THE INFLUENCE OF THE MINDLIN'S BOUNDARY CONDITIONS ON WAVE PROPAGATION IN THICK ANISOTROPIC PLATE

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Summary: *This paper deals with analytical formulation of dispersion curves for stress waves in a free (001)-cut cubic and orthotropic plate. The direction of wave propagation is arbitrary and it is defined by the angle ϕ, the angle between the [100] axis and the wave vector. The dispersion curves for Cu and carbon reinforced composite are presented. These results are compared to the uncoupled Mindlin modes. This work is very important for determination of the elastic coefficients of anisotropic materials.*

1. Introduction

The determination of the elastic coefficients of anisotropic materials is very difficult, especially if it should be nondestructive method. Comparing of dispersion curves is used in applying one of the nondestructive method of determination of the elastic coefficients. At first the dispersion curves of thick plate made from investigated material are obtained from measurement. Then the dispersion curves obtained this way are compared with computed dispersion curves of thick infinitely extended plate with free boundary conditions. The elastic coefficients in computed dispersion curves are subsequently changed until computed dispersion curves agree with dispersion curves obtained from measurement. For that reason we derived approximative dispersion relations for cubic and orthotropic plates, which are based on generalized Mindlin boundary conditions (Červená 2006).

2. Used methods of solution

To obtaining analytical dispersion relations we have used the method of partial waves. In this method, the plate wave solutions are constructed from simple exponential–type waves, which reflect back and forth between the boundaries of the plate. Every partial wave have to have the same value $k_x = k = \omega/v$, where v is the plate wave phase velocity. The solution has form $u_j = \alpha_j \exp\left[ik(x + l_z z)\right]$, $j = x, y, z$ and $l_z = k_z/k_x$, for every partial wave solution. u_j are components of displacement, k_j are components of wave vector and α_j are components of polarization of partial waves. By substitution of trial solution into Christoffel equation we obtain a system of three homogeneous linear equations for α_x, α_y a α_z (Hora 2005). Nontrivial solutions exist only when the determinant of the system vanishes. This gives a sixth order polynomial in l_z with six roots $l_z^{(n)},\ n = 1, \ldots, 6$. The allowed partial wave solutions defined by these roots correspond to the three incident and three reflected waves. The coupling between partial waves

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at the plate boundaries is given by three boundary conditions for stress $\boldsymbol{T}$. These conditions are satisfied by taking a linear combination of the six allowed partial waves. Substitution of partial waves into the boundary conditions gives a system of six homogeneous linear equations. Nontrivial solutions exist only when the determinant of the system vanishes, and this defines the dispersion relation.

For determination of approximative dispersion relations (uncoupled modes) the Mindlin's boundary conditions were used (Solie a Auld 1973)

$$T_{xz} = 0, \quad u_z = 0 \qquad \text{nebo} \qquad T_{zz} = 0, \quad u_x = 0. \tag{1}$$

These boundary conditions could be used only for uncouple modes for $\phi = 0°$ and $\phi = 45°$ (cubic) and for $\phi = 0°$ and $\phi = 90°$ (orthotropic) plate, when sagittal plane is a plane of crystal symmetry and the SH modes are uncoupled. For other angles of propagation we added to boundary conditions next boundary condition (Červená 2006):

$$u_y = 0. \tag{2}$$

5. Conclusions

In contribution are presented analytical solutions of dispersion curves for thick plate with cubic and orthotropic anisotropy and free boundary conditions. For derivation dispersion relations we used the method of partial waves (Auld 1973). In contribution there are presented dispersion relations for copper (cubic anisotropy) and for carbon reinforced composite (orthotropic anisotropy). We often used system for symbolic computing Maple for derivation of dispersion relations. The obtaining of analytical relations without this system would be very difficult. The biggest support of system Maple was its ability to split determinant of complex matrix of dimension 6 on product of two factors; one of them represents symmetrical modes and the other antisymmetrical ones of dispersion curves. Except analytical solution we derived approximation relations of dispersion curves for cubic and orthotropic plates for arbitrary direction of wave propagation.

4. Acknowledgements

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5. References

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