# Broadband dielectric terahertz metamaterials with negative permeability

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We present a design of dielectric metamaterials exhibiting a broad range of negative effective permeability in the terahertz spectral region. The investigated structures consist of an array of high-permittivity rods that exhibit a series of Mie resonances giving rise to the effective magnetic response. The spectral positions of resonances depend on the geometrical parameters of the rods and on their permittivity, which define the resonant confinement of the electromagnetic field within the rods. The electromagnetic coupling between the adjacent rods is negligible. With a suitable aspect ratio of the rods, a broadband magnetic response can be obtained. © 2009 Optical Society of America

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Metamaterials (MM) are man-made composite structures with unit cell dimensions tailored for the targeted wavelength. They permit controlling the effective electromagnetic properties and consist of patterns whose spatial details are not distinguished by the propagating field. By contrast, the internal (near) field senses the fine structure of the unit cell. In case of an internal resonance, a strong dispersion in the effective MM parameters may occur. If the resonances are well designed, MMs with simultaneously negative dielectric permittivity $\epsilon$ and magnetic permeability $\mu$ may be obtained, called left-handed media [1]. These offer possibilities to overcome the Abbe's criterion of wave optics. While negative $\epsilon$ related to the plasma resonance occurs in most metals in a broad spectral range [2], negative $\mu$ is achieved only near strong narrow magnetic resonances. However, strong natural magnetic resonances are seldom observed above gigahertz frequencies.

Most negative-index terahertz and optical MMs rely on miniaturizing microwave structures with metallic subwavelength motifs operating below their plasma frequency. Such MMs based on metallic patterns are prepared using lithographic techniques, and their resonant properties in the THz and near-IR ranges were demonstrated [3,4].

Another approach exploits the Mie resonances in dielectric resonators [5,6]. A medium made of periodically or randomly arranged spheres shows a strong dispersion in the effective $\epsilon$ or $\mu$ if the resonance is associated with an electric or magnetic mode, respectively. A set of spheres with different geometrical and/or optical properties or the use of coated spheres [7] may provide a material where both effective $\epsilon$ and $\mu$ exhibit resonances in the same spectral domain, thus representing a negative-index material. Potential candidates for dielectric composites with magnetic response are high-$\epsilon$ materials like $LiTaO_3$ or $TiO_2$, which show polaritonic resonances in the far-IR.

Most effective magnetic resonances in the gigahertz or terahertz spectral ranges reported so far are relatively narrow; the ratio of the width of the range where $\mu$ is negative to the resonance position is about 2–4% in [8,9], up to 10% in [1,3], and about 20% in [10]. One way to cover a wider band of negative $\mu$ is to dynamically shift the Mie resonance. This was experimentally demonstrated in a MM consisting of an array of rods made of $SrTiO_3$ (STO) [10]. The dielectric properties of STO in the terahertz range are controlled by the soft mode [11], which gives the possibility to tune the effective response by varying the temperature. Similar effects were observed at microwave frequencies with $(Ba,Sr)TiO_3$ [8,12].

In this Letter, we carry out calculations of the response of arrays of dielectric rods with the aim of elucidating the role of geometrical and dielectric parameters, such as aspect ratio of the rectangular section of the rods, their spacing, and dielectric losses, and compare theoretical results with our experimental data. We analyze the behavior of higher-order Mie resonances, which allows us to conceive new designs of structures exhibiting a substantially broader range of negative $\mu$.

We used samples consisting of an array of high-$\epsilon$ rods. A series of grooves were drilled in a thin STO single-crystal plate by femtosecond laser micromachining at Alphanov facility [13]. STO rods (width $a$ $=68$ $\mu$m) separated by air gaps (width $g=28$ $\mu$m) covered an area of $2.5\times 3$ mm$^2$ on the plate with a thickness $e=26$ $\mu$m; see Fig. 1(a). Details on sample preparation can be found in [10]. Transmittance spectra of the sample were measured by time-domain terahertz spectroscopy [10] with polarization of the electric field vector of terahertz radiation perpendicular to the rods. Figure 1(b) shows the amplitude transmittance where a series of Mie resonances appear as distinct minima.

Simulations were carried out by using the Maxwell equation finite-element solver HFSS [14]. We used it to calculate the spectra of complex transmittances

 

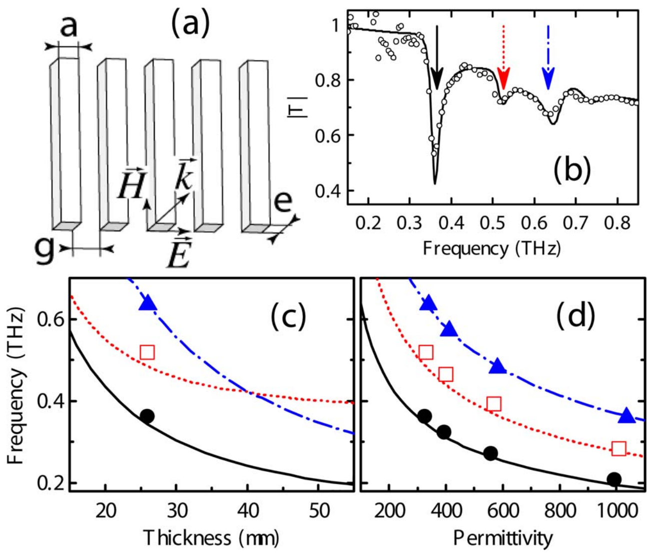


Fig. 1. (Color online) (a) Geometry of the MM structure. (b) Measured and calculated transmission amplitude spectra of the studied sample at room temperature. Mie resonances are indicated by arrows. (c) Resonant frequencies as a function of the thickness $e$. (d) Resonant frequencies as a function of Re $\epsilon$ of the rods. Symbols, experiment; lines, calculations. The resonance indicated by the dash–dotted curve has a nonmagnetic character.

and reflectances of periodic arrays of infinitely long STO rods. If not specified otherwise, we considered a frequency independent loss value of tan $\delta=2.5\%$ which is found in STO at 0.2 THz [11]. The effective MM properties ($\epsilon$ and $\mu$) were then deduced [15].

The Mie resonance frequencies essentially depend on the optical thickness of the rods in the terahertz range, i.e., on their $\epsilon$ and physical dimensions. This is shown in Figs. 1(c) and 1(d), where either $\epsilon$ or thickness $e$ of the rods is varied. We observe a very good agreement between the calculated resonant frequencies and those found experimentally (with $\epsilon$ controlled by temperature). This proves that the results of our calculations are accurate for the entire series of Mie resonances. The resonance frequency presents a hyperbolic decrease as a function of the terahertz refractive index of the rods. Despite a different shape of the MM, this trend is analogous to that found in dielectric spheres with radius $r_0$ and permittivity $\epsilon_r$, for which the resonance frequency in the long-wavelength limit and for the lowest resonance can be written as $\omega_{\text{res}}=\pi c/(r_0\sqrt{\epsilon_r})$ [5].

We also studied the influence of the aspect ratio of the rod section on the first Mie resonance frequency. In fact, this frequency is essentially determined by the smaller dimension ($a$ or $e$). While the modification of the larger dimension leads only to a very small change in the resonance position, the variation of the smaller one leads to an approximately hyperbolic tuning of the resonant frequency (cf. the relation above).

The simplest way to extend the range of negative $\mu$ is to design a structure with a set of rods with various widths. We first consider alternating rods with three different widths denoted by $a$, $b$, and $c$ (the air-gap width $g$ remains constant throughout). The results of simulations are shown in Fig. 2.

The effective $\mu$ is a superposition of individual responses of the three kinds of rods. The calculated distributions of the magnetic field (see Fig. 2) show that each rod is individually resonant and that the mutual cross couplings are negligible. However, the resonances are weaker than those observed in a MM with a uniform width of rods, since the number of the rods with each dimension is lower (cf. solid and dashed curves in Fig. 2). We also varied the air-gap width $g$ within 2–50 $\mu$m while the dimensions of the rods were fixed. This confirmed that $g$ has a minor influence on the effective magnetic properties and leads only to a shift by a few gigahertz of the entire effective response. Virtually no electric and magnetic coupling between adjacent rods occurs, even if $g$ is decreased down to 2 $\mu$m.

A much-broader continuous band of negative $\mu$ can be achieved in structures with a high aspect ratio ($a \gg e$). Here we consider structures each made of a single type of rods with a width $a$ varied over a broad range (50 to 200 $\mu$m) while $g=30$ $\mu$m and $e=20$ $\mu$m are kept constant. The transmittance and permeability spectra of such structures are shown in Figs. 3(a) and 3(b). The position of the first Mie resonance does not shift significantly; as pointed out above, it is mainly imposed by the thickness $e$ for $a \geqslant 100$ $\mu$m. By contrast, upon increasing $a$, the distance between the first- and higher-order resonances diminishes, and in the limit $a \gg e$ the resonances overlap [see Fig. 3(a)]. This leads to a broadening of the region of negative $\mu$ [Fig. 3(b)]. For example, the retrieved effective $\mu$ for $a=50$ $\mu$m is negative within 480–600 GHz, but for higher rod widths this range is much broader; namely for $a=200$ $\mu$m it spans over 250 GHz, corresponding to nearly 50% of the central frequency value.

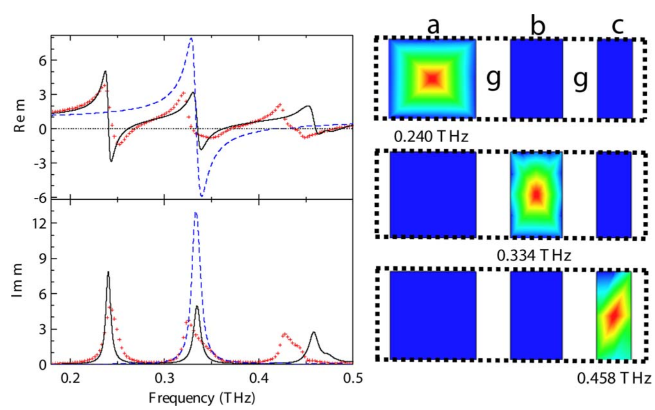


Fig. 2. (Color online) Left panel, calculated spectra of effective $\mu$ for a periodic structure of alternating rods with three different widths $a$, $b$, and $c$. Solid curve, $a=50$ $\mu$m, $b=30$ $\mu$m, $c=20$ $\mu$m, $g=20$ $\mu$m; dots, same values of $a$, $b$, $c$, but $g=2$ $\mu$m; dashed curve, $a=b=c=30$ $\mu$m, $g=20$ $\mu$m; $e=50$ $\mu$m for all structures. Right panel, sections of a unit cell of the MM with the spatial distribution of the resonant magnetic field. The ratios $H_{\max}/H_{\text{inc}}$ of the maximum and incident fields are 2.6 at 0.240 THz, 2.9 at 0.334 THz, and 2.4 at 0.458 THz.

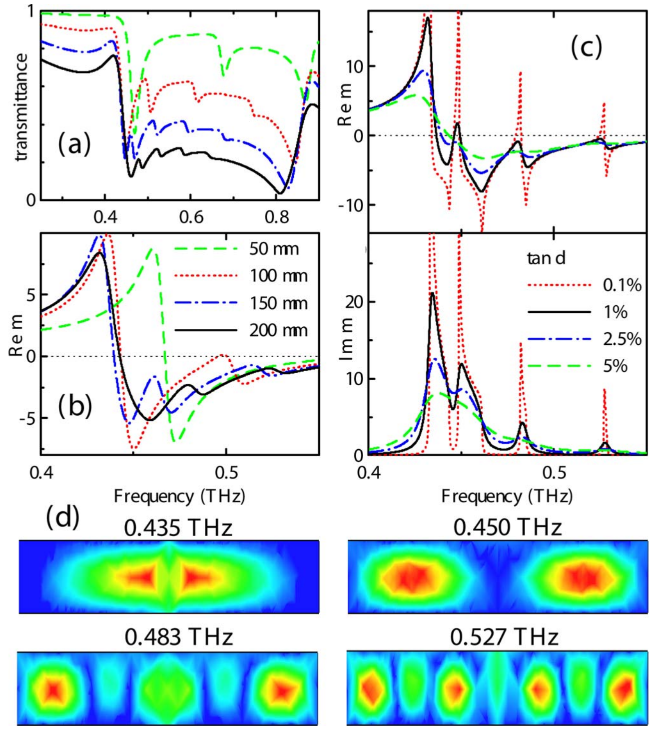


Fig. 3. (Color online) (a) Calculated amplitude transmittance for several widths $a$ of STO rods ($g=30\ \mu$m, $e=20\ \mu$m, $\tan\delta=2.5\%$) and (b) the corresponding effective $\mu$. (c) Effective $\mu$ for $a=200\ \mu$m and $\tan\delta$ from 0.1% to 5%. (d) Spatial distribution of the resonant magnetic field inside the rods for $a=200\ \mu$m and $\tan\delta=2.5\%$; the ratios $H_{\max}/H_{\mathrm{inc}}$ of the maximum and incident fields are 19.6 at 0.435 THz, 15.4 at 0.450 THz, 12.2 at 0.483 THz, and 8.7 at 0.527 THz.

Figure 3(c) shows the effective $\mu$ for $a=200\ \mu$m and values of $\tan\delta=\mathrm{Im}\,\epsilon/\mathrm{Re}\,\epsilon$ from 0.1% to 5%. We observe clear magnetic resonances resulting in peaks in the effective response. Between the peaks, the effective $\mu$ is negative over a broad frequency range only if the dielectric losses reach a sufficiently high level ($\tan\delta>1\%$). Upon the increase of $\tan\delta$, the resonances become smoother and their magnitude decreases. The imaginary part of $\mu$ shows a similar behavior; the increase in $\tan\delta$ leads to smaller effective magnetic losses at the resonances, while the mean level of losses increases proportionally to $\tan\delta$ in the range of $\mathrm{Re}\,\mu<0$ [Fig. 3(c)]. Further analysis reveals a compromise between a high negative $\epsilon$ limited by the value of $\tan\delta$ and a large bandwidth where $\mu<0$. We propose an optimum thickness of $e\approx 20\ \mu$m for $a=200\ \mu$m. Then the range of negative $\mu$ spans from 430 to 680 GHz. Figure 3(d) shows the field distribution at resonant frequencies in this case.

In conclusion, we have presented results of calculations of effective magnetic properties of terahertz MMs based on Mie resonances in dielectric rods. We focused on the relation between the geometrical parameters and the range of the negative effective permeability. This allowed us to propose two types of structures exhibiting regions with a negative $\mu$ broader than demonstrated previously: one with different widths of rods and one with a high aspect ratio $a/e$. It should be stressed that the broad band with negative $\mu$ depends on the dielectric losses, and there is an interval of optimum values within $\tan\delta\approx 1$–$5\%$. This could be a useful criterion for the choice of dielectrics for practical realization and applications.

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