Constitutive models

Part 2

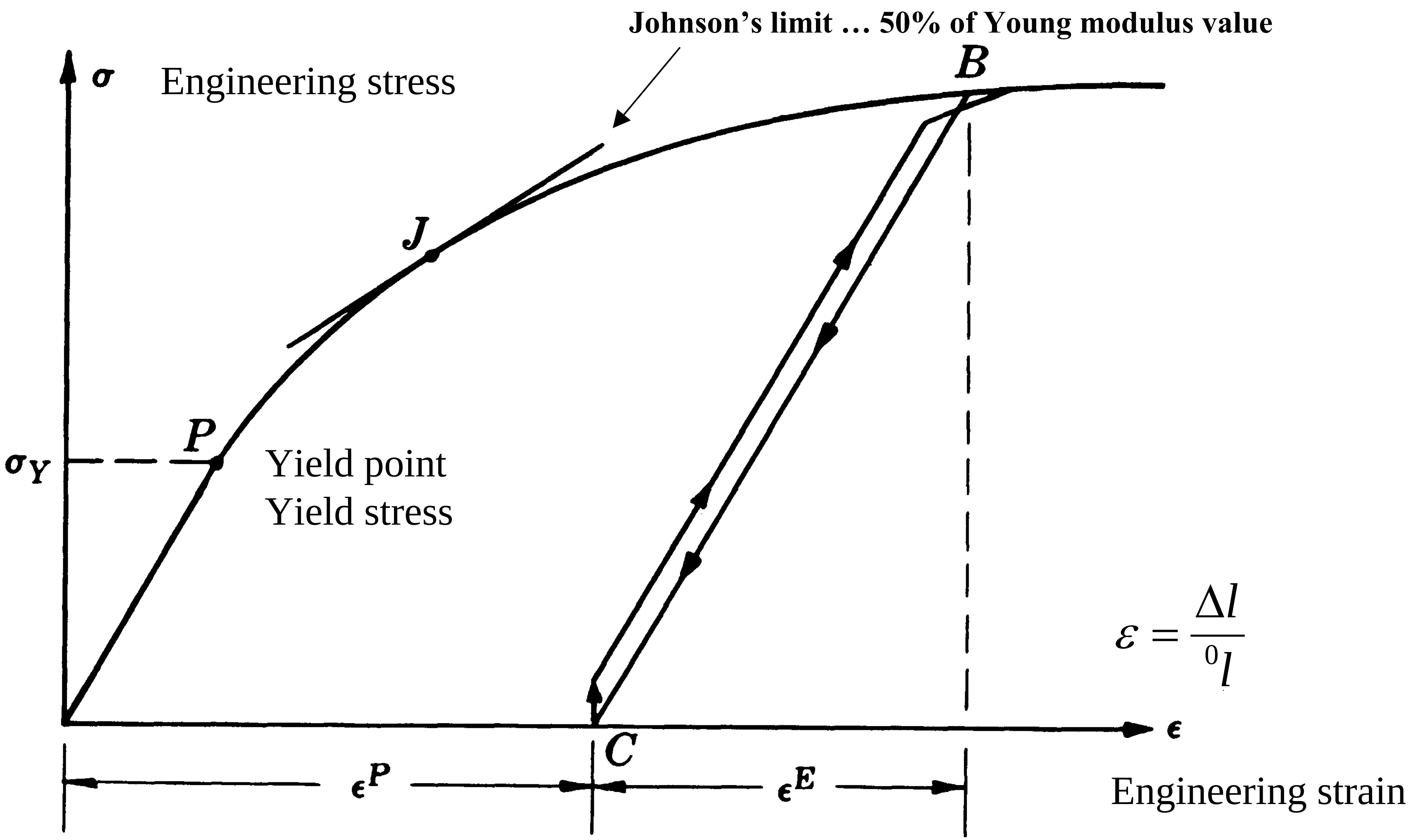
Elastoplastic

Elastoplastic material models

- Elastoplastic materials are assumed to behave elastically up to a certain stress limit after which combined elastic and plastic behaviour occurs.
- Plasticity is path dependent – the changes in the material structure are irreversible

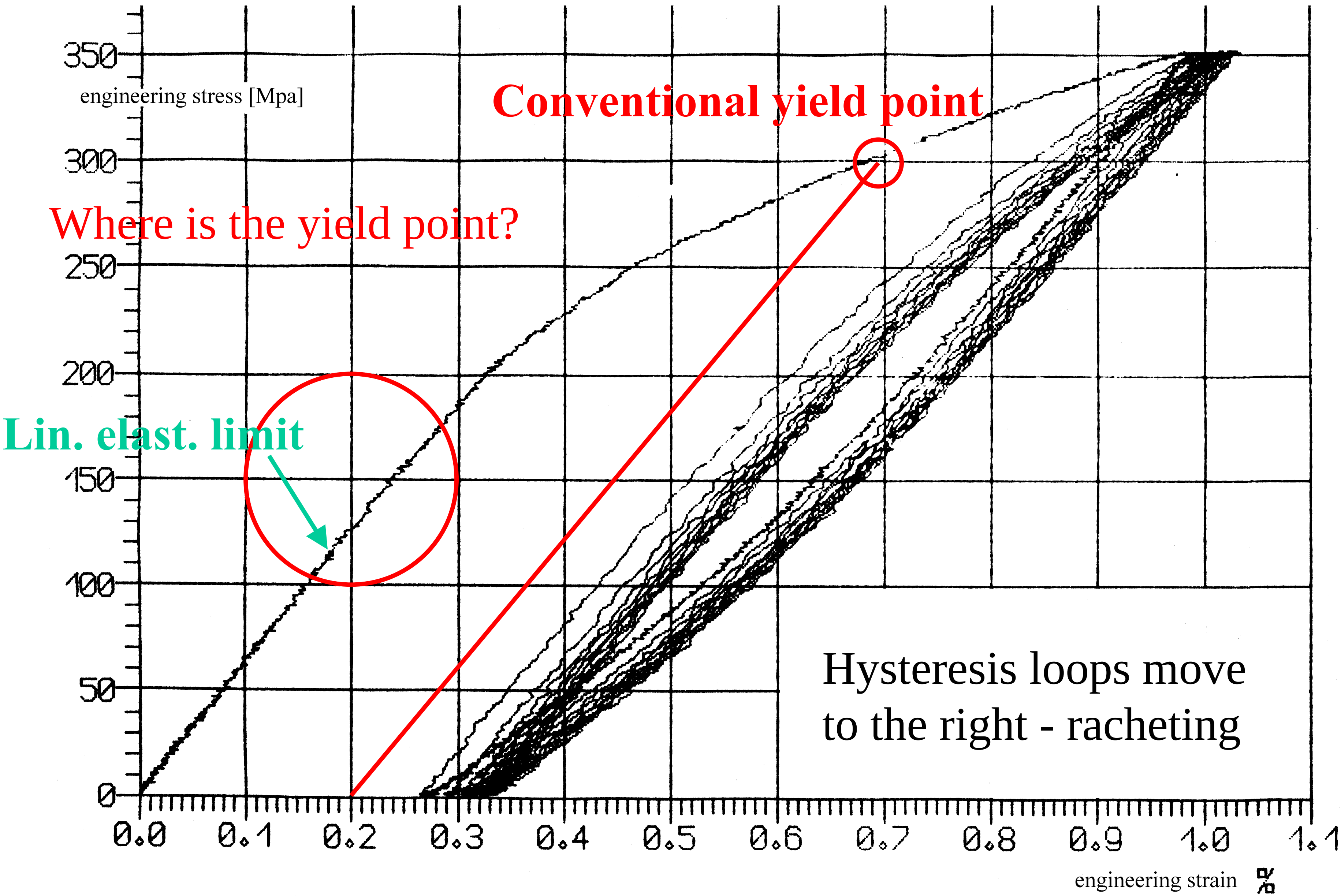
Stress-strain curve of a hypothetical material
Idealized results of one-dimensional tension test

$$\sigma = force/initial\ area$$

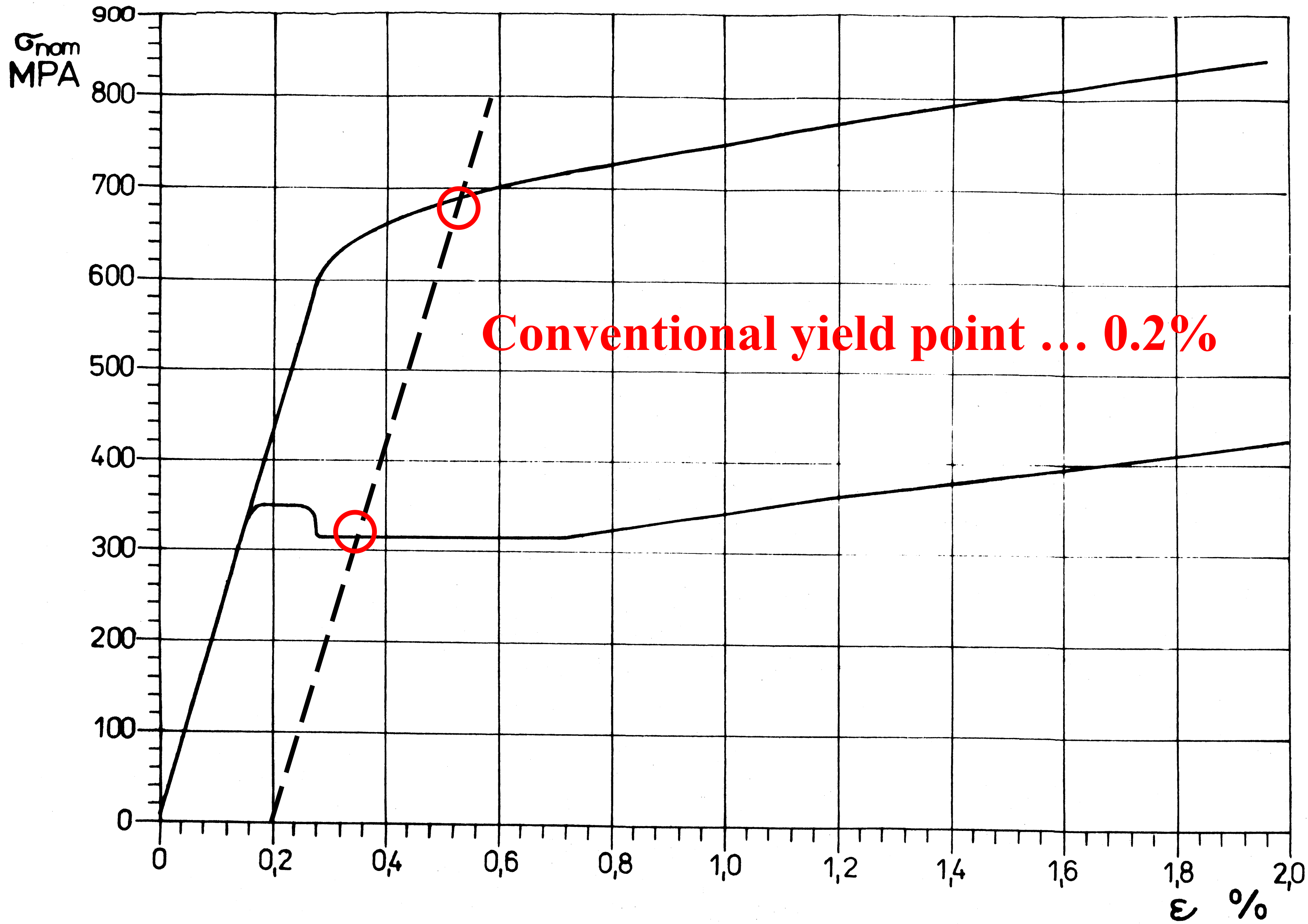


$$\varepsilon = \frac{\Delta l}{{}^{0}l}$$

Real life 1D tensile test, cyclic loading



Mild carbon steel before and after heat treatment



The **plasticity theory** covers the following fundamental points

- **Yield criteria** to define specific stress combinations that will initiate the non-elastic response – to define initial yield surface
- **Flow rule** to relate the plastic strain increments to the current stress level and stress increments
- **Hardening rule** to define the evolution of the yield surface. This depends on stress, strain and other parameters

Yield surface, function $F(\sigma_{ij}, \varepsilon_{ij}^{\mathrm{P}}, K...) = 0$

- Yield surface, defined in stress space separates stress states that give rise to elastic and plastic (irrecoverable) states
- For initially isotropic materials yield function depends on the yield stress limit and on invariant combinations of stress components
- As a simple example Von Mises … $F \equiv \sigma_{\text{effective}} - \sigma_{\text{yield}} = 0$
- Yield function, say *F*, is designed in such a way that

$$F < 0 \quad \text{stress state within the surface}$$

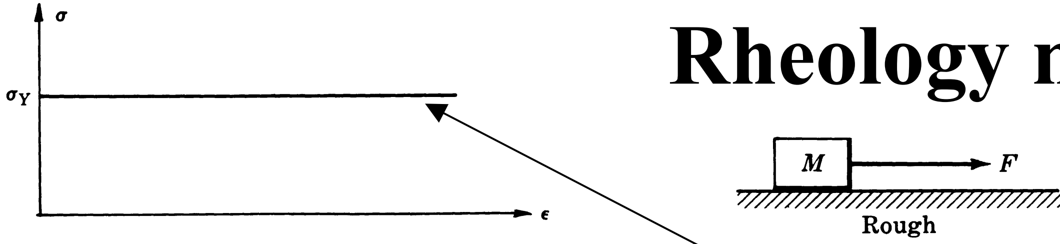
$$F = 0 \quad \text{on the surface}$$

$$F > 0 \quad \text{outside, inadmissible for analytical plasticity}$$

Three kinematic conditions are to be distinguished

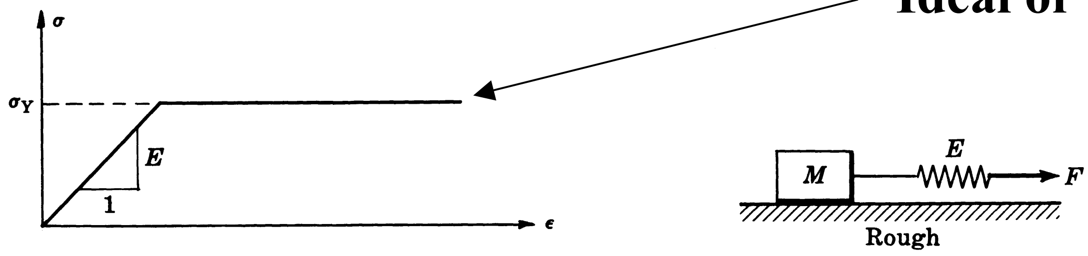
- **Small** displacements, **small** strains
  - material nonlinearity only (MNO)
- **Large** displacements and rotations, **small** strains
  - TL formulation, MNO analysis
  - 2PK stress and GL strain substituted for engineering stress and strain
- **Large** displacements and rotations, **large** strains
  - TL or UL formulation
  - Complicated constitutive models

Rheology models for plasticity

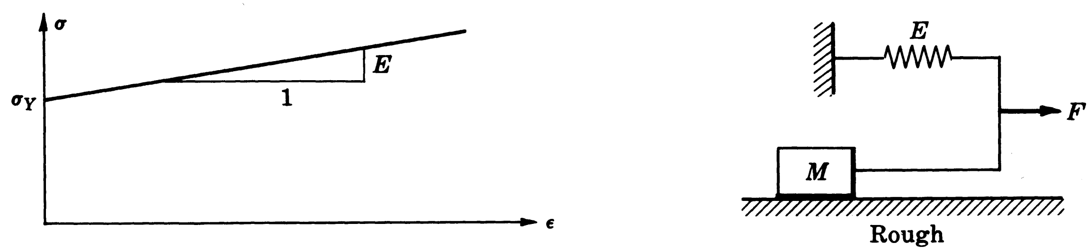


(*a*) Rigid—Perfectly Plastic

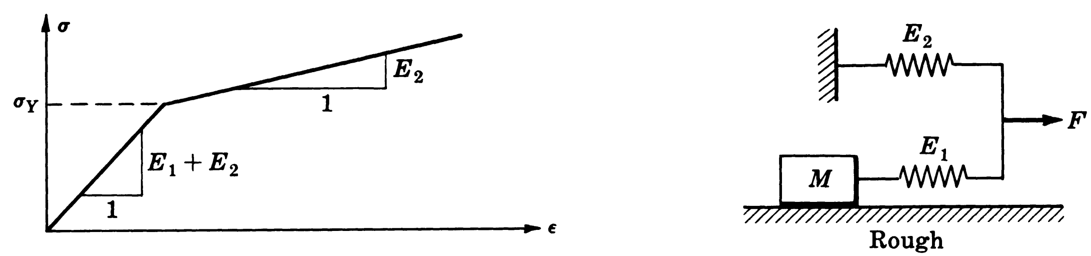
Ideal or perfect plasticity, no hardening



(*b*) Elastic—Perfectly Plastic

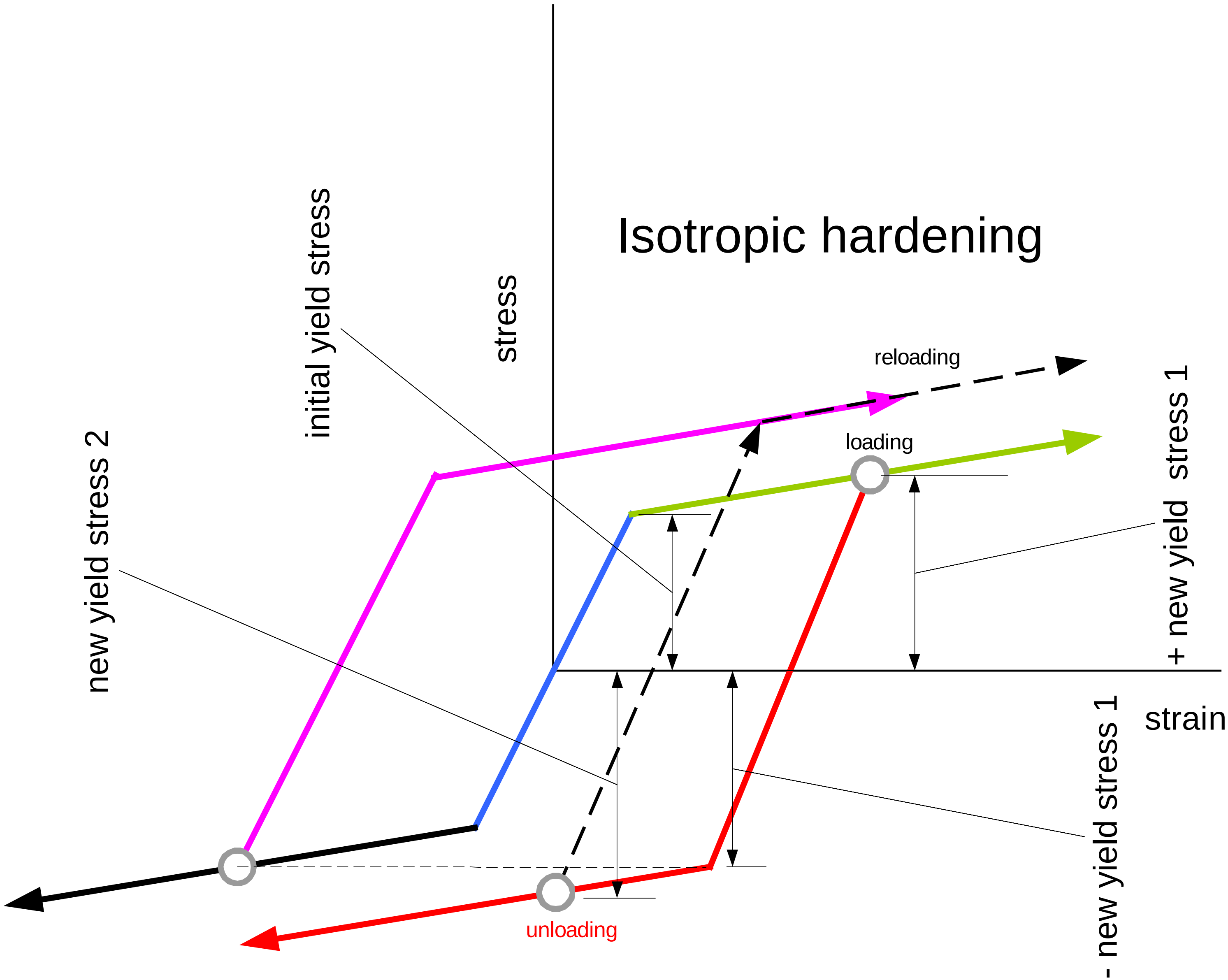


(*c*) Rigid—Linear Work Hardening

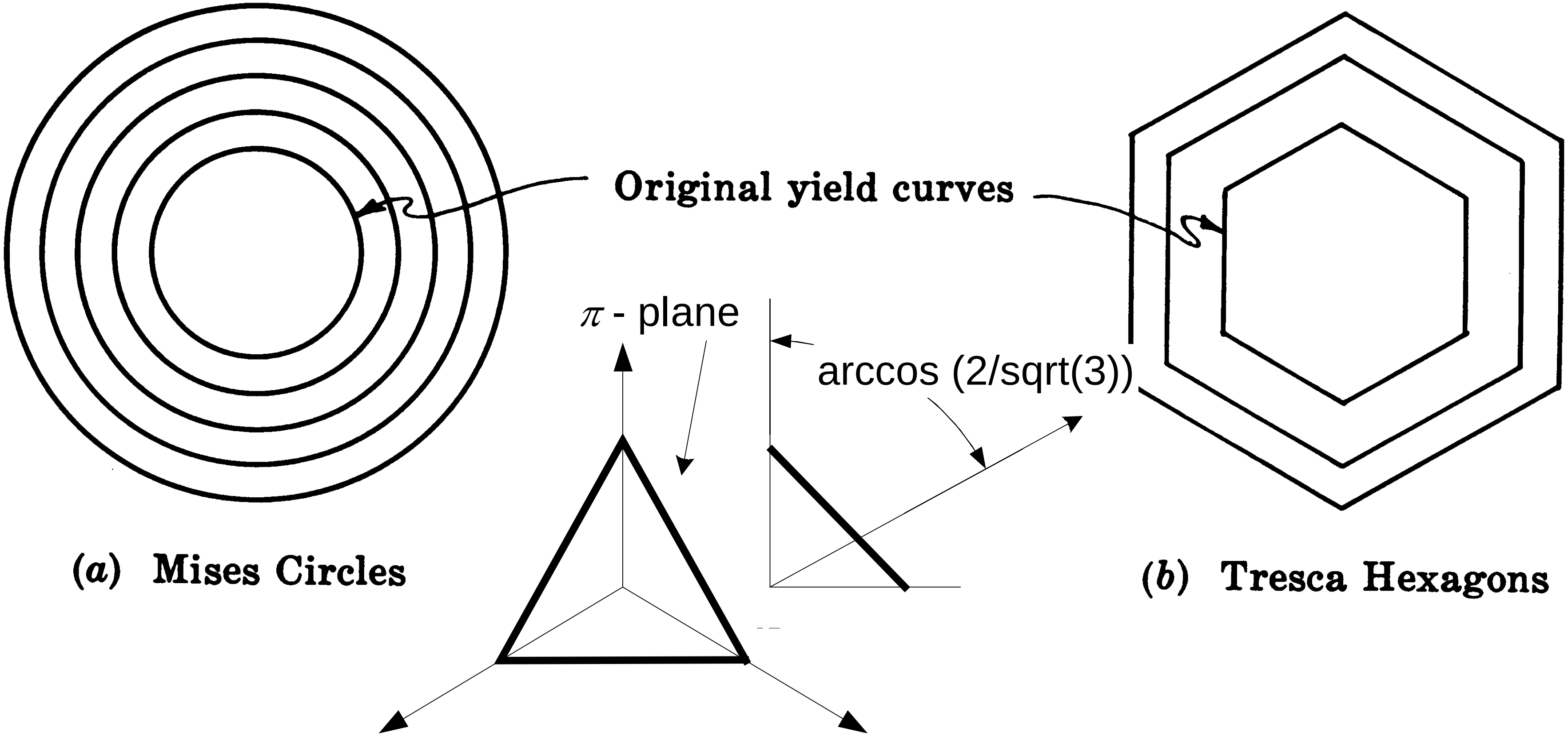


(*d*) Elastic—Linear Work Hardening

Loading, unloading, reloading and cyclic loading in 1D



Isotropic hardening in principal stress space



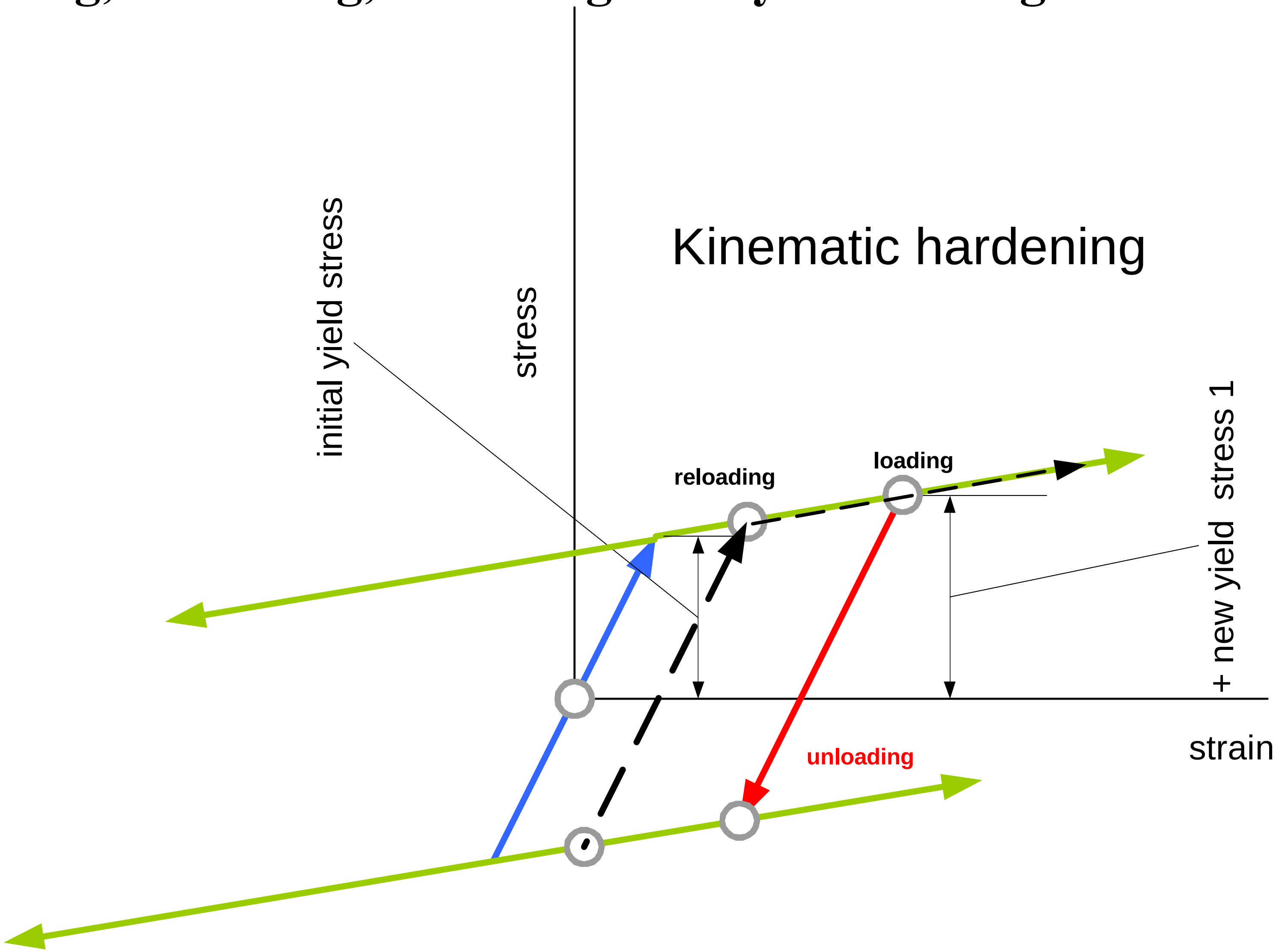
von Mises expressed by principal stresses and 1D yield stress in tension

$$F \equiv [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] - 2\sigma_Y^2 = 0$$

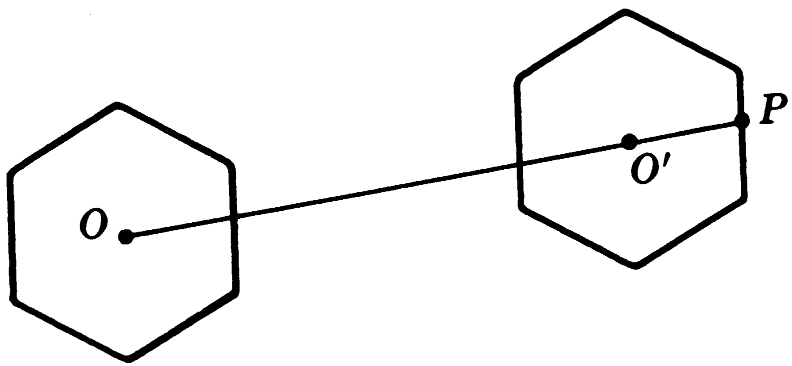
Tresca expressed by principal stresses and 1D yield stress in tension

$$F \equiv (\sigma_1 - \sigma_3) - \sigma_Y = 0, \quad \sigma_1 > \sigma_2 > \sigma_3$$

Loading, unloading, reloading and cyclic loading in 1D



Kinematic hardening in principal stress space

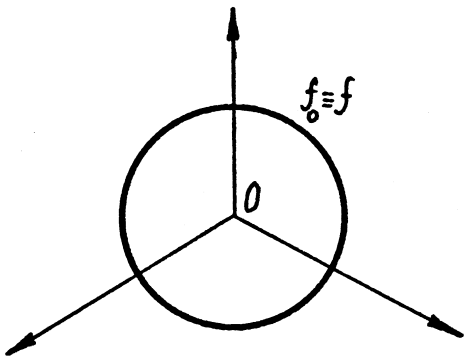


instead of $F(\sigma_{ij}) = 0$ (as in case of isotropic hardening)

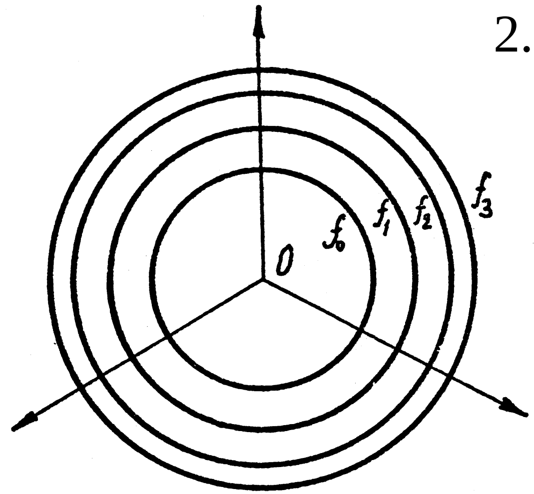
we take $F(\sigma_{ij} - \alpha_{ij}) = 0$, where $\alpha_{ij} = c\varepsilon_{ij}^{\mathrm{P}}$, c... constant

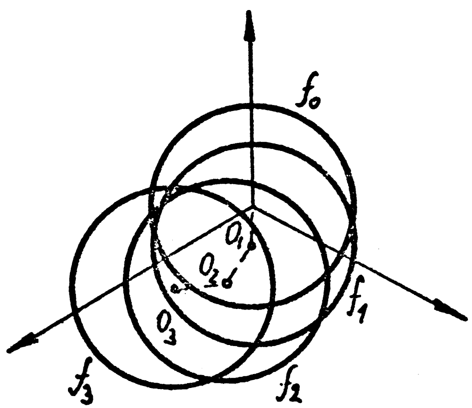
Von Mises yield condition, four hardening models

1. Perfect plasticity – no hardening

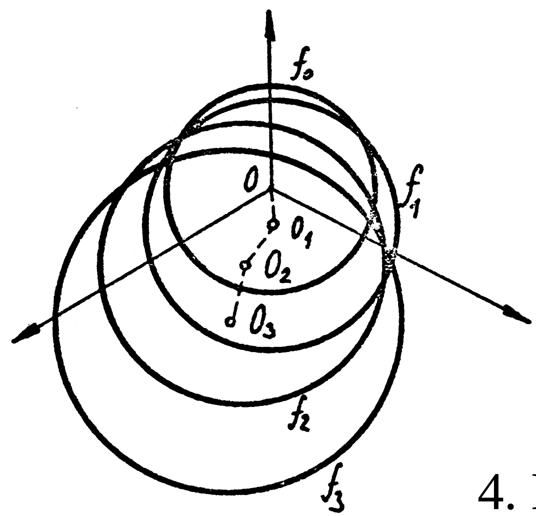


2. Isotropic hardening





3. Kinematic hardening



4. Isotropic-kinematic

Different types of yield functions

$F = F(\sigma_{ij}) \quad \cdots$ perfect plasticity

means no hardening, material starts to flow and is inclided to do so forever.

It practice it is stabilized by the 'healthy' material structure which exists around the plasticity region.

Plastic material flow is caused by motion of dislocations.

Definition of dislocations ...

Generally, the hardening depends on blocking the motion of dislocations (free flow) which depends on the permanent plastic strain $\varepsilon_{ij}^{\mathrm{P}}$.

$F = F(\sigma_{ij} - \alpha_{ij}) \quad \cdots$ kinematic hardening

where $\alpha_{ij} = c\,\varepsilon_{ij}^{\mathrm{P}}$ and c is a constant.

$F = F(\sigma_{ij}, \varepsilon_{ij}^{\mathrm{P}}) \quad \cdots$ non-isotropic hardening

hardening depends on every component of ε_{ij} in a different way

$F = F(\sigma_{ij}, K) \quad \cdots$ isotropic hardening

where $K = K(\varepsilon_{ij}^{\mathrm{P}})$ is a scalar function of $\varepsilon_{ij}^{\mathrm{P}}$, usually an invariant.

Generally, which is not general at all, we could have

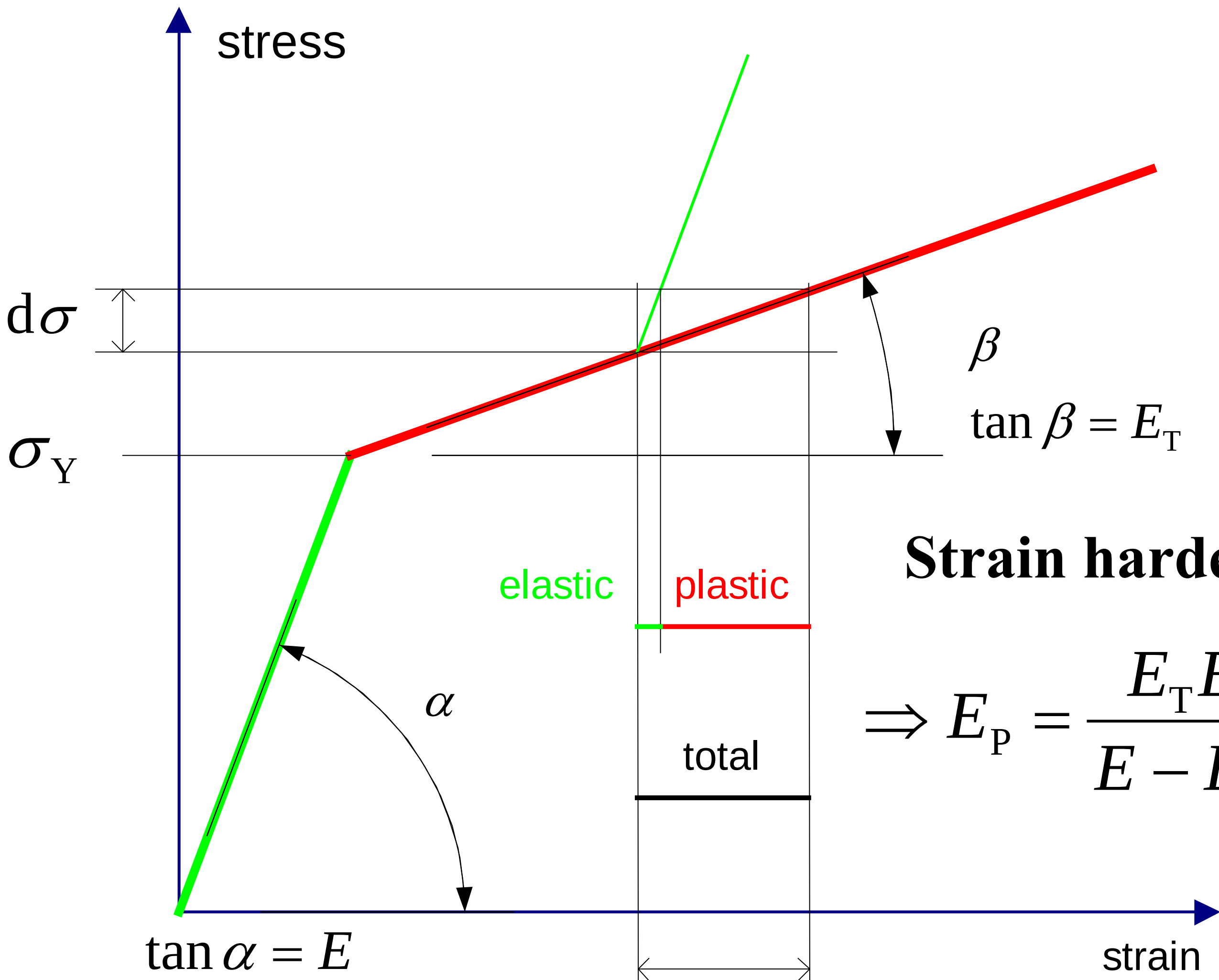
$$F = F(\sigma_{ij}, \varepsilon_{ij}^{\mathrm{P}}, K)$$

Plasticity models – physical relevance

- Von Mises
  - no need to analyze the state of stress
  - a smooth yield sufrace
  - good agreement with experiments
- Tresca
  - simple relations for decisions (advantage for hand calculations)
  - yield surface is not smooth (disadvantage for programming, the normal to yield surface at corners is not uniquely defined)
- Drucker Prager

  a more general model

1D example, bilinear characteristics



$\tan\beta = E_T$ … tangent modulus

$\tan\alpha = E$ … elastic modulus

$d\varepsilon = d\varepsilon_T = d\varepsilon_{EP}$ … means total or elastoplastic

$$d\varepsilon_T = d\varepsilon_E + d\varepsilon_P$$

$$\frac{d\sigma}{E_T} = \frac{d\sigma}{E} + \frac{d\sigma}{E_P}$$

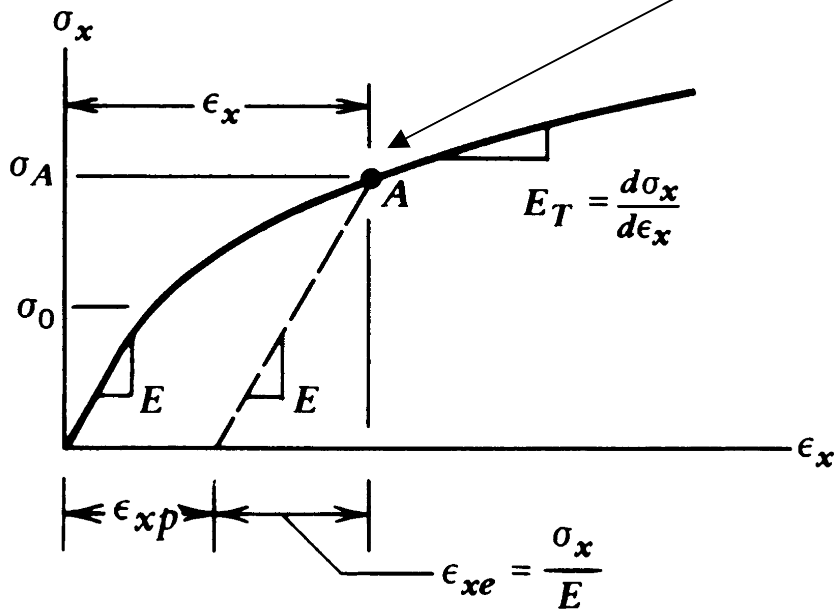
Strain hardening parameter

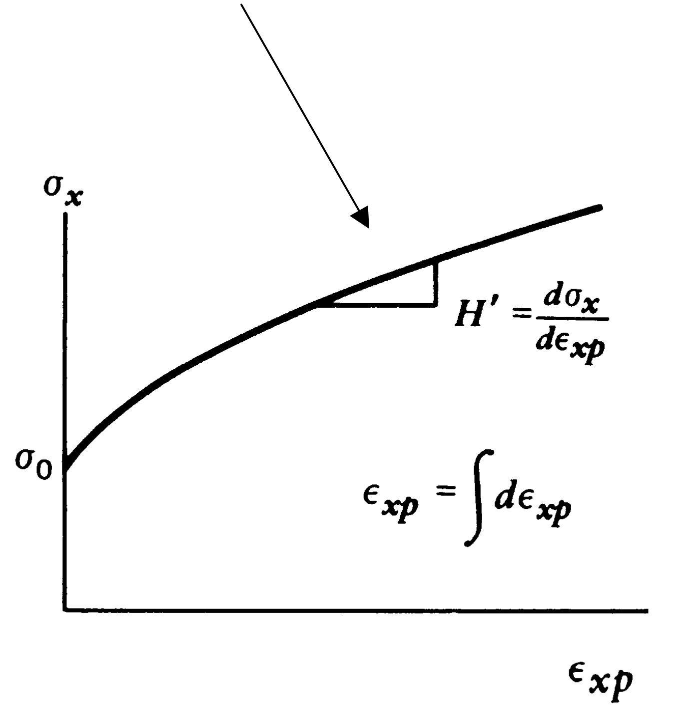
$$\Rightarrow E_P = \frac{E_T E}{E - E_T} = \frac{E_T}{1 - E_T / E} = H'$$

Strain hardening parameter again

Upon unloading and reloading the effective stress must exceed

Initial yield

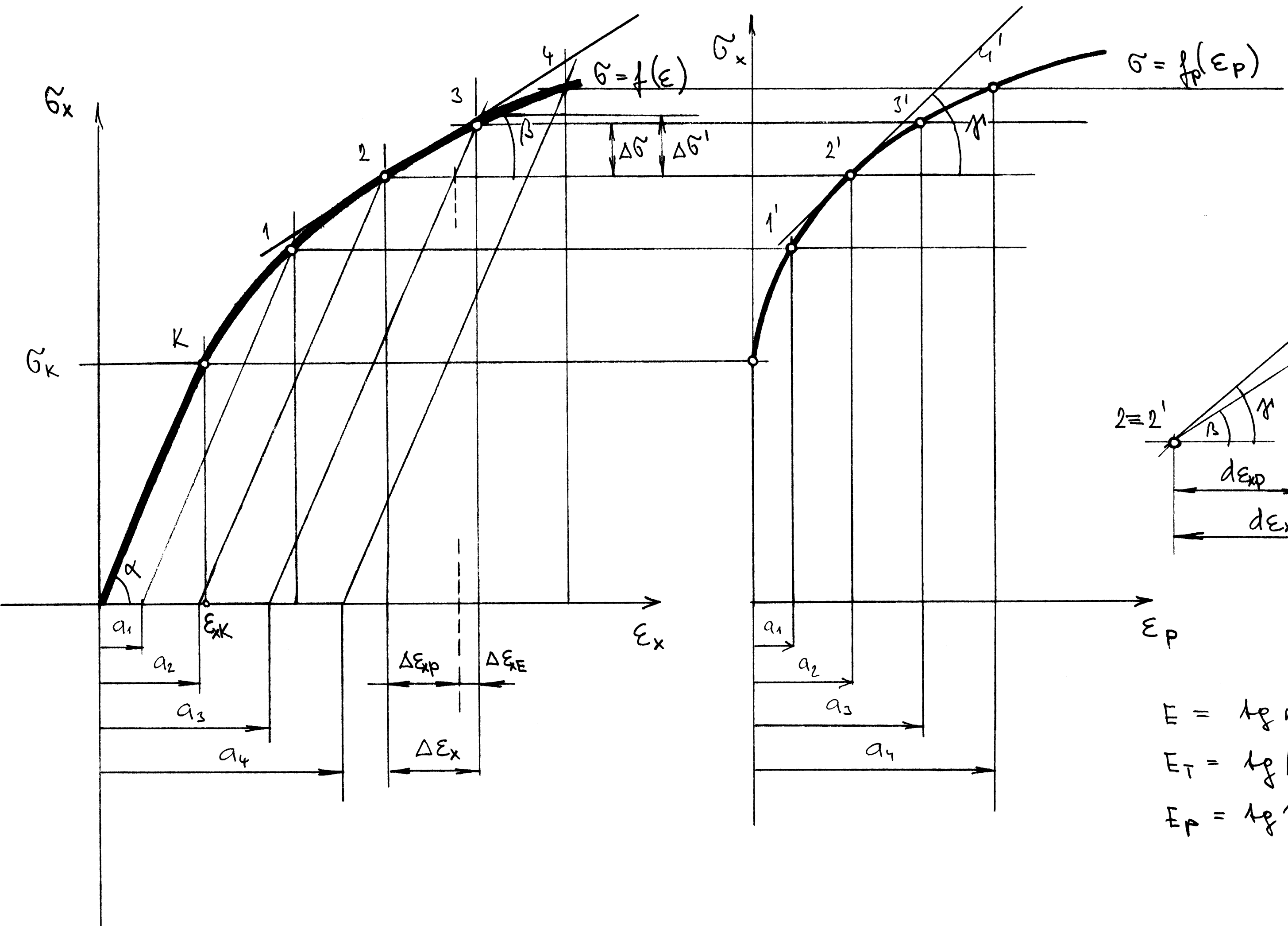




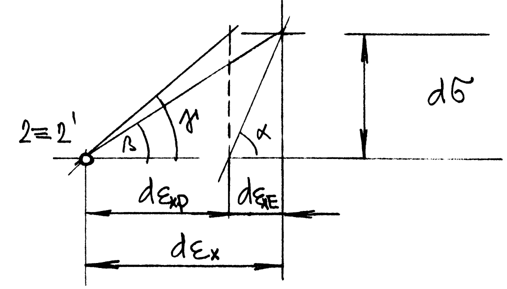
Elastic strains removed

Geometrical meaning of the strain hardening parameter is
the slope of the stress vs. plastic strain plot

How to remove elastic part



$$E_{\mathrm{P}} = \frac{E_{\mathrm{T}} E}{E - E_{\mathrm{T}}}$$

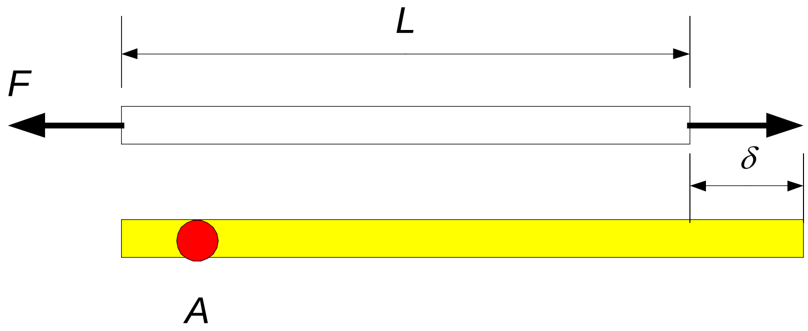


$E = \mathrm{tg}\,\alpha$ **elastic**

$E_T = \mathrm{tg}\,\beta$ **tangential modulus**

$E_P = \mathrm{tg}\,\gamma$ **plastic**

1D example, bar (rod) element elastic and tangent stiffness



Elastic stiffness

$$\sigma \le \sigma_{\mathrm{Y}}$$

$$k_{\mathrm{E}} = \frac{F}{\delta} = \frac{EA}{L}$$

Tangent stiffness

$$\sigma > \sigma_{\mathrm{Y}}$$

$$k_{\mathrm{T}} = \frac{\mathrm{d}F}{\mathrm{d}\delta} = \frac{\mathrm{d}\sigma\, A}{\mathrm{d}\varepsilon_{\mathrm{T}}\, L} = \frac{E_{\mathrm{P}}\,\mathrm{d}\varepsilon_{\mathrm{P}}\, A}{\left(\mathrm{d}\varepsilon_{\mathrm{E}} + \mathrm{d}\varepsilon_{\mathrm{P}}\right) L}$$

$$k_{\mathrm{T}} = \frac{E_{\mathrm{P}}\, A}{L} \frac{\mathrm{d}\sigma\ /\mathrm{E}_{\mathrm{P}}}{\mathrm{d}\sigma / E + \mathrm{d}\sigma / E_{\mathrm{P}}} = \frac{EA}{L}\left(1 - \frac{E_{\mathrm{P}}}{E + E_{\mathrm{P}}}\right)$$

Results of 1D experiments must be correlated to theories capable to describe full 3D behaviour of materials

- **Incremental theories** relate stress increments to strain increments
- **Deformation theories** relate total stress to total strain

Relations for incremental theories isotropic hardening example 1/9

Relation between increments and rates : $\lim_{t \to 0} \frac{\mathrm{d}\sigma}{\mathrm{d}t} = \dot{\sigma}$

Parameter only

Let the yield surface is $F(\sigma_{ij}, \varepsilon_{ij}^{\mathrm{P}}) = 0$

increment of deformation depends on F and $\dot{\sigma}_{\mathrm{eff}}$

if $F < 0$ elastic

$F = 0$ and $\dot{\sigma}_{\mathrm{eff}} < 0$ elastic

$F = 0$ and $\dot{\sigma}_{\mathrm{eff}} > 0$ elastoplastic

$F = 0$ and $\dot{\sigma}_{\mathrm{eff}} = 0$ neutral - it means that $\dot{\varepsilon}_{ij}^{\mathrm{P}} = 0$

$F > 0$ go back to yield surface

Relations for incremental theories isotropic hardening example 2/9

Flow rule is assumed in the form (Drucker, 1947)

$$\dot{\varepsilon}_{ij}^{\mathrm{P}} = \lambda \frac{\partial F}{\partial \sigma_{ij}} = \lambda \mathbf{q}$$

Eq. (i) … increment of plastic deformation has a direction normal to F while its magnitude (length of vector) is not yet known

where λ is so far unknown scalar and

$$\mathbf{q} = \{\frac{\partial F}{\partial \sigma_{11}} \quad \cdots \quad \frac{\partial F}{\partial \sigma_{31}}\}^{\mathrm{T}}$$

defines outer normal to F in six dimensional stress space

F can be expressed as a total differential

$$\mathrm{d}F = \frac{\partial F}{\partial \sigma_{ij}} \mathrm{d}\sigma_{ij} + \frac{\partial F}{\partial \varepsilon_{ij}^{\mathrm{P}}} \mathrm{d}\varepsilon_{ij}^{\mathrm{P}} = \frac{\partial F}{\partial \sigma_{ij}} \dot{\sigma}_{ij} + \frac{\partial F}{\partial \varepsilon_{ij}^{\mathrm{P}}} \dot{\varepsilon}_{ij}^{\mathrm{P}}$$

which must be zero during plastic deformations, so $\mathrm{d}F = 0$

Relations for incremental theories isotropic hardening example 3/9

Denoting $\mathbf{p} = -\{\frac{\partial F}{\partial \varepsilon_{11}^{\mathrm{P}}} \quad \cdots \quad \frac{\partial F}{\partial \varepsilon_{31}^{\mathrm{P}}}\}^{\mathrm{T}}$

the condition $\mathrm{d}F = 0$ can be expressed in the form

$$\mathbf{q}\,\mathrm{d}\sigma^{\mathrm{T}} - \mathbf{p}^{\mathrm{T}}\,\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{P}} = \mathbf{q}\,\dot{\sigma}^{\mathrm{T}} - \mathbf{p}^{\mathrm{T}}\,\dot{\boldsymbol{\varepsilon}}^{\mathrm{P}} = 0 \qquad \text{eq. (ii)}$$

stress increments are

$$\dot{\boldsymbol{\sigma}} = \mathbf{E}\,\dot{\boldsymbol{\varepsilon}}^{\mathrm{E}} = \mathbf{E}\,(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^{\mathrm{P}}) \qquad \text{eq. (iii)}$$

elastic total plastic deformations

matrix of elastic moduli

Relations for incremental theories isotropic hardening example 4/9

Combining the relations for flow rule (i),

$\mathrm{d}F = 0$ (ii) and for stress increments (iii) we get

Row vector

Column vector

$$\lambda = \frac{\mathbf{q}^{\mathrm{T}}\mathbf{E}\,\dot{\boldsymbol{\varepsilon}}}{\mathbf{p}^{\mathrm{T}}\mathbf{q} + \mathbf{q}^{\mathrm{T}}\mathbf{E}\,\mathbf{q}}$$

Still to be determined

Dot product and quadratic form … scalar

Lambda is the scalar quantity determining the magnitude of plastic strain increment in the flow rule

Relations for incremental theories isotropic hardening example 5/9

Now, for the stress increment we can write

$$\dot{\boldsymbol{\sigma}} = \mathbf{E}\dot{\boldsymbol{\varepsilon}}^{\mathrm{E}} = \mathbf{E}(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^{\mathrm{P}}) \quad \text{with} \quad \dot{\boldsymbol{\varepsilon}}^{\mathrm{P}} = \lambda\,\mathbf{q}$$

Substituting for λ we get the stress increment

as a function of total strain increment in the form

$$\dot{\boldsymbol{\sigma}} = \mathbf{E}^{\mathrm{EP}}\dot{\boldsymbol{\varepsilon}}$$

diadic product

with

$$\mathbf{E}^{\mathrm{EP}} = \mathbf{E} - \frac{\mathbf{E}\mathbf{q}\,(\mathbf{E}\mathbf{q})^{\mathrm{T}}}{\mathbf{p}^{\mathrm{T}}\mathbf{q} + \mathbf{q}^{\mathrm{T}}\mathbf{E}\mathbf{q}}$$

where **p** still has to be determined

equal to zero for perfect plasticity

Relations for incremental theories isotropic hardening example 6/9

Determination of $\mathbf{p} = -\{\frac{\partial F}{\partial \varepsilon_{11}^{\mathrm{P}}} \quad \cdots \quad \frac{\partial F}{\partial \varepsilon_{31}^{\mathrm{P}}}\}^{\mathrm{T}}$

At time t

Assume von Mises yield condition $F \equiv J_{\mathrm{D2}} - \frac{1}{3}\,{}^{t}\sigma_{\mathrm{Y}}^{2} = 0$

where $J_{\mathrm{D2}} = \frac{1}{2} s_{ij} s_{ij}$ is the second deviatoric invariant

to evaluate $\frac{\partial F}{\partial \varepsilon_{ij}^{\mathrm{P}}}$ we need $\sigma_{\mathrm{Y}} = f(\varepsilon_{ij}^{\mathrm{P}})$

Experiments suggest that

${}^{t}\sigma_{\mathrm{Y}} = f(W^{\mathrm{P}}), \quad W^{\mathrm{P}} = \int \sigma_{ij}\, \mathrm{d}\varepsilon_{\mathrm{ij}}^{\mathrm{P}} \quad \cdots$ work done by plastic increments

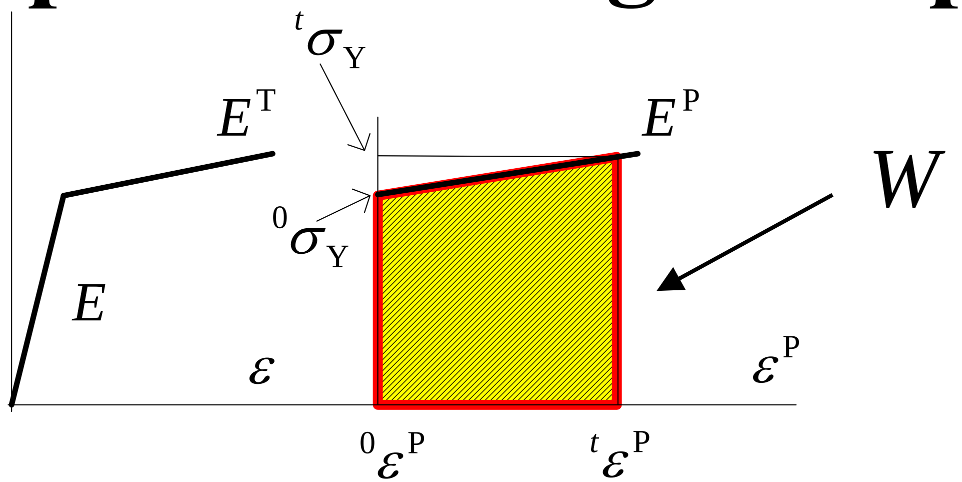
Chain rule

A new constant defined

$$\frac{\partial \mathrm{F}}{\partial \varepsilon_{\mathrm{ij}}^{\mathrm{P}}} = \frac{\partial F}{\partial\, {}^{t}\sigma_{\mathrm{Y}}} \frac{\partial\, {}^{t}\sigma_{\mathrm{Y}}}{\partial W^{\mathrm{P}}} \frac{\partial W^{\mathrm{P}}}{\partial \varepsilon_{\mathrm{ij}}^{\mathrm{P}}} = -\frac{2}{3}\,{}^{t}\sigma_{\mathrm{Y}} \frac{\partial\, {}^{t}\sigma_{\mathrm{Y}}}{\partial W^{\mathrm{P}}} \sigma_{ij} = -A\,\sigma_{ij}$$

using $\frac{\partial F}{\partial\, {}^{t}\sigma_{\mathrm{Y}}} = -\frac{2}{3}\,{}^{t}\sigma_{\mathrm{Y}}$ and $\frac{\partial W^{\mathrm{P}}}{\partial \varepsilon_{ij}^{\mathrm{P}}} = \sigma_{ij}$

Relations for incremental theories isotropic hardening example 7/9



in 1D the elastic work done $W^{\mathrm{P}} = \frac{1}{2}({}^{t}\sigma_{\mathrm{Y}} + {}^{0}\sigma_{\mathrm{Y}})\ {}^{t}\varepsilon^{\mathrm{P}}$

1D bilinear characteristics ${}^{\mathrm{t}}\sigma_{\mathrm{Y}} = ({}^{0}\sigma_{\mathrm{Y}} + E^{\mathrm{P}}\ {}^{t}\varepsilon^{\mathrm{P}})$

$$\Rightarrow W^{\mathrm{P}} = \frac{1}{2E^{\mathrm{P}}}({}^{t}\sigma_{\mathrm{Y}}^{2} - {}^{0}\sigma_{\mathrm{Y}}^{2})$$

$$\frac{\partial W^{\mathrm{P}}}{\partial {}^{t}\sigma_{\mathrm{Y}}} = \frac{{}^{t}\sigma_{\mathrm{Y}}}{E^{\mathrm{P}}} \Rightarrow A = \frac{2}{3}\ {}^{t}\sigma_{\mathrm{Y}}\ \frac{E^{\mathrm{P}}}{{}^{t}\sigma_{\mathrm{Y}}} = \frac{2}{3}E^{\mathrm{P}} = \frac{2}{3}\frac{EE^{\mathrm{T}}}{E - E^{\mathrm{T}}}$$

so finaly $\mathbf{p} = A\{\sigma_{11}\ \sigma_{22} \cdots\ \sigma_{31}\}^{\mathrm{T}}$

Relations for incremental theories isotropic hardening example 8/9

Summary. For given σ_{ij} and σ_{Y} and $\dot{\boldsymbol{\varepsilon}}_{ij}$ we can compute $\dot{\boldsymbol{\sigma}}$ as follows

$$\sigma_{\mathrm{m}} = \tfrac{1}{3}(\sigma_{11} + \sigma_{22} + \sigma_{33})$$

$$\mathbf{s} = \{s_{11}\ s_{22}\ s_{33}\ s_{12}\ s_{23}\ s_{31}\}^{\mathrm{T}} = \{\sigma_{11} - \sigma_{\mathrm{m}}\ \sigma_{22} - \sigma_{\mathrm{m}}\ \sigma_{33} - \sigma_{\mathrm{m}}\ \sigma_{12}\ \sigma_{23}\ \sigma_{31}\}^{\mathrm{T}}$$

$$\mathbf{q} = \{s_{11}\ s_{22}\ s_{33}\ 2s_{12}\ 2s_{23}\ 2s_{31}\}^{\mathrm{T}}$$

$$A = \frac{2}{3}\frac{EE^{\mathrm{T}}}{E - E^{\mathrm{T}}}$$

$$\mathbf{p} = A\{\sigma_{11}\ \sigma_{22}\ \sigma_{33}\ \sigma_{12}\ \sigma_{23}\ \sigma_{31}\}^{\mathrm{T}}$$

$$a = \mathbf{p}^{\mathrm{T}}\mathbf{q}, \quad \mathbf{b} = \mathbf{E}\mathbf{q}, \quad c = \mathbf{q}^{\mathrm{T}}\mathbf{E}\mathbf{q} = \mathbf{q}^{\mathrm{T}}\mathbf{b}$$

$$\mathbf{E}^{\mathrm{EP}} = \mathbf{E} - \frac{\mathbf{b}\mathbf{b}^{\mathrm{T}}}{a + c}$$

$$\dot{\boldsymbol{\sigma}} = \mathbf{E}^{\mathrm{EP}}\dot{\boldsymbol{\varepsilon}}$$

J2 theory, perfect plasticity 1/6

alternative notation … example of numerical treatment

$$\{\sigma\} = [E]\{\varepsilon\} \text{...Hooke's law}$$

$$\{\sigma\} = \{\sigma_{xx}\ \sigma_{yy}\ \sigma_{zz}\ \tau_{xy}\ \tau_{yz}\ \tau_{zx}\}^{\mathrm{T}}$$

$$\{\varepsilon\} = \{\varepsilon_{xx}\ \varepsilon_{yy}\ \varepsilon_{zz}\ \gamma_{xy}\ \gamma_{yz}\ \gamma_{zx}\}^{\mathrm{T}}$$

$$\sigma_{\mathrm{m}} = \tfrac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}) \quad \text{mean stress}$$

stress deviator

$$\{s\} = \{\sigma_{xx} - \sigma_{\mathrm{m}}\ \sigma_{yy} - \sigma_{\mathrm{m}}\ \sigma_{zz} - \sigma_{\mathrm{m}}\ \tau_{xy}\ \tau_{yz}\ \tau_{zx}\}^{\mathrm{T}}$$

second invariant of stress deviator

$$J_{\mathrm{D2}} = J_2 = \tfrac{1}{2}(s_{xx}^2 + s_{yy}^2 + s_{zz}^2 + 2s_{xy}^2 + 2s_{yz}^2 + 2s_{zx}^2)$$

$$\text{or} \quad J_2 = \tfrac{1}{2}\{s\}^{\mathrm{T}}[M]\{s\}, \text{ with } [M] = \mathrm{diag}(1,1,1,2,2,2)$$

J2 theory, numerical treatment …2/6

one can prove that

$$\{s\}^{\mathrm{T}}[M]\{\sigma\} = \{s\}^{\mathrm{T}}[M]\{s\}, \quad \text{since} \quad s_{xx} + s_{yy} + s_{zz} = 0$$

and also $[E][M]\{s\} = 2G\{s\}, \quad \text{with} \quad G = E/(1+\mu)$

von Mises effective stress $\sigma_{\mathrm{eff}} = \sqrt{3J_2} = \sqrt{3\{s\}^{\mathrm{T}}[M]\{s\}/2}$

yield criterion for perfectly plastic behaviour $\sigma_{\mathrm{eff}} = \sigma_{\mathrm{Y}}$

J2 theory, numerical treatment …3/6

Flow rule according to Prandtl - Reuss hypothesis

$\{\dot{\varepsilon}\} = \lambda \dfrac{\partial F}{\partial \{\sigma\}^{\mathrm{T}}} = \lambda[M]\{s\}$ … λ is so far unknown parameter

$\{\sigma\} = [E]\{\varepsilon^{\mathrm{E}}\} = [E]\{\varepsilon - \varepsilon^{\mathrm{P}}\}$ … Hooke's law

$\{\dot{\sigma}\} = [E]\{\dot{\varepsilon}\} - [E]\{\dot{\varepsilon}^{\mathrm{P}}\} = [E]\{\dot{\varepsilon}\} - 2G\{s\}$ … its time derivative, increment

no plastic deformation in elastic region can be expressed by

if $\sigma_{\mathrm{eff}} < \sigma_{\mathrm{Y}}$ then $\lambda = 0$,

else $\lambda \geq 0$

endif

Six nonlinear differential equations + one algebraic constraint (inequality)
There is exact analytical solution to this. In practice we proceed numerically

J2 theory, numerical treatment …4/6

Differentiating plasticity condition $\sigma_{\text{eff}} = \sigma_{\text{Y}}$

$$\dot{\sigma}_{\text{eff}} = \frac{\partial \sigma_{\text{eff}}}{\partial \mathbf{s}^{\text{T}}} \dot{\mathbf{s}} = \frac{3\mathbf{s}^{\text{T}}\mathbf{M}\dot{\mathbf{s}}}{2\sigma_{\text{eff}}} = 0$$

$$\Rightarrow \mathbf{s}^{\text{T}}\mathbf{M}\dot{\mathbf{s}} = 0 \text{ and also } \mathbf{s}^{\text{T}}\mathbf{M}\dot{\boldsymbol{\sigma}} = 0$$

Substituting for $\dot{\boldsymbol{\sigma}}$

$$\mathbf{s}^{\text{T}}\mathbf{M}\mathbf{E}\dot{\boldsymbol{\varepsilon}} = 2G\lambda \mathbf{s}^{\text{T}}\mathbf{M}\mathbf{s}$$

and realizing that

$$2G\lambda \mathbf{s}^{\text{T}}\mathbf{M}\mathbf{s} = 4G\lambda J_2 = 4G\lambda \sigma_{\text{eff}}^2 / 3 = 4G\lambda \sigma_{\text{Y}}^2 / 3$$

we get

$$\lambda = \frac{3\mathbf{s}^{\text{T}}\mathbf{M}\mathbf{E}\dot{\boldsymbol{\varepsilon}}}{4G\sigma_{\text{Y}}^2} = \frac{3\mathbf{s}^{\text{T}}\dot{\boldsymbol{\varepsilon}}}{2\sigma_{\text{Y}}^2}$$

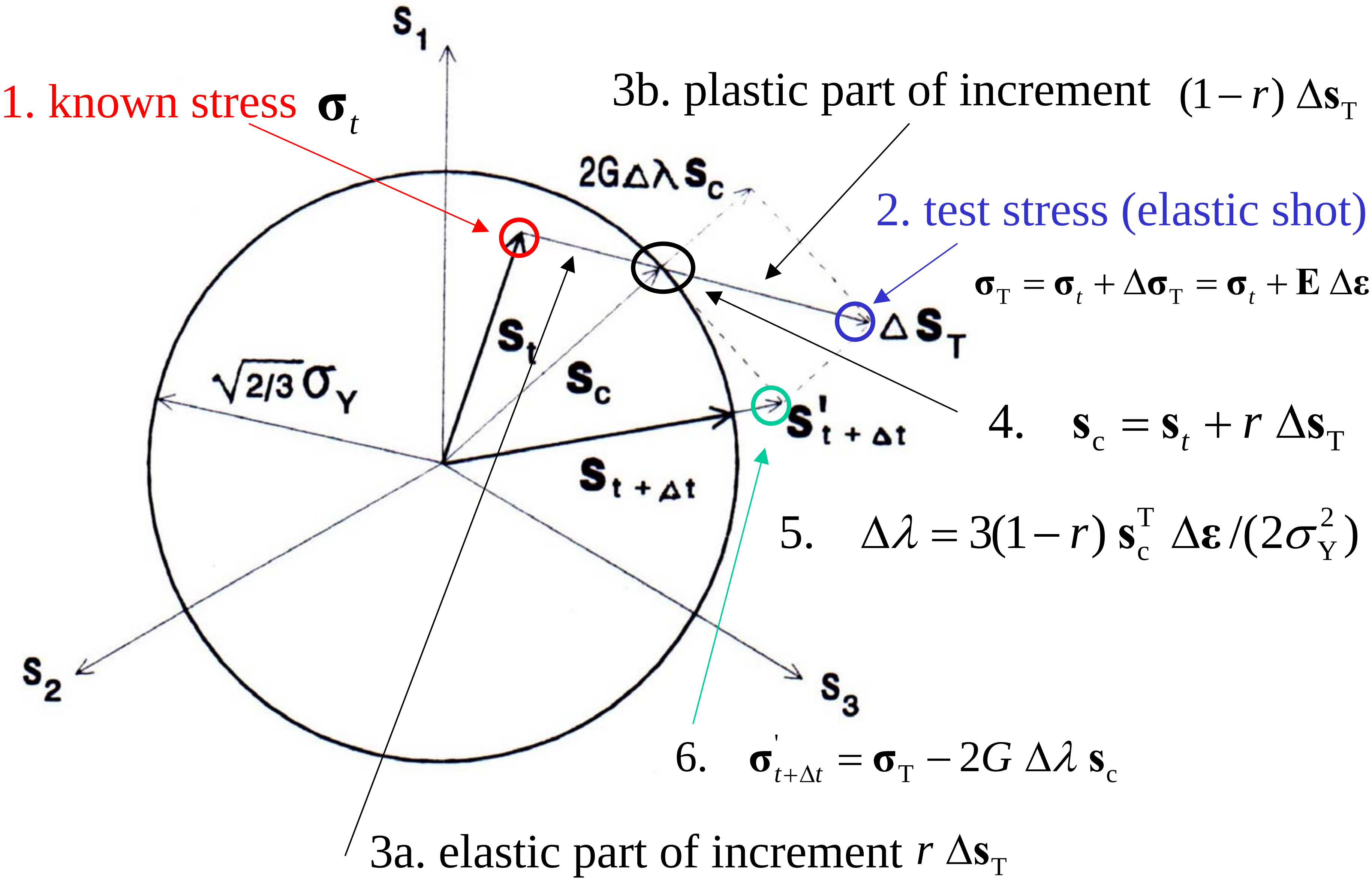
finally

$$\boxed{\dot{\boldsymbol{\sigma}} = \mathbf{E}^{\text{EP}}\dot{\boldsymbol{\varepsilon}}} \quad \text{with} \quad \mathbf{E}^{\text{EP}} = \mathbf{E} - \frac{3G}{\sigma_{\text{Y}}^2}\mathbf{s}\mathbf{s}^{\text{T}}$$

System of six nonlinear differential equations to be integrated

J2 theory, numerical treatment …5/6

predictor-corrector method, first part: predictor



J2 theory, numerical treatment …6/6

predictor-corrector method, second part: corrector

Correction

For $\mathbf{s}_{t+\Delta t} = \beta\, \mathbf{s}'_{t+\Delta t}$ find β in such a way that

$$\sigma_{\mathrm{eff}}(\mathbf{s}_{t+\Delta t}) = \sigma_{\mathrm{Y}}$$

$$\sigma_{\mathrm{eff}}(\beta\, \mathbf{s}'_{t+\Delta t}) = \sigma_{\mathrm{Y}}$$

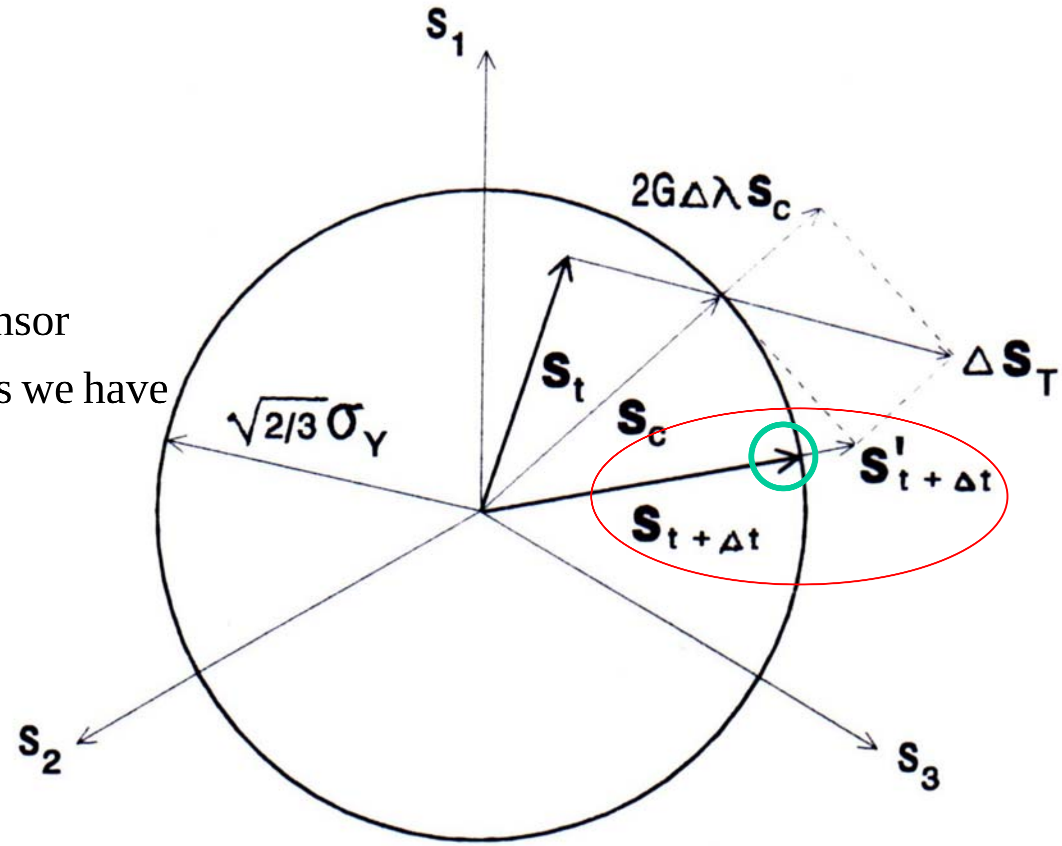
$$\beta\, \sigma_{\mathrm{eff}}(\mathbf{s}'_{t+\Delta t}) = \sigma_{\mathrm{Y}}$$

$$\Rightarrow \beta = \frac{\sigma_{\mathrm{Y}}}{\sigma_{\mathrm{eff}}(\mathbf{s}'_{t+\Delta t})}$$

$$\mathbf{s}'_{t+\Delta t} - \mathbf{s}_{t+\Delta t} = (1-\beta)\, \mathbf{s}'_{t+\Delta t}$$

and since the spherical part of the stress tensor does not enter into plasticity considerations we have

$$\boldsymbol{\sigma}_{t+\Delta t} = \boldsymbol{\sigma}'_{t+\Delta t} - (1-\beta)\, \mathbf{s}'_{t+\Delta t}$$



Secant stiffness method and the method of radial return