Computation of 2D Stratified Flows in Atmospheric Boundary Layer

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The work deals with the numerical solution of the 2D turbulent stratified flows in atmospheric boundary layer over the "sinus hills". Mathematical model for the 2D turbulent stratified flows in atmospheric boundary layer is the Boussinesq model - Reynolds averaged Navier-Stokes equations (RANS) for incompressible turbulent flows with addition of the equation of density change. The artificial compressibility method and the finite volume method have been used in all computed cases and Lax-Wendroff scheme (MacCormack form) has been used together with the Cebecci-Smith algebraic turbulence model. Computations have been performed with Reynold's numbers $10^8 \approx u_\infty = 1.5\,\frac{m}{s}$ and $5 \cdot 10^8 \approx u_\infty = 7.5\,\frac{m}{s}$ and for each Reynolds number with two density change ranges $\rho \in [1.2;\, 1.1]\,\frac{kg}{m^3}$ and $\rho \in [1.2;\, 0.9]\,\frac{kg}{m^3}$.

Mathematical model

Reynolds Averaged Navier-Stokes equations for 2D incompressible flows with addition of the equation of density change (Boussinesq model) have been used as a mathematical model:

$$
\begin{aligned}
u_x + v_y &= 0 \\
u_t + (u^2 + p)_x + (u \cdot v)_y &= (\nu + \nu_T) \cdot (u_{xx} + u_{yy}) \\
v_t + (u \cdot v)_x + (v^2 + p)_y &= (\nu + \nu_T) \cdot (v_{xx} + v_{yy}) - \frac{\rho}{\rho_0} g \\
\rho_t + u \cdot \rho_x + v \cdot \rho_y &= 0,
\end{aligned}
$$

where $(u,\, v)$ is a velocity vector, $p = \frac{P}{\rho_0}$ (P- static pressure), ρ - density (ρ_0 - initial maximal density in the bottom of the computational domain, ρ_h- initial minimal density on the top of the computational domain), ν - laminar kinematic viscosity, ν_T - turbulent kinematic viscosity computed by the Cebecci-Smith algebraic turbulence model and g - gravity acceleration. Density and pressure are changing depending on height (y-axis) as follows:

$$
\rho_\infty(y) = -\frac{\rho_0 - \rho_h}{h} \cdot y + \rho_0, \quad \frac{\partial p_\infty}{\partial y} = -\frac{\rho_\infty(y)}{\rho_0} \cdot g
$$

Boundary conditions

Inlet boundary conditions: $u = u_\infty = 1.0,\ v = v_\infty = 0,\ \rho = \rho_\infty(y)$
Outlet boundary conditions: $p' = 0$ and $(u,\, v)$ and density ρ have been extrapolated.
Boundary conditions on the wall: $u = 0,\ v = 0$, pressure perturbations and density have been extrapolated on the wall.
Boundary conditions on the upper domain boundary: $p' = 0,\ \frac{\partial u}{\partial n} = 0,\ \frac{\partial v}{\partial n} = 0,\ \rho = \rho_h$

Numerical solution

The artificial compressibility method and the finite volume method have been used in all computed cases and Lax-Wendroff scheme (MacCormack form) has been used together with the Cebecci-Smith algebraic turbulence model. All computations have been performed using pressure disturbances $p' = p - p_\infty$ instead of computing with pressure.

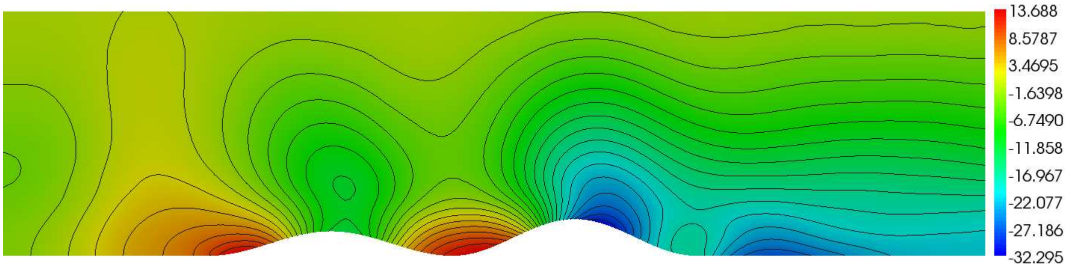

Figure 1: Sinus 10% - Sinus 15%, Contours of pressure perturbations [pa], $\rho \in [1.2;\ 1.1]\,\frac{kg}{m^3}$, $Re = 5 \cdot 10^8 \approx u_\infty = 7.5\,\frac{m}{s}$

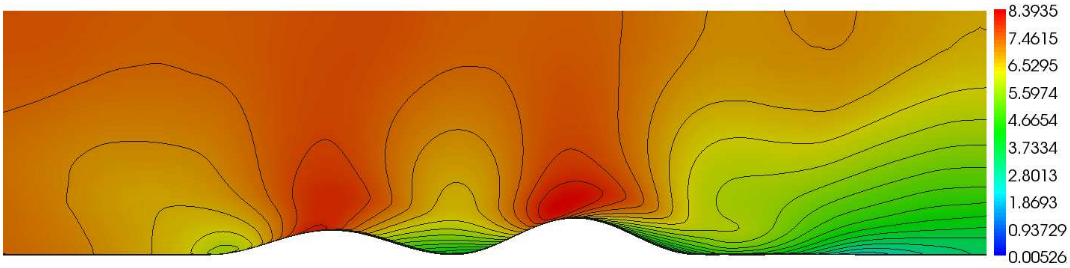

Figure 2: Sin 10% - Sinus 15%, Contours of velocity magnitude $[\frac{m}{s}]$, $\rho \in [1.2;\ 1.1]\,\frac{kg}{m^3}$, $Re = 5 \cdot 10^8 \approx u_\infty = 7.5\,\frac{m}{s}$

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