# HIGH ENERGY PARTICLES FROM MONOPOLES CONNECTED BY STRINGS

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*Monopole-antimonopole pairs connected by strings and monopole-string networks with $N > 2$ strings attached to each monopole can be formed at phase transitions in the early universe. In such hybrid defects, monopoles accelerate under the string tension and can reach ultrarelativistic Lorentz factors, $\gamma \gg 1$. We study the radiation of gauge quanta by accelerating monopoles. For monopoles with a chromomagnetic charge, we also discuss the high-energy hadron production through emission of virtual gluons and their subsequent fragmentation into hadrons. The relevant parameter for gauge boson radiation is $M/a$, where $M$ is the boson mass and $a$ is the proper acceleration of the monopole. For $M \ll a$, the gauge bosons can be considered as massless and the typical energy of the emitted quanta is $E \sim \gamma a$. In the opposite limit, $M \gg a$, the radiation power is exponentially suppressed and gauge quanta are emitted with a typical energy $E \sim \gamma M$ in a narrow range $\Delta E/E \sim (a/M)^{1/2}$. Cosmological monopole-string networks can produce photons and hadrons of extremely high energies. For a wide range of parameters these energies can be much greater than the Planck scale.*

## I. INTRODUCTION

Monopoles connected by strings can be formed in a sequence of symmetry breaking phase transitions in the early universe [1,2]. A typical sequence of this sort is

$$G \to H \times U(1) \to H \times Z_N. \tag{1}$$

For a semi-simple group $G$, the first of these phase transitions gives rise to monopoles, and at the second phase transition each monopole gets attached to $N$ strings. For $N \geq 3$, a single infinite network is formed which permeates the entire universe [3]. The magnetic fluxes of monopoles in the network are channelled into the strings that connect them. But the monopoles typically have additional, unconfined magnetic charges. The gauge fields associated with these charges may include electromagnetic or gluon fields, but may also correspond to broken gauge symmetries and have non-zero masses.

The cosmological evolution of monopole-string networks has been discussed in Ref. [4], where it was argued that the networks evolve in a self-similar manner, with a characteristic scale growing proportionally to cosmic time $t$. The typical energy of the monopoles also grows like $t$, and the monopoles become highly relativistic. Radiation of gauge quanta by accelerating monopoles is the main energy loss mechanism of the networks. Relativistic particles emitted by the monopoles (or the decay products of these particles) contribute to the spectrum of high-energy cosmic rays.

For the symmetry breaking sequence

$$G \to H \times U(1) \to H, \tag{2}$$

which can be regarded as a special case of (1) with $N = 1$, the cosmological evolution is quite different. In this case, each monopole is attached to a single string, and the second phase transition results in monopole-antimonopole (M$\bar{\text{M}}$) pairs connected by strings. If both of the phase transitions in (2) occur during the radiation era, then the average monopole separation is always smaller than the Hubble radius, and when M$\bar{\rm M}$ pairs get connected by strings and begin oscillating, they typically dissipate the bulk of their energy to friction in less than a Hubble time [2,5].

A more interesting possibility arises in the context of inflationary scenario, when monopoles are formed during inflation but are not completely inflated away. Strings can either be formed later during inflation, or in the post-inflationary epoch. In this case, the strings connecting M$\bar{\rm M}$ pairs can be very long. The correlation length of strings, $\xi$, can initially be much smaller than the average monopole separation, $d$; then the strings connecting monopoles have Brownian shapes. But in the course of the evolution, $\xi$ grows faster than $d$, due to small loop production, and to the damping force acting on the strings. Eventually, $\xi$ becomes comparable to the monopole separation, and we are left with $M\bar{M}$ pairs connected by more or less straight strings. At later times, the pairs oscillate and gradually lose their energy by gravitational radiation and by radiation of light gauge bosons (if the monopoles have unconfined magnetic charges). When the energy of a string connecting a pair is dissipated, the monopoles and antimonopoles annihilate into relativistic particles.

Although monopole-antimonopole pairs connected by strings do not typically survive until the present time, they can produce a characteristic feature in the spectrum of the stochastic gravitational wave background [6]. The magnitude of this feature depends in particular on the intensity of the gauge boson radiation, which determines the lifetime of the pairs.

The production of high energy particles by topological defects is interesting as a mechanism of emission of ultrahigh energy particles, in particular photons, nucleons and neutrinos. The energies of such particles can be much higher than have been observed until now. Particle production by cosmic strings [7] and by annihilating monopole-antimonopole pairs [8] has been previously discussed in the literatute. Here, we shall calculate the radiation power and the spectrum of gauge bosons, both massless and massive, emitted by an oscillating monopole–antimonopole pair. If the magnetic charge of the monopoles is of electromagnetic nature, then the radiation can be treated in the same way as that of an accelerating charge, and our problem reduces to a problem in classical electrodynamics. In the case of a chromomagnetic charge, the picture is more complicated. The monopoles emit heavy virtual gluons which originate a parton cascade. The hadrons are produced in the usual way beyond the confinement radius. The decay of pions results in high energy neutrino radiation. The heavy gauge bosons, such as $W$ and $Z$, can also be produced for some range of parameters of the $\mathrm{M\bar{M}}$ pair and the string.

The problem studied here is in fact a general theoretical problem: the radiation of heavy particles by an accelerating monopole. Our results can be reformulated in terms of the basic parameters of a shock problem, the proper acceleration of the monopole.

The dynamics of $\mathrm{M\bar{M}}$ pairs connected by strings is rather complicated and has not been studied in any detail in the litterature. We shall therefore concentrate on the simplest case of an oscillating pair connected by a straight string, for which the equations of motion can be solved exactly. In addition, we shall consider the case of a harmonic oscillator motion of the monopoles which has some of the features expected in more general monopole-string configurations. Taken together, the analyses of these two cases will allow us to draw a reasonably complete picture of the radiation spectrum in the general case.

After reviewing the general formalism for gauge boson radiation in the next Section, we calculate the radiation spectrum of massless gauge bosons in Section III. Radiation of massive gauge bosons is analysed in Section IV. The spectrum of hadrons resulting from radiation of gluons and their subsequent fragmentation are discussed in section V. Finally, Section VI contains some concluding remarks.

## II. GENERAL FORMALISM

The characteristic monopole radius $\delta_m$ and string thickness $\delta_s$ are determined primarily by the corresponding symmetry breaking scales, $\eta_m$ and $\eta_s$. Typically, $\delta_m \sim \eta_m^{-1}$ and $\delta_s \sim \eta_s^{-1}$. We shall assume that the two symmetery breaking scales are well separated, $\eta_s \ll \eta_m$, and thus the monopole radius is much smaller than the string thickness, $\delta_m \ll \delta_s$.

Assuming also that the string length is much greater than its thickness, we can treat monopoles as point particles and strings as infinitely thin lines. The monopole equations of motion can then be written as [9]

$$\frac{d^2x^\nu}{ds^2} = \frac{\mu}{m}\lambda^\nu, \tag{3}$$

where $\mu$ is the mass per unit length (and tension) of the Goto-Nambu string, $m$ is the monopole mass, $\tau$ is the proper time of the monopole, and $\lambda^\nu$ is a unit vector orthogonal to the monopole wordline and oriented into the string worldsheet. It follows immediately from Eq. (3) that

$$a = \frac{\mu}{m}, \tag{4}$$

is the proper acceleration of the monopoles. Eq. (3) can also be rewritten in a 3-dimensional form:

$$\gamma^3\ddot{\mathbf{x}}(t) = a\gamma^{(s)}\mathbf{n}. \tag{5}$$

where, $\gamma = (1-\dot{\mathbf{x}}^2)^{-1/2}$ is the Lorentz factor of the monopole, $\gamma^{(s)}$ is the Lorentz factor of the string at the location of the monopole, and $\mathbf{n}$ is a unit vector pointing from the monopole in the direction of the string. Eqs (3) and (5) have a simple physical meaning: in the instantaneous rest frame of the monopole, the magnitude of its acceleration is given by Eq. (4) and the direction is given by the direction of the string. The motion of the string is described by the usual Goto-Nambu equations. A simple solution of Eq. (5) describing an oscillating M$\bar{\text{M}}$ pair connected by a straight string is [10]

$$x(t) = \pm\frac{\text{sgn}(t)}{a}\left[\gamma_0 - \sqrt{1+(\gamma_0\ v_0 - a|t|)^2}\right], \tag{6a}$$

$$y(t) = z(t) = 0, \tag{6b}$$

where $v_0$ and $\gamma_0 = (1-v_0^2)^{-1/2}$ are respectively the maximal velocity and the maximal Lorentz factors of the monopoles, reached at $t=0$. The upper and lower signs correspond to M and $\bar{\text{M}}$ respectively.

The solution (6) is valid for $|t| \leq \gamma_0 v_0/a$. At $t = -\gamma_0 v_0/a$, the monopoles are at rest, with the string having its maximum length, $L = 2(\gamma_0 - 1)/a$. At $t = +\gamma_0 v_0/a$, the monopoles come to rest again, with their positions interchanged. Eq. (6) describes only half a period, $T = 4\gamma_0 v_0/a$, the other half period being obtained by exchanging the positions of the monopole and antimonopole. A peculiar feature of this solution (6) is that the monopole accelerations abruptly change direction when the monopoles meet and pass one another. Between these sign changes the monopoles move at constant proper acceleration $a$. The monopole and antimonopole meet at $t=0$ and could be expected to annihilate. However, this solution is considered as an approximation for an almost straight configuration where the monopole and antimonopole would merely come close to each other and would not collide. Besides, as we already mentioned, the monopole radii are much smaller than the string thickness, thus the monopoles are not likely to collide, even for a straight string.

To find the radiation power of gauge bosons by accelerating monopoles, we first consider radiation by ordinary gauge charges. The gauge field $A^\mu(x)$ produced by the 4-current $j^\mu(x)$ is determined by the equations

$$F^{\nu\sigma}{}_{,\nu} + M^2 A^\sigma = j^\sigma, \tag{7}$$

where $F_{\nu\sigma} = \partial_\nu A_\sigma - \partial_\sigma A_\nu$ is the field strength and $M$ is the gauge boson mass. The current corresponding to a pair of equal and opposite point charges is

$$j^\nu(x) = q\dot{x}_+^\nu \delta^{(3)}(\mathbf{x} - \mathbf{x}_+(t)) - q\dot{x}_-^\nu \delta^{(3)}(\mathbf{x} - \mathbf{x}_-(t)), \tag{8}$$

where $\mathbf{x}_+(t)$ and $\mathbf{x}_-(t)$ are the trajectories of the charges and $\dot{x}_\pm^\nu = dx_\pm^\nu/dt$. In the case of a periodic motion, the radiation power can be found from the following equations

$$P = \sum_n P_n = \sum_n \int d\Omega \frac{dP_n}{d\Omega}, \tag{9}$$

$$\frac{dP_n}{d\Omega} = -\frac{k\omega_n}{8\pi^2} j_\mu^*(\omega_n, \mathbf{k}) j^\mu(\omega_n, \mathbf{k}). \tag{10}$$

Here, $dP_n/d\Omega$ is the radiation power at frequency $\omega_n = 2\pi n/T$ per unit solid angle in the direction of $\mathbf{k}$, $|\mathbf{k}| = (\omega_n^2 - M^2)^{1/2}$, $T$ is the period of the oscillation, and

$$j^\mu(\omega_n, \mathbf{k}) = \frac{1}{T}\int_0^T dt \exp(i\omega_n t) \int d^3x \exp(-i\mathbf{k}\cdot\mathbf{x}) j^\mu(\mathbf{x}, t) \tag{11}$$

is the Fourier transform of the current. Eq. (10) is derived in the Appendix A. The derivation is similar to that of the gravitational radiation power in Weinberg's book [11].

Because of electric-magnetic duality, the same equations apply to radiation by magnetic monopoles. Hence, the radiation power from an oscillating M$\bar{\text{M}}$ pair can be found from Eqs. (8)-(11) with $q$ interpreted as the magnetic charge.

For a one dimensional motion, the current has only two non-zero components: $j^0$ and $j^1$. It is also conserved, $\nabla_\nu j^\nu = 0$, which in Fourier space can be written as

$$\omega_n j^0 = k_x j^1, \tag{12}$$

where $k_x \equiv k^1$. Combining this with Eq. (10), we have

$$\frac{dP_n}{d\Omega} = \frac{\omega_n k}{8\pi^2}(1 - \frac{k_x^2}{\omega_n^2})|j_1(\omega_n, \mathbf{k})|^2. \tag{13}$$

The motion of the M$\bar{\text{M}}$ pair in Eq. (6) has the properties

$$x_+(t) = -x_-(t) = -x_+(-t) \tag{14a}$$

$$x_+(t) = -x_+(t + T/2). \tag{14b}$$

In such a case, it is easily seen that the system radiates only in odd modes, and that for $n$ odd, we have

$$j_1(\omega_n, \mathbf{k}) = \frac{8q}{T} \int_0^{T/4} \dot{x}(t)dt \cos(\omega_n t) \cos[k_x x_+(t)]. \tag{15}$$

Before we proceed with the calculation of the radiation spectrum, we shall briefly comment on the limits of applicability of the classical treatment of the radiation process that we adopt in this paper. The problem of radiation by a monopole pulled by a contracting string is equivalent to that of a monopole moving in a longitudinal magnetic field, or of an electric charge moving in a longitudinal electric field. Such processes in strong fields are characterized [12], [13] by the invariant $\chi$ given by

$$\chi = (q/m^3)\sqrt{(F_{\mu\nu}p_\nu)^2} = f/m^2, \tag{16}$$

where $q$, $m$ and $p_\nu$ are respectively the charge, mass and momentum of the radiating particle and $f$ is the force acting on the particle in its instantaneous rest frame. In the case of a monopole connected to a string the corresponding parameter is $\chi = \mu/m^2 = a/m$. The case $\chi \ll 1$ corresponds to the quasiclassical regime [12]. Hence, our classical treatment of radiation is justified provided that $\mu \ll m^2$.

## III. RADIATION OF MASSLESS GAUGE BOSONS

We begin by calculating the radiation of massless gauge bosons by an oscillating $M\bar{M}$ pair described by Eq. (6). In this case, Eqs. (13), (15) for the radiation power can be transformed to

$$P_n = \frac{2q^2a^2}{\pi\gamma_0^2 v_0^2} \int_0^1 (1-u^2)[S_n(u)]^2 du, \tag{17}$$

$$S_n(u) = \int_0^{\frac{n\pi}{2v_0}(1-\frac{1}{\gamma_0})} \cos(u\xi) \sin\left(\sqrt{\left(\frac{n\pi}{2v_0} - \xi\right)^2 - \frac{n^2\pi^2}{4\gamma_0^2 v_0^2}}\right) d\xi. \tag{18}$$

where we have introduced new integration variables $u = k_x/\omega_n$ and $\xi = n\pi a x_+(t)/2\gamma_0 v_0$. These integrals cannot be evaluated analytically. However, the high-frequency asymptotic behaviour can be obtained as an expansion in powers of $1/n$. This can be done simply by repeatedly integrating by parts in Eq. (18) [10].

The leading term of the expansion is

$$S_n(u) \approx \frac{2v_0}{n\pi\gamma_0^2}\frac{1+3u^2v_0^2}{(1-u^2v_0^2)^3} \tag{19}$$

and the corresponding power spectrum is

$$P_n \approx A\left(\frac{qa\gamma_0}{n}\right)^2, \tag{20}$$

where

$$A = \frac{8}{\pi^3}\int_0^1 (1-x^2)\frac{[\gamma_0^2+3x^2(\gamma_0^2-1)]^2}{[\gamma_0^2-x^2(\gamma_0^2-1)]^6}dx. \tag{21}$$

For ultrarelativistic monopoles, $\gamma_0 \gg 1$, this can be approximated as

$$P_n \approx \frac{16}{5\pi^3}\left(\frac{qa\gamma_0}{n}\right)^2. \tag{22}$$

The $1/n$ expansion is valid as long as the first neglected term is small compared to the leading term (22). A somewhat lengthy but staightforward calculation gives the condition $n \gg \gamma_0^2$. At lower frequencies, the $n$-dependence is rather complicated and analytic approximations are difficult to obtain.

We computed the radiation spectrum numerically by integrating Eqs. (17), (18) for various values of $\gamma_0$. A generic example of this spectrum, obtained for $\gamma_0 = 10$, is plotted in Figure 1. It exhibits the expected behaviour (22) at high frequencies and is approximately flat at lower frequencies. The transition happens around $n \sim \gamma_0^2$ which is the characteristic frequency $\overline{\omega} \sim \gamma_0^2\omega_1$, at which most of the power is radiated. We note that in the rest frame of the monopole it is

$$\omega_{rest} \sim \overline{\omega}/\gamma_0 \sim \gamma_0\omega_1 \sim a. \tag{23}$$

The total power radiated by the system can be evaluated directly using the standard formulas of electrodynamics,

$$P = \frac{(qa)^2}{4\pi}\int_0^1 du(1-u^2)\int_0^1 d\xi([\gamma_0^2(u-v_0\xi)^2+1-u^2]^{-3/2}+[\gamma_0^2(u+v_0\xi)^2+1-u^2]^{-3/2})^2. \tag{24}$$

When the monopole and antimonopole reach ultrarelativistic speeds, their radiation becomes focused in their respective forward directions, which are opposite, and thus interference between the monopole and antimonopole fields is negligible. This means that the total power radiated by the system is close to twice the power radiated by the monopole alone. The radiation intensity of a single charge $q$ moving with proper acceleration $a$ is [14] $P = q^2a^2/6\pi$. Thus, for high velocities of the monopole and antimonopole $\gamma_0 \gg 1$, the total power tends to

$$P \approx \frac{1}{3\pi}(qa)^2. \tag{25}$$

The numerically calculated power has been plotted as a function of $\gamma_0$ in Figure 2 and indeed quickly tends to this asymptotic limit.

The monopole trajectory (6) that we studied so far is rather special. As we already mentioned, the monopole accelerations change discontinuously as the monopoles pass one another. It is this feature that is responsible for the power-law (rather than exponential) falloff of $P_n$ at large $n$. To see how the character of the spectrum is changed in a more generic case when the acceleration varies smoothly, we considered a simple harmonic-oscillator-type motion of the monopoles,

$$x_+(t) = x_0 \sin \Omega t = -x_-(t). \tag{26}$$

The period of this motion is $T = 2\pi/\Omega$, the maximum velocity reached by the monopoles is $v_0 = \Omega x_0$, and the maximum Lorentz factor is $\gamma_0 = (1-v_0^2)^{-1/2}$. The parameters $x_0$

and $\Omega$ should of course satisfy $v_0 = \Omega x_0 < 1$, but we can shoose $v_0$ to be arbitrarily close to 1. The proper acceleration of the monopoles in Eq. (26) is

$$a(t) = -\frac{\Omega^2 x_0 \sin \Omega t}{[1 - \Omega^2 x_0^2 \cos^2 \Omega t]^{3/2}}. \tag{27}$$

Its maximum value determined by condition $\dot{a}(t_m) = 0$ is reached at $\sin \Omega t_m = \pm 1/\sqrt{2}\gamma_0 v_0$, if $v_0 > 1/\sqrt{3}$. It is given by

$$a_m = \frac{2}{3\sqrt{3}} \gamma_0^2 \Omega. \tag{28}$$

For ultrarelativistic monopoles, $\gamma \gg 1$, the acceleration remains of this order of magnitude, $a \sim \gamma_0^2 \Omega$, only for a brief interval of time,

$$\Delta t \sim \left( \frac{a_m}{|\ddot{a}_m|} \right)^{1/2} \sim (\gamma_0 \Omega)^{-1}. \tag{29}$$

Since $P \propto a_m^2$ this part of the oscillation period gives the dominant contribution to the radiation power in the massless case.

The power radiated in massless gauge bosons by the harmonic solution (26) can be computed from the standard formulas (13) and (15). It gives

$$P_n = \frac{2}{\pi} (n\Omega q)^2 \int_0^1 du (\frac{1}{u^2} - 1) \mathrm{J}_n^2 (n u v_0), \tag{30}$$

where $\mathrm{J}_n$ is the $n$-th Bessel function.

For large frequency modes $n \gg 1$ and a parameter $x < 1$, the Bessel function can be expanded using the formula

$$\mathrm{J}_n^2(nx) \simeq \frac{1}{2n\pi} \frac{\mathrm{e}^{-2n(1-x^2)^{3/2}/3}}{\sqrt{1-x^2}} \tag{31}$$

to get the simplified expression for the power

$$P_n \approx n \left( \frac{\Omega q}{\pi} \right)^2 \int_0^1 du (\frac{1}{u^2} - 1) \frac{\mathrm{e}^{-2n(1-u^2 v_0^2)^{3/2}/3}}{\sqrt{1 - u^2 v_0^2}}. \tag{32}$$

At high frequencies, the radiation is focused in the monopoles forward direction, which corresponds to $u \sim 1$. This translates in (32) to the steep fall off of the exponential.

For not too high frequency modes $1 \ll n \lesssim \gamma_0^2$, the expression of the Fourier transform of the current can be somewhat simplified

$$\begin{aligned} j_0(n\Omega, \mathbf{k}) &= \frac{8q}{T} \int_0^{T/4} dt \sin(n\Omega t) \sin[nu(1 - 1/\gamma_0^2) \sin(\Omega t)], \\ &\approx \frac{4q}{\pi} \int_0^{\pi/2} dv \sin(nv) \sin[nu \sin v], \end{aligned} \tag{33}$$

where $u = k_x/\Omega$. This corresponds basically to taking $v_0 = 1$ in $j_0$. Thus, in this frequency range, the approximation (32) takes the form

$$P_n \approx n \left(\frac{\Omega q}{\pi}\right)^2 \int_0^1 \frac{du}{u^2} \sqrt{1-u^2} \exp\left(-\frac{2n}{3}(1-u^2)^{3/2}\right). \tag{34}$$

A simple change of variable $\xi = 2n(1-u^2)^{3/2}/3$ enables us to evaluate the integral:

$$\begin{aligned} P_n &\approx \frac{(\Omega q)^2}{2\pi^2} \int_0^{2n/3} \frac{d\xi \mathrm{e}^{-\xi}}{[1-(3\xi/2n)^{2/3}]^{3/2}} \\ &\approx \frac{(\Omega q)^2}{2\pi^2} \int_0^{\infty} d\xi \mathrm{e}^{-\xi} = \frac{(\Omega q)^2}{2\pi^2}. \end{aligned} \tag{35}$$

So the spectrum is expected to be flat in this frequency range.

For very high frequency modes $n \gg \gamma_0^3$, we can expand the argument of the exponential in (32) around $u = 1$, and simply take all the other slower varying terms as being constant. This gives

$$\begin{aligned} P_n &\approx \frac{2}{\pi^2} (\Omega q)^2 n \mathrm{e}^{-2n/3\gamma_0^3} \int_0^{+\infty} v dv \mathrm{e}^{-2nv_0^2 v/\gamma_0} \\ &\approx \frac{\gamma_0^3 \Omega^2 q^2}{2\pi^2 v_0^4} \frac{\mathrm{e}^{-2n/3\gamma_0^3}}{n}. \end{aligned} \tag{36}$$

The radiation spectrum (30) was computed numerically for various values of $\gamma_0$. A generic example of this spectrum obtained for $\gamma_0 = 25$ is given in Figure 3. As found in (35), the low frequency behaviour is flat, up to frequencies $\bar{\omega} \sim a_m \gamma_0$. The expected high frequency behaviour (36) then sets in at higher frequencies. Compared to the spectrum obtained for the exact solution (6), this spectrum is very similar: flat at lower frequencies then decaying fast. But, for the harmonic solution, the flat part has a smaller value, extends farther in frequencies and the fall off at higher frequencies is much faster.

The total power radiated by the two monopoles is easy to evaluate when they are ultrarelativistic $\gamma_0 \gg 1$. In this limit, as explained previously, interference between the monopole and antimonopole fields is negligible and the total power is just the sum of the power emitted by the monopole and antimonopole separately. The radiation intensity for a single charge moving with a proper acceleration (27) is [14] $P = 9q^2 a_m^2/16\gamma_0$. Thus, for ultrarelativistic monopoles $\gamma_0 \gg 1$, the total power tends to

$$P \approx \frac{9}{8\gamma_0} q^2 a_m^2 = \frac{1}{6}\gamma_0^3 \Omega^2 q^2. \tag{37}$$

Contrary to the previous case, Eq. (25), this has an extra factor $\gamma_0$ in the denominator. It comes from the fact that for ultrarelativistic monopoles, the main contribution to the total power comes from the interval of time $\Delta t$ in (29) when their acceleration is near its maximum $a_m$ so that $P \sim q^2 a_m^2 \Delta t/T \sim q^2 a_m^2/\gamma_0$. This total power is also consistent with the picture of a spectrum consisting of a low frequency flat behaviour $P_n \sim A\gamma_0^{-4}$ up to $n \sim \gamma_0^3$ as shown in (35), and then a negligible exponentially decreasing high frequency behaviour which can be ignored. This gives a useful approximation for the spectrum of massless gauge bosons,

$$P(E) = \frac{(\Omega q)^2}{2\pi^2} \equiv P_0 \quad \text{for} \quad E < \frac{\pi^2}{3}\gamma_0^3 \Omega \equiv E_0, \tag{38a}$$

$$P(E) = 0 \qquad \text{for} \quad E > E_0, \tag{38b}$$

which has the same total power as the exact spectrum and will be a good approximation for large $\gamma$-factors.

It is instructive to compare the results we obtained in this Section with the case of synchrotron radiation by a charge moving at a constant speed in a circular orbit [14]. The power spectrum in this case exhibits a peak around $\bar{\omega} \sim \gamma^3 \Omega$, where $\gamma$ is the Lorentz factor and $\Omega$ is the angular frequency of the rotating charge. The proper acceleration of the charge is $a \approx \gamma^2\Omega$, so once again the characteristic frequency is $\bar{\omega} \sim \gamma a$.

## IV. RADIATION OF MASSIVE GAUGE BOSONS

We will now consider the radiation of massive gauge bosons of mass $M$. For simplicity, we will consider only the harmonic oscillation solution (26) in this section.

The power radiated in massive gauge bosons is still given by the general formula (13) but now $k = \omega_n u_n$, where we have introduced

$$u_n = \sqrt{1 - M^2/\omega_n^2} \tag{39}$$

and $\omega_n = n\Omega$. The current and its Fourier transform remain the same as in the massless case, and thus we obtain a very similar formula

$$P_n = 2\frac{q^2\Omega^2}{\pi}n^2 \int_0^{u_n} du(\frac{1}{u^2} - 1)\mathrm{J}_n^2(nuv_0), \tag{40}$$

where only the upper bound in the integral has changed. The massless gauge boson radiation (40) is obviously recovered in the limit $M \to 0$ or $n \to \infty$. Using the asymptotic expression of the Bessel function (31), valid for $n \gg 1$, we obtain

$$P_n \approx \frac{q^2}{\pi^2}\Omega^2 n \int_0^{u_n} \frac{du}{\sqrt{1 - u^2v_0^2}} \left(\frac{1 - u^2}{u^2}\right) \exp\left(-\frac{2}{3}n(1 - u^2v_0^2)^{3/2}\right) \tag{41}$$

In the case of massive gauge bosons with $M \gg a_m$, where $a_m$ is the maximal acceleration, both functions in front of the exponent can be taken at value $u = u_n$ and after integration we obtain

$$P_n \approx \frac{q^2}{2\pi^2}\frac{\gamma_0^2 M^2}{v_0^2}\left(1 - \frac{M^2}{n^2\Omega^2}\right)^{-3/2}\left(1 + \frac{\gamma_0^2 v_0^2 M^2}{n^2\Omega^2}\right)^{-1}\frac{1}{n^2}\exp\left(-f(n)\right), \tag{42}$$

where

$$f(n) = \frac{2}{3}\frac{n}{\gamma_0^3}\left(1 + \frac{\gamma_0^2 v_0^2 M^2}{n^2\Omega^2}\right)^{3/2}. \tag{43}$$

The function (43) has a minimum

$$f_m = \sqrt{3}\frac{Mv_0}{\gamma_0^2\Omega} \tag{44}$$

which is reached at

$$n = n_m = \sqrt{2}\frac{\gamma_0 v_0 M}{\Omega}. \tag{45}$$

The second derivative of $f(n)$ at $n = n_m$ is

$$f_{nn} = \frac{2}{\sqrt{3}}\frac{\Omega}{\gamma_0^4 v_0 M}. \tag{46}$$

The spectrum (42) has a maximum at $n \approx n_m$.

At very large $n \gg (M/a_m)n_m$ we obtain the spectrum (37) of a massless case[1], as it must be according to (40).

At $n_m \ll n < (M/a_m)n_m$ the energy flux is exponentially suppressed as

$$P_n \approx \frac{q^2}{2\pi^2}\frac{\gamma_0^2 M^2}{v_0^2}\frac{1}{n^2}\exp(-\frac{2}{3}n/\gamma_0^3). \tag{47}$$

At $M/\Omega \ll n \ll n_m$ the energy flux is also exponentially suppressed as

$$P_n \approx \frac{q^2}{2\pi^2}\frac{\Omega^2}{v_0^4}\exp(-\frac{2}{3}M^3 v_0^3/n^2\Omega^3). \tag{48}$$

In the region of the maximum $n_m$, the spectrum is

$$P_n \approx \frac{q^2}{12\pi^2}\frac{\Omega^2}{v_0^4}\exp\left(-\sqrt{3}\frac{v_0 M}{\gamma_0^2\Omega}\right)\exp\left(-\frac{1}{2}f_{nn}(n-n_m)^2\right). \tag{49}$$

Therefore, in the case we are considering $M \gg a_m$, the characteristic energy of emitted gauge bosons is

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[1]This can not be seen directly from (42) as the dominant term in this regime was already neglected. To get this extra term, the factor $(1-u^2)$ in (41) must be kept and integrated with the exponential term

$$E \approx n_m \Omega \approx \gamma_0 M, \tag{50}$$

while in the massless case it is $E \approx \gamma_0 a_m$.

The power spectra (40) have been computed numerically and are shown in Figure 4 for various values of $M/a_m$. Note that our analytical formulae given above are valid only for $M/a_m \gg 1$. One can see that at large values of n these spectra tend to the massless spectrum shown by a solid line.

The total radiated power, obtained by integrating (49)

$$P \approx \sqrt{\frac{2\pi}{3}} \frac{q^2}{6\pi^2} \frac{\gamma_0^3 \Omega^2}{v_0^4} \sqrt{M v_0 / a_m} \exp\left(-\frac{2}{3}\frac{M v_0}{a_m}\right), \tag{51}$$

is exponentially suppressed in the case $M \gg a_m$.

The formulae above are valid for harmonically oscillating monopoles with a frequency $\Omega$. We can reformulate these results for the instantaneous radiation power from a monopole moving with a proper accelleration $a$ and Lorentz factor $\gamma$. This is possible to do because for $M \gg a_m$ the radiation is strongly dominated by four very short bursts during the period $T$. The time dependence of the radiated power is determined mainly by the factor

$$P(t) \propto \exp\left(-\frac{2}{3}\frac{Mv}{a(t)}\right),$$

and the bursts occur at the maxima of the proper acceleration $a(t)$. Expanding $a(t)$ near the maximum, $a(t) \approx a_m + \frac{1}{2}\ddot{a}_m (t - t_m)^2$, and using Eq.(27) we obtain an estimate for the duration of the bursts

$$\tau \sim \left(\frac{a_m}{|\ddot{a}_m|}\frac{3a_m}{Mv_0}\right)^{1/2} \sim \frac{1}{v_0 \Omega \gamma_0}\left(\frac{a_m}{Mv_0}\right)^{1/2}. \tag{52}$$

It is $(a_m/Mv_0)^{1/2}$ times shorter than the corresponding duration in the massless case (29).

The fact that the dominant part of the radiation comes out in short bursts at $a \approx a_m$ suggests the following *ansatz* for the instantaneous power:

$$P(a_m, \gamma, E) \approx k \frac{T}{4\tau} P_{har}(E, \Omega). \tag{53}$$

Here, $\gamma = \sqrt{\frac{2}{3}}\gamma_m$ is the Lorentz factor of a harmonically oscillating monopole at the moment when it reaches $a = a_m$, $k \sim 1$ is a numerical coefficient, and $P_{har}(E, \Omega)$ is given by (49). We expect Eq.(53) to be valid for $E$ near the maximum of the spectrum at least for $|E - E_m|/E_m \ll 1$. Integrating the instantaneous spectrum (53) over the period $T$, with $a(t)$ and $\gamma(t)$ for the harmonic oscillator motion, and comparing with the averaged oscillator spectrum (49) we determine the normalizing constant, $k = 2\sqrt{2}/(3\sqrt{\pi})$. The total power and the spectrum for a relativistic ($\gamma \gg 1$) monopole moving with proper acceleration $a$ and Lorentz-factor $\gamma$ are thus given by

$$P(a) = \frac{\sqrt{3}}{8\pi} q^2 a M \exp\left(-\frac{2}{3}\frac{M}{a}\right), \tag{54}$$

$$P(a, \gamma, E) = \frac{q^2}{12\pi\sqrt{\pi}} \frac{a}{\gamma} \left(\frac{M}{a}\right)^{1/2} \exp\left(-\frac{2}{3}\frac{M}{a}\right) \exp\left(-f_{EE}(E - E_m)^2\right), \tag{55}$$

where

$$E_m = \sqrt{3}\gamma M. \tag{56}$$

$$f_{EE} = \frac{4}{27}\frac{1}{M a \gamma^2}, \tag{57}$$

Eqs.(56) and (57) indicate that the gauge quanta are emitted with a characteristic energy $E \sim \gamma M$ in a narrow range

$$\Delta E/E \sim (a/M)^{1/2}. \tag{58}$$

In the case of a vector field $A_\mu(x)$ considered above, there is a compensation in the energy-momentum tensor between the two terms corresponding to two nonvanishing components of the field (e.g $A_0$ and $A_3$). Such a compensation is absent for a scalar field $\phi(x)$ with an interaction $L(x) = q\phi(x)j(x)$, where

$$j(x) = \int d\tau \delta^{(4)}(x - x(\tau)) \tag{59}$$

and $\tau$ is the proper time along the trajectory of the source. In this case, a similar analysis, for $M \gg a$, gives the following expression for the instantaneous radiation power of a monopole moving with acceleration $a$ and Lorentz-factor $\gamma \gg 1$:

$$P_s = \frac{3\sqrt{3}}{8\pi} q^2 \gamma^2 a M \exp\left(-\frac{2}{3}\frac{M}{a}\right). \tag{60}$$

## V. PRODUCTION OF HIGH ENERGY HADRONS.

We shall consider here the production of high energy hadrons by an accelerating monopole through the most relevant process for this purpose: emission of "heavy" off-mass-shall gluons.

The GUT monopole has [15] a chromomagnetic charge, which is screened beyond the confinement radius. The monopole-gluon coupling constant is $1/\alpha_{QCD}$. Production of off-mass-shall gluons occurs in the same way as the radiation of massive gauge bosons (Section IV). The virtualities of gluons $Q^2$ higher than $a^2$ are exponentially suppressed and thus the typical vituality and energy of the emitted gluon is $a^2$ and $\gamma a$, respectively. A virtual gluon decays into two partons, thus starting a parton cascade, which is described by the Gribov-Lipatov-Alterelli-Parisi equation [16]. Beyond the confinement radius, this cascade is converted into a hadronic jet. This picture is quite similar to hadron production in $e^+e^-$ annihilation or in deep-inealastic scattering. We expect it to apply as long as $a > \Lambda$, where $\Lambda = 0.234\ GeV$ is the QCD scale.

The spectrum of hadrons $h$ radiated by a monopole moving with a proper acceleration $a$ can be expressed with the help of a fragmentation function, $W_h(a, E_g, x)$ as

$$N_h(E_h) = \int_{E_h}^{E_g^{max}} dE_g N_g(E_g) W_h(a, E_g, x)/E_g, \tag{61}$$

where $x = E_h/E_g$ and $N_g(E_g)$ is the spectrum of heavy gauge bosons calculated in Section V. Actually one can use the massless spectrum when considering virtualities less than $a^2$.

In the rest frame of the monopole, the typical virtuality and energy of the gluon are, respectively, $a^2$ and $a$, similar to the virtual photon in $e^+e^-$-annihilation. We shall use the fragmentation function based on the calculations of ref. [17] (see also more general expression in ref. [18]) which depends only on the parameter $x$:

$$W_h(x) = \frac{K_h}{x} \exp\left(-\frac{\ln^2 x/x_0}{\sigma_0}\right), \tag{62}$$

where $K_h$ is the normalization constant, taken from the condition

$$\int_0^1 x W_h(x) dx = f_h, \tag{63}$$

where $f_h$ is the fraction of energy transferred to hadrons of type $h$, $x_0 = \sqrt{\Lambda/a}$, and

$$\sigma_0 = \frac{1}{12b}\sqrt{\frac{2\pi}{3\alpha_s^3}}. \tag{64}$$

The value $b = (1/12\pi)(33 - 2f)$ determines the QCD coupling constant in the one-loop approximation as $\alpha_s^{-1} = b\ln(a^2/\Lambda^2)$, where $f$ is the number of guark flavors unfrozen at a given value of $a$. Numerically, $\sigma_0 = 0.121\sqrt{b}\ln^{3/2}(a^2/\Lambda^2)$. Note that the fragmentation function (62) differs significantly from the function introduced by Hill [8] which was subsequently used in much of the literature on cosmic rays from topological defects.

The fragmentation function $W_h(x)$ has a maximum at

$$x = x_m = x_0 \exp(-\sigma_0/2). \tag{65}$$

The origin of this maximum is related to the effect of coherent radiation of soft gluons by a jet of partons [17], [18]. At small $x$ the size of emitted gluons is larger than the width of the parton beam. A jet therefore radiates soft gluons as a single source with a color charge equal to the algebraic sum of the color charges of partons in the beam.

For ultrarelativistic monopoles, the approximation (38) for the spectrum of massless gauge bosons can be used instead of the exact spectrum. Then, the integration in Eq. (61) for the hadron spectrum can be performed and we obtain, for $E_h \leq E_0$,

$$N_h(E_h) = \frac{P_0}{E_0} f_h \frac{E_0}{E_h} \frac{\mathrm{e}^{-\sigma_0^2/4}}{x_0} \frac{\phi(\ln(x_0)/\sqrt{\sigma_0}) - \phi(\ln(E_0 x_0/E_h)/\sqrt{\sigma_0})}{\phi(\sqrt{\sigma_0}/2)}, \tag{66}$$

where we have introduced the auxiliary function

$$\phi(x) = \int_x^{+\infty} \mathrm{e}^{-t^2} dt. \tag{67}$$

We obviously have $N_h(E_0) = 0$, while the low energy asymptotic behavior of the spectrum is:

$$N_h(E) \approx \frac{P_0}{E_0} f_h \frac{\mathrm{e}^{-\sigma_0^2/4}}{x_0} \frac{\phi(\ln(x_0)/\sqrt{\sigma_0})}{\phi(\sqrt{\sigma_0}/2)} \frac{E_0}{E_h}. \tag{68}$$

A numerical example for the hadron (nucleon) spectrum, which corresponds to $a = 100\ GeV$ and $a = 10\ TeV$ is shown in figure 5. The value of $f_h$ can be taken as 0.1, which is roughly valid for nucleons. The shape of the spectrum depends only on the parameter $a$.

## VI. CONCLUDING REMARKS

We have analyzed the radiation of massless and massive gauge bosons by accelerating monopoles. In the massless case, we calculated the radiation spectrum for an oscillating $M\bar{M}$ pair connected by a straight string and for a monopole undergoing a harmonic oscillator motion. Combining this with the standard analysis of the synchrotron radiation, we arrive at the following qualitative picture.

For a monopole of magnetic charge $q$ moving with a proper acceleration $a$, the total radiated power is $P = q^2 a^2/6\pi$. In our case, $a = \mu/m$, where $\mu$ is the string tension and $m$ is the monopole mass. In the instantaneous rest frame of the monopole, the characteristic energy of the emitted quanta is $\bar{E}_{rest} \sim a$, and thus in the observer's frame it is

$$\bar{E} \sim \gamma a, \tag{69}$$

where $\gamma$ is the Lorentz factor of the monopole. For $E \gg \bar{E}$ the spectral power is strongly suppressed, while for $E < \bar{E}$ the spectrum is somewhat dependent on the type of motion (for example, the spectrum is flat for a harmonic oscillator and has a maximum at $E \sim \bar{E}$ for a circular motion).

The same picture applies in the case of massive gauge bosons, as long as their mass is sufficiently small, $M \ll a$. In the opposite limit of large mass, $M \gg a$, the radiation power is exponentially suppressed [see Eq.(51)], and gauge quanta are emitted with a characteristic energy $\bar{E} \sim \gamma M$ in a narrow range $\Delta E/E \sim (a/M)^{1/2}$. The interesting case for practical applications is the production of hadrons by an accelerating monopole. It effectively occurs through radiation of gluons due to the chromodynamic charge of the monopole. As in the general case of heavy quanta, the radiation of gluons with virtualities $|Q|^2 > a$ is exponentially suppressed and therefore most gluons are produced with $|Q|^2 \leq a$, where the massless limit is approximately valid. Beyond the confinement radius, high-energy gluons fragment into hadrons; the corresponding fragmentation function is given by Eq.(62).

The obtained formulae for radiation of accelerating monopole can be straightforwadly generalized to any other accelerating particle.

In a monopole-string network, with $N$ strings attached to each monopole, the proper acceleration of a monopole is determined by the vector sum of the tension forces exerted by the strings. By order of magnitude it is still given by $a \sim \mu/m$. The cosmological evolution of monopole-string networks is expected to be characterized by a single length scale,

$$\xi(t) \sim \kappa t, \tag{70}$$

where $\kappa = const$ and $t$ is the cosmic time. This scale gives the average distance between monopoles and the average length of string segments. The typical energy of a monopole is

$$E_m \sim \mu\xi \sim \mu\kappa t. \tag{71}$$

The corresponding Lorentz factor is $\gamma \sim (\mu/m)\kappa t$, and from Eq.(69) the typical energy of quanta emitted at time $t$ is

$$\bar{E} \sim (\mu^2/m^2)\kappa t. \tag{72}$$

Assuming that radiation of gauge quanta is the dominant energy loss mechanism of the networks, one finds [4] that the parameter $\kappa$ is given by

$$\kappa \sim \mu/\alpha m^2, \tag{73}$$

where $\alpha = e^2/4\pi$ and $e$ is the gauge coupling. Networks can also lose energy by production of closed loops of string and of small nets. The effect of these mechanisms is hard to estimate without numerical simulations of the network dynamics. For the time being, Eq.(73) should be regarded as a lower bound on $\kappa$, the upper bound being due to causality, $\xi \lesssim t$. Hence,

$$\mu/\alpha m^2 \lesssim \kappa \lesssim 1. \tag{74}$$

It is clear from Eqs.(71),((72) that the energies of monopoles themselves and of the quanta they emit can get arbitrarily large at sufficiently late times. In particular, the emitted particles can have super-Planckian energies, $E \gg m_p$, where $m_p$ is the Planck mass. To give a quantitative estimate, we note that the string tension $\mu$ and the monopole mass $m$ can be expressed in terms of the corresponding symmetry breaking scales as $\mu \sim \eta_s^2$, $m \sim 4\pi\eta_m/e$. These scales are expected to satisfy

$$\eta_{EW} \lesssim \eta_s < \eta_m < m_p, \tag{75}$$

where $\eta_{EW} \sim 10^2 GeV$ is the electroweak symmetry breaking scale. It follows from Eqs.(72),(74) that at the present time

$$\bar{E}/m_p \gtrsim 10^{56}(\eta_s^6/\eta_m^4 m_p^2), \tag{76}$$

and thus the emitted particles are super-Planckian provided that

$$\eta_s^3/\eta_m^2 m_p \gg 10^{-28}. \tag{77}$$

This covers a wide range of parameter values. For example, one could have GUT-scale monopoles, $\eta_m \sim 10^{16}\ GeV$, and intermediate-scale strings, $\eta_s \gtrsim 10^9\ GeV$, or electroweak-scale strings and light monopoles with $\eta_m \lesssim 10^7\ GeV$.

The interaction of super-Planckian particles with the microwave background and with cosmic magnetic fields, the resulting cosmic ray fluxes, and some other defect models that can give rise to such particles will be discussed elsewhere [19]. Conventional astrophysical acceleration mechanisms are not capable to account for particles of such tremendous energy, and if they are ever observed, it appears that topological defects would be the only possible explanation.

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## APPENDIX A: GAUGE BOSON RADIATION SPECTRUM

The electromagnetic radiation of a periodic electromagnetic source has been extensively studied and is well understood [14]. However, for the purpose of this work we will need a generalization of the standard formulae to the case of radiation of *massive* gauge bosons.

We consider a gauge boson field $A^\mu$ of mass $M$ coupled to a source term $j^\mu$,

$$\mathcal{L} = -\frac{1}{2}F^{\mu\nu}F_{\mu\nu} + M^2 A^\mu A_\mu + A_\mu j^\mu. \tag{A1}$$

To this Lagrangian correspond the equations of motion

$$(\Box + M^2)A^\mu = j^\mu, \tag{A2}$$

and the energy momentum tensor for the bosons

$$T_{\mu\nu} = -2(\partial_\mu A_\lambda)(\partial_\nu A^\lambda) - \mathcal{L}g^{\mu\nu}. \tag{A3}$$

The power radiated in bosons can be found from

$$P(t) = r^2 \int d\Omega S_r(t), \tag{A4}$$

where

$$S_r = -2(\partial_0 A_\lambda)(\partial_r A^\lambda) \tag{A5}$$

is the radial outgoing energy flux and $\partial_r$ is a derivative with respect to the radial distance $r = |\mathbf{x}|$. For a periodic motion, this power can be averaged over a period to get

$$P = r^2 \int \frac{dt}{T} \int d\Omega S_r(t), \tag{A6}$$

In the case of a massless gauge bosons $M = 0$, the equation (A2) can be solved exactly using the Lienhart-Wiechert potentials, which greatly simplifies the problem. For a massive gauge boson there is no such solution. However, Eq. (A2) is linear and can be solved in Fourier space. We therefore introduce the Fourier transforms

$$A^\mu(\omega_n, \mathbf{k}) = \frac{1}{T}\int dt \int d^3x e^{i(\omega_n t - \mathbf{k}\cdot\mathbf{x})} A^\mu(t, \mathbf{x}), \tag{A7}$$

$$j^\mu(\omega_n, \mathbf{k}) = \frac{1}{T}\int dt \int d^3x e^{i(\omega_n t - \mathbf{k}\cdot\mathbf{x})} j^\mu(t, \mathbf{x}), \tag{A8}$$

where $T$ is the period of the source and $\omega_n = 2\pi n/T$. Eq. (A2) becomes

$$(k^2-\omega_n^2+M^2)A^\mu(\omega_n\mathbf{k})=j^\mu(\omega_n,\mathbf{k}). \tag{A9}$$

This equation is trivially solved to get

$$A^\mu(\mathbf{x})=\sum_n \mathrm{e}^{-i\omega_n t}A_n^\mu(\mathbf{x}) \tag{A10}$$

$$A_n^\mu(\mathbf{x})=\int\frac{d^3k}{(2\pi)^3}\mathrm{e}^{i\mathbf{k}\cdot\mathbf{x}}\frac{j^\mu(\omega_n,\mathbf{k})}{k^2-k_0^2\pm i\varepsilon}, \tag{A11}$$

where we have introduced

$$k_0^2=\omega_n^2-M^2. \tag{A12}$$

The expression (A11) for $A_n^\mu(\mathbf{x})$ can be greatly simplified in the particular case we are interested in, where it is evaluated at a point $\mathbf{x}$ far from the source so that at any point of the source $\mathbf{x}'$, $|\mathbf{x}|\gg|\mathbf{x}'|$:

$$A_\mu(\omega_n,\mathbf{x})=\int\frac{dt}{T}\int d^3x' j^\mu(t,\mathbf{x}')\int\frac{k^2dk}{k^2-k_0^2\pm i\varepsilon}\int\sin\theta d\theta d\phi \mathrm{e}^{ik|\mathbf{x}-\mathbf{x}'|\cos\theta} \tag{A13}$$

$$=\int\frac{d^3x'\mathrm{e}^{ik_0|\mathbf{x}-\mathbf{x}'|}}{4\pi|\mathbf{x}-\mathbf{x}'|}j^\mu(t,\mathbf{x}')\approx\frac{\mathrm{e}^{ik_0r}}{4\pi r}j^\mu(\omega_n,k_0\mathbf{n}), \tag{A14}$$

where we have introduced $r=|\mathbf{x}|$ and $\mathbf{n}=\mathbf{x}/r$. The radial energy flux (A5) can then be expressed to first order in $1/r$ as

$$S_r=-\frac{1}{8\pi^2r^2}\sum_{n,m}(\omega_nk_0)\mathrm{e}^{i(m-n)\omega_nt}j^\mu(\omega_n,k_0\mathbf{n})j_\mu^*(\omega_m,k_0\mathbf{n}). \tag{A15}$$

Then the total power (A6) becomes

$$P=\sum_n P_n, \tag{A16}$$

$$P_n=\frac{k_0\omega_n}{8\pi^2}j^\mu(\omega_n,k_0\mathbf{n})j_\mu^*(\omega_n,k_0\mathbf{n}) \tag{A17}$$

where $P_n$ is obviously the power radiated by the mode $n$.

[1] T.W.B. Kibble *J. of Phys.* **A9**, 1387 (1976).

[2] For a review of topological defects in cosmology, see A. Vilenkin and E.P.S. Shellard, *Cosmic strings and other topological defects* (Cambridge University Press 1994).

[3] M. Aryal, A.E. Everett, T. Vachaspati and A. Vilenkin, *Phys. Rev.* **D34**, 434 (1986).

[4] T. Vachaspati and A. Vilenkin, *Phys. Rev.* **D35**, 1131 (1987).

[5] R. Holman, T.W.B. Kibble and S.-J. Rey, *Phys. Rev. Lett.* **69**, 241 (1992).

[6] X. Martin and A. Vilenkin *Phys.Rev. Lett.* **77**, 2879 (1996).

[7] E. Witten, *Nucl. Phys.* **B249**, 557 (1985); J.P. Ostriker, C. Thompson and E. Witten, *Phys. Lett.* **180B**, 230 (1986); C.T. Hill, D.N. Schramm and T.P. Walker, Phys. Rev. **D36**, 1007 (1987); R.H. Brandenberger, *Nucl. Phys.* **B293**, 812 (1987).

[8] C.T. Hill, *Nucl. Phys.* **B224**, 469 (1983); P. Bhattacharjee and G. Sigl, *Phys. Rev.* **D51**, 4079 (1995).

[9] B. Carter "Covariant mechanics of simple and conducting Strings and Membranes", in *The formation and evolution of Cosmic strings*, ed. by G. Gibbons, S. Hawking and T. Vachaspati, pp 143-178 (Cambridge U.P. 1990).

[10] X. Martin and A. Vilenkin, *Phys. Rev. D*, in press [gr-qc 9612008].

[11] S. Weinberg "*Gravitation and Cosmology*", pp 260-266 (J. Wiley & Sons 1972).

[12] V.I.Ritus and A.I.Nikishov, Proc. P.N.Lebedev Physical Institute (Trudy FIAN) v.111, pp 1-278, 1979.

[13] V.B.Berestetskii, E.M.Lifshitz and L.P.Pitaevskii, Quantum Electrodynamics, pp 449-455

(Pergamon Press 1980.

[14] Landau, Lifschitz "*The classical theory of fields*", pp 170-173 (Pergamon Press 1971).

[15] J.Ellis, in "Magnetic Monopoles" (eds R.A.Carrigan and W.P.Trower) NATO Advanced Study Institute, pp 17 - 40, 1982, Plenum Press.
J.Preskill, ibid pp 11-126.

[16] V.N.Gribov and L.N.Lipatov, Sov. Journal Nucl. Phys. 15, 438, 1972. G.Altarelli and G.Parisi, Nucl. Phys. B 126, 298, 1977.

[17] R.K.Ellis, W.J.Stirling and B.R.Webber, "QCD and Collider Physics" in preparation, 1997.

[18] Yu.L.Dokshitzer, V.A.Khose, A.H.Mueller and S.I.Troyan, "Basics of Pertubative QCD", Editions Frontiers, 1991.

[19] V. Berezinsky and A. Vilenkin (unpublished).

FIG. 1. A log-log plot of the gauge quanta radiation spectrum $P_n/(qa)^2$ in the case $\gamma_0 = 10$ (solid line) with its high frequency approximation (22) (dashed line). The corresponding spectrum for the harmonic solution has been added (dotted line) for comparison.

FIG. 2. Total power of massless gauge quanta emission $P/(qa)^2$ as a function of $\gamma_0$. It quickly tends to a limit of $1/3\pi$.

FIG. 3. A log-log plot of the gauge quanta radiation spectrum $P_n/(qa)^2$ from the harmonic solution (26) in the case $\gamma_0 = 25$ (solid line) with its high frequency (36) (long dashed line) and low frequency (35) (dashed line) approximations.

FIG. 4. A log-log plot of the massive gauge quanta radiation spectrum $P_n/(qa)^2$ for $\gamma_0 = 25$ and $M/a_m$ taking the values 0.1 (long dash), 0.5 (dash), 1 (dash-dot) and 5 (dot). The massless case from figure (3) has also been added (solid line).

FIG. 5. A log-log plot of the hadron spectrum (66) for $a = 100\ GeV$ (solid line) and $a = 10\ TeV$ (dashed line) with their low energy approximations (68) (respectively dotted line, dash-dotted line).