The *p*-Spin Interaction Spin-glass: A Prototype for Glassy Dynamics

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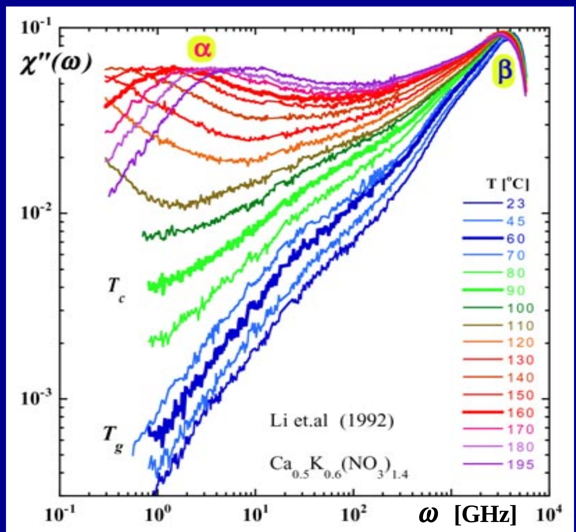
Two Experiments
Dynamic Mean Field Equations and Numerical Solution
Stability Criteria and Phase Diagram
Crossover Scaling Analysis, Reparametrisation Invariance
Cage Relaxation and Critical Slowing Down in Supercooled Liquids

Critical slowing down: α-relaxation

Dielectric loss $\chi''(\omega)$

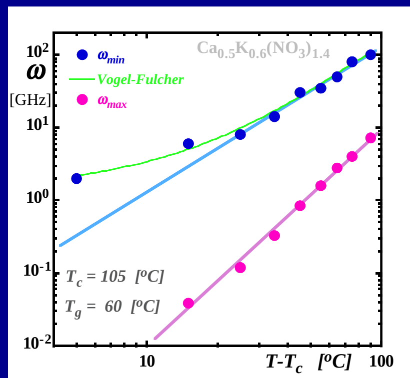
„CKN“ ionic glass

G.Li, W.M.Du, X.K.Chen, H.Z.Cummins, N.J.Tao: Phys.Rev. **A 45,** 3867 (1992)

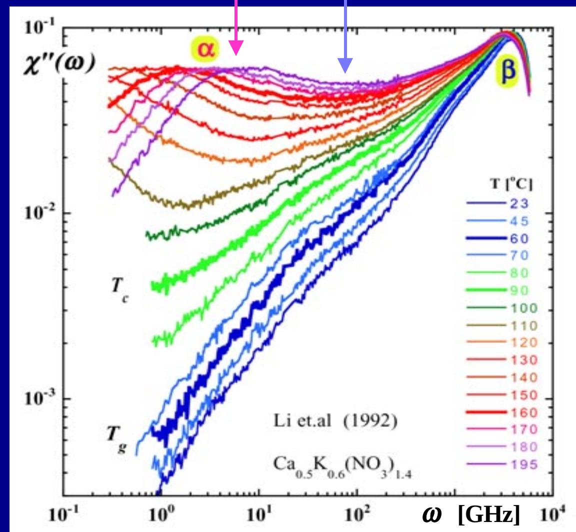


Critical slowing down: α-relaxation

$T_c > T_g$



ω_{max} ω_{min}



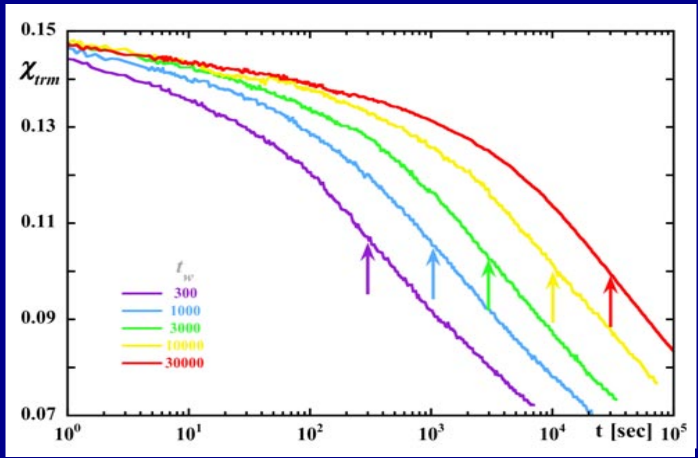
$\omega_{max} \sim (T-T_c)^{-\varphi}$

$\omega_{min} \sim (T-T_c)^{-\varphi'}$

Aging: Field cooled magnetisation $Ag_{1-x}Mn_x$

quench at $t = 0$ with field on
field off at t_w
observe magnetisation at $t_w + t$

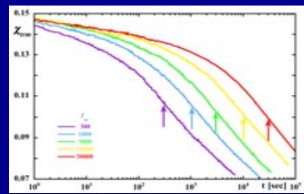
Ph.Refregier, E.Vincent
J.Hammann, M.Ocio:
J.Phys(France)
48,1533 (1987)

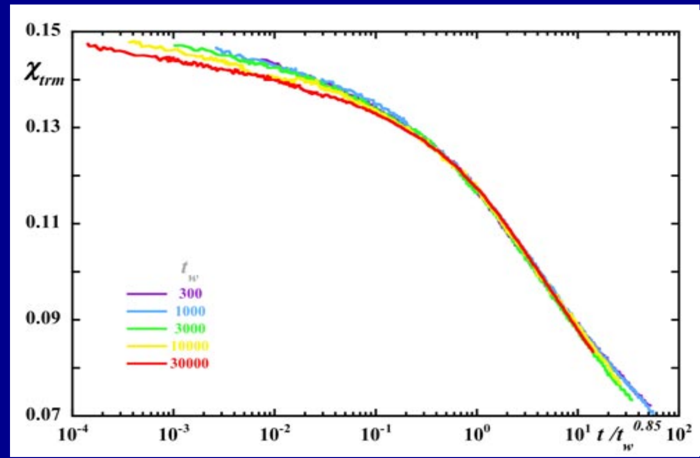


Aging: Field cooled magnetisation $Ag_{1-x}Mn_x$

$$\chi(t+t_w , t_w) \Rightarrow \bar{\chi}(t/t_w^{\mu})$$

$\mu \sim 0.85$





Spin glass models

Hamiltonian

$$H = \sum_i U(\varphi_i) - \sum_i h_i \varphi_i - \tfrac{1}{2}\sum_{i,j} J_{ij}\varphi_i\varphi_j - \tfrac{1}{3!}\sum_{i,j,k} J_{i,j,k}\,\varphi_i\varphi_j\varphi_k \cdots$$

Ising model (Glauber dynamics)

$$U(\varphi) = \tfrac{1}{2}U_o\,(1-\varphi^2)^2$$

Spherical model

$$\left\langle \varphi_i(t)^2 \right\rangle = 1 \qquad U(\varphi) = \tfrac{1}{2}\,\mu\,\varphi^2$$

Quenched disorder (Gaussian)

$$\overline{J^2_{i_1\cdots i_p}} = \frac{1}{N^{p-1}}\,V_p \qquad V(q) = \tfrac{1}{2}\sum_p V_p\,q^p$$

Spin glass models: Dynamics

Langevin equation

$$\frac{\mathrm{d}\varphi_i(t)}{\mathrm{d}t} = -\frac{\partial H(\varphi(t))}{\partial\varphi_i(t)} + \zeta_i(t)$$

Thermal noise, fluctuating forces

$$\langle \zeta_i(t)\,\zeta_j(t')\rangle = 2T\,\delta_{ij}\delta(t-t')$$

Response function

$$r(t_m,t_w) = \overline{\delta\,\langle\varphi_i(t_m)\rangle\,/\delta h_i(t_w)}$$

Correlation function

$$q(t_m,t_w) = \overline{\langle\varphi_i(t_m)\,\varphi_i(t_w)\rangle}$$

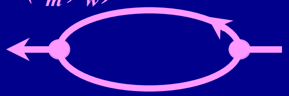
Spherical p–spin model:

T.R.Kirkpatrik, D.Thirumalai: Phys.Rev. **B 36**,5388 (1987)

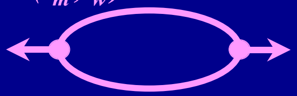
A.Crisanti, H.Horner, H.J.Sommers: Z.Physik B **92**,257 (1993)

L.F.Cugliandolo, J.Kurchan: Phys.Rev.Lett. **71**,1 (1993)

$L(t_m, t_w)$



$M(t_m, t_w)$



Dynamic mean field equations

$$\left(\partial_{t_m} + \mu(t_m)\right) r(t_m, t_w) = \int_{t_w}^{t_m} \mathrm{d}s\, L(t_m; s)\, r(s; t_w)$$

$$\left(\partial_{t_m} + \mu(t_m)\right) q(t_m, t_w) = \int_{t_m}^{t_w} \mathrm{d}s\, L(t_m; s)\, q(s; t_w) + \int_0^{t_w} \mathrm{d}s \left\{ L(t_m; s)\, q(t_w; s) + M(t_m; s)\, r(t_w; s) \right\}$$

Self-energies

$$L(t_m, t_w) = V''\left(q(t_m; t_w)\right) r(t_m; t_w)$$

$$M(t_m, t_w) = V'\left(q(t_m; t_w)\right)$$

Exact for long ranged interactions.

Approximation for short ranged interactions:

$$q(\vec{k}, t_m, t_w) \approx c(\vec{k})\, q(t_m, t_w) \cdots$$

Spherical p–spin model: $T > T_c$

Equilibrium: FDT-solution $r(t) = -\beta\, \partial_t\, q(t)$

Mode coupling theory for supercooled liquids

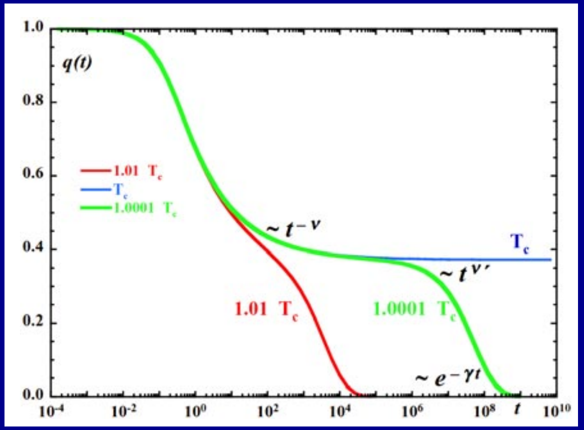
W.Götze, L.Sjörgren

J.Phys.C **21,** 3407 (1988)

Rep.Prog.Phys. **55**, 241 (1992)

$$q(\vec{k}, t) \approx c(\vec{k})\, q(t)$$

Mode coupling equations are identical to dynamic mean field equations assuming FDT



Spherical p–spin model: $T > T_c$

Equilibrium: FDT-solution $r(t) = -\beta\, \partial_t\, q(t)$

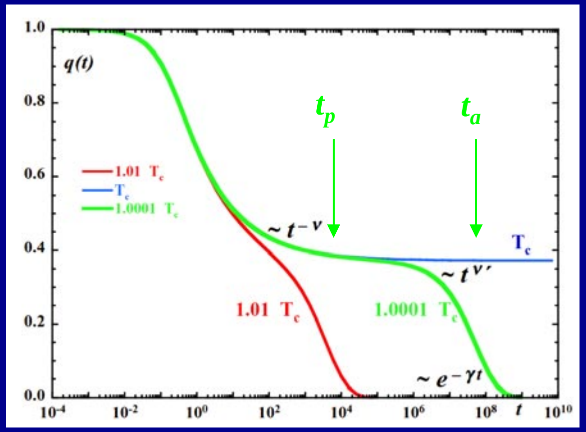
Dynamical critical exponents ν ν'

$$\frac{\Gamma(-\nu)^2}{\Gamma(-2\nu)} = \frac{\Gamma(\nu')^2}{\Gamma(2\nu')}$$

Critical slowing down

$$t_p \sim (T - T_c)^{-\frac{1}{2\nu}}$$

$$t_a \sim (T - T_c)^{-\frac{\nu+\nu'}{2\nu\nu'}}$$



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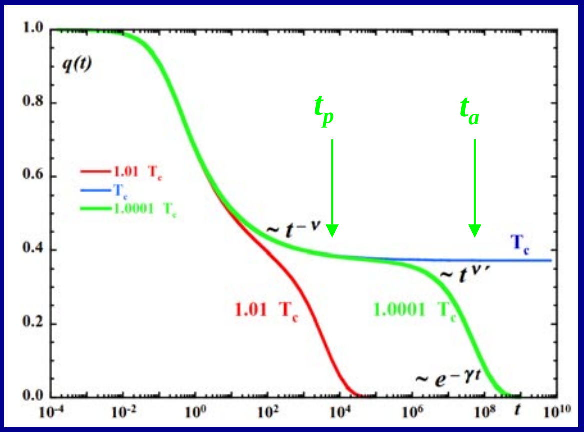
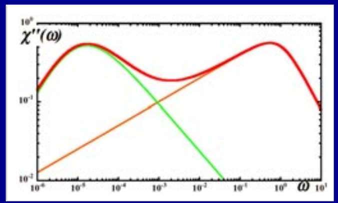


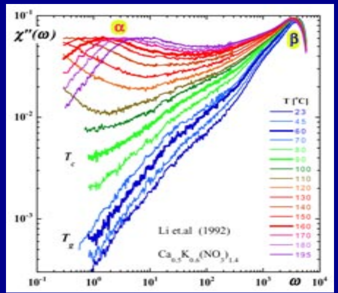
Spherical *p*–spin model: $T > T_c$

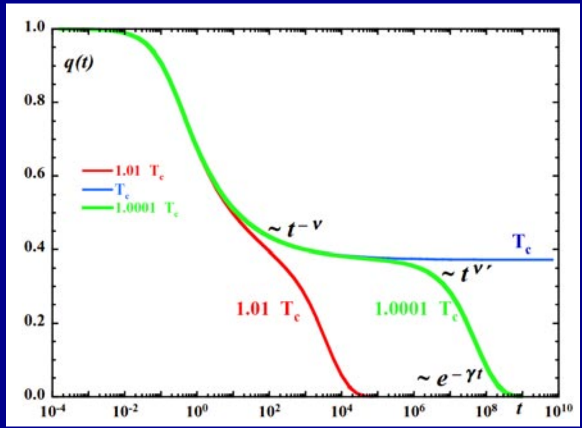
Susceptibility $\chi''(\omega)$

$$r(t) = -\beta\,\partial_t\,q(t)$$

$$\chi''(\omega) = \int_0^\infty \mathrm{d}t\, r(t)\,\sin(\omega t)$$







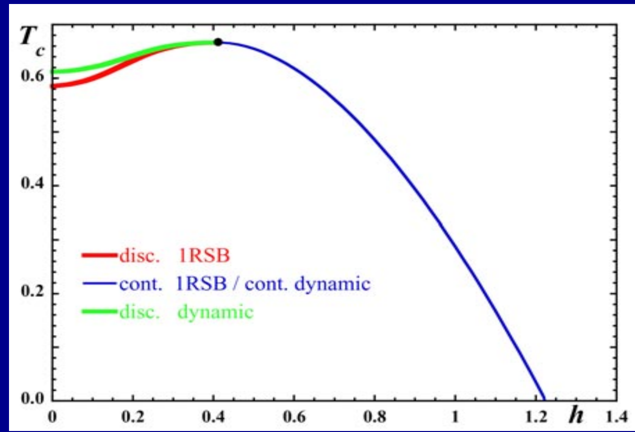
Spherical *p*–spin model: Freezing transition $T_c(h)$

Stability criterium:

$$\frac{\mathrm{d}}{\mathrm{d}\,t}\,q(t) \le 0$$

Discontinuous transition for $h < h^*$

Continuous transition for $h > h^*$



H.Horner :
Z.Physik **B 86**, 291 (1992)

A.Crisanti, H.Horner, H.J.Sommers:
Z.Physik **B 92**,257 (1993)

$T_c > T_{RSB}$

$T_{c,MC} > T_{Glas}$

Spherical *p*–spin model:

Out of equilibrium dynamics for $T < T_c$

Regularisation: long time scale $\tau_\infty \longrightarrow \infty$

- finite N $\qquad \tau_\infty \sim \exp N^{1/3}$
- slow cooling $\qquad T(t) = (1 - t/\tau_\infty)\, T_c$
- **aging** $\qquad \tau_\infty = t + t_w$
- **relaxing bonds,** **cage dynamics** **(supercooled liquids)** $\qquad \overline{J(t)\, J(t')} \sim \exp\left((t-t')/\tau_\infty\right)$

Slow cooling:
H.Horner:
Europhys. Lett. **2** 487 (1986)

Relaxing bonds:
H.Horner:
Z. Phys. **B57**, 29 (1984)

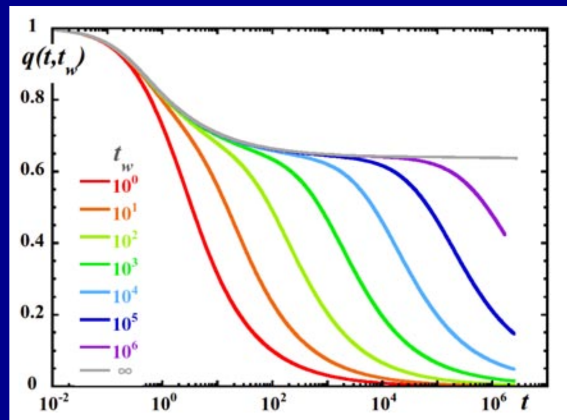
Spherical *p*–spin model: Aging

Numerical solution of the dynamic mean field equations

Coupled non-linear integro-differential equations for 2-time functions

Initial decay towards plateau
no t_w dependence

Final decay
scaling with $t_w^{1-\eta}$



Violation of the FDT

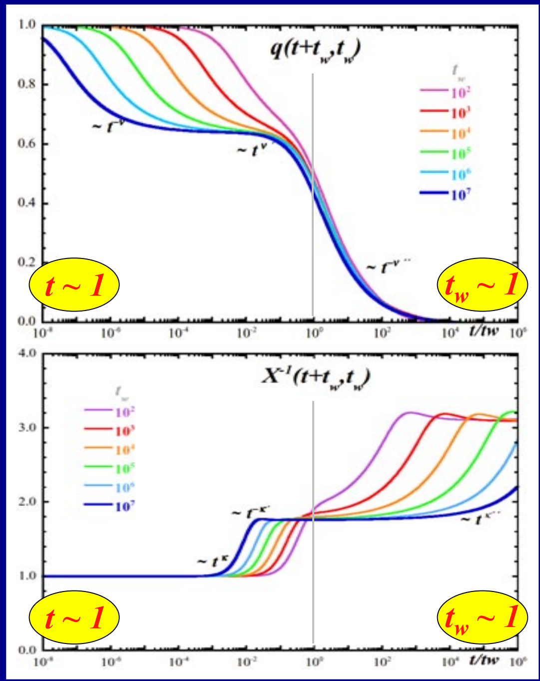
Non ergodicity parameter X

$$\partial_{t_w} q(t_m, t_w) = \beta\, X(t_m, t_w)\, r(t_m, t_w)$$

Effective temperature:

$$T_{\text{eff}}(t_m, t_w) = X^{-1}(t_m, t_w)\, T$$

L.F.Cugliandolo, J.Kurchan, L.Peliti
Phys.Rev. **E55**, 3898 (1997)



Crossover scaling analysis:

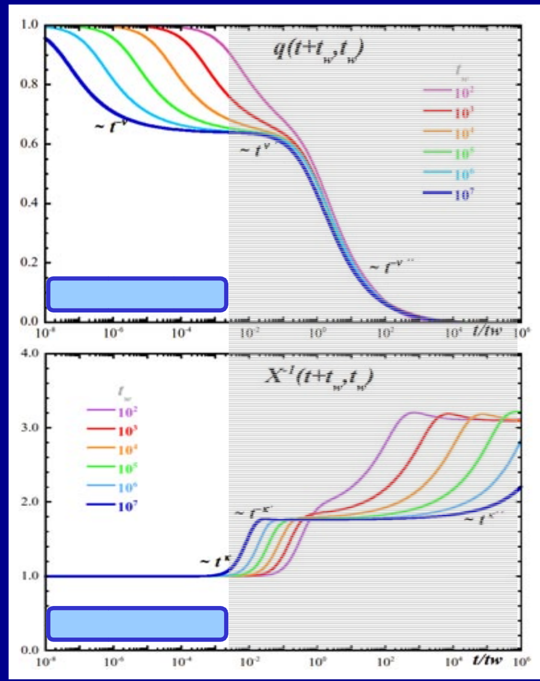
FDT-regime:

$$\partial_{t_w} q(t_m, t_w) = \beta\, r(t_m, t_w)$$

Equilibrium within a valley

FDT-solution for

$t = t_m - t_w < t_p(t_w)$



Plateau-regime:

$t \sim t_p(t_m)$

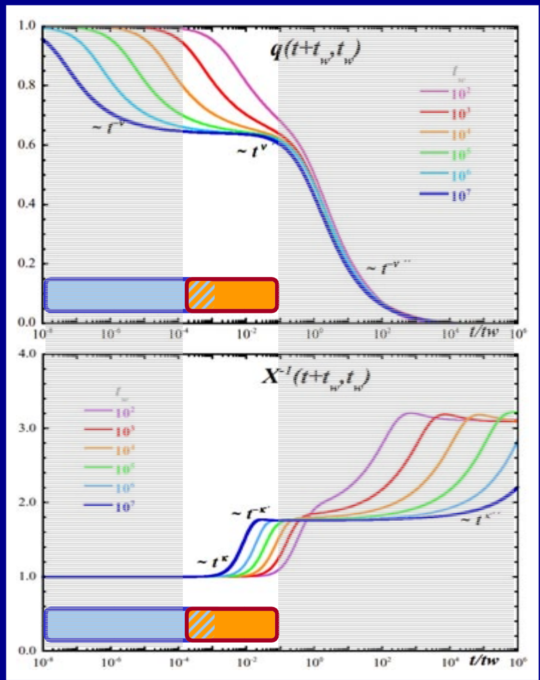
$$q(t_m, t_w) \approx q_c$$

Onset of nonergodicity

$$\kappa = 3\,\nu - 1$$
$$\kappa' = 3\,\nu' - 1$$

Correction to leading order:

$$q(t_m, t_w) = q_c + t_p^{-\nu}\, \hat{q}_p(t/t_p)$$



QFDT-regime

Non ergodicity parameter X

$$X(t_m, t_w) \approx X_c$$

for $t_p(t_w) < t < t_a(t_w)$

Scaling function $\Psi(x)$

$$q(t_m, t_w) \rightarrow \Psi\big(h(t_m)/h(t_w)\big)$$

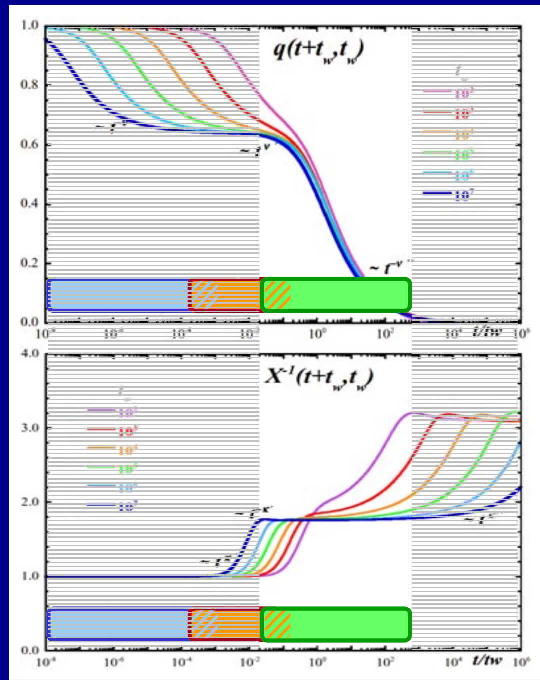
"gauge invariance":

$h(t)$ **arbitrary (monotonous)**

H.Sompolinsky, A.Zippelius:
Phys.Rev.Lett. **47**, 359 (1982)

H.Horner:
Z. Phys. **B57**, 29 (1984)

L.F.Cugliandolo, J.Kurchan:
Phys.Rev.Lett. **71**, 1 (1993)



QFDT-regime

Non ergodicity parameter X

$$X(t_m,t_w)\approx X_c$$

for $t_p(t_w) < t < t_a(t_w)$

Scaling function $\Psi(x)$

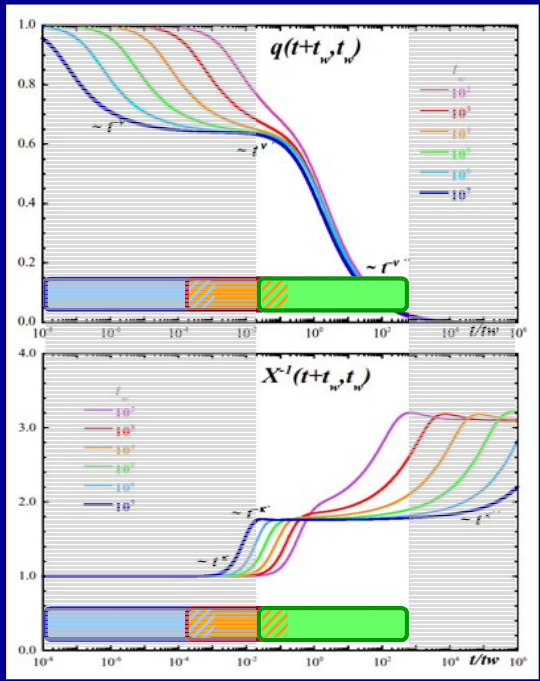
$$q(t_m,t_w)\to\Psi\big(h(t_m)/h(t_w)\big)$$

"gauge invariance":

$h(t)$ arbitrary (monotonous)

Correction: $\Xi(x)$

$$X(t_m,t_w)\to X_c+t_a^{-\frac{\nu\kappa'}{\nu+\nu'}}\,\Xi\big(h(t_m)/h(t_w)\big)$$



More scaling-regimes

$t > t_a(t_w)$ $t_w >> 1$

$t_w \sim 1$

Matching:

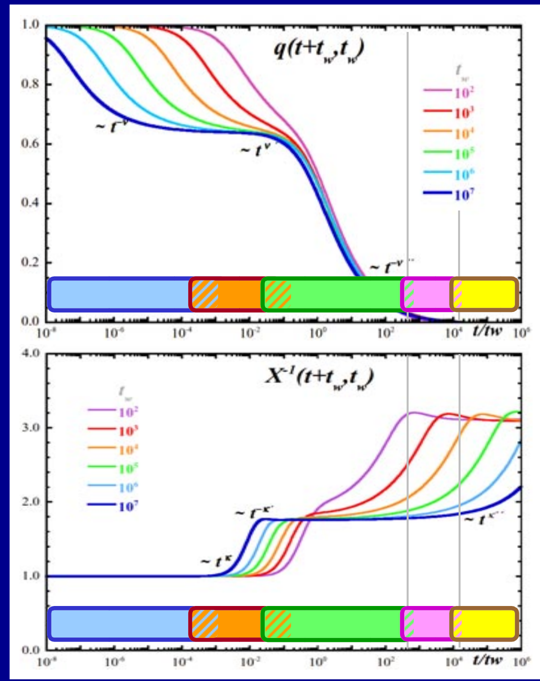
$$t_p(t_w)\sim t_a(t_w)^{\frac{\nu}{\nu+\nu'}}$$

$$h(t)=e^{\frac{1}{\eta}t^{\eta}}$$

$$t_a(t_w)\sim t_w^{1-\eta}$$

$$\eta=\frac{\nu\,(3\nu'-1)}{2(\nu+\nu')+\nu(3\nu'-1)}$$

Stability of QFDT-phase: $\nu' > 1/3$

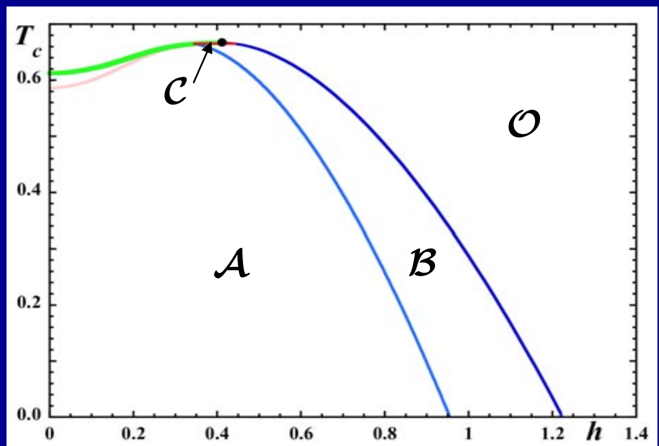


Spherical *p*–spin model: Dynamic phase diagram

$\mathcal{O}$ Paramagnet

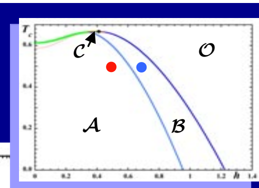
$\mathcal{A}$ QFDT-phase

$\mathcal{B}$ Hierarchy of
$\mathcal{C}$ time scales

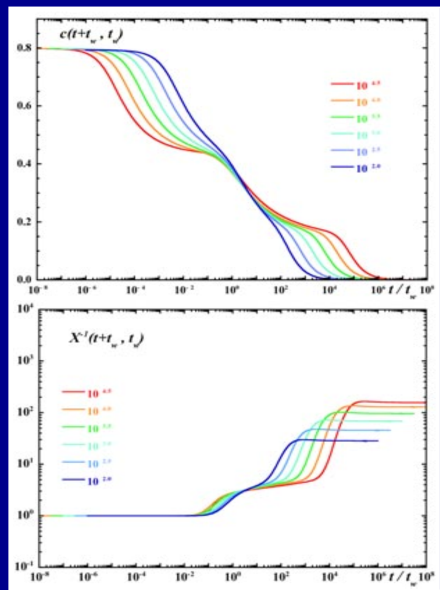


H.Horner:
Z.Physik **B100**, 243 (1996)
and to be published

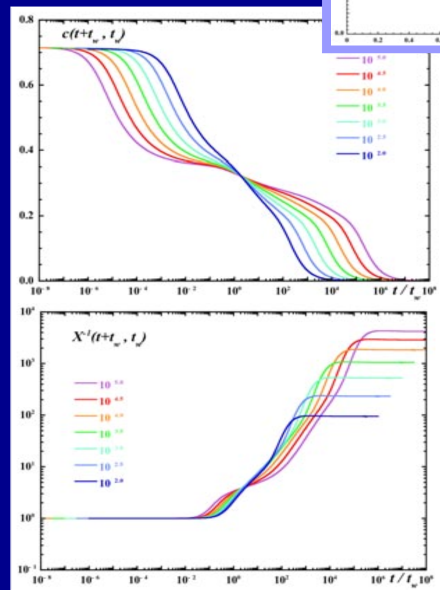
Spherical *p*–spin model: $\mathcal{A}$ $\mathcal{B}$ transition



$\mathcal{A}$



$\mathcal{B}$



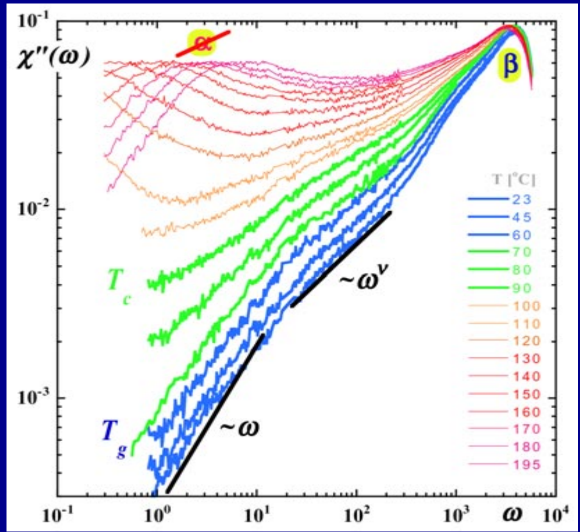
Critical slowing down: CKN-glass $T < T_c$

No α–peak

$\chi''(\omega) \sim \omega$

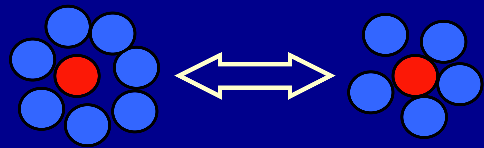
→ $r(t) \sim e^{-\gamma t}$

Coupling to transverse currents
within mode coupling theory
W.Götze, L.Sjörgren
J.Phys.C **21,** 3407 (1988)
Rep.Prog.Phys. **55**, 241 (1992)



Cage relaxation in supercooled liquids:

Mode coupling theory: anharmonic density fluctuations



Bond-correlation function: $\overline{J(t)\,J(t')} = C_J(t-t') \sim \mathrm{e}^{-(t-t')/\tau_c}$

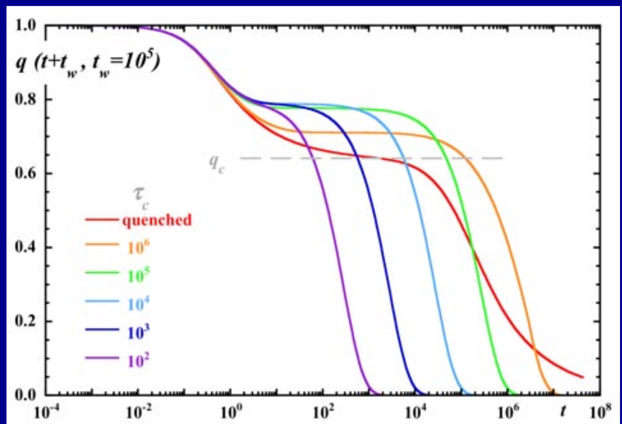
Bond-response function:

spin glasses: quenched disorder $G_J(t) = 0$

glasses: cage relaxation in equilibrium, FDT $G_J(t) = -\beta\,\partial_t\, C_J(t)$

Cage relaxation in supercooled liquids:

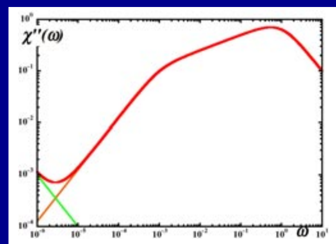
Correlation function: $t_w = 10^5$ $\tau_c = 10^2 \cdots 10^6$

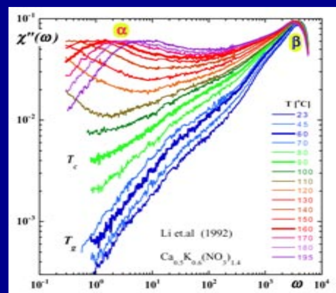


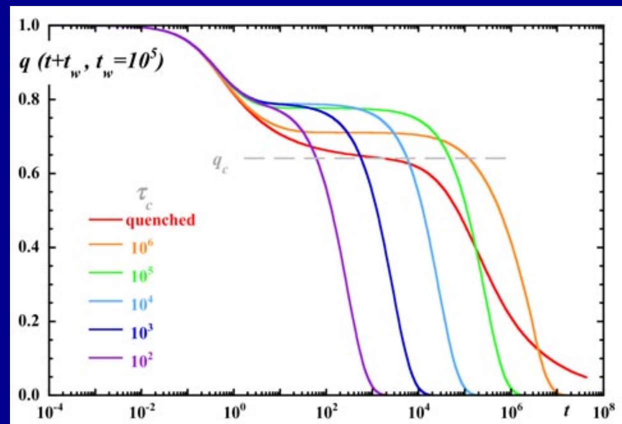
Cage relaxation in supercooled liquids:

Time and frequency scales for $t_w >> \tau_c$ **:** $t_o \sim (T_c - T)^{-\frac{1}{\nu}}$

$\omega_o \sim t_o^{-1}$ $\omega_{min} \sim \tau_c^{-\frac{1}{2}}\, t_o^{-\frac{1}{2}(1-\nu)}$ $\omega_c \sim \tau_c^{-1}$





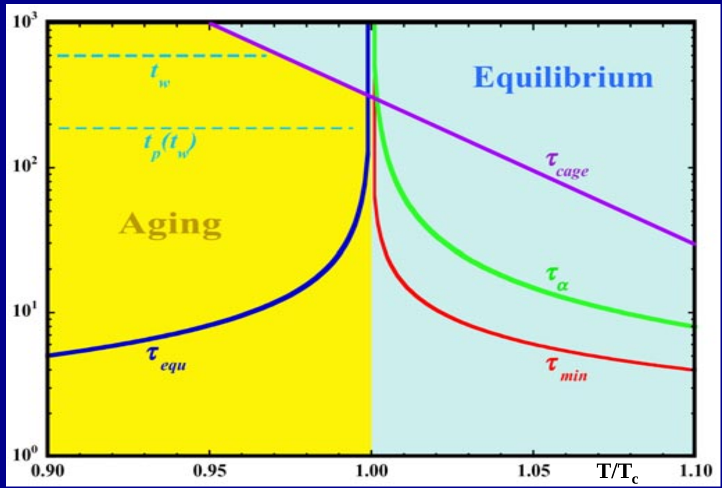


Cage relaxation in supercooled liquids:

Time scales:

Aging
for
$t_w < \tau_{cage}$ →

τ_{equ}
for
$t_w > \tau_{cage}$ →



Outlook

Solution of dynamics:	numerics and crossover scaling
Power laws:	self-induced criticality
Dynamics vs. replica:	different phase diagrams
	different states?
dynamics:	$\lim_{t\to\infty} \lim_{N\to\infty}$?
replica:	$\lim_{N\to\infty} \lim_{t\to\infty}$?
Cage relaxation in glasses:	restoration of equilibrium aging

Miscellaneous problems

- **Memory effects**
- **Off equilibrium mode coupling theory**
- **Mean field theory for finite range interactions**
- **Off equilibrium dissipative quantum dynamics**

?