BOUND STATES IN MONOPOLES: SOURCES FOR UHECR ?

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Bound states in monopoles are studied through a simplified, Witten-like model. As the overall structure is determined in full details, it is shown in particular that only those states having a vanishing angular momentum are allowed; for those, the energy spectrum is derived numerically and an approximation is set up that allows an easy description in terms of wavefunctions, usefull for further applications. The monopoles are then proposed as candidates of ultra high energy cosmic rays, an hypothesis that should soon be testable through the Pierre Auger Observatory.

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I. INTRODUCTION

Among the various topological defects that may have formed during phase transitions in the early universe [1], monopoles are absolutely unavoidable in Grand Unified Theories (GUT), according to which there was a time when all symmetries were unified in a single semi-simple gauge group, since electromagnetism is an exact (unbroken) symmetry still now. They are however usually considered as a sufficient nuisance [2] (in the sense that they would very rapidely come to dominate the universe) that inflation is generally invoked right after their time of formation so that they are completely diluted [3] and thus unobservable. These results are based on the by-now very standard view that they originated at the GUT phase transition, i.e., at an energy around $10^{15}-10^{16}$ GeV. Many ways out however have been proposed other than inflationnary scenarios, among which the Langacker-Pi mechanism [4] which relies upon using cosmic strings to connect the monopoles and anti-monopoles pairs, thereby effectively enhancing considerably the decay probability, thus reducing the remnant monopole density. Although the model, in its original presentation, suffers from many drawbacks, it has at least the advantage of proposing another solution, not involving inflation and letting open other alternative possibilities. Domain walls have also been used to sweep them away [5]; in all cases, undesirable monopoles are gotten rid of by means of higher dimensional topological defects [6].

Yet another alternative possible solution to the monopole excess problem is the simplest one, although completely overlooked until recently: it consists in noting that the monopole density is in fact proportional to the fourth power of the energy scale η at which the symmetry breaking during which monopoles were generated took place, given the monopole mass $m_{_M}$ is essentially η times the inverse of the corresponding coupling constant, i.e., ~ 137 in the case of electromagnetism (as ought to be the case):

$$\Omega_{_M} h^2 \simeq 10^{11}\left(\frac{\eta}{10^{14}\text{GeV}}\right)^3\left(\frac{m_{_M}}{10^{16}\text{GeV}}\right), \qquad (1)$$

(here we note $\Omega_{_M}$ the monopole density in units of the critical closure density, and h the Hubble constant in units of 100 km $\cdot$ s$^{-1}\cdot$ Mpc^{-1}) so that it suffices to lower η to $\lesssim 10^9$ Gev in order to cure the density problem. This is the solution we shall adopt here, for it might also provide a useful explanation for the high energy cosmic ray mystery [7]. (Note in this regard that one can also simply assume the monopole overdensity problem to be cured somehow and investigate anyway the possibility that they give rise to high energy cosmic rays [8].) This way, the possibility that monopoles are still present in the universe is still reasonnable.

Once the mass scale is fixed, the relevant physics still needs be properly clarified in order to enable one to study the interactions between monopoles and other particles. In this regard, models have been suggested where bound states of scalar [9], gauge vector bosons [10] or fermions [11] can form in SO(3) t'Hooft-Polyakov monopoles [12]. Those modify the scattering solutions and can enhance greatly the cross sections by means of the Callan-Rubakov effect [13]. It is the same kind of approach we wish to present here, although using a simplified Witten like model (generally used to describe current-carrying cosmic strings [14,15] or domain walls [16]) and a completely different method for calculating the bound state energy levels. Our model, contrary to the other proposed models having bound states, presents the advantage (apart from its simplicity) that the bound states are present for dynamical reasons, i.e., they do not exist for all values of the underlying parameters. This means that we can consider that particles may be trapped in the monopole even though they might be-long to representations that are not directly related to that of the monopole fields. Besides, we shall work using the Julia–Zee fully numerical solution [17] and not the explicit Bogomol'nyi–Prasad–Sommerfield (BPS) analytic limit [18].

In what follows, we shall first define our model and show that bound states may be present in monopoles, provided their angular momentum vanish, exactly as in the Callan-Rubakov mechanism [13]. This not-quite-obvious result follows from the requirement that bound states do exist in the monopole, which in turn seriously constrains the possible couplings between the fields involved.

Having explored the full microscopic structure of a monopole having bound states, we go on to investigate the possibility that, in a fashion similar to Witten superconducting cosmic string models [14], through the existence of vortons [19] (rotating superconducting cosmic string loops configurations), these monopoles could be the source of Ultra High Energy Cosmic Rays (UHECR). Their acceleration, once they are coupled to electromagnetism, poses no particular problem as long as an external magnetic field exists in the accelerating region under consideration. Similarly to the model based on vortons [20], they can propagate along huge (cosmologically speaking) distances, and thus have the ability to reach us fairly easily. Their interaction cross-section with air nuclei can be obtained through the Callan-Rubakov effet [13] and is thus evaluated to be hadronic in nature. The expected properties of the resulting UHECR distribution are essentially those derived in the vorton case [20] and can therefore reproduce the existing data. More data, thanks for instance to the Pierre Auger Observatory project [21], will give a definite answer concerning this possibility.

II. THE MONOPOLE STATE

The model we shall use in what follows involves the symmetry breaking of an SO(3) invariance by means of a Higgs field Φ belonging to the **3** representation of SO(3), coupled to a complex scalar field Σ which we assume, for the sake of simplicity, not to be coupled to the gauge field $\mathbf{A}^\mu$ of SO(3). This assumption should of course be modified when one wants to evaluate the long-range electromagnetic interaction of the resulting monopole with other particle, but we shall show latter on how this can be achieved. With a metric convention having positive signature, we have the model

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}(D_\mu\Phi^a)^\dagger(D^\mu\Phi_a) - \frac{1}{2}(\partial_\mu\Sigma)^\star(\partial^\mu\Sigma) - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a \\ &-\frac{\lambda_\phi}{4}(\Phi_a\Phi^a - \eta^2)^2 - f(\Phi_a\Phi^a - \eta^2)|\Sigma|^2 \\ &-\frac{m_\sigma^2}{2}|\Sigma|^2 - \frac{\lambda_\sigma}{4}|\Sigma|^4, \end{aligned} \tag{2}$$

where the covariant derivative is

$$D_\mu\Phi_a \equiv \partial_\mu\Phi_a - q\varepsilon_{ab}{}^{c}A^b_\mu\Phi_c, \tag{3}$$

and the gauge field strength

$$F^a_{\mu\nu} \equiv \partial_\mu A^a_\mu - \partial_\nu A^a_\mu + q\varepsilon^a{}_{bc}A^b_\mu A^c_\nu. \tag{4}$$

A static monopole configuration then has the form [12,17], in spherical coordinates $x^i \equiv (r,\theta,\phi)$

$$\Phi^a = \eta h(r)\frac{x^a}{r}, \tag{5}$$

$$A^a_i = -\frac{1-K(r)}{qr^2}\,\varepsilon^a{}_{ij}x^j, \quad A^a_0 = 0. \tag{6}$$

The field equations for the configuration (5) and (6), with the Σ field not taken into account, i.e. for the ordinary 't Hooft–Poyakov monopole, read

$$\frac{1}{r^2}\frac{d}{dr}(r^2\frac{dh}{dr}) = \frac{2}{r^2}hK^2 + \lambda_\phi\eta^2 h(h^2-1) \tag{7}$$

$$\frac{d^2K}{dr^2} = q^2\eta^2 Kh^2 + \frac{1}{r^2}K(K^2-1), \tag{8}$$

with boundary conditions

$$h(0) = K(\infty) = 0, \quad h(\infty) = K(0) = 1. \tag{9}$$

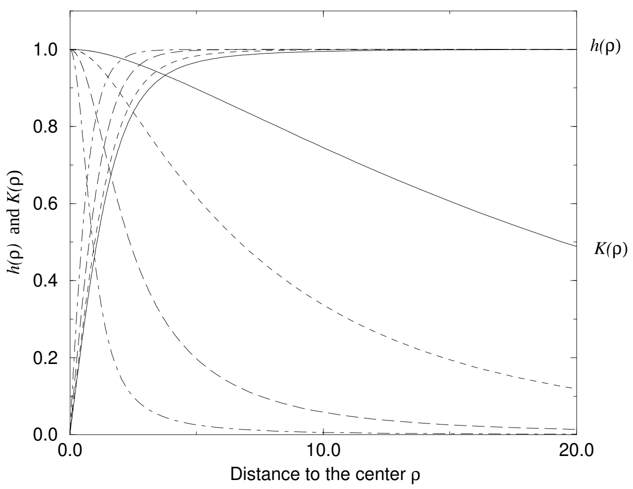

FIG. 1. Amplitude of the field functions $h(\rho)$ and $K(\rho)$ vs the rescalled unit of length $\rho = r\lambda_\phi^{1/2}\eta$. Curves are shown for $q^2/\lambda_\phi = 10^{-2}$ (solid line), 0.1 (dashed line), 1 (long dashed line) and 10 (dot-dashed line).

These equations have been solved numerically for various values of the only relevant parameter q^2/λ_ϕ and produce the characteristic curves on Figure 1, where h and K are shown as functions of the rescalled unit of length $\rho = r\sqrt{\lambda_\phi}\eta$.

The total energy of the monopole is expressible simply in terms of q^2/λ_ϕ and the Higgs field mass $m_{_H} = \sqrt{\lambda_\phi}\eta$ as

$$E_{_M} = 4\pi m_{_H} \int \rho d\rho \Big\{ \frac{K'^2}{(q^2/\lambda_\phi)\rho^4} + \frac{1}{2}h'^2 + \frac{K^2 h^2}{\rho^2} + \frac{1}{4}(h^2-1)^2 + \frac{(1-K^2)^2}{2(q^2/\lambda_\phi)\rho^4} \Big\}, \tag{10}$$

which is shown on Figure 2 as a function of q^2/λ_ϕ.

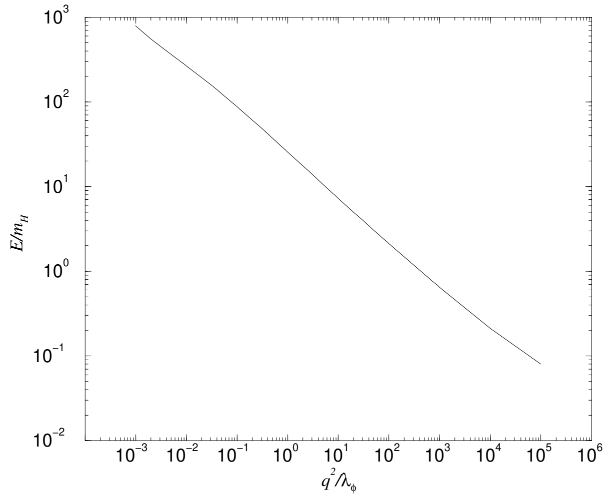

FIG. 2. Variation of the total monopole energy as a function of q^2/λ_ϕ.

III. THE SCALAR CONDENSATE

In this background, we now investigate the behaviour of the bosonic field Σ by first looking at the field equation in which the nonlinar term is omitted. Separating space and time variables in the form

$$\Sigma(x^\mu) = \sigma(r)\mathrm{e}^{i\omega t} Y_{\ell m}(\theta,\phi), \tag{11}$$

the field equation for Σ in the monopole background gives the Schrödinger-like eigenvalue equation for the amplitude of this field as

$$-\Delta\sigma + V(r)\sigma = \omega^2\sigma, \tag{12}$$

where

$$V(r) = [\frac{\ell(\ell+1)}{r^2} + 2f\eta^2(h^2-1) + m_\sigma^2] \tag{13}$$

which will admit bound state eigensolutions ($\omega^2 < m_\sigma^2$) provided that the potential V satisfies either

$$V < 0 \Longrightarrow \ell = 0 \ \ \& \ \ m_\sigma^2 < 2f\eta^2, \tag{14}$$

or

$$\exists R \in [0,\infty[\ ; \ \left.\frac{dV}{dr}\right|_{r=R} = 0 \ \text{ and } \ V(R) < 0. \tag{15}$$

In the latter case, denoting by a prime a derivative with respect to the rescalled distance ρ, one finds that the minimum of the potential would be for ρ such that

$$\frac{\lambda_\phi}{2f}\ell(\ell+1) = hh'\rho^3. \tag{16}$$

On Figure 3 is plotted the right hand side of this relation against ρ for three orders of magnitude of the relevant underlying parameter q^2/λ_ϕ, and it is seen that in general this function is of order unity throughout its range of variation. Hence, in order for the condition (16) to be satisfied, it is necessary that the left hand side be also of order unity. Assuming λ_ϕ and f to have comparable values (they are both quartic interaction term coupling constants), this leads to the constraint

$$\ell(\ell+1) \sim 1, \tag{17}$$

which shows that no bound state is expected for large angular momentum. For this reason, we restricted our analysis to vanishing angular momentum states $\ell = 0$.

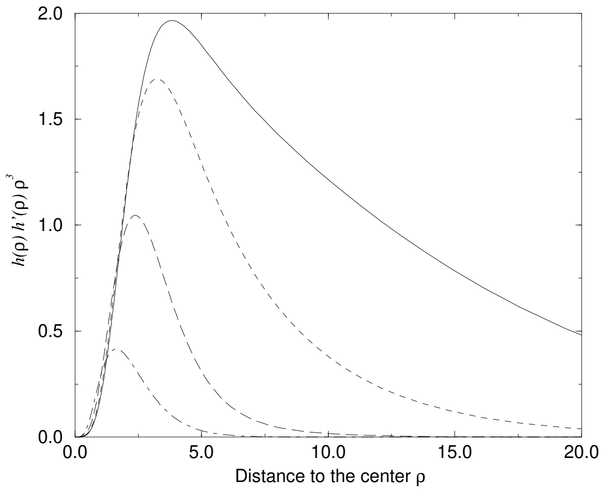

FIG. 3. Value of the function $\rho^3 h(\rho) dh/d\rho$ which determines the possibility of bound states for nonzero values of the angular momentum ℓ vs the rescalled unit of length $\rho = r\lambda_\phi^{1/2}\eta$. As on Figure 1, curves are shown for $q^2/\lambda_\phi = 10^{-2}$ (solid line), 0.1 (dashed line), 1 (long dashed line) and 10 (dot-dashed line).

Yet another way to convince oneself that the bound states should be restricted to those with vanishing angular momentum consists in investigating the full, non perturbative, field equation for Σ, still within the separated form (11), that is, assuming the resulting state to be still an eigenstate of the angular momentum. The existence of the non linear term would then imply $|Y_{\ell m}|^2$ to be a constant, i.e., independent of both angular variables θ and ϕ. Thus, only the case $\ell = 0$ can satisfy the non linear equation, so that the background σ field has vanishing angular momentum.

We now therefore consider again the ansatz (11) with no spherical harmonics included and in the region of parameter space where a condensate might form, i.e. we assume $m_\sigma^2 < 2f\eta^2$. Setting $\omega = 0$ yields the actual vacuum state as the solution of the field equations

$$\frac{1}{r^2}\frac{d}{dr}(r^2\frac{dh}{dr}) = \frac{2}{r^2}hK^2 + \lambda_\phi \eta^2 h(h^2-1) + 2f\eta^2 h\sigma^2, \quad (18)$$

$$\frac{d^2K}{dr^2} = q^2\eta^2 K h^2 + \frac{1}{r^2}K(K^2-1), \quad (19)$$

$$\frac{1}{r^2}\frac{d}{dr}(r^2\frac{d\sigma}{dr}) = [m_\sigma^2 + 2f\eta^2(h^2-1)]\sigma + \lambda_\sigma \sigma^3, \quad (20)$$

with boundary conditions for σ as

$$\frac{d\sigma}{dr}(0) = 0, \quad \lim_{r\to\infty} \sigma = 0. \quad (21)$$

Rescalling the field σ through

$$Y(\rho) = \sqrt{\lambda_\sigma}\frac{\sigma}{m_\sigma} \quad (22)$$

and defining the dimensionless parameters as

$$\alpha_1 = \frac{m_\sigma^2}{\lambda_\sigma \eta^2}, \quad \alpha_2 = \frac{f m_\sigma^2}{\lambda_\sigma \lambda_\phi \eta^2}, \quad \alpha_3 = \frac{m_\sigma^4}{\lambda_\sigma \lambda_\phi \eta^4}, \quad (23)$$

allows a numerical calculation of the vacuum solution. Such a solution, obtained by means of a Successive Over Relaxation method [22], is shown for a special set of parameters $\{\alpha_i\}$ on Figure 4 (here and in what follows, the background parameter q^2/λ_ϕ has been fixed to the arbitrary value 0.1).

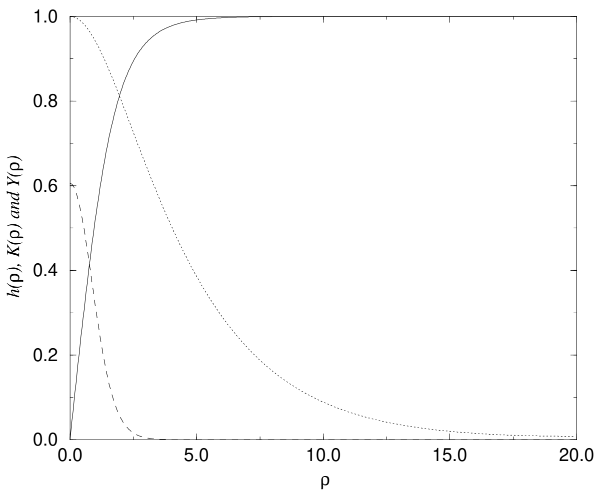

FIG. 4. Fields function $h(\rho)$ [full line], $K(\rho)$ [dotted line] and $Y(\rho)$ [dashed line] for the set of parameters $\alpha_1 = 0.1$, $\alpha_2 = 0.6$ and $\alpha_3 = 0.6$.

These fields represent the vacuum state out of which the bound state solutions can be calculated.

IV. QUANTUM THEORY

In order to calculate quantum effects related to the monopole solution, we turn to the standard solitonic approach [23]. We shall for now on consider the Higgs and gauge vector fields to represent a fixed background in which the charged scalar field Σ evolves. This means we describes its dynamics through an effective Lagrangian density

$$\mathcal{L}_\Sigma = -\frac{1}{2}|\partial_\mu \Sigma|^2 - f(\Phi_a\Phi^a - \eta^2)|\Sigma|^2 - \frac{m_\sigma^2}{2}|\Sigma|^2 - \frac{\lambda_\sigma}{4}|\Sigma|^4, \quad (24)$$

and the localized solutions are derivable from the effective potential $V[\Sigma]$

$$L_\Sigma = \int d^3x\, \mathcal{L}_\Sigma = \int d^3x \frac{1}{2}\frac{\partial \Sigma}{\partial t} - V[\Sigma], \quad (25)$$

$$V[\Sigma] = \int d^3x \left\{\frac{1}{2}(\nabla\Sigma)^2 + \frac{\lambda_\sigma}{4}|\Sigma|^4 + \left[f(\Phi_a\Phi^a - \eta^2) + \frac{m_\sigma^2}{2}\right]|\Sigma|^2\right\}. \quad (26)$$

The classical solution Σ_c derived earlier is then obtained by minimizing $V[\Sigma]$:

$$\frac{\delta V}{\delta \Sigma} = 0 \Longrightarrow \Delta\Sigma_c = [2f(\Phi_a\Phi^a - \eta^2) + m_\sigma^2]\Sigma_c + \lambda_\sigma \Sigma_c^3, \quad (27)$$

where, because of the $U(1)$ symmetry in the scalar field Σ, the later could have been chosen real.

Expanding $V[\Sigma]$ around the classical solution and noting respectively

$$\tilde{\eta} = \frac{1}{\sqrt{2}}(\psi + \psi^\star) \tag{28}$$

and

$$\tilde{\mu} = \frac{1}{\sqrt{2}}(\psi - \psi^\star), \tag{29}$$

with ψ the quantum perturbation from the classical solution $\Sigma = \Sigma_c + \psi$, one gets

$$\begin{aligned} V[\Sigma] &= V[\Sigma_c] + V_{int}[\tilde{\eta},\tilde{\mu}] \qquad (30)\\ &- \frac{1}{2}\int d^3x\, \tilde{\eta}\Big[\frac{\Delta}{2} - f(\Phi_a\Phi^a - \eta^2) - \frac{m_\sigma}{2} - \frac{3}{2}\Sigma_c\Big]\tilde{\eta}\\ &- \frac{1}{2}\int d^3x\, \tilde{\mu}\Big[\frac{\Delta}{2} - f(\Phi_a\Phi^a - \eta^2) - \frac{m_\sigma}{2} - \frac{1}{2}\Sigma_c\Big]\tilde{\mu}, \end{aligned}$$

where $V_{int}[\tilde{\eta},\tilde{\mu}]$ comprises the interaction terms between $\tilde{\eta}$ and $\tilde{\mu}$, originating from the self coupling $|\Sigma|^4$. These terms are not explicitely developed here since we are only interested in the bound state solutions around the monopole, i.e., the stationnary solutions on the basis of which the system can be quantized. We are therefore looking for the eigenmodes of the second derivatives of V with respect to the fields $\tilde{\eta}$ and $\tilde{\mu}$, which are then seen to satisfy the Schrödinger-like equations

$$\Big[-\Delta + 2f(\Phi^a\Phi_a - \eta^2) + m_\sigma^2 + 3\Sigma_c^2\Big]\tilde{\eta}_i = \omega_i^2\tilde{\eta}_i, \tag{31}$$

and

$$\Big[-\Delta + 2f(\Phi^a\Phi_a - \eta^2) + m_\sigma^2 + \Sigma_c^2\Big]\tilde{\mu}_j = \Omega_j^2\tilde{\mu}_j. \tag{32}$$

The eigenmodes $\tilde{\eta}_i$ are of two different kinds: either ω_i belongs to a discrete set, which is the case if $\omega_i \le m_\sigma$, so that $\tilde{\eta}_i$ represents a bound state localized around the monopole, or $\omega_i > m_\sigma$ can be parametrized by a continuous index q that can be identified with a momentum. In the later case, the corresponding modes are diffusion, i.e. asymptotically free states, with momentum q. Similar considerations obviously apply also to the states $\tilde{\mu}_j$.

On the basis of this set of eigenmodes, and following Goldstone and Jackiw [24], one can in principle build the quantum theory of the monopoles by constructing a Fock space as follows. The original monopole state, i.e., the classical solution with no bound state, can be boosted to aquire an arbitrary momentum P, thereby generating the set $\{|P\rangle\}$. The same can be done for monopoles with any occupation numbers in the bound states $\{|P, n_i^{(\eta)}, n_j^{(\mu)}\rangle\}$. Finally, the Fock space is completed by inclusion of the diffusion states labelled by the momenta of the various ingoing and outgoing particles $\{|q_1^{(\eta)},\cdots,q_N^{(\eta)},q_1^{(\mu)},\cdots,p_M^{(\mu)}\rangle\}$. Thus, an arbitrary quantum state belongs to the set

$$\begin{aligned} \mathcal{F} = \Big\{&|P, n_i^{(\eta)}, n_j^{(\mu)}, q_1^{(\eta)},\cdots,q_N^{(\eta)},q_1^{(\mu)},\cdots,p_M^{(\mu)}\rangle,\\ &n_i^{(\eta)}, n_j^{(\mu)} \in \mathbf{N}, q_\alpha^{(\eta)}, q_\beta^{(\mu)} \in \mathbf{R}^3\Big\}, \end{aligned} \tag{33}$$

and this space is assumed disconnected from the ordinary free particle space of the theory without a monopole. Cross sections will then be calculable by computing matrix elements of the quantum field $|\Sigma|^4$, which is seen to involve essentially space integrals of the eigenfunctions. Specific such calculations and their application to primordial cosmology will be the subject of a further work, and for now on we shall concentrate on the actual structure of both the self potential and the bound states.

V. THE SELF POTENTIALS; BOUND STATES

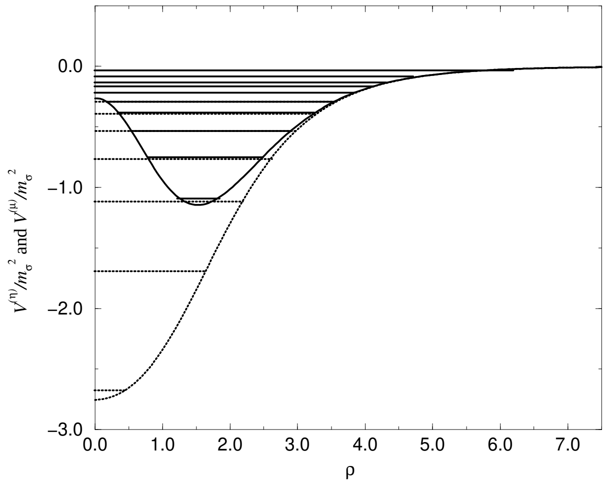

FIG. 5. Confining potential of the first $[V^{(\mu)}(r)]$ and second $[V^{(\eta)}(r)]$ kind in units of the $\Sigma-$field mass squared m_σ^2 for the set of parameters $\alpha_1 = 0.1$, $\alpha_2 = 0.2$ and $\alpha_3 = 0.1$ as a function of the rescalled distance to the monopole core ρ. In this case, the minimum of $V^{(\mu)}(r)$ is located at $\rho = 0$ while that of $V^{(\eta)}(r)$ is for $\rho = \rho_m \neq 0$. The bound states eigenvalues are indicated as the straight lines. The full lines correspond to $V^{(\mu)}$ and its associated eigenvalues, while the dotted lines stand for $V^{(\eta)}$.

The purpose of this section is to exhibit the various possible situations, depending on the underlying microscopic parameters, in which the field Σ can evolve. To achieve this, we rewrite the eigenmode equations for $\tilde{\eta}$ and $\tilde{\mu}$ in the form

$$\begin{cases} [-\Delta + V^{(\eta)}(r)]\tilde{\eta} = (\omega^2 - m_\sigma^2)\tilde{\eta}, \\ [-\Delta + V^{(\mu)}(r)]\tilde{\mu} = (\omega^2 - m_\sigma^2)\tilde{\mu} \end{cases}, \tag{34}$$

which define the self potentials $V^{(\eta)}(r)$ and $V^{(\mu)}(r)$ by comparison with Eqs. (31) and (32). Some characteristic shapes and amplitudes of these potentials are shown on Figure 5, in rescalled units (the square of the carrier mass), together with the corresponding eigenvalues in the same units.

The figure reveals the existence of two different kinds of self potentials, respectively called of the first and second kind, depending on the underlying microscopic parameters. The first kind is characterized by an absolute minimum in the monopole core and corresponds to a weak coupling where f is small, while the second kind, having a local maximum at $r = 0$ and a minimum for some nonzero value of the distance to the monopole core, reflects the existence of a strong coupling. As a rule, and as could have been expected, the energy eigenstates are more bound for confining potentials of the first kind that for second kind. Therefore, one can expect the lifetime of the corresponding configurations to be strongly dependent on the parameters.

The figure 6 illustrates some wavefunctions living in self potentials of the first and second kinds.

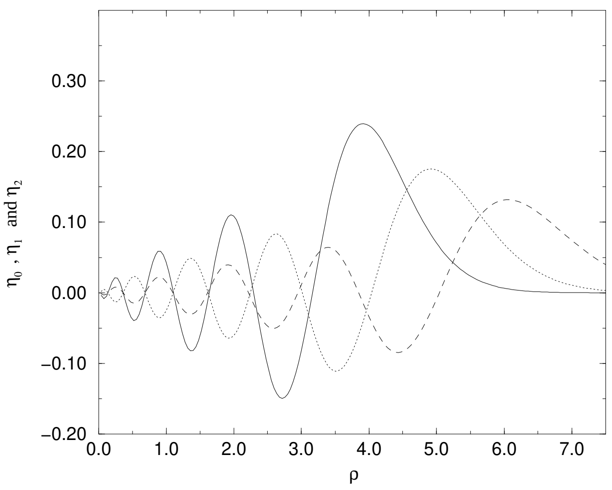

FIG. 6. An illustrative example of some wavefunctions living in a self potential of the second kind. The set of parameters has been chosen as on figure 5.

These wavefunctions are all that is needed to fully clarify the internal structure of the monopole. The expected spectrum would have line features as obtained by reversing the axes on Fig. 5. We therefore turn to these cross-sections before discussing the acceleration and propagation mechanisms of monopoles.

VI. MONOPOLE-PROTON INTERACTIONS

Contrary to the case of an ordinary (structureless) monopole, the configurations we have been studying have the ability to interact through deep-inelastic scattering with air nucleus, a proton say to simplify matters. Indeed, a particle trapped inside the monopole core can be scattered off the monopole, effectively ionizing it. As in the vorton case [20], three possible interactions can take place, depending on the proton energy (we keep working in the monopole center-of-mass frame): elastic interaction, which was discussed in [7] and which will presumably yield an unobservable shower accompanied with Čerenkov radiation, excitation interaction whereby a charge carrier will use the proton energy to move to a higher energy level, and finally ionization. The last two cases are the most important as one expects them to yield an observable signal; they will be compared to existing data and used for making predictions in the Pierre Auger Observatory [21].

Once a trapped particle has been moved up to an excited state, it will decay into the minimum energy state, for instance in radiating a photon*, assuming there does exist such a channel. Calling $\Delta E = E_n - E_0 \sim m_\sigma$ the energy difference between the two eigenenergies (see, e.g. Fig. 5), such an observed primary photon would have energy, seen on earth

$$E_{obs} = \gamma \Delta E, \tag{35}$$

with $\gamma = \varepsilon / m_{_M}$ the monopole Lorentz factor. Here, ε is the total monopole energy, while $m_{_M} \sim \eta/e$ is its mass. It should be remarked that the monopole mass can be much larger than the carrier mass, in practice as high as a hundred times, so that the monopole itself must be accelerated to much higher energies than the observed 10^{20} eV. We shall see however in the following section that this is not a very serious constraint. If the energy is sufficient to ionize the monopole, the observed energy will be also of the order of that given on Eq. (35), with ΔE simply equal to E_0, the minimum energy state. In order that the reaction actually takes place, it is necessary that the incident proton, which, as seen from the monopole frame, is having the same Lorentz factor γ, has enough energy to excite the corresponding state. Therefore, $\Delta E = \gamma m_p$, with m_p the proton mass, so that

$$\gamma = (\frac{\varepsilon}{m_p})^{1/2} \sim 10^{6.5}. \tag{36}$$

Such a number would seem to imply states in a monopole at energies of the order of a few thousands TeV, and therefore far below the cosmological limit $m_{_M} \lesssim 10^9$ GeV. This leaves a large range of possibilities for the ratio η/m_σ.

Various cases have to be taken into account if one wants to actually understand the data in terms of bound states in monopoles. First of all, the determination of

*The term "photon" shall be used here in the generic sense of an unbound decay product of our $\Sigma-$field. In case Σ is effectively coupled to electromagnetism, this "photon" would be an actual photon.

the energy levels, as discussed above, although specifically relevant to our case of a bosonic condensate, is essentially the same in the case of fermionic bound states. However, in the latter case, the Pauli exclusion principle would apply and the overall distribution of populated states would be different: as all bosons would preferentially occupy the same state (the lowest energy state), fermions on the other hand would fill all the states. The expected spectrum would therefore be qualitatively different. The solid prediction here is the same as that for the vorton solution to the UHECR problem [20], namely that of the existence of a line spectrum, the exact form of which needing a specific model to be determined. If such a detection was indeed achieved, the method presented above would prove usefull to actually determine the relevant particle physics model through a fit of the data.

Identifying the primary as a monopole is also possible through its interactions with air nuclei. In other words, one needs to know the monopole atom cross section, or, as emphasized above, the monopole proton cross section. Such a calculation is feasible in principle by making use of the quantum theory developed in Section IV and V, and will have to be done in case a ray spectrum is indeed observed in the Pierre Auger Observatory [21]. For the time being, a rough evaluation will be enough.

A monopole such as described here interacts with a fermionic field such as a proton through possible exitations of the A_0 modes, the so-called dyonic modes. This is a resonant exitation which effectively confines the fermion in the core of the monopole for a long time, making the cross-section essentially hadronic (the proton is then seen by the monopole as a quasi-bound state). Once trapped, the proton (or a quark therein) has time to interact with the bound states. As a result [13], the interaction cross-section we are seeking is given by:

$$\sigma_{M_P} \sim q^{-2} m_{_P}^{-2}, \tag{37}$$

numerically of the order of a few hundreds mb. Such a large cross-section implies that most of the monopoles arriving in the atmosphere would be detectable, provided they carry bound states.

VII. ACCELERATION AND PROPAGATION

Once the internal structure of a monopole is known, it must be shown that they have the ability to be detected as UHECR. It is the purpose of this section to show that indeed many astrophysical sites have the ability to accelerate them. Moreover, there is no GZK cutoff for them since they share the vorton features of being effectively highly charged, a fact that is compensated by their huge mass: the electromagnetic energy losses are similar to that of a heavy nucleus having an electric charge $Z = 1/2q = 137/2$ [7], but are reduced by inverse powers of the mass [20].

Accelerating a magnetic monopole in the presence of a magnetic field is a very easy task: even the galactic magnetic field could do it [7] as the kinetic energy $E_{_M}$ of a monopole in a magnetic field $B \sim 10^{-6}$ G with a coherence length $D \sim 300$ pc would be of order

$$E_{_M} = q_{_M} BL \sim 6 \times 10^{19} \text{ eV} \left(\frac{B}{3 \times 10^{-6}\text{G}} \right) \left(\frac{D}{300\text{pc}} \right), \tag{38}$$

with $q_{_M}$ the magnetic charge (inversely proportional to the electric coupling constant). Using Eq. (38), one gets table I for the maximum energy acquired by a magnetic monopole (adapted from [25]).

Magnetic fields, distance and energy

object	B (G)	D (pc)	$E_{_M}$ (eV)
Neutron Star	$10^{9.5} - 10^{13}$	10^{-12}	$10^{20} - 10^{24}$
White Dwarf	$10^4 - 10^8$	10^{-9}	$10^{18} - 10^{22}$
AGN	$10^3 - 10^{4.5}$	10^{-3}	$10^{23} - 10^{24}$
SNR	$10^{-5} - 10^{-4}$	10	$10^{19} - 10^{20}$
RG lobes	$10^{-5} - 10^{-4}$	10^5	$10^{23} - 10^{24}$

TABLE I. Characteristic values of magnetic fields B and associated coherence lengths D for astrophysical objects candidates for accelerating UHECR. The third column gives the kinetic energy range that can be obtained for a monopole in such a field.

Monopoles are however believed not to be bound to small objects such as stars (See Ref. [7] and references therein). Therefore, among the various candidates presented on Table I, only Active Galactic Nuclei (AGN) and Radio Galaxy (RG) lobes have a chance to be the likely accelerators of monopoles. In both cases, the situation is such that the regions where the magnetic field is important correspond also to regions where other particles would be accelerated. These regions are believed to be filled essentially with electromagnetic radiation together with very low hadronic density (in the sense that protons in such a medium would dominantly see the radiation and would otherwise have a mean free path, with respect to proton-proton interactions, larger than the acceleration zone). In turn, this radiation is responsible for degrading the accelerated proton energies, but is far below the threshold for interacting efficiently with monopoles. Therefore, the mean free path of the monopoles is larger than the actual size of the acceleration region. As a result, one does not expect any kind of cutoff in the injection spectrum.

Finally, one needs to know the expected number of such events. Let us assume for the sake of the argument that AGN are the most likely accelerator candidates. To estimate the flux Φ_{AGN}, we set ϕ_{AGN} the number of monopoles emitted per unit time by a characteristic AGN, N_{AGN} the number of AGNs up to a distance which we arbitrarily assume corresponds to a redshift $z=1$ and $\langle D\rangle$ the mean distance to the AGN under consideration. One then obtains

$$\Phi_{AGN} = \phi_{AGN} N_{AGN} \langle D\rangle^{-2}. \tag{39}$$

The quantity ϕ_{AGN} is

$$\phi_{AGN} = \epsilon\, t_0^{-1} N_M, \tag{40}$$

with ϵ the ratio between the power used to accelerate monopoles and the electromagnetic luminosity, i.e., a measure of the efficiency of our mechanism, while t_0 is the characteristic time for emitting a monopole. N_M is the number of monopoles present in the accelerating zone of the AGN. Typically $N_M = \alpha n_M L_{AGN}^3$, where α is the ratio between the hadronic (ρ_b) and monopole (ρ_M) densities; n_M is the mean number density of monopoles in the universe and L_{AGN} the characteristic size of the acceleration region. Standard monopole formation mechanisms give [2]

$$n_M \simeq 10^{-19}\left(\frac{m_M}{10^{11}\mathrm{Gev}}\right)^3 \quad \mathrm{cm}^{-3}, \tag{41}$$

which implies

$$N_M \simeq 10^{26}\alpha\left(\frac{m_M}{10^{11}\mathrm{Gev}}\right)^3\left(\frac{L_{AGN}}{10^{-3}\mathrm{pc}}\right)^3. \tag{42}$$

Considering the monopoles to be relativistics so that their velocity is essentially that of light, the escape time t_0 can be assumed to be the size L_{AGN}, so that Eqs. (42) and (40) combine into

$$\phi_{AGN} \simeq 10^{21}\epsilon\,\alpha\left(\frac{m_M}{10^{11}\mathrm{Gev}}\right)^3\left(\frac{L_{AGN}}{10^{-3}\mathrm{pc}}\right)^2 \quad \mathrm{s}^{-1}. \tag{43}$$

The sphere with $z=1$ contains a baryonic mass $M_B = 5.8\times 10^{54}h^{-1}$g, with h the Hubble constant H_0 in units of 75 km·s^{-1}Mpc^{-1}. The total number of galaxies in such a radius is therefore $N_g = 3\times 10^{10}h^{-1}M_{10}^{-1}$, M_{10} being the mass of the galaxy in units of 10^{10} solar masses, with N_{AGN} approximately a tenth of this value [27]. The expected flux Φ_M is now given by

$$\Phi_{AGN} \simeq 3\times 10^{19}\epsilon\,\alpha_6\, M_{10}\, m_{11}^3\, L_{-3}^2\left(\frac{\langle D\rangle}{1\,\mathrm{Gpc}}\right)^{-2}\mathrm{cm}^{-2}\cdot\mathrm{s}^{-1}, \tag{44}$$

with $\alpha_6 = \alpha/10^6$, $m_{11} = m_M/10^{11}$GeV and $L_{-3} = L_{AGN}/10^{-3}$pc. In Eq. (44), it should be remarked that the efficiency ϵ can exceed unity and in fact presumably depends on the energy scale η.

VIII. CONCLUSIONS

Monopoles represent the most generic prediction of GUT models and as such are always presented as a possible cosmological nuisance. This is because they are usually postulated to form at the GUT phase transition, in which case their remnant density in the universe would be proportional to the fourth power of this energy scale [2], a density considerably larger than the critical density today. Going back to the original idea of Kephart and Weiler [7], we consider instead their usefulness in explaining the mystery of the UHECRs by assuming the simplest of all solution to the monopole problem, namely that they are produced at a phase transition taking place at a temperature scale no higher than 10^9 GeV, implying a monopole mass $m_M \lesssim 10^{11}$ GeV. In this case, one can safely consider that magnetic monopoles do exist in reasonnable number in the universe and their study becomes less academic.

Monopoles as such can hardly interact with air protons to yield air showers as they must be topologically stable: their expected signal would be an extremely difficult to observe Čerenkov shower [7]. However, they are obviously easily accelerated to energies much higher even than the world record observed until now of $E_{W.R.} \sim 3\times 10^{20}$ eV [26]. One is therefore tempted to consider them as candidates for explaining those data. The point we want to make here is that a possibility is left opened when one considers the nature of monopole.

A monopole is a solitonic configuration of a winding localised Higgs field. In most reasonnable theories, such a Higgs field not only permit the symmetry breaking, and thus the appearance of topological defects, but serves also as a means to provide masses to various particles to which they couple. This in turn implies that these particles might get trapped in the monopole core, forming bound states that might interact with the air protons in such a way as to be effectively expelled from the monopole. Moreover, they behave as a neutron in a nucleus: they are stable in the form of bound states, but unstable otherwise. The observations can then be explained in supposing that monopoles regularly hit the earth atmosphere with tremendous energies, releasing only part of it in such "ionising" processes. The unstable particles thus obtained are relativistic, although not highly with a Lorentz fact not exceeding 10^7 and can initiate air showers.

We have developped a model in which, to make things simple, the bound states are formed by means of the condensation of a scalar field. Such a field can be charged, a point that would modify our analysis by corrections of order $e^2 \sim 1/137$, but it should be reminded that the leading effects we studied come from the topological defect itself and are thus of order $e^{-2} \sim 137$; the correction we neglected is therefore some four orders of magnitude smaller. Given this approximation, we have examined in detail the internal structure of the charged monopole and derived the expected energy levels that might hopefully be measured in a precise UHECR experiment, as the latter should, in our model, yield a line spectrum. If such an observation was done, the formalism we have presented would allow a straightforward computation of the free parameters and a reconstruction of the underlying theory. Whether or not the Pierre Auger Observatory [21] project will fulfill this task is yet an open question and depends essentially on the spacing of the energy levels; for some cases, one expects that it will.

Our main result is that such a model satisfies all the present observational constraints: their expected flux is of the correct order of magnitude and they interact strongly with the atmosphere. This last feature comes from the presence of bound states and the Callan-Rubakov effect [13]. A model based on such topological defects would have the advantages of the non-acceleration scenarios (bottom-up models) since they would easily propagate, together with the advantages of acceleration mechanisms, as they need point sources in order to be observed. Note that this is compatible with the most recent data (observed doublets and triplets of events within 2.5°) [28] implying localized acceleration. These very data render the previous mechanism for UHECR using monopoles [7] very unprobable.

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