ENERGY DISSIPATED AT THE SHOCK WAVE DURING ITS PROPAGATION IN WATER

Miloš, Müller
Technical university of Liberec, Department of Power Engineering equipment, Hálkova 6, 461 17 Liberec 1.

Abstract

The shock wave propagation in liquids is usually calculated without considering the energy dissipation. However in some situations, when the initial shock wave pressure is in order of 1000 MPa, the dissipation effect can be important. This paper presents the possibility of calculation of the energy dissipated at the shock wave during its propagation in water. The dissipated energy and the temperature increase at the shock wave are expressed as a function of shock wave peak pressure. The NIST data [2] are used for the fitting of coefficients of the equation of state, which is applied for the calculation of the shock wave energy dissipation.

The equation of state

The most common equation of state (EOS) for liquids is Tait´s equation, which represents the dependence between the liquid density and the pressure. In the presented case, it is convenient to use the EOS in isentropic form as [3]

$$\frac{\rho(p,s)}{\rho(p=0,s)}=\left(\frac{B(s)+p}{B(s)+p_0}\right)^{\frac{1}{n}}, \tag{1}$$

where $n=7$ and $B(s)$ can be expressed as function of the sound velocity c_0 and the density ρ_0 at pressure p_0 as

$$B(s)=\frac{\rho_0 c_0}{n}. \tag{2}$$

$$c_0(T)=k_{4c}T^4+k_{3c}T^3+k_{2c}T^2+k_{1c}T+k_{0c} \tag{3}$$

$$\rho_0(T)=k_{4\rho}T^4+k_{3\rho}T^3+k_{2\rho}T^2+k_{1\rho}T+k_{0\rho}. \tag{4}$$

For the calculation of the dissipated energy, the specific heat as function of temperature evaluated at the pressure p_0 is required as

$$c_p(T)=k_{4cp}T^4+k_{3cp}T^3+k_{2cp}T^2+k_{1cp}T+k_{0cp}. \tag{5}$$

The material data used for the calculations was obtained from NIST web database [2]. The coefficients for the Eqs. (3)-(5) are given in the Tab.1.

	k_0	k_1	k_2	k_3	k_4
c_0	-1.3036E+04	1.4695E+02	-5.6297E-01	9.8018E-04	-6.5999E-07
c_p	4.0409E+04	-4.3075E+02	1.9216E+00	-3.8150E-03	2.8284E-06
ρ_0	1.1209E+03	2.3785E+01	-9.8675E-02	1.8172E-04	-1.2888E-07

Tab. 1. Coefficients for the Eqs. (3), (4) and (5).

Enthalpy dissipated at the shock wave

If the shock wave passes a position in liquid, which had temperature T_0 and pressure p_0, the temperature and pressure in that place increase up to the values p and T as it can be seen in Fig. 1. The specific enthalpy increment experienced by the fluid ΔH is given in from Rankine-Hugoniot condition [1] as

$$\Delta H = \frac{p}{2}\left(\frac{1}{\rho} + \frac{1}{\rho_0}\right). \tag{6}$$

Having passed the shock wave the pressure in the liquid reaches again the pressure p_0 along an adiabatic curve but the temperature returns to a higher value T_1 due to the dissipation process. The estimation of the temperature T_1 is the key for obtaining the dissipated energy. Based on the consideration, the enthalpy increment ΔH can be evaluated as a sum of the undissipated and dissipated enthalpy as

$$\Delta H = h + h_{dis}, \tag{7}$$

which can be expressed using the equation of state (1) according to the Fig. 1. as

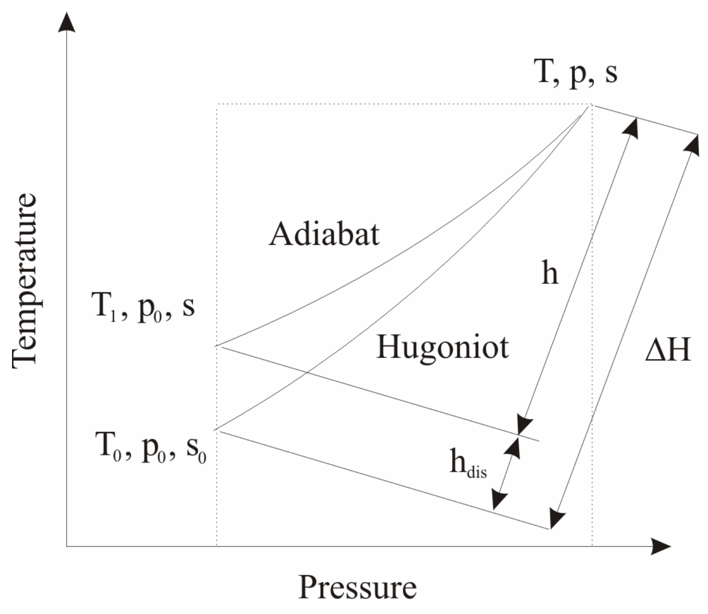

Fig. 1. Distribution of the total energy at the shock wave.

$$h = \frac{2c_1^2}{n-1}\left(\left(\frac{\rho}{\rho_1}\right)^{\frac{1}{n}} - 1\right), \tag{8}$$

The dissipated enthalpy can be evaluated from the specific heat data, which is known as an explicit function of the temperatures T_0 and T_1 as

$$h_{dis} = \int_{T_0}^{T_1} c_p(T_0, T_1)\,dT\,. \tag{9}$$

Having imposed Eqs. (6) and (8) into Eq. (7) one obtain

$$\frac{h_{dis}}{c_1^{\ 2}} = -\frac{2}{n-1}\left[\left(\frac{\rho}{\rho_1}\right)^{n-1} - 1\right] + \frac{p}{2\rho_1}\left(\frac{\rho_1}{\rho} + \frac{\rho_1}{\rho_0}\right). \tag{10}$$

The dissipated enthalpy is eliminated from this equation using (9), where the heat capacity is obtained from (5). Finally, the ratio ρ_1/ρ can be eliminated using Eqs. (1) and (2) as

$$p = \frac{c_1^2\rho_1}{n}\left[\left(\frac{\rho}{\rho_1}\right)^{n-1} - 1\right]. \tag{11}$$

For given pressure p the temperature T_1 is obtained from the Eq. (10). This temperature is then used in the Eq. (6) for the calculation of the dissipated enthalpy h_{dis}.

T_0=293K	NIST			Richardson[3]
p [MPa]	ΔT_1 [K]	T_1 [K]	h_{dis} [J/kg]	h_{dis} [J/kg]
0	0.0	293.0	0.0	0.0
250	0.2	293.2	1046.0	x
500	1.2	294.2	5152.0	5570.0
1000	5.4	298.4	22480.0	23450.0
1500	11.5	304.5	47978.0	49350.0
2000	18.8	311.8	78484.0	80050.0
2500	26.9	319.9	112547.0	115000.0
3000	35.6	328.6	148720.0	152500.0
3500	44.7	337.7	186806.0	192000.0
4000	54.0	347.0	225771.0	233000.0

Tab. 2. Dissipated enthalpy calculated as a function of the pressure at the shock wave.

The explicit formula for energy dissipated at the shock wave as function of pressure has the form

$$h_{dis}(p) = 3.0917x10^{-2} p^4 - 4.7538p^3 + 2.8574x10^2 p^2 - 1.5626x10^2 p\,. \quad (12)$$

Conclusion

The paper presented a possibility of calculation of shock wave energy dissipated at the shock wave in water. For the solution, the dissipated enthalpy based on the new material data derived from NIST material database was derived. The presented model for the energy dissipation can be used for any liquids when the coefficients for Eqs. (2) – (5) are known. The comparison of the dissipated energy calculated by Richardson in [3] and the presented data shows very good agreement.

Acknowledgement
This work is supported by grant no. 4674788501 of the Ministry of Education and by grant no. 101/07/1612 of the Grant Agency of the Czech Republic.

References

[1] R.H. Cole, Underwater explosion, Princeton Univ. Press, Princeton, 1948

[2] P. J. Linstrom, W. G. Malard, NIST Chemistry WebBook, NIST Standard Reference Database Number 69, National Institute of Standards and Technology, Gaithersburg MD, 2005, (http://webbook.nist.gov).

[3] J. M. Richardson, A. B. Arons, R. R. Halverson, Hydrodynamic Properties of Sea Water at the Front of a Shock Wave, The Journal of Chemical Physics 15 (11) (1947) 785-794.