DISPERSION MODE SEPARATION IN ULTRASONIC SIGNAL PROCESSING

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Abstract

This paper report on a technique for the analysis of propagating multimode signals. The method involves a 2-D Fourier transformation of the time history of the waves received at a series of equally spaced positions along the propagation path. The output of the transform being presented using an isometric projection which gives a 3-D view of the wave number dispersion curves. The time history of the waves was obtained by the commercial finite element (FE) code, MARC. The results of numerical studies and the dispersion curves of Lamb waves propagating in the 2.0-mm-thick steel plate are presented. The results are in good agreement with analytical predictions and show the effectiveness of using the 2-D Fourier transform method to identify and measure the amplitudes of individual Lamb modes.

Key words: guided waves, Fourier transform

1 Introduction

The application of the conventional ultrasonic methods, such as pulse-echo, has been limited to testing relatively simple geometries or interrogating the region in the immediate vicinity of the transducer. A new ultrasonic methodology uses guided waves to examine structural components. The advantages of this technique include: its ability to test the entire structure in a single measurement; and its capability to test inaccessible regions of complex components.

The propagation of guided waves in a complex structure is a complicated process that is difficult to understand and interpret. The current research develops the mechanics fundamentals that models this propagation.

One approach to modeling guided wave propagation phenomena is to analytically solve the governing differential equations of motion and their associated boundary conditions. This

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procedure already has been done for simple geometries and perfect specimens without defects (see [Gra75] and [Mik78]). However, these equations become intractable for more complicated geometries or for a non-perfect specimen.

Another approach to this problem is a numerical solution. There are three main numerical methods which can be used for this problem: The finite difference method (FDM), the finite element method (FEM) or the boundary element method (BEM). The FDM was the first numerical method to be applied to investigate the propagation of stress waves. The BEM has the advantage that just the surface of the specimen needs to be discretized; the numerical problem itself is therefore reduced by one dimension. On the other hand, the primary advantage of the FEM is that there are numerous commercial FEM codes available, thus eliminating any need to develop actual code.

The objective of this research is to compare known analytical solution of guided wave propagating problem in a thick plate with its numerically obtained solution.

2 The FEM numerical modeling

Temporal and spatial resolution of the finite element model is critical for the convergence of these numerical results. Choosing an adequate integration time step, Δt, is very important for the accuracy of the solution. In general, the accuracy of the model can be increased with increasingly smaller integration time steps. With time steps that are too long, the high frequency components are not resolved accurately enough. On the other hand, too small time steps are a waste of calculation time. Therefore, a compromise must be found. According to our experiences, this compromis is 20 points per cycle of the highest frequency component; this gives accurate solutions in an efficient manner. This rule is expressed as:

$$\Delta t = \frac{1}{20 f_{\text{max}}} \ , \tag{1}$$

where f_{max} is the highest frequency of interest. By determining the highest frequency for waves propagating through the structure¸ and using Eq. (1), a time step,Δt, is calculated that is small enough to model the temporal behavior of the propagation. If the input function gets close to a step function, the ratio given in Eq. (1) might not provide sufficient temporal resolution. In some cases, this ratio has to be increased up to ten times. Also, the needed time step can alternatively be related to the time the fastest possible wave needs to propagate between successive nodes in the mesh.

The size of the elements are chosen in a manner so that the propagating waves are spatially resolved. According to our experiences, it is needed that more than 20 nodes per wavelength be used. This rule can be expressed as:

$$l_{\text{e}} = \frac{\lambda_{\text{min}}}{20} \ , \tag{2}$$

where l_{e} is the element length and λ_{min} is the shortest wavelength of interest. If highly accurate numerical results are needed, Eq. (2) might not be sufficient, and a higher level of discretization might be required.

Eqs. (1) and (2) show that for high frequency wave propagation problems, enormous computer resources are needed. Computing such problems leads to high values of f_{max}, and also small values of λ_{min}, which means a very dense mesh and very small integration time steps.

In order to understand the behavior of the FEM applied to the solution of guided wave problems, a relatively simple geometry is considered: a 2 mm thick and 100 mm long steel plate as in [MJQ99]. This geometry has the advantage of a known analytical solution (the Rayleigh-Lamb equation, see following section). The FEM program used for this work was MARC ver. K7.3.2 with pre- and post-processor MENTAT ver. 3.2.0. that was installed on workstation SGI OCTANE (processor R 10000, 195 MHz, 256 MB RAM, 4+9 GB HD). The FE model formulated to solve this configuration with the material properties and the resulting wave speeds are summarized in Table 1. The upper left corner of this plate, which is modeled with square shaped elements (l_{e} = 0.1 mm), is loaded with a displacement boundary condition in the x- and y-directions. Figure 1 shows the applied displacements on the different nodes in the upper left corner of the plate. The time function of these displacements is triangle pulse; width of 0.2 μs. This load has no practical meaning, but its frequency content is appropriate for exciting high frequency waves. The goal of this model is to show dispersion effects up to a frequency, f, of 5 MHz. According to the recommendations, this transient problem is solved with a integration time step, $\Delta t = 10^{-8}$ s. The central difference approximation was used to obtain the time marching solution.

Geometric properties			**Material properties**		
Width	2	[mm]	Young's modulus	2.10^{11}	[Pa]
Length	100	[mm]	Poisson's number	0.29	[-]
Element type	4-node, plane stress		Density	7850	[kg/m^3]
Element length	0.1	[mm]	Compresion wave	5778	[m/s]
Elements	20000	[-]	Shear wave	3142	[m/s]
Nodes	21021	[-]	Rayleigh wave	2909	[m/s]

Table 1: FEM model and material properties

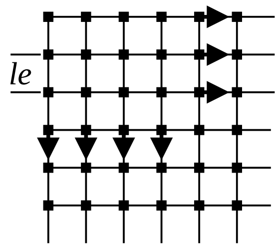

Figure 1: The displacement direction of applied load.

3 Dispersion curves

In the analysis that follows, the material is assumed to be linear elastic, isotropic, homogeneous, nonpiezoelectric, and nonabsorbing. Assuming a harmonic wave propagating in a plate with the coordinate system shown in Figure 2, the displacement on the surface, $\mathbf{u}(x,t)$, may be described by a general analytic expression given by Brekhovskikh [Bre60] as,

$$\mathbf{u}\left(x,t\right) = \mathbf{A}\left(\omega\right) e^{i(\omega t - kx - \theta)} \ , \tag{3}$$

where $\mathbf{A}(\omega)$ is a frequency-dependent amplitude constant, $\omega = 2\pi f$ is the angular frequency, the wave number $k = \omega/c$, c is the phase velocity, and θ denotes the phase.

Lamb waves are two-dimensional propagating vibrations in free plates, with displacements that may be symmetric (symmetric modes) or antisymmetric (antisymetric modes) with respect to the middle of the plate, and are eigensolutions of characteristic equations, hence the term free or normals modes. The velocities of all Lamb waves are dispersive and in any plate of thickness, $2d$, at a particular frequency, f, there will be a finite number of propagating modes, which may be determined from the number of real roots of the Rayleigh-Lamb equation. The phase velocities of Lamb waves as a function of the wave number may be obtained by solving the following transcendental equations:

$$\frac{\tan \frac{2\pi}{\lambda} d\sqrt{\left(c/c_2\right)^2 - 1}}{\tan \frac{2\pi}{\lambda} d\sqrt{\left(c/c_1\right)^2 - 1}} + \left(4\frac{\sqrt{\left[\left(c/c_1\right)^2 - 1\right]\left[\left(c/c_2\right)^2 - 1\right]}}{\left[2 - \left(c/c_2\right)^2\right]^2}\right)^{\pm 1} = 0 \ . \tag{4}$$

The $+1$ and -1 signs relate to symmetric and antisymmetric Lamb waves, respectively, and c_1 and c_2 are the bulk longitudinal and shear waves velocities, respectively. The group velocity, $c_g = \partial\omega/\partial k$, may be calculated once the phase velocity as a function of the wavelength is known. Figure 3 shows the predicted dispersion curves for the first ten symmetric and antisymmetric Lamb waves. Figure 3a) and b) shows the dispersion curves evaluated by Eq. (4). Figure 3d) shows the dispersion curves of Lamb waves propagating in 2.0-mm-thick steel plate, where $c_1 = 5778$ m/s and $c_2 = 3142$ m/s.

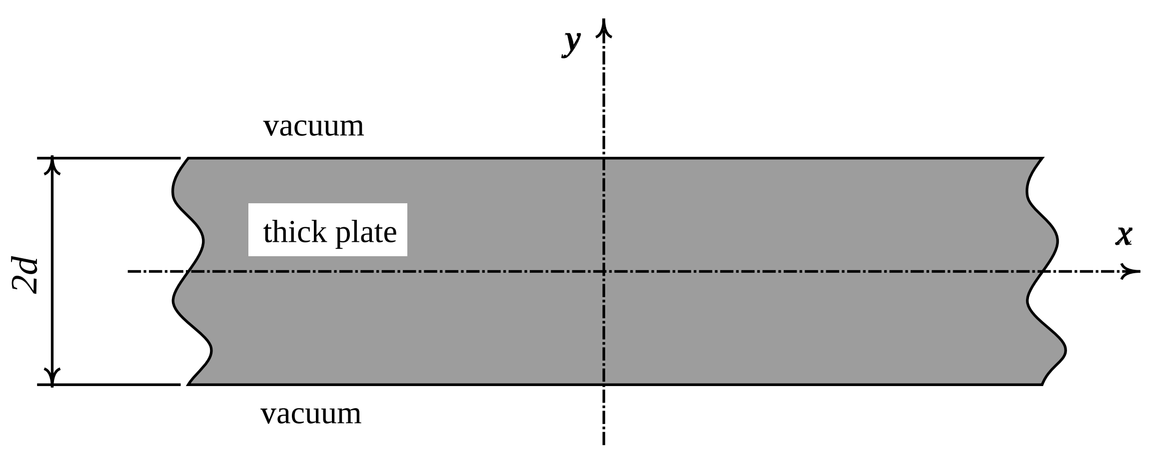

Figure 2: Schematic representation of the plate geometry and coordinate system used.

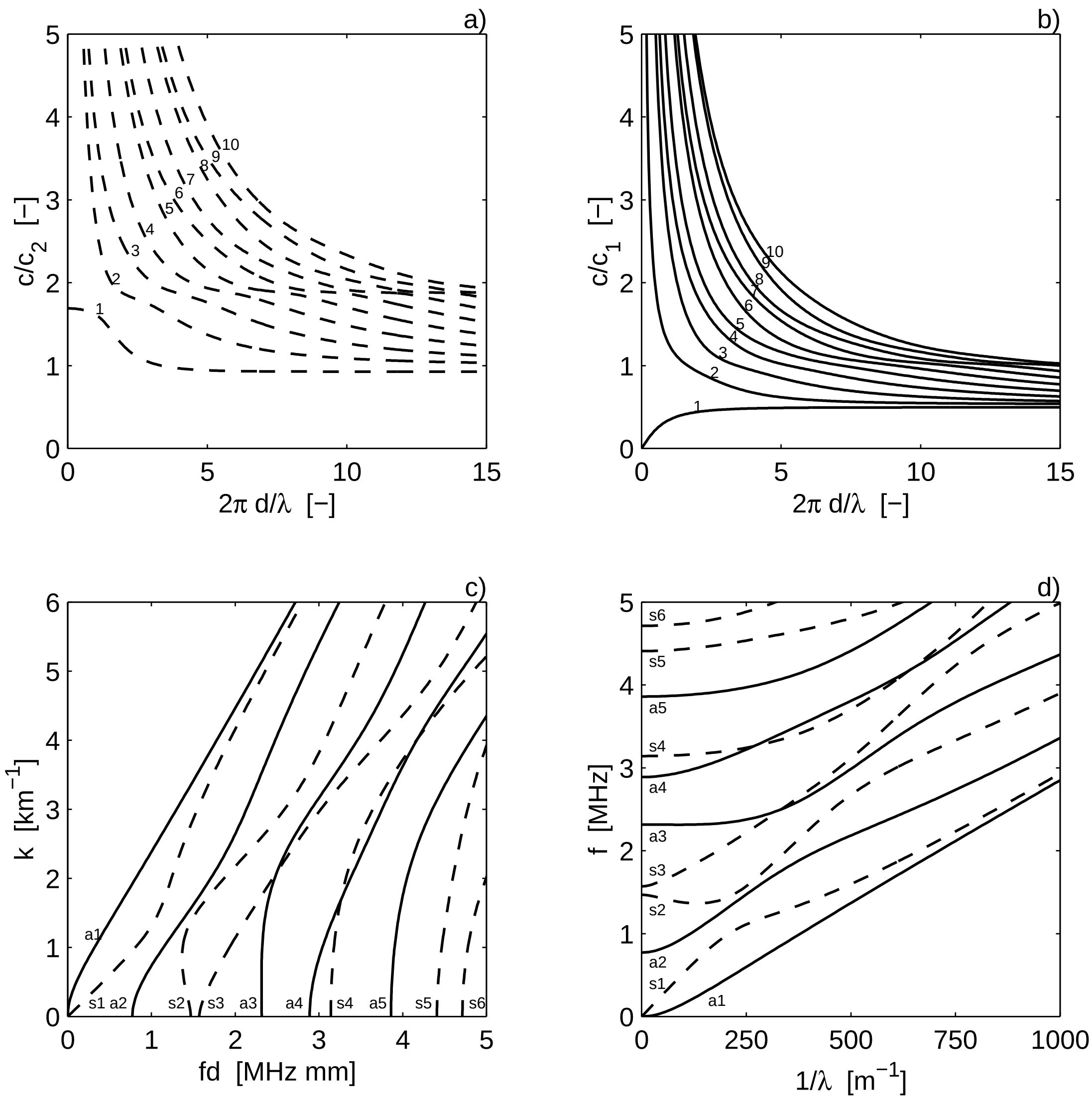

Figure 3: Lamb wave dispersion curves.

4 A two-dimensional spectral method

The key problem associated with the quantitative measurement of the characteristics of propagating Lamb waves is that more than one mode can exist at any given frequency. The 2-D FFT method described in [AC91] is an extension of the one-dimensional phase spectrum method developed by Sachse and Pao [SP78] for the measurement of the velocity of stress waves.

Propagating Lamb waves are sinusoidal in both the frequency and spatial domains, as can be seen from Eq. (3). Therefore, a temporal Fourier transform may be carried out to go from the time to frequency domain, then a spatial Fourier transform may be carried out to go to the frequency-wave-number domain, where the amplitudes and wave numbers of individual modes may be measured.

Applying spatial Fourier methods in practice to data gained experimentally or numerically requires us to carry out a two-dimensional Fourier transform of Eq. (3) giving

$$H(k,f) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} u(x,t)\, e^{-i(kx+\omega t)} dx dt \ . \tag{5}$$

The discrete two-dimensional Fourier transform may be defined in a similar way to the one-dimensional DFT. The result of this transformation will be a two-dimensional array of amplitudes at discrete frequencies and wave numbers. As in the one-dimensional case, aliasing must be avoided by sampling the data at a sufficiently high frequency in time and wave number in space. Usually the signal will be not periodic within the temporal and spatial sampling windows and leakage will occur. Window functions such as the Hann window may be used to reduce this leakage, and zeros may be padded to the end of the signal to enable the frequency and wave number of the maximum amplitude to be determined more accurately.

The algorithm:

1. Create the array (in column order) from experimentally or numerically gained the time histories of the waves received at a series of equally spaced positions along the propagation path.

2. Carry out a temporal Fourier transform of each column to obtain a frequency spectrum for each position. At the stage, an array with the spectral information for each position in its respective column is obtained.

3. Carry out a spatial Fourier transform of each row formed by the components at a given frequency to obtain the amplitude-wave-number-frequency information.

For demonstration, only the x displacements at nodes on the upper surface of the plate are considered for the 2D-FFT. The spatial and temporal sampling rate for the 2D-FFT must be chosen high enough to avoid aliasing for the frequency and wave number range under consideration. Since the upper frequency limit under consideration is 5 MHz, a sampling rate of $\Delta T = 10^{-7}$ s is used. From a FEM point of view, the element length $l_e = 0.1$ mm leads to accurate results for $\lambda > \lambda_{min} = 2$ mm [see Eq. (2)]. This results in a maximum value for $1/\lambda = 500$ m^{-1}. Therefore a spatial sampling step of $\Delta x = 0.5$ mm is used. This means that only solutions of every fifth surface node are needed for the 2D-FFT. In oder to get a signal without reflections from the right end of the plate¸the time signal is windowed (Hann window) with a cut-off time of 17 μs. This cut-off time corresponds to the time a longitudinal wave needs to travel from one end of the plate to the other. The solution for times larger then 17 μs are ignored and replaced by zero in order to increase the frequency resolution of the time FFT (zero padding).

Figure 4 shows the pseudocolor plot of $1/\lambda - f-$spectrum. It can be seen clearly from this figure that only certain $1/\lambda - f-$combinations have meaningful amplitudes; these values are solutions to Eq. (4). The exact solutions of Eq. (4) are plotted as solid lines. Note that there are also some spurious peaks caused by data sampling and numerical errors.

For this FEM model, the recommended ratio $\lambda/l_e = 20$ is reached for $1/\lambda = 500$ m^{-1}. The fact that there is good agreement even for higher values of $1/\lambda$ leads to the conclusion that this wavelength limit is not that critical. However, the ratio between the integration time step, Δt, and the frequency, f_{max}, is much more critical; the numerical solutions get worse, the closer the ratio $1/(\Delta T f_{max})$ gets to the recommended value of 20. In summary, this model of a plate shows that a commercial FEM can be used to model the dispersive nature of guided waves.

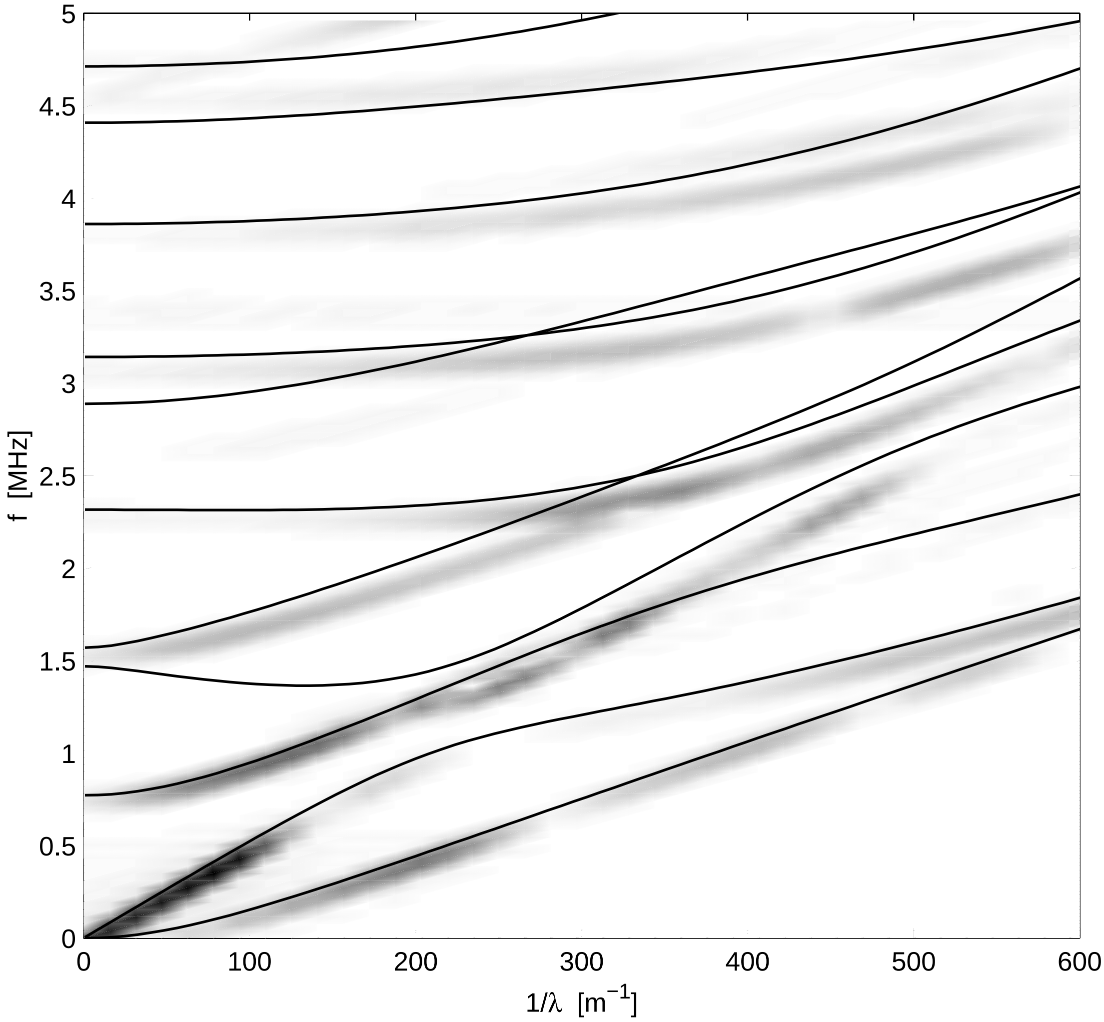

Figure 4: The pseudocolor plot of $1/\lambda - f-$spectrum.

5 Conclusion

This research clearly illustrates the effectiveness of using the FEM to model 2D guided wave propagation. This research establishes the foundation mechanics to numerically solve guided wave propagation in complex structures and uses the powerful post-processing capabilities of a commercial FEM code to study and interpret guided wave propagation phenomena.

An investigation into the influence of the two most important FEM parameters, the mesh density (element length) and the time step size between solution points (integration time step), is completed by studying a problem where a wall established analytical solution is available, a thick plate. The FEM solution converges for certain values of element length and integration time step. This optimization is critical in order to avoid unnecessarily high hardware requirements and enormous total calculation times. The highest wave frequency affects the integration time step while the shortest wavelength influences the element length. The numerical results are in complete agreement with the analytical solution.

An additional advantage of the FEM model is that the numerical results can be elegantly presented using the post-processing, graphical capabilities inherent to the program. For example, a snapshot of the displacement field or a color plot of the stress distributions can give new insights into wave propagation phenomena.

Acknowledgements

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