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Solving Poisson equation over planar NURBS domains with isogeometric analysis

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Recently, Hughes et al. in [3] proposed a concept of the so-called isogeometric analysis – the method for numerical solving of partial differential equations based on finite element method and geometric modeling. The main advantages of isogeometric analysis are an exact representation of objects of interest and the possibility to omit generation of a triangular (quadrangular) mesh which is very time-consuming.

In this talk, we focus on solving Poisson equation over a planar NURBS domain with Dirichlet boundary condition. The problem is closely related to the following geometrical problem. Let a planar domain boundary be given by either one closed NURBS curve, or several connected NURBS curves forming a closed domain. We need to find a planar NURBS surface (i.e., control points, their weights and corresponding knot vectors) describing the domain bounded by these boundary NURBS curves (see Fig. 1). This problem has attracted attention of researchers in the last years, especially in connection with isogeometric analysis (see e.g. [2, 4, 6]). It was revealed that the choice of inner control points of NURBS description of such a domain can significantly influence results and convergence rates of the follow-up isogeometric analysis. We will present several algorithms which can be used to solve this problem and we will demonstrate examples of different NURBS descriptions (and parameterizations) of a given domain.

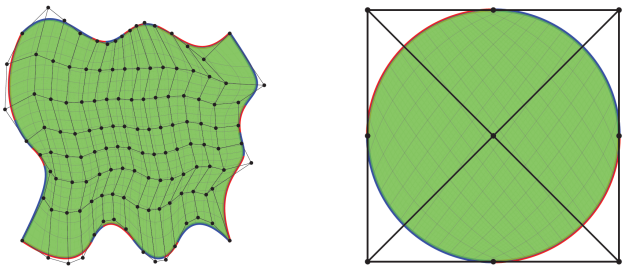

Figure 1: Left: NURBS domain given by 4 boundary NURBS curves; Right: NURBS description of a disc.

This problem can be also generalized for spatial domains, i.e., NURBS solids, which is far more difficult to solve. The first attempts to finding volumetric NURBS

parameterizations for special classes of boundary NURBS surfaces have been presented recently, e.g. paper [1] deals with class of swept volumes which covers a number of interesting free-form shapes, or paper [5] presents a method based on discrete volumetric harmonic functions which uses triangle mesh of a domain boundary and also tetrahedral mesh of a domain interior. Unfortunately, a general algorithm which would return volumetric NURBS parameterization of an arbitrary closed NURBS surface representing a domain boundary is not known at present.

Further, we will also present an implementation of an isogeometric analysis-based solver in `MATHEMATICA` for Poisson equation over planar NURBS domains with Dirichlet boundary condition which we plan to generalize for Neumann and Robin boundary conditions and more general elliptic partial differential equations in the future.

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