Is there a *mean-field* theory of Anderson localization?

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Brief history of disordered electron problems

1958: *Anderson*—single quantum particle in a **random potential**.

$\downarrow$

1960s: Disorder driven Metal-Insulator transition (LD) propagated by the work of *Mott* (mobility edge, localization in 1D systems, ...).

$\downarrow$

1970s: *Thouless's* scaling picture $g = G/G_Q = E_{\rm Th}/\Delta \Rightarrow$ **Scaling theory of localization** (*GoF*) $\Rightarrow$ **Weak localization** (*GLK*); **Effective Field Theories**: bosonic (*Wegner*); fermionic (*ELK*) and SUSY (*Efetov*) nonlinear σ-models (1980s).

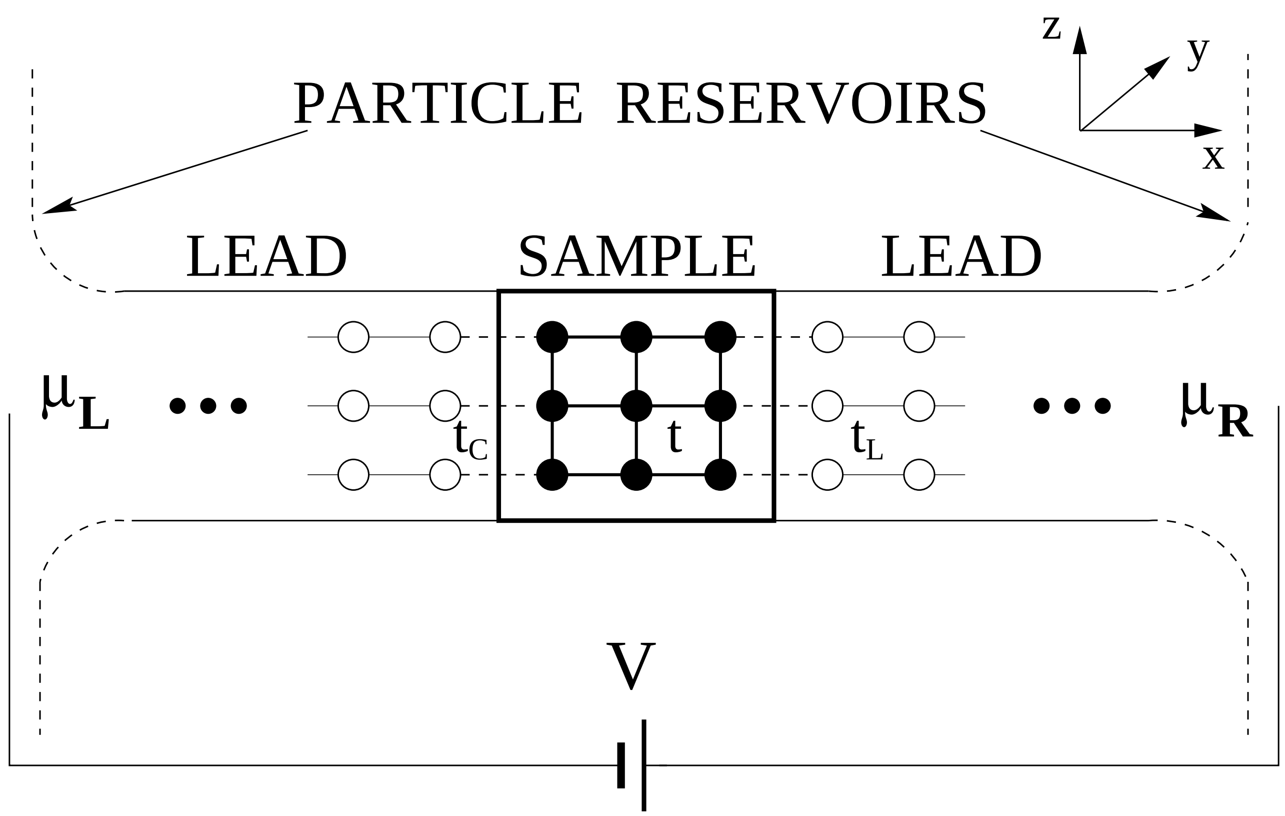
$\downarrow$

1980s: Mesoscopic Physics $\Leftrightarrow$ phase-coherent systems (at nanoscale and low temperatures): weak localization, universal conductance fluctuations (*Altshuler, Webb, Lee, ...*), conductance quantization, Aharonov-Bohm effects, persistent currents, ...

Standard Model

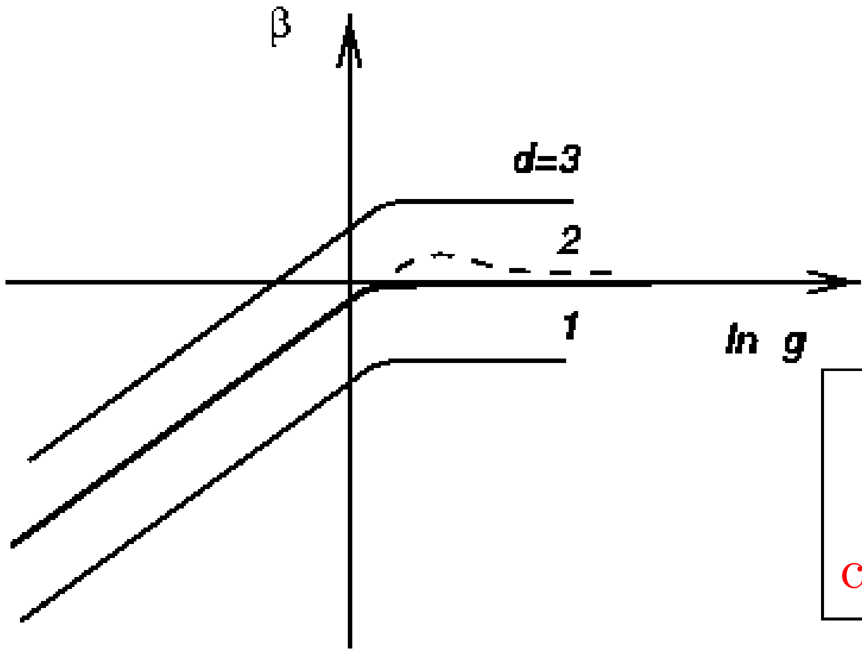
Hamiltonian: $\hat{H} = \sum_{\mathbf{m}} \varepsilon_{\mathbf{m}} |\mathbf{m}\rangle\langle\mathbf{m}| + \sum_{\langle \mathbf{m},\mathbf{n}\rangle} t_{\mathbf{mn}} |\mathbf{m}\rangle\langle\mathbf{n}|.$

- Random variable simulates **disorder**: $\varepsilon_{\mathbf{m}} \in [-W/2, W/2]$.



- Numerically **exact** results via quantum transport techniques.

Why is LD transition in $d > 3$ still an unsolved problem?

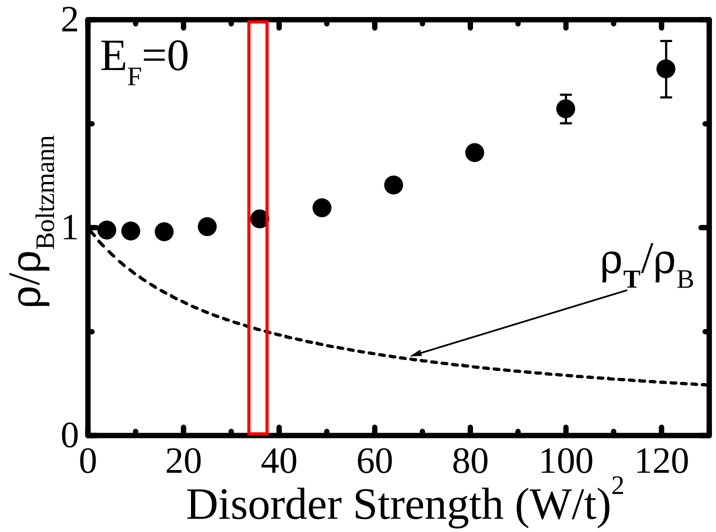


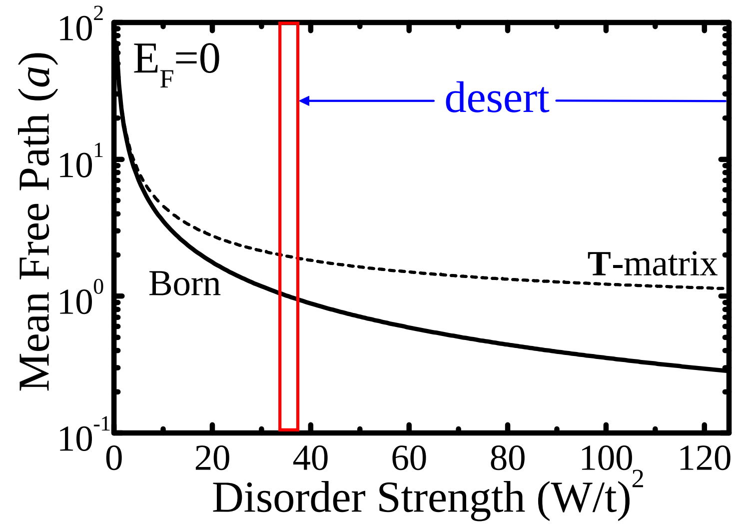
$$\text{metallic: } g(L) \propto L^{d-2}$$
$$\text{localized: } g(L) \propto e^{-L/\xi}$$
$$\text{critical: } g(L) = g_c,\ \xi = |W - W_c|^{-\nu}$$

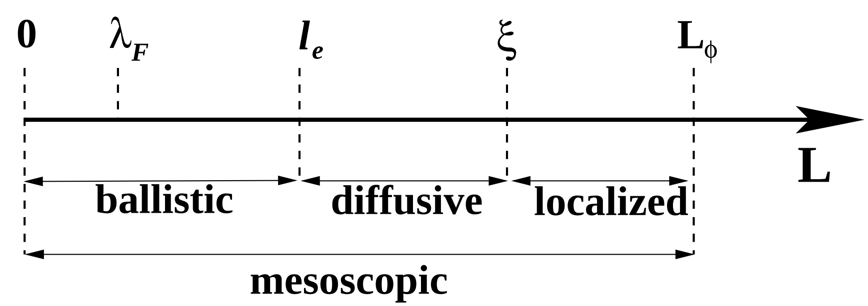
- **1D**: All states are localized for arbitrarily weak disorder, $\xi = 4\beta\ell$.
- **2D**: Weak localization correction diverges: $\sigma = \sigma_0 - (e^2/\pi h)\ \ln(L/\ell)$
(exponentially large ξ at weak disorder, $\xi(E_F = 0) \simeq 1 + 5.2 \cdot 10^4/W^4$).
- **3D** (and beyond): Genuine LD transition $\lim_{\omega\to 0, T\to 0, \Omega\to\infty} \sigma(E_F) = 0$
takes place at **strong disorder** [$W_c \approx 16.5t$ at $E_F = 0$] $\Leftrightarrow$ **strong coupling limit** in the effective-field-theoretical language.

Quantum transport through the Standard Model

- Scaling theory $g_c \sim 1$ versus plausible $k_F \ell \sim 1$ onset of criticality.



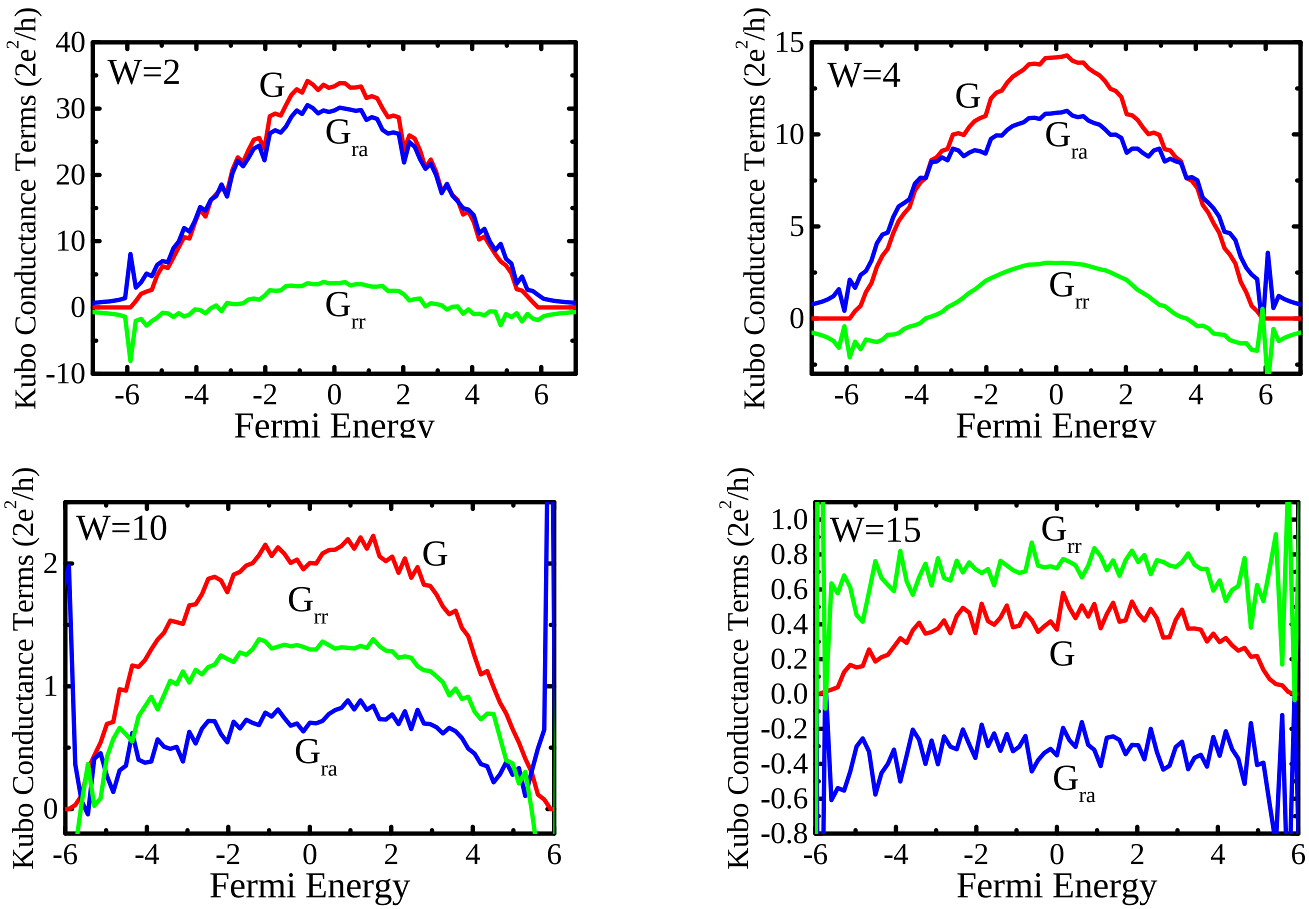




- Boltzmann breaks down $\ell < a$ for $W \geq 6 \Rightarrow$ **no small parameter** (*sine qua non* for standard analytics, $d = 2 + \epsilon$) in the **deserted** regime.

Kubo *ante* Anderson localization

More nonperturbative insights: **Kubo** vs. **Kubo** vs. **Landauer**

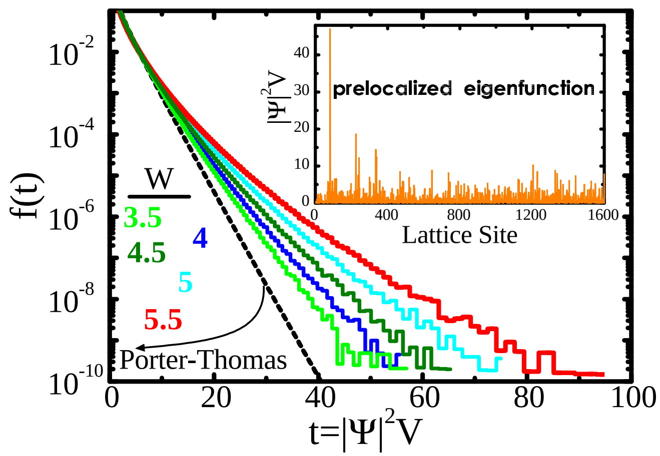


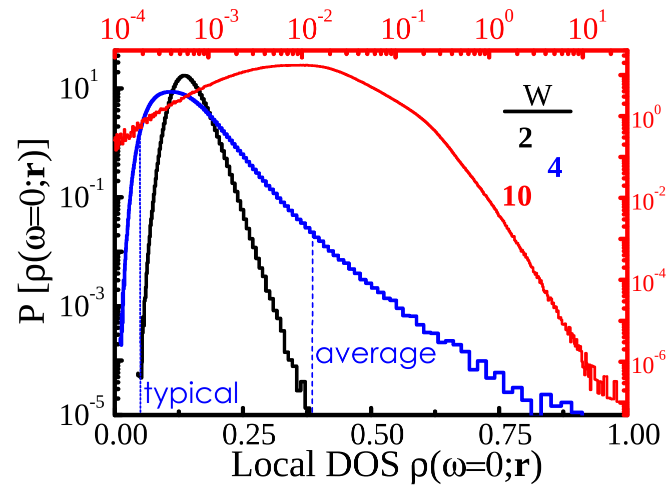
$$G = \frac{4e^2}{hL_x^2}\mathrm{Tr}\left[\hbar\hat{v}_x\,\mathrm{Im}\,\hat{G}\,\hbar\hat{v}_x\,\mathrm{Im}\,\hat{G}\right] = \frac{2e^2}{hL_x^2}\mathrm{Tr}\left[\hbar\hat{v}_x\,\hat{G}^r\,\hbar\hat{v}_x\,\hat{G}^a\right] + G_{rr} = \frac{2e^2}{h}\mathrm{Tr}\,\mathbf{t}\mathbf{t}^\dagger$$

Toward *unconventional* Order Parameter theory

I Naive attempts at mean-field theory fail badly—conventional OP:

- $M = \langle m(\mathbf{r}) \rangle \sim |T - T_c|^\beta$ as $T \to T_c$ (thermal phase transitions).
- *However*: $\langle \mathrm{Im}\, G(\mathbf{r}, \mathbf{r}) \rangle$ or $\langle Q \rangle$-matrix of NLσM are analytic at W_c.
- **Transport quantities** $\propto \langle G^r G^a \rangle$ exhibit criticality!





II Mesoscopic fluctuations $\Rightarrow$ broad distributions of physical quantities in open (g, $\rho(\mathbf{r})$, τ_c, $\ldots$) or closed ($|\Psi(\mathbf{r})|$, α, K_n, $\ldots$) phase-coherent systems.

- Critical eigenfunctions $[L < \xi_c,\ g(\xi_c) \sim \mathcal{O}(1)]$ are **multifractals**: $\left\langle \sum_{\mathbf{r},\alpha} |\Psi_\alpha(\mathbf{r})|^{2q} \delta(E - E_\alpha) \right\rangle \propto L^{-d^*(q)(q-1)} \Rightarrow$ additional set of critical exponents $d - d^*(q)$ is introduced by **fractal dimensions** $d^*(q) < d$.

A possible way out: *Typical* Medium Theory

$\rightarrow$ **A quest for:** nonperturbative theory of quantity resisting mesoscopic fluctuations $\leftarrow$ hint: local picture of localization (*Anderson* 1958; *Abou-Chacra, Anderson, Thouless* 1971).

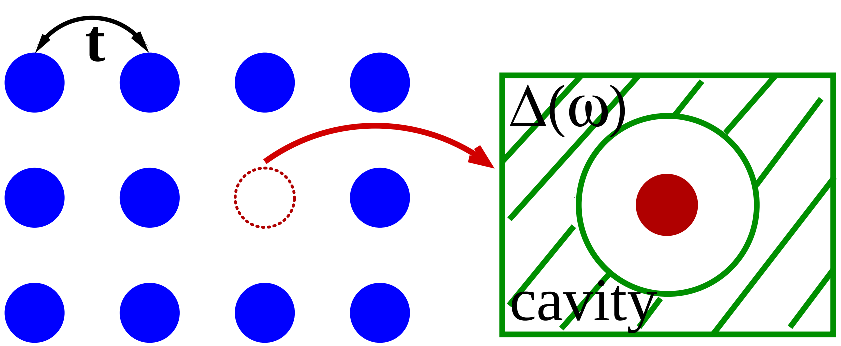
- Typical values evade far tails (*Shapiro* 1987): $\mathcal{P}(X; L; \{\alpha_n\}) \approx F(X; \alpha_L)$.

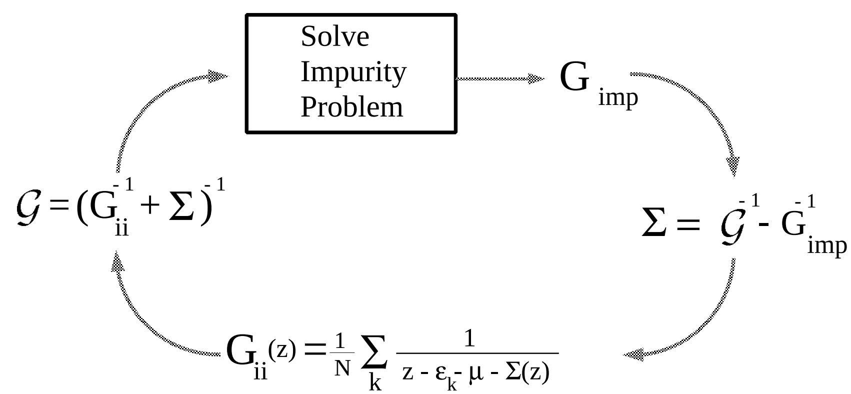
- NLσM and Anderson model on Bethe lattice $\rightarrow$ whole distribution function $P[\rho(\mathbf{r})]$ is an Order Parameter (*Mirlin, Fyodorov* 1994).

- Multifractal scaling supports **typical LDOS** as **the** Order Parameter:

$$\rho_{\text{typ}} = \exp\langle \ln(\rho(\mathbf{r})\rangle \sim L^{d-\alpha_0} \Rightarrow \rho_{\text{typ}} \sim \xi^{\beta}, \; \beta = \nu(\alpha_0 - d)$$

$\rightarrow$ **TMT calculates ρ_{typ} self-consistently in a DMFT-like fashion**.



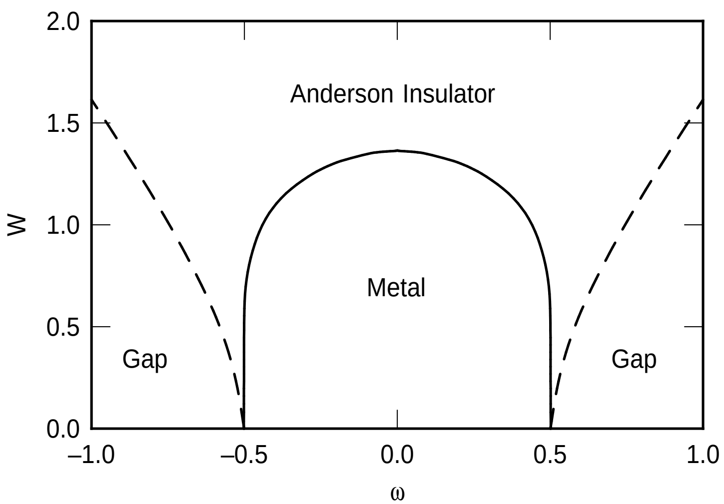


TMT Formalism

single site + typical medium defined by the self-energy $\Sigma(\omega)$

- Local Green functions: $G(\omega, \varepsilon_i) = [\omega - \varepsilon_i - \Delta(\omega)]^{-1}$.
- "Cavity function": $\Delta(\omega) = \Delta_0(\omega - \Sigma(\omega))$ with $\Delta_0(\omega) = \omega - G_0(\omega)^{-1}$.
- Lattice $G_0(\omega) = \int_{-\infty}^{+\infty} d\omega' \, D(\omega')/(\omega - \omega')$ as the Hilbert transform of bare DOS.
- Typical LDOS + self-consistency $G_{\mathrm{em}}(\omega) = G_0(\omega - \Sigma(\omega)) = G_{\mathrm{typ}}(\omega)$:

$$\rho_{\mathrm{typ}} = \exp\left[\int d\varepsilon_i P(\varepsilon_i) \ln \rho(\omega, \varepsilon_i)\right], \; G_{\mathrm{typ}} = \int_{-\infty}^{+\infty} d\omega' \, \frac{\rho_{\mathrm{typ}}(\omega')}{\omega - \omega'}$$



Example: $D(\omega) = 4/\pi\sqrt{1 - (2\omega)^2}$

$$\rho(\omega = 0; W) = \left(\frac{4}{\pi}\right)^2 (W_c - W)$$

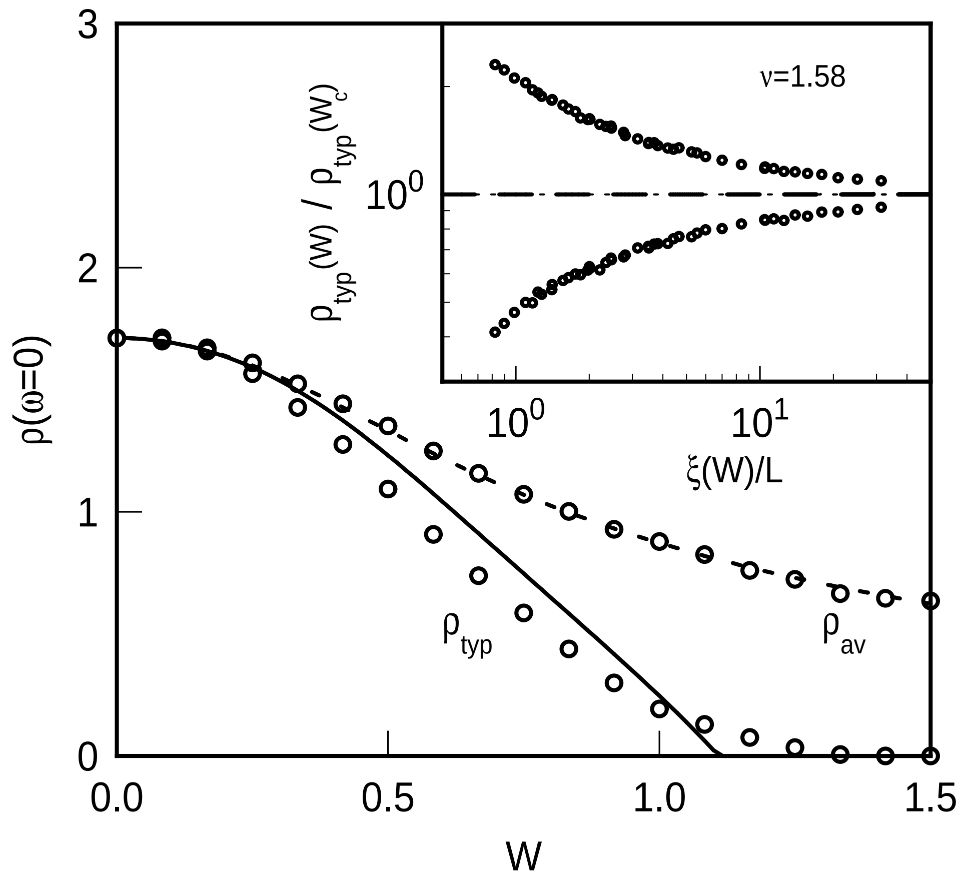
$$\beta = 1.0, \quad W_c \approx 1.36$$

Gist of the talk: ``Experiment'' (numerical) vs. TM Theory

- "Experimental" $\rho_{\mathrm{typ}}(\omega;\mathbf{r}) = \exp\langle \ln(-G(\omega;\mathbf{r},\mathbf{r})/\pi)\rangle$ from **exact** Green function

$$\hat{G}(\omega) = [\omega - \hat{H} - \hat{\Sigma}_L(\omega) - \hat{\Sigma}_R(\omega)]^{-1} \rightarrow G(\omega;\mathbf{r},\mathbf{r}) = \langle \mathbf{r}|\hat{G}(\omega)|\mathbf{r}\rangle$$

for a finite size systems in two-probe geometry (lattices up to 16^3):



← **Phase diagram**

solid line≡TMT vs. circles≡numerics

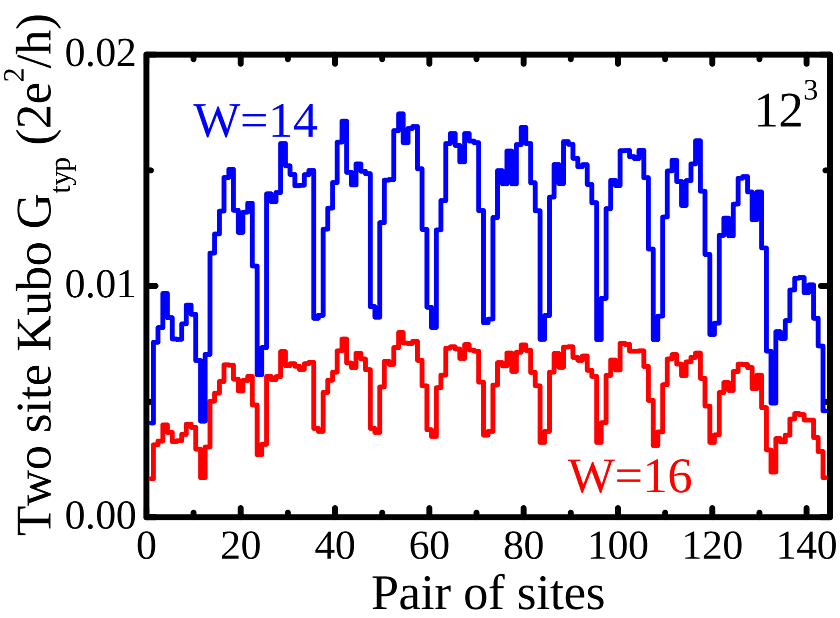
No adjustable parameters

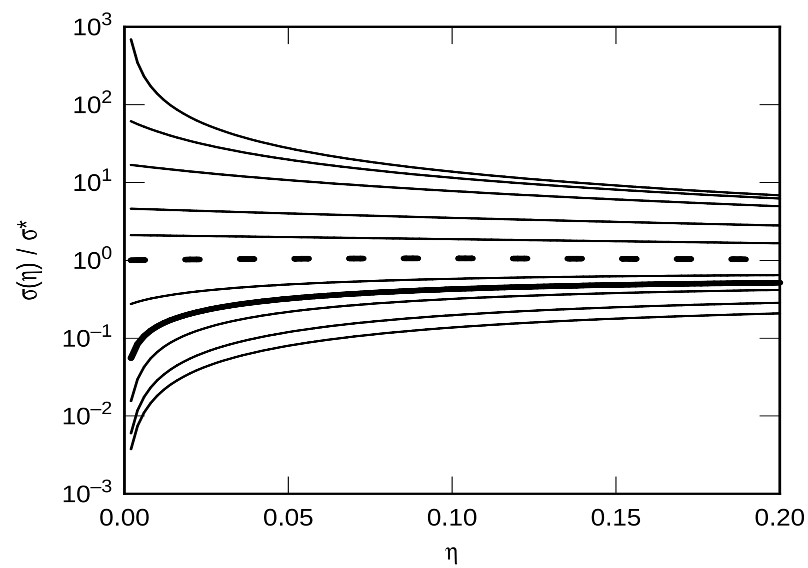
TMT in the band center:
$\beta = 1.0$, $W_c \approx 13.2$

Exact in the band center:
$\beta = \nu \approx 1.58$, $W_c \approx 16.5$

Quantum transport close to LD transition

- Can Kubo formula for conductivity be reduced to a two-site computation: $\sigma = \Lambda \langle \mathrm{Im}\, G_{12} \mathrm{Im}\, G_{21} - \mathrm{Im}\, G_{11} \mathrm{Im}\, G_{22} \rangle$?
- Is Λ is finite at the transition?
- $\mathrm{Im}\, G_{ij}$ from two-sites embedded in the typical medium.

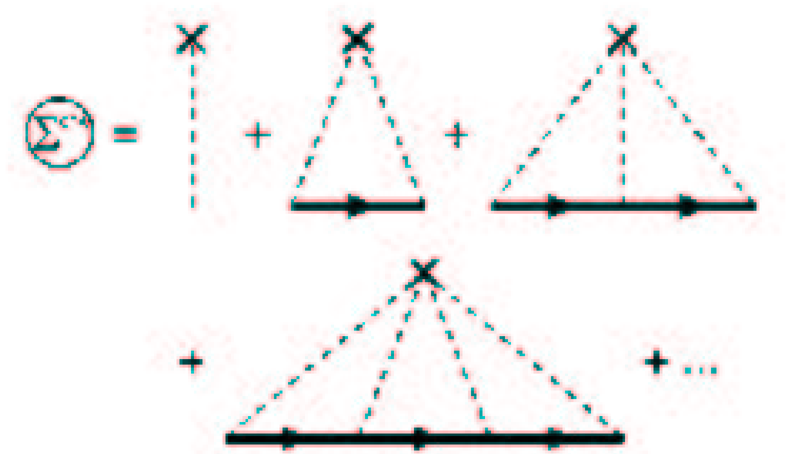




- Inelastic scattering rate $\Sigma \rightarrow \Sigma - i\eta$ mimics finite-temperature effects.

Foretaste: Nonperturbative physics

- *Past* : Standard DMFT → CPA disorder + strong interactions ($d = \infty$ limit)



DMFT for $U = 0 \to$ CPA
Nonlocal corrections → DCA
No localization!

- *Present* : Mean-field treatment of interactions + numerical (Bethe lattice) of LD.

- *Future*: Mean-Field (**order parameter**) treatment of both interactions and localization → standard model: $\hat{H} = \sum_{ij\sigma} (\varepsilon_i \delta_{ij} - t_{ij}) c^\dagger_{i,\sigma} c_{j,\sigma} + U \sum_i n_{i,\uparrow} n_{i,\downarrow}$

$$S_{\text{eff}}(i) = \sum_\sigma \int_0^\beta d\tau \int_0^\beta d\tau' \, c^\dagger_{i,\sigma}(\tau)[\delta(\tau - \tau')(\partial_\tau + \varepsilon_i - \mu) + \Delta_i(\tau, \tau')] c_{i,\sigma}(\tau') + U \int_0^\beta d\tau \, n_{i,\uparrow}(\tau) n_{i,\downarrow}(\tau)$$

Recipe: (a) Find local $G(\omega_n, \varepsilon_i)$ from $S_{\text{eff}}(i) \Leftrightarrow$ ensemble of auxiliary AI problems in the bath $\Delta(\tau, \tau')$; (b) Typical disorder average of $\operatorname{Im} G(\omega_n, \varepsilon_i) \to$ Hilbert trans.

C O N C L U S I O N

• **Anderson localization**—tantalizingly simply formulated problem without satisfactory solution (predict critical exponents of LD transition that will agree with extensive numerics).

• Is LD transition in $d = 3$ just a simple pile-up of known (perturbative) quantum interference effects in $d = 2 + \epsilon$?

• TMT offers analytical self-consistent scheme to obtain **typical LDOS** as **the unconventional Order Parameter** $\rightarrow$ **unconventional** Mean-Field Theory (*no upper critical dimension*) $\Leftrightarrow$ Order Parameter Theory.

• **Strongly** correlated electrons in **strong** disorder $\rightarrow$ DMFT + typical medium philosophy offers a genuine **nonperturbative** route.

R E F E R E N C E S

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