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The third problem for the Stokes system in bounded domain

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Abstract. The third problem for the Stokes system is studied on a bounded domain in R^3 with Ljapunov connected boundary. We construct a solution of this problem in the form of appropriate potentials and determine unknown source densities via integral equation systems on the boundary of the domain. The solution is given explicitly in the form of a series. Then we study the integral equation which we obtain using the direct integral equation method. Again, we prove the applicability of the successive approximation method for solving this integral equation.

Keywords: Stokes system; third problem; single layer potential; double layer potential; integral equation method; successive approximation

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Abbreviated title: The third problem for the Stokes system

1 Introduction

One traditional way how to study boundary value problems for the Stokes system is the integral equation method (see [20], [26], [1], [3], [14], [15], [16], [17], [27]). The most papers and books study the Dirichlet problem (see for example [33], [19], [5], [10], [24], [25],[22], [8], [12], [29]). Lately the Neumann problem for the Stokes system has been also studied (see [13], [11], [2], [21], [18]).

In the present paper we study the third problem for the Stokes system

$$\nabla p - \Delta \mathbf{u} = 0 \quad \text{in} \quad G, \qquad \nabla \cdot \mathbf{u} = 0 \quad \text{in} \quad G, \tag{1}$$

$$T(\mathbf{u}, p)\mathbf{n}^G + A\mathbf{u} = \mathbf{g} \quad \text{on} \quad \partial G \tag{2}$$

using methods of hydrodynamical potential theory. Here $G \subset R^3$ is a bounded domain with connected boundary ∂G of class $C^{1,\alpha}$, $0 < \alpha < 1$, $\mathbf{n}^G$ is the outward unit normal vector of G, $\mathbf{u} = (u_1, u_2, u_3)$ is a velocity field, p is a pressure and

$$T(\mathbf{u}, p) \equiv [\nabla u + (\nabla u)^T] - pI$$

is the corresponding stress tensor. (Here I denotes the identity matrix.) The matrix function A has all entries $a_{ij} \in C^\alpha(\partial G, R^3)$, $A(\mathbf{x})$ is a symmetric matrix of type 3×3 for each $\mathbf{x}$. We suppose moreover that there are a nonnegative continuous function $c(\mathbf{x})$ and a constant C such that

$$c(\mathbf{x})|\mathbf{v}|^2 \le \mathbf{v} \cdot A(\mathbf{x})\mathbf{v} \le C|\mathbf{v}|^2 \tag{3}$$

for all $\mathbf{v} \in R^3$ and $\mathbf{x} \in \partial G$. We shall suppose moreover that there is $\mathbf{z} \in \partial G$ such that $c(\mathbf{z}) > 0$. Remark that

$$\nabla p = \left(\frac{\partial p}{\partial x_1}, \frac{\partial p}{\partial x_2}, \frac{\partial p}{\partial x_3} \right), \quad \Delta \mathbf{u} = \sum_{j=1}^{3} \frac{\partial^2 \mathbf{u}}{\partial x_j^2}, \quad \nabla \cdot \mathbf{u} = \sum_{j=1}^{3} \frac{\partial u_j}{\partial x_j}.$$

We shall show that for each $\mathbf{g} \in C^\alpha(\partial G, R^3)$ there is unique classical solution of the problem (1), (2). We also show that for each $\mathbf{g} \in L^s(\partial G, R^3)$, $1 < s < \infty$, there is unique L^s-solution of the problem (1), (2), i.e. a solution of the Stokes system (1) such that the nontangential maximal functions of p, $\mathbf{u}$, $\nabla \mathbf{u}$ are in $L^s(\partial G, R^1)$ and the condition (2) is fulfilled in the sense of the nontagential limit (see § 2).

We prove the existence of a solution using the indirect integral equation method. We look for a velocity $\mathbf{u}$ in the form of a hydrodynamical single layer potential $E_G \mathbf{\Psi}$ with an unknown density $\mathbf{\Psi}$ and a pressure p in the form of the corresponding pressure $Q_G \mathbf{\Psi}$. (For the definition of these potentials see §3.) This method is an analogy of the method for studying of the Neumann problem for the Laplace equation. If one looks for a solution of the Neumann problem for the Laplace equation with the boundary condition f in the form of a harmonic potential $S\varphi$ with an unknown density φ then one obtains the integral equation $(1/2)\varphi + K'_\Delta \varphi = f$. It is a classical result that for G convex we can obtain a solution of this integral equation using the successive approximation method. In 2001 O. Steinbach and W. L. Wendland proved a charming result that this is true also for a bounded domain $G \subset R^3$ with connected Lipschitz boundary and a boundary condition f from the Sobolev space $H^{1/2}(\partial G)$ (see [31]). Later they used this result in studying the Neumann problem for the Laplace equation by the more popular direct integral equations method (see [30], [9]). This method utilizes the representation of the solution in the form $u = Sf + D_\Delta u$, where $D_\Delta u$ is the harmonic double layer potential with density u. This leads to the integral equation $(1/2)u + K_\Delta u = Sf$. Since both integral equations are adjoint they deduced that we can obtain a solution of the integral equation $(1/2)u + K_\Delta u = Sf$ using the successive approximation method. The author studied in [21] classical and L^s-solutions the Neumann problem for the Stokes system on domains with boundary of class $C^{1,\alpha}$ using the indirect integral equation method. A solution has been looked for in the form of a hydrodynamical single layer potential. For a bounded domain with connected boundary it was shown that a solution of the corresponding integral equation $(1/2)\mathbf{\Psi} + K'\mathbf{\Psi} = \mathbf{g}$ can be obtained by the successive approximation method. In the present paper we look for classical and L^s solutions of the Robin problem for the Stokes system in the form of a hydrodynamical single layer potential $E_G \mathbf{\Psi}$ with an unknown density $\mathbf{\Psi}$. It is shown that the unique solution of the corresponding integral equation $(1/2)\mathbf{\Psi} + K'\mathbf{\Psi} + AE_G\mathbf{\Psi} = \mathbf{g}$ can be obtained by the successive approximation method. Then we turn to the direct integral equation method. This method depends on the representation of the solution by $\mathbf{u} = E_G[T(\mathbf{u}, p)\mathbf{n}^G] + D_G \mathbf{u} =$

$E_G[\mathbf{g} - A\mathbf{u}] + D_G\mathbf{u}$, $p = Q_G[T(\mathbf{u}, p)\mathbf{n}^G] + P_G\mathbf{u} = Q_G[\mathbf{g} - A\mathbf{u}] + P_G\mathbf{u}$ (for the definitions of the corresponding potentials see § 3). This representation leads to the integral equation $(1/2)\mathbf{u} + K\mathbf{u} + E_G A\mathbf{u} = E_G\mathbf{g}$. We shall show that the unique solution of this integral equation can be obtained by the successive approximation method.

2 Formulation of the problem

Starting from now, throughout the paper $G \subset R^3$ denotes a bounded domain with connected boundary ∂G of class $C^{1,\alpha}$, $0 < \alpha < 1$, and $G^e := R^3 \setminus \operatorname{cl} G$ denotes its complement with $\partial G^e = \partial G$. Here $\operatorname{cl} G$ denotes the closure of G and ∂G the boundary of G.

If $K = R^n$ or $K = C^n$ we denote by $C^0(H, K)$ the space of all continuous functions from H to K. Similarly $C^\beta(H, K)$ denotes the space of all K-valued β-Hölder functions on H for $0 < \beta < 1$. If $k \in N$ then $C^k(H, K)$ denotes the space of all functions the derivatives of which up to the order k are from $C^0(H, K)$ and $C^\infty(H, K) = \cap\{C^k(H, K); k \in N\}$. If $1 \le q < \infty$ then $L^q(H, K)$ denotes the space of all K-valued Borel measurable functions $\mathbf{f}$ for which $|\mathbf{f}|^q$ is integrable in H.

$(\mathbf{u}, p)$ is called a classical solution of the problem (1), (2) if $p \in C^1(G, R) \cap C^0(\operatorname{cl} G, R)$, $\mathbf{u} \in C^2(G, R^3) \cap C^1(\operatorname{cl} G, R^3)$ satisfy (1), (2).

We shall also study some classes of strong solutions of the third problem for the Stokes system with a boundary condition $\mathbf{g} \in L^s(\partial G, R^3)$.

If $\mathbf{x} \in \partial G$, $a > 0$ denote the non-tangential approach regions of opening a at the point $\mathbf{x}$ by

$$\Gamma_a(\mathbf{x}) := \{\mathbf{y} \in G; |\mathbf{x} - \mathbf{y}| < (1 + a)\operatorname{dist}(\mathbf{y}, \partial G)\}.$$

We fix $a > 0$ large enough such that $\mathbf{x} \in \operatorname{cl} \Gamma_a(\mathbf{x})$ for every $\mathbf{x} \in \partial G$. We shall write $\Gamma(\mathbf{x}) = \Gamma_a(\mathbf{x})$. If now $\mathbf{v}$ is a vector function defined in G we denote the non-tangential maximal function of $\mathbf{v}$ on ∂G by

$$\mathbf{v}^*(\mathbf{x}) := \sup\{|\mathbf{v}(\mathbf{y})|; \mathbf{y} \in \Gamma(\mathbf{x})\}.$$

If $\mathbf{x} \in \partial G$ then

$$\mathbf{v}(\mathbf{x}) = \lim_{\substack{\mathbf{y} \to \mathbf{x} \\ \mathbf{y} \in \Gamma(\mathbf{x})}} \mathbf{v}(\mathbf{y})$$

is the non-tangential limit of $\mathbf{v}$ with respect to G at $\mathbf{x}$.

If $\mathbf{g} \in L^s(\partial G, R^3)$, $1 < s < \infty$, then we call $\mathbf{u}, p$ an L^s-solution of the problem (1), (2) if $\mathbf{u} \in C^2(G)$, $p \in C^1(G)$ satisfy (1); $\mathbf{u}^*, (\nabla \mathbf{u})^*, p^* \in L^s(\partial G, R^1)$; for almost all $\mathbf{x} \in \partial G$ there exist the non-tangential limits of $\mathbf{u}$, $\nabla \mathbf{u}$ and p at $\mathbf{x}$ and the condition (2) is fulfilled in the sense of the nontangential limit a.e. on ∂G.

3 Hydrodynamical potentials

For $\mathbf{x} = [x_1, x_2, x_3] \in R^3$and $j, k = 1, 2, 3$ define

$$E_{j,k}(\mathbf{x}) = \frac{1}{8\pi}\left[\delta_{jk}\frac{1}{|\mathbf{x}|} + \frac{x_j x_k}{|\mathbf{x}|^3}\right] \tag{4}$$

where δ_{jk} is the Kronecker delta. For $\mathbf{\Psi} = [\Psi_1, \Psi_2, \Psi_3] \in L^1(\partial G, R^3)$ define the hydrodynamical single layer potential with density $\mathbf{\Psi}$ by

$$E_G\mathbf{\Psi}(\mathbf{x}) = \int\limits_{\partial G} E(\mathbf{x} - \mathbf{y})\mathbf{\Psi}(\mathbf{y})\; d\mathbf{y}$$

and the corresponding pressure

$$Q_G\mathbf{\Psi}(\mathbf{x}) = \int\limits_{\partial G} \frac{(\mathbf{x} - \mathbf{y}) \cdot \mathbf{\Psi}(\mathbf{y})}{4\pi|\mathbf{x} - \mathbf{y}|^3}\; d\mathbf{y}.$$

Then $E_G\mathbf{\Psi} \in C^\infty(R^3 \setminus \partial G, R^3)$, $Q_G\mathbf{\Psi} \in C^\infty(R^3 \setminus \partial G, R^1)$. If we put $\mathbf{u} = E_G\mathbf{\Psi}$, $p = Q_G\mathbf{\Psi}$ then $\mathbf{u}$, p solve the Stokes system (1). If $\mathbf{\Psi} \in \mathcal{C}^0(\partial G, R^3)$ then $E_G\mathbf{\Psi} \in C^0(R^3, R^3)$ and $E_G\mathbf{\Psi} \in C^\alpha(\partial G, R^3)$. If, in addition, $\mathbf{\Psi} \in C^\alpha(\partial G, R^3)$ then $\nabla E_G\mathbf{\Psi}$ and $Q_G\mathbf{\Psi}$ can be continuously extended onto $\mathrm{cl}\, G$ and onto $\mathrm{cl}\, G^e$, too (see [26]).

If $\mathbf{\Psi} \in L^s(\partial G, R^3)$, $1 < s < \infty$, then the nontangential maximal operators $(E_G\mathbf{\Psi})^*, (\nabla E_G\mathbf{\Psi})^*, (Q_G\mathbf{\Psi})^* \in L^s(\partial G, R^1)$ and there is a constant M dependent only on G and s such that

$$\|(E_G\mathbf{\Psi})^* + (\nabla E_G\mathbf{\Psi})^* + (Q_G\mathbf{\Psi})^*\|_{L^s(\partial G)} \le M\|\mathbf{\Psi}\|_{L^s(\partial G)}. \tag{5}$$

Moreover, there are the nontangential limits of $E_G\mathbf{\Psi}$, $\nabla E_G\mathbf{\Psi}$ and $Q_G\mathbf{\Psi}$ a.e. on ∂G and $E_G\mathbf{\Psi}(\mathbf{x})$ is the nontangential limit of $E_G\mathbf{\Psi}$ for almost all $\mathbf{x} \in \partial G$ (see [21] or [20]).

Remark that for the unit outward normal vector $\mathbf{n}^G$ we have $E_G\mathbf{n}^G = 0$ in R^3, $Q_G\mathbf{n}^G = -1$ in G, $Q_G\mathbf{n}^G = 0$ in G^e. (Compare [33].)

For p, $\mathbf{u} = (u_1, u_2, u_3)$ define the stress tensor

$$T(\mathbf{u}, p) = 2\hat{\nabla}\mathbf{u} - pI, \tag{6}$$

where I denotes the identity matrix and

$$\hat{\nabla}\mathbf{u} = \frac{1}{2}[\nabla\mathbf{u} + (\nabla\mathbf{u})^T]$$

is the strain tensor, with $(\nabla\mathbf{u})^T$ as the matrix transposed to $\nabla\mathbf{u} = (\partial_j u_k)$, $(k, j = 1, 2, 3)$.

If $\mathbf{\Psi} \in L^s(\partial G, R^3)$, $1 < s < \infty$ denote

$$K'\mathbf{\Psi}(\mathbf{x}) = \frac{3}{4\pi} \lim_{\epsilon \searrow 0} \int\limits_{\partial G \setminus B(\mathbf{x};\epsilon)} \frac{[(\mathbf{y}-\mathbf{x}) \cdot \mathbf{n}^G(\mathbf{x})][(\mathbf{x}-\mathbf{y}) \cdot \mathbf{\Psi}(\mathbf{y})](\mathbf{x}-\mathbf{y})}{|\mathbf{x}-\mathbf{y}|^5} \, d\mathbf{y}$$

whenever this limit has a sense. (Here $B(\mathbf{x};\epsilon)$ denotes the open ball with the center $\mathbf{x}$ and the radius ϵ.) Then K' is a compact linear operator in the spaces $L^s(\partial G, R^3)$, $1 < s < \infty$, and $C^\alpha(\partial G, R^3)$ (see [20], [21] and [33]). If $\mathbf{\Psi} \in C^\alpha(\partial G, R^3)$ then

$$T(E_G\mathbf{\Psi}, Q_G\mathbf{\Psi})\mathbf{n}^G = \left(\frac{1}{2}I + K'\right)\mathbf{\Psi}, \tag{7}$$

where I denotes the identity operator (see [26]). If $\mathbf{\Psi} \in L^s(\partial G, R^3)$ then (7) holds true in the sense of the nontangential limit at almost all point of ∂G (see [21], [20], or [4]).

For $\mathbf{y} \in \partial G$ and $\mathbf{x} \in R^3 \setminus \{\mathbf{y}\}$ denote

$$D_{k,j}(\mathbf{x}, \mathbf{y}) = -\frac{3}{4\pi} \frac{(x_k - y_k)(x_j - y_j)(\mathbf{x}-\mathbf{y}) \cdot \mathbf{n}^G(\mathbf{y})}{|\mathbf{x}-\mathbf{y}|^5} \tag{8}$$

for $j, k = 1, 2, 3$.

For $\mathbf{\Psi} = [\Psi_1, \Psi_2, \Psi_3] \in L^1(\partial G, R^3)$ define the hydrodynamical double layer potential with density $\mathbf{\Psi}$ by

$$(D_G\mathbf{\Psi})(\mathbf{x}) = \int\limits_{\partial G} D(\mathbf{x}, \mathbf{y})\mathbf{\Psi}(\mathbf{y}) \, d\mathbf{y}, \qquad \mathbf{x} \in R^3 \setminus \partial G \tag{9}$$

and by

$$(P_G\mathbf{\Psi})(\mathbf{x}) = -\frac{1}{2\pi} \int\limits_{\partial G} \left\{ 3\,\frac{(x_j - y_j)(\mathbf{x}-\mathbf{y}) \cdot \mathbf{n}^G(\mathbf{y})}{|\mathbf{x}-\mathbf{y}|^5} - \frac{\mathbf{n}_j^G(\mathbf{y})}{|\mathbf{x}-\mathbf{y}|^3} \right\} \mathbf{\Psi}(\mathbf{y}) \, d\mathbf{y} \tag{10}$$

the corresponding pressure. Then $\mathbf{u} = D_G\mathbf{\Psi} \in C^\infty(R^3 \setminus \partial G, R^3)$, $p = P_G\mathbf{\Psi} \in C^\infty(R^3 \setminus \partial G, R^3)$ solve the Stokes system (1) in $R^3 \setminus \partial G$.

If $\mathbf{\Psi} \in L^s(\partial G, R^3)$, $1 < s < \infty$ denote

$$K\mathbf{\Psi}(\mathbf{x}) = -\lim_{\epsilon \searrow 0} \int\limits_{\partial G \setminus B(\mathbf{x};\epsilon)} D(\mathbf{x}, \mathbf{y})\mathbf{\Psi}(\mathbf{y}) \, d\mathbf{y}$$

whenever this limit has a sense. Then K is a compact linear operator in the spaces $L^s(\partial G, R^3)$, $1 < s < \infty$, and $C^\alpha(\partial G, R^3)$ (see [20], [21] or [33]). Remark that K and K' are adjoint operators. If $\mathbf{\Psi} \in C^\alpha(\partial G, R^3)$ then

$$\lim_{\substack{\mathbf{z} \to \mathbf{x} \\ \mathbf{z} \in G}} D_G\mathbf{\Psi}(\mathbf{x}) = \frac{1}{2}\mathbf{\Psi}(\mathbf{z}) - K\mathbf{\Psi}(\mathbf{z}), \quad \lim_{\substack{\mathbf{z} \to \mathbf{x} \\ \mathbf{z} \in G^e}} D_G\mathbf{\Psi}(\mathbf{x}) = -\frac{1}{2}\mathbf{\Psi}(\mathbf{z}) - K\mathbf{\Psi}(\mathbf{z}). \tag{11}$$

If $\mathbf{\Psi} \in L^s(\partial G, R^3)$, $1 < s < \infty$ then $(D_G \mathbf{\Psi})^* \in L^s(\partial G, R^1)$ and the relation (11) holds in the sense of the nontangential limit at almost all points of ∂G (see [20] and [33]).

4 The integral representation

We shall prove the existence of a solution of the problem (1), (2) using the indirect integral equation method, i.e. we shall look for a solution in the form

$$\mathbf{u} = E_G \mathbf{\Psi}, \qquad p = Q_G \mathbf{\Psi} \tag{12}$$

with an unknown density $\mathbf{\Psi}$. If we look for a classical solution for the boundary condition $\mathbf{g} \in C^\alpha(\partial G, R^3)$ we have $\mathbf{\Psi} \in C^\alpha(\partial G, R^3)$. If we look for an L^s-solution for $\mathbf{g} \in L^s(\partial G, R^3)$, $1 < s < \infty$, we have $\mathbf{\Psi} \in L^s(\partial G, R^3)$. Using the boundary properties of potentials we obtain that $(\mathbf{u}, p)$ is a solution of the of the problem (1), (2) if and only if $\frac{1}{2}\mathbf{\Psi} + K'\mathbf{\Psi} + AE_G\mathbf{\Psi} = \mathbf{g}$.

Lemma 4.1. *The operator $K' + AE_G$ is compact in $C^\alpha(\partial G, R^3)$ and in the spaces $L^s(\partial G, R^3)$, $1 < s < \infty$.*

Proof. Suppose that $X = C^\alpha(\partial G, R^3)$ or $X = L^s(\partial G, R^3)$, $1 < s < \infty$. The operator K' is compact in X by [21], Lemma 9. The operator E_G is compact in $L^s(\partial G, R^3)$, $1 < s < \infty$, by [22], Lemma 4.4. The operator E_G is a bounded linear operator from $C^0(\partial G, R^3)$ to $C^\alpha(\partial G, R^3)$ by [26]. Since the $J : \mathbf{\Psi} \mapsto \mathbf{\Psi}$ is a compact operator from $C^\alpha(\partial G, R^3)$ to $C^0(\partial G, R^3)$, the operator $E_G = E_G J$ is a compact operator in $C^\alpha(\partial G, R^3)$ as a composition of a bounded operator and a compact operator. Since $\mathbf{\Psi} \mapsto A\mathbf{\Psi}$ is a bounded linear operator in X, the operator AE_G is a compact linear operator in X (see [28], Corollary 4.5 and [28], Theorem 4.6). Therefore $K' + AE_G$ is a compact linear operator in X.

Lemma 4.2. *Let $(\mathbf{u}, p)$ and $(\mathbf{v}, q)$ be two solutions of (1). Suppose moreover that $\mathbf{u}, \mathbf{v} \in C^1(\operatorname{cl} G, R^3)$ and $p, q \in C^0(\operatorname{cl} G, R^1)$. Then*

$$\int_{\partial G} (\mathbf{u} - i\mathbf{v}) \cdot \{[T(\mathbf{u}, p)\mathbf{n}^G + A\mathbf{u}] + i[T(\mathbf{v}, q)\mathbf{n}^G + A\mathbf{v}]\} \, d\mathbf{y}$$

$$= 2 \int_G [|\hat{\nabla}\mathbf{u}|^2 + |\hat{\nabla}\mathbf{v}|^2] \, d\mathbf{y} + \int_{\partial G} [\mathbf{u} \cdot A\mathbf{u} + \mathbf{v} \cdot A\mathbf{v}] \, d\mathbf{y} \geq 0.$$

If

$$\int_{\partial G} (\mathbf{u} - i\mathbf{v}) \cdot \{[T(\mathbf{u}, p)\mathbf{n}^G + A\mathbf{u}] + i[T(\mathbf{v}, q)\mathbf{n}^G + A\mathbf{v}]\} \, d\mathbf{y} = 0. \tag{13}$$

then $\mathbf{u} \equiv 0$, $\mathbf{v} \equiv 0$.

Proof. Green's formula (compare [21], Lemma 3 or [32]), Fubini's theorem and the symmetry of matrix A give

$$\int\limits_{\partial G} (\mathbf{u} - i\mathbf{v}) \cdot \{[T(\mathbf{u}, p)\mathbf{n}^G + A\mathbf{u}] + i[T(\mathbf{v}, q)\mathbf{n}^G + A\mathbf{v}]\} \; d\mathbf{y}$$

$$= 2 \int\limits_G [|\hat{\nabla}\mathbf{u}|^2 + |\hat{\nabla}\mathbf{v}|^2] \; d\mathbf{y} + \int\limits_{\partial G} [\mathbf{u} \cdot A\mathbf{u} + \mathbf{v} \cdot A\mathbf{v}] \; d\mathbf{y}$$

$$\geq 2 \int\limits_G [|\hat{\nabla}\mathbf{u}|^2 + \hat{\nabla}\mathbf{v}|^2] \; d\mathbf{y} + \int\limits_{\partial G} c(\mathbf{y})[|\mathbf{u}|^2 + |\mathbf{v}|^2] \; d\mathbf{y} \geq 0.$$

Suppose that (13) holds true. Then $\hat{\nabla}\mathbf{u} = 0$ in G, $c(\mathbf{y})|\mathbf{u}(\mathbf{y})|^2 = 0$ on ∂G. Since $\hat{\nabla}\mathbf{u} = 0$ in G, the vector function $\mathbf{u}$ is a rigid body motions, i.e $\mathbf{u}(\mathbf{y}) = B\mathbf{y} + \mathbf{b}$ with a vector $\mathbf{b}$ and a skew-symmetric matrix $B = (b_{jk})$ (see [21], Lemma 6). We have supposed that there is $\mathbf{z} \in \partial G$ such that $c(\mathbf{z}) > 0$. According to the continuity of c there is $r > 0$ such that $c(\mathbf{y}) > 0$ for $\mathbf{y} \in B(\mathbf{z}; r) \cap \partial G$. Since $c(\mathbf{y})|\mathbf{u}(\mathbf{y})|^2 = 0$ on ∂G we infer that $\mathbf{u}(\mathbf{y}) = 0$ on $B(\mathbf{z}; r) \cap \partial G$. Suppose first that $b_{jk} \neq 0$ for some j, k. Since B is a skew-symmetric matrix we have $b_{kj} = -b_{jk}$ and thus $k \neq j$. Put $L_j = \{\mathbf{y} = [y_1, y_2, y_3]; b_{j1}y_1 + b_{j2}y_2 + b_{j3}y_3 + b_j = 0\}$, $L_k = \{\mathbf{y} = [y_1, y_2, y_3]; b_{k1}y_1 + b_{k2}y_2 + b_{k3}y_3 + b_k = 0\}$. Since $b_{jk} = -b_{kj} \neq 0$, $b_{jj} = b_{kk} = 0$, the hyperplanes L_j, L_k are different. Since $u_j(\mathbf{y}) = b_{j1}y_1 + b_{j2}y_2 + b_{j3}y_3 + b_j$, $u_k(\mathbf{y}) = b_{k1}y_1 + b_{k2}y_2 + b_{k3}y_3 + b_k$ and $\mathbf{u}(\mathbf{y}) = 0$ on $B(\mathbf{z}; r) \cap \partial G$, we infer that $B(\mathbf{z}; r) \cap \partial G \subset L_j \cap L_k$. But $B(\mathbf{z}; r) \cap \partial G$ cannot be a subset of the line $L_j \cap L_k$. This means that $b_{jk} = 0$ for all j, k. Hence $\mathbf{u}$ is constant. Since $\mathbf{u}(\mathbf{y}) = 0$ on $B(\mathbf{x}; r) \cap \partial G$ we deduce that $\mathbf{u} \equiv 0$. Similarly for $\mathbf{v}$.

Theorem 4.3. *The operator $\frac{1}{2}I + K' + AE_G$ is continuously invertible in $C^\alpha(\partial G, R^3)$ and in $L^s(\partial G, R^3)$, $1 < s < \infty$. Fix $\mathbf{g} \in L^s(\partial G, R^3)$, $1 < s < \infty$. Then there is unique L^s-solution $\mathbf{u}$, p of the problem (1), (2). Putting $\mathbf{\Psi} = (\frac{1}{2}I + K' + AE_G)^{-1}\mathbf{g}$ we have $\mathbf{u} = E_G\mathbf{\Psi}$, $p = Q_G\mathbf{\Psi}$. If $\mathbf{g} \in C^\alpha(\partial G, R^3)$ then $\mathbf{\Psi} \in C^\alpha(\partial G, R^3)$ and $\mathbf{u}$, p form a classical solution of the problem (1), (2).*

Proof. Suppose first that $\mathbf{u}$, p form a classical solution of the problem (1), (2) with $g \equiv 0$. Since

$$\int\limits_{\partial G} \mathbf{u} \cdot [T(\mathbf{u}, p)\mathbf{n}^G + A\mathbf{u}] \; d\mathbf{y} = 0$$

Lemma 4.2 gives that $\mathbf{u} \equiv 0$. Since $\mathbf{u}$, p solve (1) we have $\nabla p = \Delta \mathbf{u} = 0$. So, p is constant. From the boundary condition we get $0 = T(\mathbf{u}, p)\mathbf{n}^G + A\mathbf{u} = -p$.

Let now $\mathbf{\Psi} \in C^\alpha(\partial G, R^3)$ be such that $(\frac{1}{2}I + K' + AE_G)\mathbf{\Psi} = 0$. Then $\mathbf{u} = E_G\mathbf{\Psi}$, $p = Q_G\mathbf{\Psi}$ solve the problem (1), (2) with $\mathbf{g} \equiv 0$. We have proved that $\mathbf{u} = 0$, $p = 0$. Therefore

$$0 = T(\mathbf{u}, p)\mathbf{n}^G + A\mathbf{u} = T(\mathbf{u}, p)\mathbf{n}^G = \frac{1}{2}\mathbf{\Psi} + K'\mathbf{\Psi}.$$

According to [21], Lemma 11 we have $\mathbf{\Psi} = 0$. Since $K' + AE_G$ is a compact operator in $C^\alpha(\partial G, R^3)$ and in $L^s(\partial G, R^3)$, the operator $\frac{1}{2}I + K' + AE_G$ has the same kernel in both spaces by [23], Lemma 2.1. Since the operator $\frac{1}{2}I + K' + AE_G$ is one-to-one in $C^\alpha(\partial G, R^3)$ and in $L^s(\partial G, R^3)$, the Riesz-Schauder theorem gives that $\frac{1}{2}I + K' + AE_G$ is continuously invertible in $C^\alpha(\partial G, R^3)$ and in $L^s(\partial G, R^3)$ (see [34], Chapter X, §5, Theorem 1). If $\mathbf{\Psi} = (\frac{1}{2}I + K' + AE_G)^{-1}\mathbf{g}$ and $\mathbf{u} = E_G\mathbf{\Psi}$, $p = Q_G\mathbf{\Psi}$, then $\mathbf{u}$, p solve the problem (1), (2).

We now show the uniqueness of an L^s-solution. If $\mathbf{u}$, p form a solution of the problem (1), (2) then $\mathbf{u}$, p solve the Neumann problem for the Stokes system

$$\nabla p - \Delta \mathbf{u} = 0 \quad \text{in} \quad G, \qquad \nabla \cdot \mathbf{u} = 0 \quad \text{in} \quad G,$$

$$T(\mathbf{u}, p)\mathbf{n}^G = \mathbf{h} \quad \text{on} \quad \partial G$$

where $\mathbf{h} = \mathbf{g} - A\mathbf{u}$. According [21], Theorem 4 and [21], Theorem 5 there is $\mathbf{\Psi} \in L^s(\partial G, R^3)$ such that $\mathbf{u} = E_G\mathbf{\Psi}$, $p = E_G\mathbf{\Psi}$. If $\mathbf{g} = 0$ then $(\frac{1}{2}I + K' + AE_G)\mathbf{\Psi} = 0$ and the invertibility of $\frac{1}{2}I + K' + AE_G$ forces $\mathbf{\Psi} = 0$. Therefore $\mathbf{u} = E_G\mathbf{\Psi} = 0$, $p = Q_G\mathbf{\Psi} = 0$.

5 Solution of the integral equation

In this section we estimate the spectrum of the operator $\frac{1}{2}I + K' + AE_G$ and calculate a solution of the equation $(\frac{1}{2}I + K' + AE_G)\mathbf{\Psi} = \mathbf{g}$ using the successive approximation method. For this reason we shall rewrite this equation onto the equation $\mathbf{\Psi} - [I - \gamma^{-1}(\frac{1}{2}I + K' + AE_G)]\mathbf{\Psi} = \gamma^{-1}\mathbf{g}$ and show that there is an equivalent norm $\| \ \|$ such that $\|I - \gamma^{-1}(\frac{1}{2}I + K' + AE_G)\| < 1$.

We need the following Lemma 5.1, Lemma 5.2 proved in [22] and Lemma 5.3 proved in [7]. If $c = c_1 + ic_2$ is a complex number, denote by $\overline{c} = c_1 - ic_2$ its complex conjugate.

Lemma 5.1. *Let $\Psi \in C^\alpha(\partial G, C^3)$. Then we have*

$$\int\limits_{\partial G} \overline{\mathbf{\Psi}} \cdot E_G\mathbf{\Psi} \, d\mathbf{y} = 2 \int\limits_{R^3 \setminus \partial G} |\hat{\nabla} E_G \mathbf{\Psi}|^2 \, d\mathbf{x} \geq 0.$$

If

$$\int\limits_{\partial G} \overline{\mathbf{\Psi}} \cdot (E_G\mathbf{\Psi}) \, d\mathbf{y} = 0,$$

then $Q_G\mathbf{\Psi}$ is constant on each component of $R^3 \setminus \partial G$ and $E_G\mathbf{\Psi} = 0$ in R^3.

Lemma 5.2. *If we denote*

$$\beta_0 := \max_{j=1,2,3} \sup_{\mathbf{x}\in\partial G} \int\limits_{\partial G} \sum_{k=1}^{3} |E_{jk}(\mathbf{x}-\mathbf{y})| \; d\mathbf{y}, \tag{14}$$

then

$$\|E_G\|_{L^2(\partial G)} \le \beta_0 < \infty. \tag{15}$$

If $\Psi \in L^2(\partial G, C^3)$*, then*

$$\int\limits_{\partial G} |E_G \boldsymbol{\Psi}|^2 \; d\mathbf{y} \le \beta_0 \int\limits_{\partial G} \boldsymbol{\Psi} \cdot (E_G \overline{\boldsymbol{\Psi}}) \; d\mathbf{y}. \tag{16}$$

If M *is a positive constant with*

$$\int\limits_{\{\mathbf{y}\in\partial G; |\mathbf{x}-\mathbf{y}|<r\}} 1 \; d\mathbf{y} \le M r^2 \tag{17}$$

for each $\mathbf{x} \in \partial G$ *and* $0 < r < \operatorname{diam} \partial G$*, then*

$$\beta_0 \le M \operatorname{diam} \partial G.$$

(Here $\operatorname{diam} \partial G = \sup\{|\mathbf{x}-\mathbf{y}|; \mathbf{x}, \mathbf{y} \in \partial G\}$ *is the diameter of* ∂G*.)*

Lemma 5.3. *Let* X *be a complex Banach space. Denote by* $\mathcal{N}$ *the set of all norms on* X *equivalent to the original norm. If* S *is a bounded linear operator in* X *denote by* $\sigma(S)$ *the spectrum of* S *and*

$$r(S) = \sup\{|\lambda|; \lambda \in \sigma(S)\}$$

the spectral radius of S*. Then*

$$r(S) = \inf_{\|\ \|\in\mathcal{N}} \|S\|.$$

Lemma 5.4. *Let* β_0 *be given by (14) and* C *is the constant from the inequality (3). If* $\boldsymbol{\Psi} \in C^\alpha(\partial G, R^3)$ *then*

$$0 \le \int\limits_{\partial G} (E_G \boldsymbol{\Psi}) \cdot \left(\frac{1}{2} I + K' + A E_G\right) \boldsymbol{\Psi} \; d\mathbf{y} \le (1 + C\beta_0) \int\limits_{\partial G} \boldsymbol{\Psi} \cdot E_G \boldsymbol{\Psi} \; d\mathbf{y}.$$

Proof. According to Lemma 4.2, Lemma 5.1 and Lemma 5.2 we have

$$0 \le \int\limits_{\partial G} E_G \boldsymbol{\Psi} \cdot \left(\frac{1}{2} I + K' + A E_G\right) \boldsymbol{\Psi} \; d\mathbf{y} = 2 \int\limits_{G} |\hat{\nabla} E_G \boldsymbol{\Psi}|^2 \; d\mathbf{y} + \int\limits_{\partial G} (E_G \boldsymbol{\Psi}) \cdot (A E_G \boldsymbol{\Psi}) \; d\mathbf{y}$$

$$\leq 2 \int\limits_{R^3 \setminus \partial G} |\hat{\nabla} E_G \mathbf{\Psi}|^2 \, d\mathbf{y} + \int\limits_{\partial G} C |E_G \mathbf{\Psi}|^2 \, d\mathbf{y} \leq (1 + C\beta_0) \int\limits_{\partial G} \mathbf{\Psi} \cdot E_G \mathbf{\Psi} \, d\mathbf{y}.$$

Proposition 5.5. *Let β_0 be given by (14). Then $\sigma(\frac{1}{2}I + K' + AE_G) \subset (0, 1 + C\beta_0\rangle$ in $C^\alpha(\partial G, C^3)$ and in $L^s(\partial G, C^3)$ for $1 < s < \infty$.*

Proof. Suppose that $X = C^\alpha(\partial G, C^3)$ or $X = L^s(\partial G, C^3)$, $1 < s < \infty$. Let $\lambda \in \sigma(\frac{1}{2}I + K' + AE_G) \setminus \{\frac{1}{2}\}$. Since $K' + AE_G$ is a compact operator in X by Lemma 4.1, Riesz-Schauder theorem gives that λ is an eigenvalue of $\frac{1}{2}I + K' + AE_G$ in X (see [34], Chapter X, §5, Theorem 1). Since $K' + AE_G$ is a compact operator in $C^\alpha(\partial G, C^3)$ and in $L^s(\partial G, C^3)$, the kernel of $\frac{1}{2}I + K' + AE_G - \lambda I$ is the same in both spaces (see [23], Lemma 2.1). It means that there is $\mathbf{\Psi} \in C^\alpha(\partial G, C^3)$ such that $(\frac{1}{2}I + K' + AE_G)\mathbf{\Psi} = \lambda \mathbf{\Psi}$.

Suppose first that

$$\int\limits_{\partial G} \mathbf{\Psi} \cdot E_G \overline{\mathbf{\Psi}} \, d\mathbf{y} = 0.$$

Then $E_G \mathbf{\Psi} = 0$ in R^3 and $Q_G \mathbf{\Psi} = a_1$ in G, $Q_G \mathbf{\Psi} = a_2$ in G^e (see Lemma 5.1). According to [21], Lemma 10 there is a constant d such that $\mathbf{\Psi} = d\mathbf{n}^G$. Since $E_G \mathbf{n}^G = 0$, $Q_G \mathbf{n}^G = -1$ in G, we have $(\frac{1}{2}I + K' + AE_G)\mathbf{\Psi} = d\mathbf{n}^G = \mathbf{\Psi}$ and $\lambda = 1$.

Let now

$$\int\limits_{\partial G} \mathbf{\Psi} \cdot E_G \overline{\mathbf{\Psi}} \, d\mathbf{y} \neq 0.$$

According to Lemma 4.2

$$\lambda \int\limits_{\partial G} \mathbf{\Psi} \cdot E_G \overline{\mathbf{\Psi}} \, d\mathbf{y} = \int\limits_{\partial G} (E_G \overline{\mathbf{\Psi}}) \cdot \left(\frac{1}{2}I + K' + AE_G\right) \mathbf{\Psi} \, d\mathbf{y}$$

$$= \int\limits_{\partial G} (E_G \overline{\mathbf{\Psi}}) \cdot [T(E_G \mathbf{\Psi}, Q_G \mathbf{\Psi})\mathbf{n}^G + AE_G \mathbf{\Psi}] \, d\mathbf{y} \geq 0.$$

Since

$$\int\limits_{\partial G} \mathbf{\Psi} \cdot (E_G \overline{\mathbf{\Psi}}) \, d\mathbf{y} > 0$$

by Lemma 5.1, we obtain

$$\lambda = \int\limits_{\partial G} (E_G \overline{\mathbf{\Psi}}) \cdot \left(\frac{1}{2}I + K' + AE_G\right) \mathbf{\Psi} \, d\mathbf{y} \left[\int\limits_{\partial G} \mathbf{\Psi} \cdot (E_G \overline{\mathbf{\Psi}}) \, d\mathbf{y}\right]^{-1} \geq 0.$$

The invertibility of the operator $\frac{1}{2}I + K' + AE_G$ (see Theorem 4.3) forces that $\lambda > 0$. Since the eigenvalue λ is real we can suppose that $\mathbf{\Psi} \in C^\alpha(\partial G, R^3)$.

Lemma 5.4 gives

$$\lambda = \int\limits_{\partial G} (E_G \mathbf{\Psi}) \cdot \left(\frac{1}{2} I + K' + AE_G\right) \mathbf{\Psi}\ d\mathbf{y} \left[\int\limits_{\partial G} \mathbf{\Psi} \cdot (E_G \mathbf{\Psi})\ d\mathbf{y}\right]^{-1} \leq (1 + C\beta_0).$$

Theorem 5.6. *Let $X = C^\alpha(\partial G, R^3)$ or $X = L^s(\partial G, R^3)$, $1 < s < \infty$, β_0 be given by (14). Fix $\gamma > (1 + C\beta_0)/2$. Denote $L = I - \gamma^{-1}(\frac{1}{2} I + K' + AE_G)$. Then there is an equivalent norm $\| \ \|$ on X such that $\|L\| < 1$. Moreover,*

$$\left(\frac{1}{2} I + K' + AE_G\right)^{-1} = \gamma^{-1} \sum_{j=0}^{\infty} L^j$$

in X. Let now $\mathbf{g} \in X$. Fix $\mathbf{\Psi}_0 \in X$. For nonnegative integer j put

$$\mathbf{\Psi}_{j+1} = L \mathbf{\Psi}_j + \gamma^{-1} \mathbf{g}. \tag{18}$$

Then there is

$$\mathbf{\Psi} = \lim_{j \to \infty} \mathbf{\Psi}_j$$

in X, $[(1/2)I + K' + AE_G]\mathbf{\Psi} = \mathbf{g}$ and

$$\|\mathbf{\Psi} - \mathbf{\Psi}_j\| \leq C q^j [\|\mathbf{g}\| + \|\mathbf{\Psi}_0\|] \tag{19}$$

for arbitrary j. Here constants $C > 0$, $0 < q < 1$ do not depend on $\mathbf{g}$ and $\mathbf{\Psi_0}$.

Proof. Put $Y = C^\alpha(\partial G, C^3)$ for $X = C^\alpha(\partial G, R^3)$, $Y = L^s(\partial G, C^3)$ for $X = L^s(\partial G, R^3)$. According to Proposition 5.5 we have $\sigma(\frac{1}{2} I + K' + AE_G) \subset (0, 1 + C\beta_0\rangle$ in Y and thus $\sigma(L) \subset \langle 1 - \gamma^{-1}(1 + C\beta_0), 1) \subset (-1, 1)$. Since $r(L) < 1$, Lemma 5.3 gives that there is an equivalent norm $\| \ \|$ such that $q = \|L\| < 1$. It is a classical result that

$$(I - L)^{-1} = \sum_{j=0}^{\infty} L^j$$

(see [6], Proposition 9.106), there is

$$\mathbf{\Psi} = \lim_{j \to \infty} \mathbf{\Psi}_j,$$

$(I - L)\mathbf{\Psi} = \gamma^{-1}\mathbf{g}$ (see [6], Theorem 9.128). According to [6], Theorem 9.128 we have

$$\|\mathbf{\Psi} - \mathbf{\Psi}_j\| \leq \frac{q^j}{1-q} \|\mathbf{\Psi}_1 - \mathbf{\Psi}_0\| = \frac{q^j}{1-q} \|(L - I)\mathbf{\Psi}_0 + \gamma^{-1}\mathbf{g}\| \leq q^j \frac{q + 1 + \gamma^{-1}}{1-q} [\|\mathbf{g}\| + \|\mathbf{\Psi}_0\|].$$

Since $[(1/2)I + K' + AE_G] = \gamma(I - L)$ we obtain the proposition of the theorem.

6 Direct boundary integral equation method

Let $(\mathbf{u}, p)$ be an L^s-solution the problem (1), (2), where $1 < s < \infty$. Denote by $\mathbf{u}$ the nontangential limit of $\mathbf{u}$ and by p the nontangential limit of p. Then

$$\mathbf{u} = E_G[T(\mathbf{u}, p)\mathbf{n}^G] + D_G\mathbf{u}, \quad p = Q_G[T(\mathbf{u}, p)\mathbf{n}^G] + P_G\mathbf{u} \qquad \text{in } G,$$

$$\frac{1}{2}\mathbf{u} + K\mathbf{u} = E_G[T(\mathbf{u}, p)\mathbf{n}^G] \qquad \text{on } \partial G$$

(see [21], Proposition 1). According to the boundary condition (2) we get

$$\mathbf{u} = E_G(\mathbf{g} - A\mathbf{u}) + D_G\mathbf{u} \qquad \text{in } G, \tag{20}$$

$$\frac{1}{2}\mathbf{u} + K\mathbf{u} + E_G A\mathbf{u} = E_G\mathbf{g} \qquad \text{on } \partial G. \tag{21}$$

So, we must solve the equation (21). If $\mathbf{g} \in C^\alpha(\partial G, R^3)$ then $(\mathbf{u}, p)$ is a classical solution of the problem (1), (2) and we can solve the equation (21) in $C^\alpha(\partial G, R^3)$.

Theorem 6.1. *Let $X = C^\alpha(\partial G, R^3)$ or $X = L^s(\partial G, R^3)$, $1 < s < \infty$, β_0 be given by (14). Fix $\gamma > (1 + C\beta_0)/2$. Denote $M = I - \gamma^{-1}(\frac{1}{2}I + K + E_G A)$. Then there is an equivalent norm $\| \ \|$ on X such that $\|M\| < 1$. Moreover,*

$$\left(\frac{1}{2}I + K + E_G A\right)^{-1} = \gamma^{-1}\sum_{j=0}^{\infty} M^j$$

in X. Let now $\mathbf{f} \in X$. Fix $\mathbf{\Psi}_0 \in X$. For nonnegative integer j put

$$\mathbf{\Psi}_{j+1} = M\mathbf{\Psi}_j + \gamma^{-1}\mathbf{f}.$$

Then there is

$$\mathbf{\Psi} = \lim_{j\to\infty} \mathbf{\Psi}_j$$

in X, $[(1/2)I + K + E_G A]\mathbf{\Psi} = \mathbf{f}$ and

$$\|\mathbf{\Psi} - \mathbf{\Psi}_j\| \le Cq^j[\|\mathbf{f}\| + \|\mathbf{\Psi}_0\|]$$

for arbitrary j. Here constants $C > 0$, $0 < q < 1$ do not depend on $\mathbf{f}$ and $\mathbf{\Psi_0}$.

Proof. For $X = C^\alpha(\partial G, R^3)$ put $Y = C^\alpha(\partial G, R^3)$, for $X = L^s(\partial G, R^3)$ put $Y = L^s(\partial G, C^3)$. According to Proposition 5.5 we have $\sigma(\frac{1}{2}I + K' + AE_G) \subset (0, 1 + C\beta_0\rangle$ in $L^t(\partial G, C^3)$ for each $t \in (1, \infty)$. Since $(\frac{1}{2}I + K + E_G A)$ is the adjoint operator of $(\frac{1}{2}I + K' + AE_G)$ we infer that $\sigma(\frac{1}{2}I + K + E_G A) \subset (0, 1 + C\beta_0\rangle$ in $L^s(\partial G, C^3)$ for each $s \in (1, \infty)$. Since $K + E_G A$ is a compact operator in $C^\alpha(\partial G, C^3)$, Riesz-Schauder theorem gives that every $\lambda \in \sigma(\frac{1}{2}I + K + E_G A) \setminus \{1/2\}$ in $C^\alpha(\partial G, C^3)$ is an eigenvalue (see [34], Chapter X, §5, Theorem 1).

Thus $\sigma(\frac{1}{2}I+K+E_GA) \subset (0,1+C\beta_0\rangle$ in $C^\alpha(\partial G, C^3)$. Since $\sigma(M) \subset (-1,1)$ in Y there is an equivalent norm $\| \ \|$ on Y such that $q = \|M\| < 1$ (see Lemma 5.3). According to [6], Proposition 9.106)

$$(I-M)^{-1} = \sum_{j=0}^{\infty} M^j$$

Moreover, there is

$$\mathbf{\Psi} = \lim_{j\to\infty} \mathbf{\Psi}_j,$$

$(I-M)\mathbf{\Psi} = \gamma^{-1}\mathbf{g}$ by [6], Theorem 9.128. According to [6], Theorem 9.128 we have

$$\|\mathbf{\Psi}-\mathbf{\Psi}_j\| \leq \frac{q^j}{1-q}\|\mathbf{\Psi}_1-\mathbf{\Psi}_0\| = \frac{q^j}{1-q}\|(L-I)\mathbf{\Psi}_0+\gamma^{-1}\mathbf{g}\| \leq q^j\,\frac{q+1+\gamma^{-1}}{1-q}[\|\mathbf{g}\|+\|\mathbf{\Psi}_0\|].$$

Since $[(1/2)I+K+E_GA] = \gamma(I-M)$ we obtain the proposition of the theorem.

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