The hyperelastic model of urethral tissue based on the microcontinual approach

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We use the microcontinual approach [1] in description of a hyperelastic material whose actual state at the macroscopic scale is characterized by a minimal value of its (macroscopic) stored-energy. That means that each "particle" (a material point of the standard continuum description) is a deformable, elastic corpuscle whose micro-deformation is supposed to be homogeneous. The (macroscopic) deformation gradient, $\boldsymbol{F}$, thus describes a relative change of distances of surrounding particles while a *micro-deformation gradient* $\boldsymbol{G}_{mic}$ describes own deformations of individual particles – see Fig. 1.

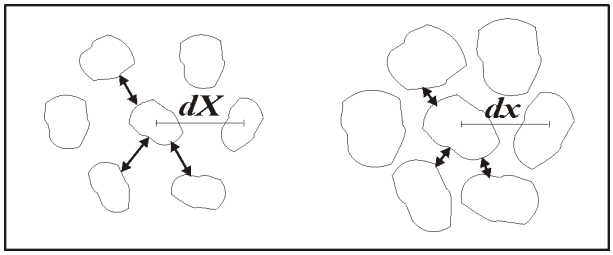

Figure 1
Individual particles can move and deform independently. Their relative displacements are described by the (macroscopic) deformation gradient $\boldsymbol{F}$, their own deformation by a *micro-deformation* gradient $\boldsymbol{G}_{mic}$ and their relative volume change (which is independent on the macroscopic volume change, det $\boldsymbol{F}$) *by* the term det $\boldsymbol{G}_{mic}$.

To guarantee stability of these micro-deformations the elastic energy stored in each corpuscle has to be a quasiconvex function [2], [3] of its micro-deformation gradient $\boldsymbol{G}_{mic}$. It leads to an explicit dependence of the density of a microscopic stored-energy w_{mic} (i.e. energy stored within individual particles and their mutual interactions defined at the microscopic scale) on a relative change of the *volume* of individual particles, det $\boldsymbol{G}_{mic}$, i.e.

$$w_{mic}(\boldsymbol{G}_{mic,} \det \boldsymbol{G}_{mic,} \boldsymbol{F})$$

(there is also a dependence on $\boldsymbol{F}$ because of the dependence of w_{mic} on mutual positions of particles). It is worth stressing that *this* volume change need not be constant if the material is *incompressible* at macroscopic scales. The occurrence of the term det $\boldsymbol{G}_{mic}$ then yields a nontrivial nonlinear behaviour in dependence on the way in which the microstructure "communicates" with changes at macroscopic scales. In the concrete, if the microstructure is fixed (no reaction on a change of the microstructure exists) the *macroscopic* stored-energy, w, is defined by

$$w(\boldsymbol{F}) = w_{mic}(\boldsymbol{G}_{mic}^{(fix)}, \det \boldsymbol{G}_{mic}^{(fix)}, \boldsymbol{F}) \qquad (1)$$

and the dependence on microstructure (as well as the term det $\boldsymbol{G}_{mic}$ alone) plays only a role of some “constant” correction. For example, in a quadratic approximation of the microscopic stored-energy the material behaves as a kind of the Neo-Hooke one.

However, when the microstructure can flexible adapt to changes of macroscopic conditions it continually minimizes its microscopic energy compatible with varying macroscopic arrangement, i.e. the *macrosocpic* stored-energy may be defined as follows

$$w(\boldsymbol{F}) = \min_{\boldsymbol{G}_{mic}} w_{mic}(\boldsymbol{G}_{mic}, \det \boldsymbol{G}_{mic}, \boldsymbol{F}). \qquad (2)$$

Then the ‘micro-volume’ term plays the crucial role even in a simplest quadratic approximation of w_{mic}. In the concrete, it leads to an essential decreasing of macroscopic stiffness of the material [4] and, in turn, macroscopic behaviour of such a material is extremely sensitive on a change of microstructural parameters, which is a typical feature of muscle tissues.

To illustrate the approach we form a model of a deformable pipe from a (macroscopically) incompressible material approaching by its geometrical parameters the human (female) urethra. Let us approximate the cross-section of the urethra as an annulus with radii R_a (inner) and R_b (outer) exposed to pressures p_a (internal) and p_b (external) so that $Dp \equiv p_a - p_b > 0$. Since the deformation is supposed to be radial we can describe it by the function $r(R)$ where R is the point in the reference configuration. The principal values of the (macroscopic) strain tensor are r' ($\equiv dr/dR$) and r/R. The model is formed at microscopic scales by finding a reasonable, quasiconvex microscopic stored-energy – in fact, the energy coming from structural arrangements of individual fibers of the urethral tissue. It depends on some *microstructural parameters* describing approximately the arrangement of collagen fibers and actin and myosin filaments – see Fig. 2.

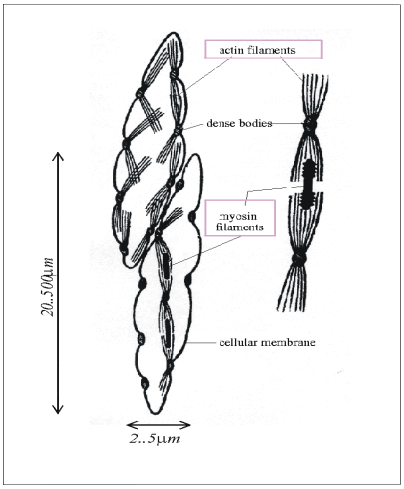

Figure 2
The structure of smooth muscular cells with actin and myosin filaments whose mutual movement control the muscular work. The microstructural parameters of our models are introduced to describe, at least approximately, these arrangement.

For example, the simplest case of such a quasiconvex micro-energy function in two-dimensional case is a quadratic function including the quadratic dependence on det $\boldsymbol{G}_{mic}$, say

$$w_{mic} \approx \frac{E}{2}\left[\left(\frac{r'-a_1 l_1}{1-l_1}-1\right)^2+\left(\frac{r/R-a_2 l_2}{1-l_2}-1\right)^2\right]+K(a_1 a_2-1)^2,$$

where microstructural parameters l_i correspond in a way to the actin-myosin relative displacement within individual muscle cells in radial and circular directions, α_i are principal values of the micro-deformation gradient $\boldsymbol{G_{mic}}$ and E, K are some parameters which should be determined by mechanical measurements (it is worth noting that the last term coming from the dependence of the micro-volume deformation plays the crucial role in our model). However, mechanical measurements are usually done on samples of "dead" tissue. To overcome this problem we accept the hypothesis that the microstructure of a living tissue is rather adaptable to macroscopic changes while the "dead" one is approximately fixed – it corresponds to a fixation of some fibers when living processes were stopped[1]. Then, by using the model (1) with fixed microstructure, we are able to find correct values of the parameters E, K (by comparing the results obtained on the model with fixed microstructure with experimental data) as a function of the measured mechanical parameters (such as the Young's modulus) and, then, use them in the model describing the *living* tissue.

If now the microstructure is supposed to fully adapt a macroscopic arrangement by finding a value of micro-deformation (given by the variables a_i) minimizing the energy w_{mic} at each (macroscopic) point (for any macroscopic deformation $\boldsymbol{F}$) we use the model (2) and obtain a nontrivial, nonlinear (macroscopic) stored-energy $\boldsymbol{w(F)}$. Using this energy in the numerical solution of the pressure dependence on the pipe deformation we obtain an interesting dependence of the pressure in the urethra on the microstructural parameters l_i.

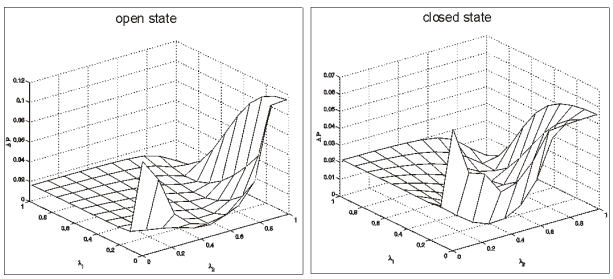

Figure 3
The dependence of the pressure Dp on the micro-structural parameters l_1 and l_2 for two deformation cases corresponding to the open state of the urethra (during micturition) and the closed one for fixed values of the mechanical coefficients E and K (E = 0.17 MPa and K = 1 MPa for the both figures).

This dependence was calculated especially at two states – that one describing the state when the urethra is closed and that corresponding to the opening of the urethra during micturition – see Fig. 3. Thus for the both states we could find the microstructural parameters l_i giving the measured values of the pressure [5].

We form a more complicated models both postulating more general microscopic stored-energy functions (e.g. working with more microstructural parameters) and taking into account the layered structure of the urethral wall with different orientations of the actin filaments. These results enable us to study the effect of microstructural changes within the urethral tissue (e.g. during the muscle stimulation or after some pathological changes) on macroscopic, mechanical behavior of the urethra.

Acknowledgements
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References
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[1] This assumption is supported by the measuring results themselves: A measurement of the Young modulus on a sample of "dead" urethral tissue gives the Young's modulus $E_Y \approx 0.25$ MPa. Taking into account characteristic values of the urethra deformation found in urethral dynamics (R_a ~ 1mm, R_b ~ 3mm, deformation ~ 2mm) we get by a simple calculation that the pressure Dp in a pipe from such a material must be in order of ~ 10^5 Pa. However, pressure measurements [5] show that $Dp \sim 5 \cdot 10^3$ Pa! We see that the living tissue must be essentially softer that a sample of material removed from the tissue.