphoid properties, respectively, and not on the framework in question. Therefore, the reader is referred to [5].

Proposition 1 *For any undirected graph G and any basic assignment on $\mathbf{X}_V$ it holds true that*

$$(G) \Longrightarrow (L) \Longrightarrow (P). \tag{2}$$

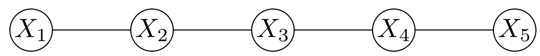
Example 2 Let X_1, X_2 and X_3 be three binary variables with values in $\mathbf{X}_1, \mathbf{X}_2$ and $\mathbf{X}_3$, respectively, $\mathbf{X}_i = \{a_i, \bar{a}_i\}, i = 1,2,3$, on the graph



and the basic assignment m be defined as in Example 1.

From the results obtained in that example, we can see that this basic assignment satisfies the pairwise Markov property (P), implied by $\{1\} \perp\!\!\!\perp \{3\}|\{2\}$ $[m]$, but does not satisfy the local Markov property (L), due to the fact that $\{1\} \perp\!\!\!\perp \{2,3\}|\emptyset$ $[m]$ is not true. ♢

Example 3 Let X_1, X_2, X_3, X_4 and X_5 be five binary variables with values in $\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3, \mathbf{X}_4$ and $\mathbf{X}_5$ respectively, $\mathbf{X}_i = \{a_i, \bar{a}_i\}, i = 1, \ldots, 5$, be defined on the graph



and the basic assignment m have the only focal element, namely

$$A = \{(a_1, a_2, \bar{a}_3, \bar{a}_4, \bar{a}_5), (\bar{a}_1, \bar{a}_2, \bar{a}_3, a_4, a_5)\}.$$

Therefore, all marginal basic assignments on any subspace of $\mathbf{X}_1 \times \mathbf{X}_2 \times \mathbf{X}_3 \times \mathbf{X}_4 \times \mathbf{X}_5$ have also only one focal element, namely for $m^{\downarrow 2345}$, $m^{\downarrow 1345}$, $m^{\downarrow 1245}$, $m^{\downarrow 123}$, $m^{\downarrow 234}$, $m^{\downarrow 345}$, $m^{\downarrow 12}$, $m^{\downarrow 13}$, $m^{\downarrow 24}$, $m^{\downarrow 2}$ and $m^{\downarrow 3}$

$$\begin{aligned}
A^{\downarrow 2345} &= \{(a_2, \bar{a}_3, \bar{a}_4, \bar{a}_5), (\bar{a}_2, \bar{a}_3, a_4, a_5)\}, \\
A^{\downarrow 1345} &= \{(a_1, \bar{a}_3, \bar{a}_4, \bar{a}_5), (\bar{a}_1, \bar{a}_3, a_4, a_5)\}, \\
A^{\downarrow 1245} &= \{(a_1, a_2, \bar{a}_4, \bar{a}_5), (\bar{a}_1, \bar{a}_2, a_4, a_5)\}, \\
A^{\downarrow 123} &= \{(a_1, a_2, \bar{a}_3), (\bar{a}_1, \bar{a}_2, \bar{a}_3)\}, \\
A^{\downarrow 234} &= \{(a_2, \bar{a}_3, \bar{a}_4), (\bar{a}_2, \bar{a}_3, a_4)\}, \\
A^{\downarrow 345} &= \{(\bar{a}_3, \bar{a}_4, \bar{a}_5), (\bar{a}_3, a_4, a_5)\}, \\
A^{\downarrow 12} &= \{(a_1, a_2), (\bar{a}_1, \bar{a}_2\}, \\
A^{\downarrow 13} &= \{(a_1, \bar{a}_3), (\bar{a}_1, \bar{a}_3)\}, \\
A^{\downarrow 24} &= \{(a_2, \bar{a}_4), (\bar{a}_2, a_4)\}, \\
A^{\downarrow 2} &= \{a_2, \bar{a}_2\} = \mathbf{X}_2
\end{aligned}$$

and

$$A^{\downarrow 3} \quad = \quad \{\bar{a}_3\},$$

respectively, are corresponding focal elements.

Therefore we can easily check that

$$m^{\downarrow 2}(A^{\downarrow 2}) = m^{\downarrow 12}(A^{\downarrow 12}) \cdot m^{\downarrow 2345}(A^{\downarrow 2345}), \tag{3}$$

$$m(A) \cdot m^{\downarrow 13}(A^{\downarrow 13}) = m^{\downarrow 123}(A^{\downarrow 123}) \cdot m^{\downarrow 1345}(A^{\downarrow 1345}), \tag{4}$$

and

$$m(A) \cdot m^{\downarrow 24}(A^{\downarrow 24}) = m^{\downarrow 124}(A^{\downarrow 124}) \cdot m^{\downarrow 1245}(A^{\downarrow 1245}).$$

These equalities imply (due to the fact that A is the only focal element) that $\{1\} \perp\!\!\!\perp \{3,4,5\}|\{2\}$, $\{2\} \perp\!\!\!\perp \{4,5\}|\{1,3\}$ and $\{3\} \perp\!\!\!\perp \{1,5\}|\{2,4\}$, respectively. Analogous to (3) and (4) we can check that also $\{5\} \perp\!\!\!\perp \{1,2,3\}|\{4\}$ and $\{4\} \perp\!\!\!\perp \{1,2\}|\{3,5\}$, respectively, hold, which means that this basic assignment satisfies local Markov property (L). Nevertheless, since

$$m(B) \cdot m^{\downarrow 3}(B^{\downarrow 3}) = 0 \quad \neq \quad 1 = m^{\downarrow 123}(B^{\downarrow 123}) \cdot m^{\downarrow 345}(B^{\downarrow 345}),$$

where

$$B \quad = \quad \{(0,0,0,0,0), (1,1,0,0,0), (0,0,0,1,1), (1,1,0,1,1)\},$$

it does not satisfy (G). $\diamondsuit$

The following theorem is a counterpart of the well-known theorem by Pearl and Paz [6] in the framework of evidence theory.

Theorem 4 *If a basic assignment m on $\mathbf{X}$ is such that (A5) holds true for $K \perp\!\!\!\perp L|M[m]$, then*

$$(G) \Longleftrightarrow (L) \Longleftrightarrow (P).$$

Due to the results obtained in the previous section (Theorem 3), we immediately get the following

Corrolary 1 *Let m be a basic assignment such that $m(A) > 0$ if and only if $A = \times_{i \in N} A_i$, where A_i is a focal element on $\mathbf{X}_i$. Then*

$$(G) \Longleftrightarrow (L) \Longleftrightarrow (P).$$

5 Summary and Conclusions

The paper started with a brief discussion, based on recently published results, why random sets independence is the most appropriate independence concept in evidence theory. Then we have compared two generalizations of random sets independence — conditional noninteractivity and the new concept of conditional independence. We showed that although from the viewpoint of formal properties satisfied by these concepts conditional noninteractivity seems to be slightly better than conditional independence (for more details see [11]), from the viewpoint of multidimensional models the latter is superior to the former, as it is consistent with marginalization. Finally, we introduced Markov properties of basic assignments and demonstrated the relationship among them.

There are still some problems to be solved. First, it is the question of the possibility to weaken the sufficient condition in order (A5) would be still satisfied. Another problem is the relationship between Markov properties and factorization of basic assignments with respect to a graph — i.e. the analogy of results from probabilistic [3] and possibilistic [8] frameworks.

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