Unconventional scaling at dirty superconducting quantum phase transitions

Motivation
The dirty p-wave superconducting quantum phase transition
Unconventional critical behavior
Soft-mode coupling: general aspects

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Motivation

recent experiments on ferromagnetic superconductors

several weak itinerant ferromagnets (UGe_2, $ZrZn_2$) show superconductivity

superconductivity is likely of the p-wave triplet type

superconductivity is easily destroyed by disorder

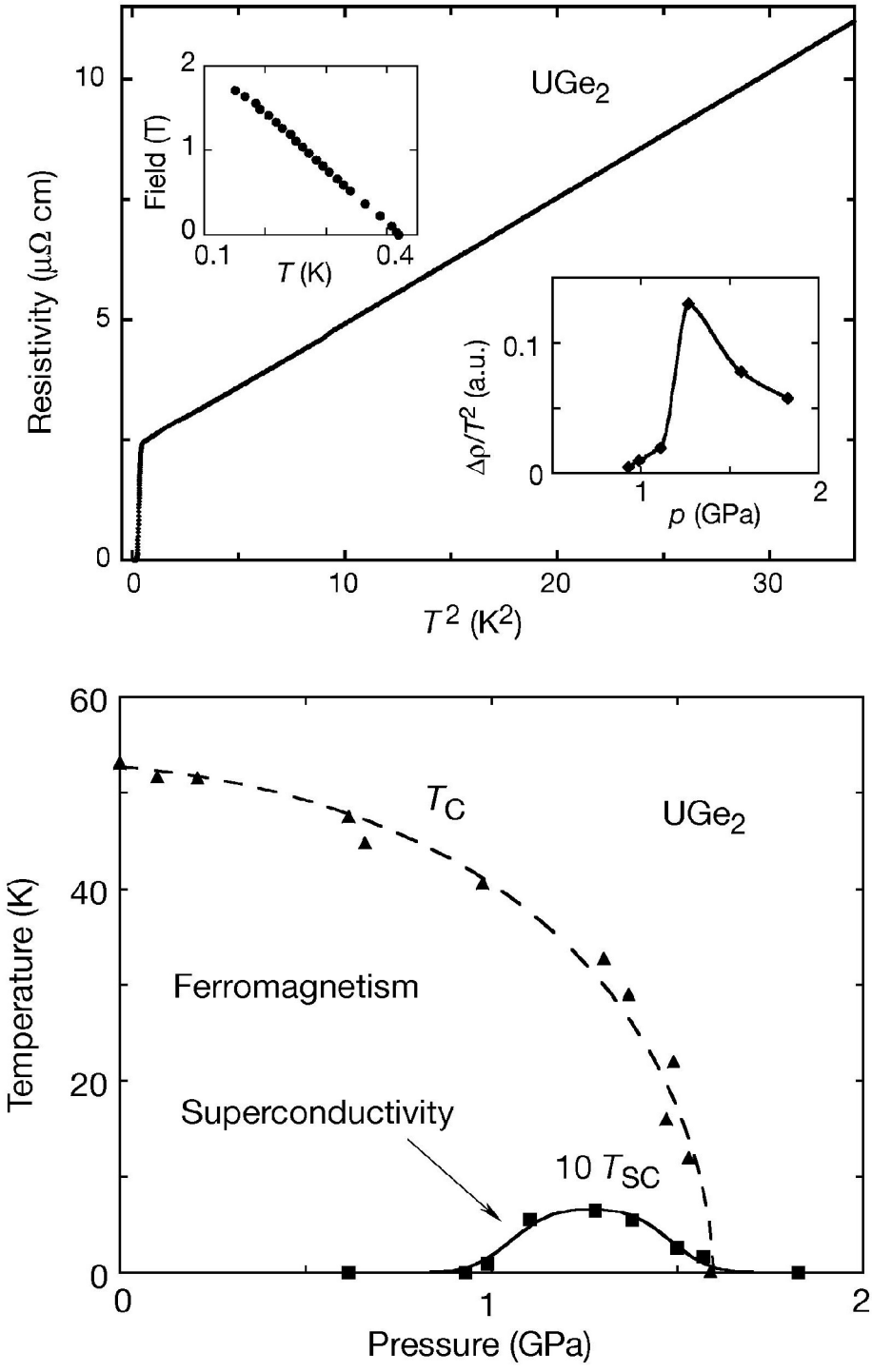
$\Rightarrow$ superconducting quantum phase transition accessible in experiment

general point of view:

superconducting quantum phase transitions are effected by coupling between critical and non-critical soft modes

leads to non-local order-parameter theories

influence of angular momentum on the mode coupling effects



UGe_2, after Saxena et al., Nature 406, 587 (2000)

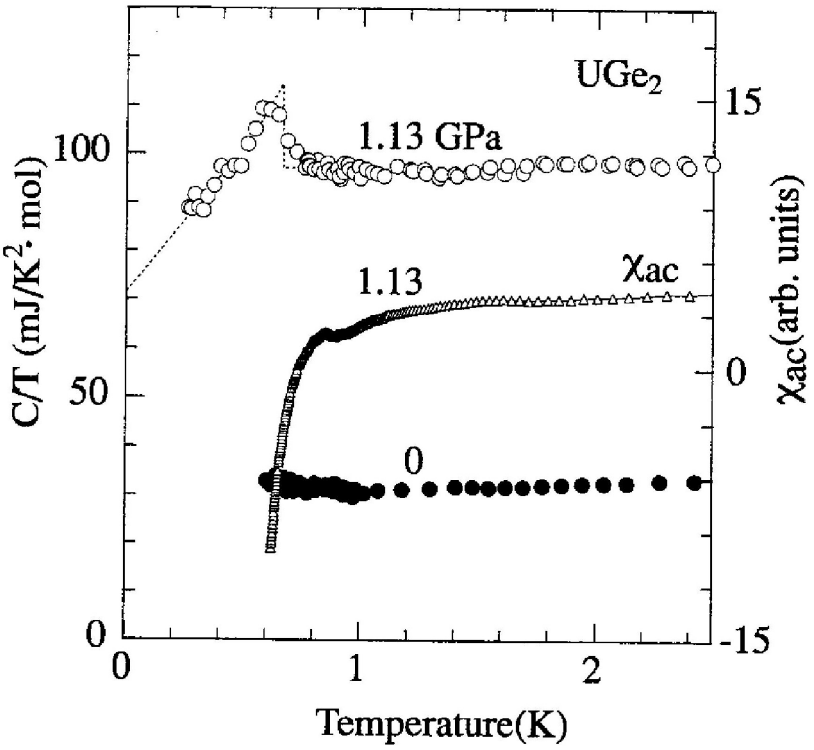
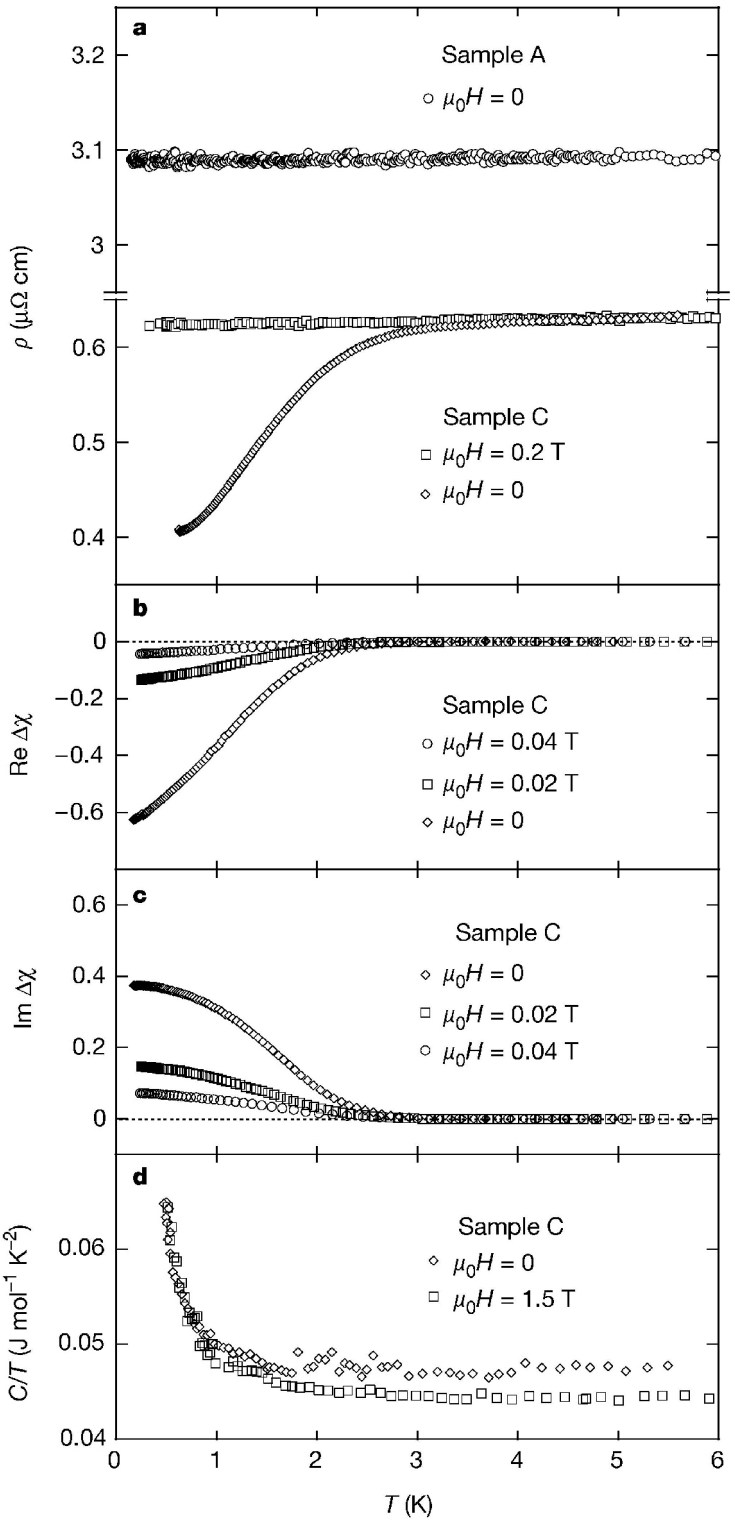
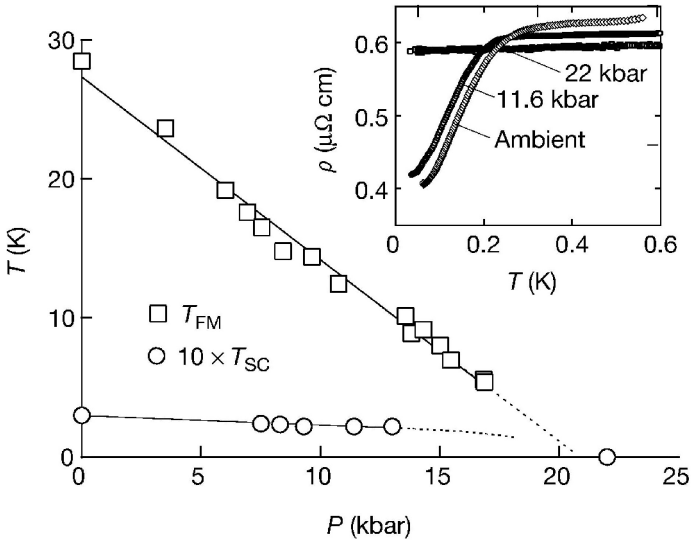


Figure 4. Temperature dependence of the specific heat in the form of C/T at 0 and 1.13 GPa, togehter with the ac susceptibility at 1.13 GPa in UGe_2.

after Tateiwa et al., J.Phys. Condens. Mat. 13, L17 (2001)



$ZrZn_2$, after Pfleiderer et al., Nature 412, 58 (2001)d



Superconductivity and disorder

Anderson's theorem:

conventional (s-wave) superconductivity is only weakly influenced by (weak) non-magnetic impurities

reason: time-reversal invariance is not broken, pairing between dirty eigenstate and its time-reversed counterpart

at large disorder: Anderson's theorem breaks down due to interplay of disorder and many-body effects

Exotic (non-s-wave) superconductivity:

Anderson's theorem not valid
impurity scattering mixes gap and node regions of the Fermi surface

exotic superconductivity will be destroyed when mean free path l_s reaches coherence length ξ_0

In exotic superconductors: quantum phase transition at weak disorder

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Landau-Ginzburg-Wilson theory of the p-wave superconducting quantum phase transition

minimal microscopic model

$$H = H_0 + H_{dis} + \tfrac{1}{2} \sum_{kpq\,\sigma\sigma'} \sum V_{kp}\; a_\sigma^+(k+q)\, a_{\sigma'}^+(-k)\, a_{\sigma'}(-p)\, a_\sigma(p+q)$$

H_0 free electrons

H_{dis} disorder, nonmagnetic random potential

V_{kp} p-wave Cooper channel interaction, $V_{kp} = \Gamma\, (e_k \cdot e_p)$

p-wave Cooper channel density: $n^+_{\sigma\sigma'}(q) = \sum e_k\; a_\sigma^+(k+q)\, a_{\sigma'}^+(-k)$

introduce replicas, Hubbard-Stratonovich transformation of the interaction, formally integrate out fermionic degrees of freedom

Landau-Ginzburg-Wilson functional

$$F(\varphi^*,\varphi) = \sum_{\alpha}\sum_{ij}\sum_{k\omega} \varphi_i^{\alpha *}(q,\omega)\left[1 - \Gamma\, \chi_c^{(2)}(q,\omega)\right] \varphi_j^{\alpha}(q,\omega) \pm \sum_n \Gamma^n\, \chi_c^{(2n)}\, (\varphi^*\varphi)^n$$

$\chi_c^{(2n)}$ Cooper density correlation functions of dirty metal ($H_0 + H_{dis}$)

Cooper correlation functions of disordered metal

clean electrons:

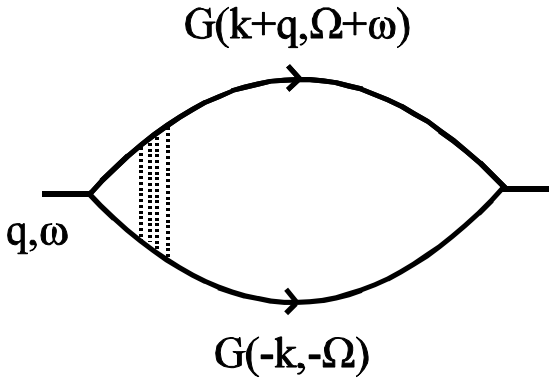
$\chi_c^{(2)}(q=0,\omega=0,T) \sim N_F \log(1/T)$ for all pairing angular momenta

$\Rightarrow$ system will be superconducting at T=0 for arbitrarily weak attractive interaction in any of the angular momentum channels

dirty electrons:

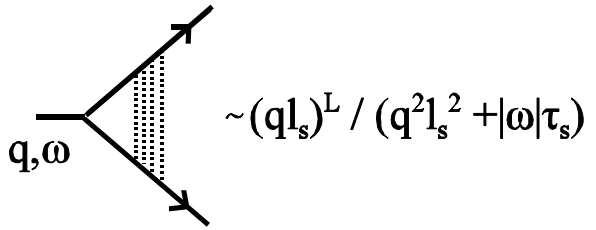
s-wave pairing:
$\chi_c^{(2)}(q=0,\omega=0,T) \sim N_F \log(1/T)$

Anderson's theorem



higher angular momenta:
$\chi_c^{(2)}(q=0,\omega=0,T=0) \sim N_F \log(k_F l_S)$

quantum phase transition at a finite strength of the attractive interaction



Momentum and frequency dependence at T=0

- p-wave case
- small q and ω
- small disorder (large l_S)

2-point Cooper correlation function:

$$\chi_c^{(2)}(q,\omega) = N_F \log(k_F l_S) - N_F |\omega| \tau_s - N_F q^2 \left[1 + \log(1/q)\right]$$

4-point Cooper correlation:

replica-diagonal piece: $\chi_c^{(4,\mathrm{diag})} \sim N_F \tau_s^2 (q l_s)^4 / (q^2 l_s^2 + |\omega| \tau_s)^2 \sim N_F \tau_s^2$

replica-off-diagonal piece: $\chi_c^{(4,\text{off-d})} \sim N_F^2 k_F^{-3}$

τ_s scattering time

$l_s = v_F \tau_s$ scattering length

typical BCS logarithms still exist in the dirty p-wave case but they are shifted to higher order terms

Resulting order parameter field theory

$$F(\varphi^*,\varphi) = \sum_{\alpha} \sum_{ij} \sum_{q\omega} \varphi_i^{\alpha*}(q,\omega) \left[\delta_{ij}(t + a_2\, q^2 + |\omega|) - a_{log}\, q_i q_j \log(l_s q) \right] \varphi_j^{\alpha}(q,\omega)$$

$$+ u \int d^dx\, d\tau \sum_{\alpha} \sum_{ij} \varphi_i^{\alpha*}(x,\tau)\varphi_i^{\alpha}(x,\tau)\varphi_j^{\alpha*}(x,\tau)\varphi_j^{\alpha}(x,\tau)$$

$$+ v \int d^dx\, d\tau\, d\tau' \sum_{\alpha\beta} \sum_{ij} \varphi_i^{\alpha*}(x,\tau)\varphi_i^{\alpha}(x,\tau)\varphi_j^{\beta*}(x,\tau')\varphi_j^{\beta}(x,\tau')$$

$$t = \left[1 - \Gamma\, N_F\, \log(k_F l_s)\right] / (\Gamma\, N_F\, \tau_s)$$

$$a_2 \sim a_{log} \sim l_s^2 / \tau_s$$

$$u \sim 1 / N_F$$

$$v \sim 1 / (k_F^3\, \tau_s^2)$$

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Renormalization group analysis

$$F(\varphi^*,\varphi) = \sum_{\alpha}\sum_{ij}\sum_{q\omega} \varphi_i^{\alpha*}(q,\omega)\left[\delta_{ij}(t + a_2 q^2 + |\omega|) - a_{log}\, q_i q_j \log(l_s q)\right]\varphi_j^{\alpha}(q,\omega)$$

$$+ u\int d^dx\, d\tau \sum_{\alpha}\sum_{ij} \varphi_i^{\alpha*}(x,\tau)\varphi_i^{\alpha}(x,\tau)\varphi_j^{\alpha*}(x,\tau)\varphi_j^{\alpha}(x,\tau)$$

$$+ v\int d^dx\, d\tau\, d\tau' \sum_{\alpha\beta}\sum_{ij} \varphi_i^{\alpha*}(x,\tau)\varphi_i^{\alpha}(x,\tau)\varphi_j^{\beta*}(x,\tau')\varphi_j^{\beta}(x,\tau')$$

Gaussian fixed point

mean-field static critical exponents
dynamical exponent z=2
strong logarithmic corrections from the $q^2 \log(q)$ term

Tree-level renormalization group analysis (power counting)

[u] = 4-d-z = 2-d < 0 quantum fluctuations are irrelevant
[v] = 4-d >0 disorder fluctuations are relevant

Gaussian fixed point is unstable
asymptotic critical behavior will be non-mean field like

Asymptotic critical behavior

order parameter field theory is identical to that of itinerant antiferromagnetic quantum phase transition

perturbative RG gives conventional (finite-disorder) fixed point
perturbative fixed point is unstable against rare regions

$\Rightarrow$ runaway flow to large disorder in all of parameter space

Possible scenarios

(i) non-perturbative finite-disorder FP, power-law scaling $\omega \sim q^z$
(ii) infinite-randomness fixed point, activated scaling $\log(\omega) \sim q^\psi$
(iii) destruction (smearing) of the sharp phase transition

numerical simulations of the antiferromagnet and results for random quantum Ising models suggest

asymptotic critical behavior may be of infinite-randomness type

Gaussian crossover region

When does the (replica-off-diagonal) disorder term become important?
dimensionless coupling constant ratio: $v/(tu) > 1$

$$|\Gamma-\Gamma_C|/\Gamma_C < \Gamma_C N_F \exp[-1/(\Gamma_C N_F)]$$

v important in exponentially narrow region around quantum critical point

In contrast:
logarithmic terms become important for $\xi > l_S$

$$|\Gamma-\Gamma_C|/\Gamma_C < \Gamma_C N_F$$

wide Gaussian crossover region showing mean-field critical behavior, z=2 and logarithmic corrections to power-law scaling

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Soft-mode coupling and angular momentum

singular behavior of the vertices in the order parameter field theory is caused by coupling between order parameter and additional soft modes

principal difference between clean and dirty systems:

clean systems: soft modes in all angular momentum channels
dirty systems: soft modes only in only L=0 channel

in dirty systems the singularities in the order parameter field theory are suppressed with increasing L

example: superconducting quantum phase transition:

s-wave: $\Gamma^{(2)}(q,\omega) = t + 1/\log[1/(q^2 l_s^{\,2} + |\omega|\tau_s)]$
p-wave: $\Gamma^{(2)}(q,\omega) = t + q^2 \log(1/q^2 l_s^{\,2}) + |\omega|\tau_s$
d-wave: $\Gamma^{(2)}(q,\omega) = t + q^2 + q^4 \log(1/q^2 l_s^{\,2}) + |\omega|\tau_s$

Conclusions

- exotic (non-s-wave) superconductivity is destroyed by weak nonmagnetic disorder => disorder driven quantum phase transition
- the order parameter field theory of this transition is generically non-local due to coupling to additional soft modes
- with increasing angular momentum the singularity is shifted to higher order, i.e. more irrelevant terms
- the p-wave case is marginal, the mode coupling effects lead to logarithmic corrections
- asymptotically the renormalization group flow is towards large disorder infinite-randomness fixed point ???
- the asymptotic critical region is very narrow
- outside is a wide Gaussian crossover region characterized by mean-field exponents and logarithmic corrections to scaling