Representing Subjective Probabilities

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- Differing theories of propositional attitudes = different foundations of subjective probability

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- If someone believes *p*, then they are disposed to assent to *p*
  - Upon hearing *p* uttered in appropriate circumstances they will utter *yes*
- This is also an *eliminativist* account

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  - Caveat: "But this method is applicable only in cases where the believed proposition is one that can eventually be decided to the satisfaction of both parties, that that the bet can be settled." Quine 1987 18-19, although this applies to all sentences, including tautologies, for Quine.

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- We're not going to go to follow this debate, but go around it

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    - i.e. it can be a proposition, a sentence, a sentence-analogue, etc.

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  - For now, we suppose we're trying to figure out what's going on inside a robot

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- How simple?

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- But we don't require that the completion be classical

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- For the purposes of this paper we will assume that the algebra is Boolean

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(A) Additivity (Independence of disjoint events)
If $A \cap C = \emptyset = B \cap C$ and $A \precsim B$ then $A \cup C \precsim B \cup C$

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  For each $A, B \in E$, such that B^c and A^c are defined in $\mathcal{S}$ if $A \precsim B$ then either $B^c \precsim A^c$ or A^c and B^c are incomparable.

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- Subject is able to compare subjective events to the ‘reference‘ events which provide a reference measure for the subjective events.
- They allow us to substitute familiar quantities like length, volume, area, for the decidedly less familiar quantity of degree of belief.

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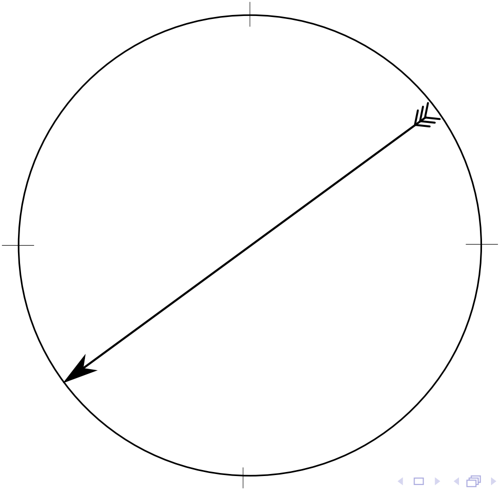

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(RQP) Reference qualitative probability:
The relation $\precsim_{ref}$ on the algebra $\mathcal{G}$ defined as
$a \precsim_{ref} b \equiv_{def} l(a) \leq l(b)$ satisfies the axioms of qualitative probability.

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  - (ii) (Closure) - for each $A \in F$ the sets $\{x | A \precsim_X x\}$ and $\{x | A \succsim_X x\}$ are closed
  - (iii) (Preservation of $\precsim$) if $A \precsim B, A \succsim_X a$ and $B \precsim_X b$ then $a \precsim_{ref} b$

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(i) There is a partial probability function p on $\mathcal{F}$ that respects the ordering $\precsim$

(ii) $\mathcal{G}$ is a completion of the partial structure $\mathcal{F}$ and there is a probability function p‘ on $\mathcal{G}$ such that p‘ restricted to $\mathcal{F}$ is p.

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- And it's more fundamental, since we can add in sanctions on top of the reference experiment