Doppler Institute's microconference

Analytic and algebraic methods in physics IV

October 21st, 2008, Villa Lanna, Prague

(on the occasion of 70th birthday of Prof. Miloslav Havlíček)

9.15	OPENING
1st morning session - chair **Pavel Exner**	
9.15 - 10.00	**Armin Uhlmann** (Universität Leipzig) *On the skeleton of the quantum world*
10.00 - 10.45	**Roman Kotecký** (Warwick University and Charles University) *Entropic barriers for Potts antiferromagnets at zero temperature*
10.45 - 11.15	coffee break
2nd morning session - chair **Jiří Patera**	
11.15 - 12.00	**Edita Pelantová** (Doppler Institute, Prague) *Graded contractions of Lie algebras and their representations*
12.00 - 12.45	**Wolfgang Lassner** (Lausitz University of Applied Sciences, Senftenberg) *From canonical realizations to Groebner bases methods for non-commutative algebras*
12.45 - 13.15	**Čestmír Burdík** (Doppler Institute, Prague) *Boson realizations after 30 years*
13.15 - 14.30	LUNCH
1st afternoon session - chair **Armin Uhlmann**	
14.30 - 15.15	**Pavel Winternitz** (CRM Montreal) *Superintegrable systems and the Painleve transcendents*
15.15 - 16.00	**Patrick Moylan** (Penn State, Philadelphia) *Positive energy representations and deformations of semisimple Lie algebras*
16.00 - 16.30	**Severin Pošta** (Doppler Institute, Prague) *Nonstandard deformed enveloping algebras and some of their properties*
16.30 - 17.00	coffee break
2nd afternoon session - chair **Jiří Tolar**	
17.00 - 17.45	**Miloš Tater** (Institute of Nuclear Physics, Řež) *Non-Hermitian spectral effects in a PT-symmetric waveguide*
17.45 - 18.30	**Pavel Exner** (Doppler Institute, Prague) *On the spectrum coming from "bending" a chain quantum graph*
19.00 - 22.00	DINNER

Abstracts

Čestmír Burdík

Title: *Boson realizations after 30 years*

Abstract: We remember the general construction of boson realizations for the algebra A_2 and B_2 starting from Verma modules and show new applications for the separation of variables in Gaudin models.

Pavel Exner

Title: *On the spectrum coming from "bending" a chain quantum graph*

Abstract: The theory of quantum graphs contains many open questions. One of them concerns geometrically induced properties of such graphs if we regard them as embedded in a Euclidean space. In this talk, describing a common work with Pierre Duclos and Ondřej Turek, we analyze a simple nontrivial example of a planar chain graph consisting of an infinite family of identical rings with a delta-coupling in the contact points. In the case of a straight periodic chain the spectrum has generically open gaps; we show that a "bending" perturbation gives rise to a discrete spectrum in the gaps and analyze its properties. We also relate all that to an event in which jubilee played an essential role.

Roman Kotecký

Title: *Entropic barriers for Potts antiferromagnets at zero temperature*

Abstract: A possibility that Potts antiferromagnets at zero temperature feature a long range order has been an intriguing conjecture for quite a long time. Mathematically, the existence of this phase transition amounts to an easily formulated claim about a non-trivial structure of the uniform distribution on the set of all proper colourings of a particular graph—usually a regular lattice. This claim is represented as a non-unicity of corresponding Gibbs states.

The talk is an account of the quest for an example with such phase transition. The task and various attempts at its solution will be explained, finishing with a proof that 3-state Potts antiferromagnet on the diced lattice indeed has a long range order at zero (as well as small non-vanishing) temperature. The main idea is to argue for non-unicity from an appropriate evaluation of entropic barriers between distinct Gibbs states. The talk is based on joint papers with J. Salas and A. Sokal.

Patrick Moylan

Title: *Positive energy representations and deformations of semisimple Lie algebras*

Abstract: The study of invariant convex cones of positivity in semisimple Lie algebras has been much studied in recent years. We illustrate some results in this area by way of example using the conformal Lie algebra, and present important physical applications. In particular we consider a positive energy, infinitesimally unitary representation of the conformal Lie algebra, and with the help of the scale generator we construct an isomorphic copy of the anti-de Sitter subalgebra, which isomorphic copy depends upon a parameter alpha. We study this deformation of the deSitter subalgebra, relate it to a deformation of the Poincar Lie algebra first described in P. Bozek, M. Havlicek, O. Navratil, Preprint: Universitas Carolina Pragensis, NCITF/1, 1985 (see: M. Havlicek, P. Moylan, J. Math. Phys., 34, 11, 5320-5332, (1993)), and obtain useful information on the Poincar content of the representation of the conformal algebra.

Wolfgang Lassner

Title: *From canonical realizations to Groebner bases methods for non-commutative algebras*

Abstract: (Some algebraic problems are reviewed which have been solved by concepts of symbolic computation)

Despite constructive methods like the use of a generalized trace formula to generate canonical realizations of Lie algebras, one has to execute simple but tedious algebraic calculations in Weyl algebras in order to check and prove the resulting formulas. Former versions of general purpose computer algebra systems (CAS) were not very well prepared for such types of non-commutative calculations. However, symbol representations of Weyl algebras could use two algorithmic facilities most effectively implemented in almost all CAS, i.e. polynomial algebra and differentiation. This technique works also for localizations of Weyl algebras as well as of enveloping algebras. A more serious computational problem occurs in quotient fields of Weyl algebras or more general in enveloping fields of Lie algebras. The main difficulty of calculations in these non-commutative fields of fractions consists in transforming left fractions into right fractions. An algorithmic solution was found by an extension of the concept of Groebner basis and of the Buchberger Algorithm to enveloping algebras.

When symmetry algebras of differential equations are determined by CAS packages as finite dimensional Lie algebras they should be identified with respect to known classification tables. The problem whether two Lie algebras given by their structure constants are isomorphic or not is equivalent to the existence of solutions of certain polynomial equations. Systems of such equations could be solved by Groebner basis methods even in a parallelized version.

Edita Pelantová

Title: *Graded contractions of Lie algebras and their representations*

Abstract: We discuss differences between gradings and group gradings of a Lie algebra. The role of automorphisms for classification of group gradings is explained. We recall the notion of graded contraction L^ε of Lie algebra L and describe a method for construction of certain type of representations of L^ε. The starting point of our construction is a representation r of the original Lie algebra L which is compatible with the grading. The compatibility of irreducible representations of the Lie algebra $sl(n,\mathbb{C})$ is studied for both non-equivalent $\mathbb{Z}_2$-gradings of this algebra.

Severin Pošta

Title: *Nonstandard deformed enveloping algebras and some of their properties*

Abstract: We present some examples of nonstandard deformed enveloping algebras - those which are not quantum groups - and discuss its various properties, such as questions about representations, center, homomorphisms etc.

Miloš Tater

Title: *Non-Hermitian spectral effects in a $\mathcal{PT}$-symmetric waveguide*

Abstract: A planar waveguide of constant width with $\mathcal{PT}$-symmetric Robin boundary conditions is presented. When the boundary coupling function is compactly supported perturbation of a homogeneous coupling, the continuous spectrum is positive and independent of such perturbation. The residual spectrum is empty. If the perturbation is small in the supremum norm, sufficient conditions for existence or absence of eigenvalues are derived. A numerical study of the eigenvalues shows a non-Hermitian features reflected in an unusual dependence on various boundary-coupling parameters.

Armin Uhlmann

Title: *On the skeleton of the quantum world*

Abstract: The (or better "a") quantum world is a huge hierarchy of quantum systems. The physical content of one of its systems is largely determined by its place within it, i. e. how it is embedded in larger ones. I restrict myself to the finite cases: All quantum systems considered can be described by *-algebras $\mathcal{B}(\mathcal{H})$, $\dim\mathcal{H}<\infty$ and their *-subalgebras. (Indeed, It is not easy to do better!)

By this point of view, intrinsic properties of a quantum system should not depend on its position within the quantum world. (For example observables, states, quantum properties of and measurements within a quantum system, ...)

States of a quantum system can be changed by actions in other quantum systems, whether in larger systems (POVM-measurements) or in causal independent systems (because of entanglement.)

It is may aim to illustrate these points of view by some examples and to conclude with some general observations.

Pavel Winternitz

Title: *Superintegrable systems and the Painleve transcendents*

Abstract: A review is given of superintegrable systems with third order integrals of motion in classical and quantum mechanics. The quantum case is very rich and involves potentials expressed in terms of Painleve transcendents. Energy levels and wave functions for these systems can be calculated using the tools of supersymmetry theory and the representation theory of polynomial algebras.