# Numerical modelling of axially impacted rod with a spiral groove

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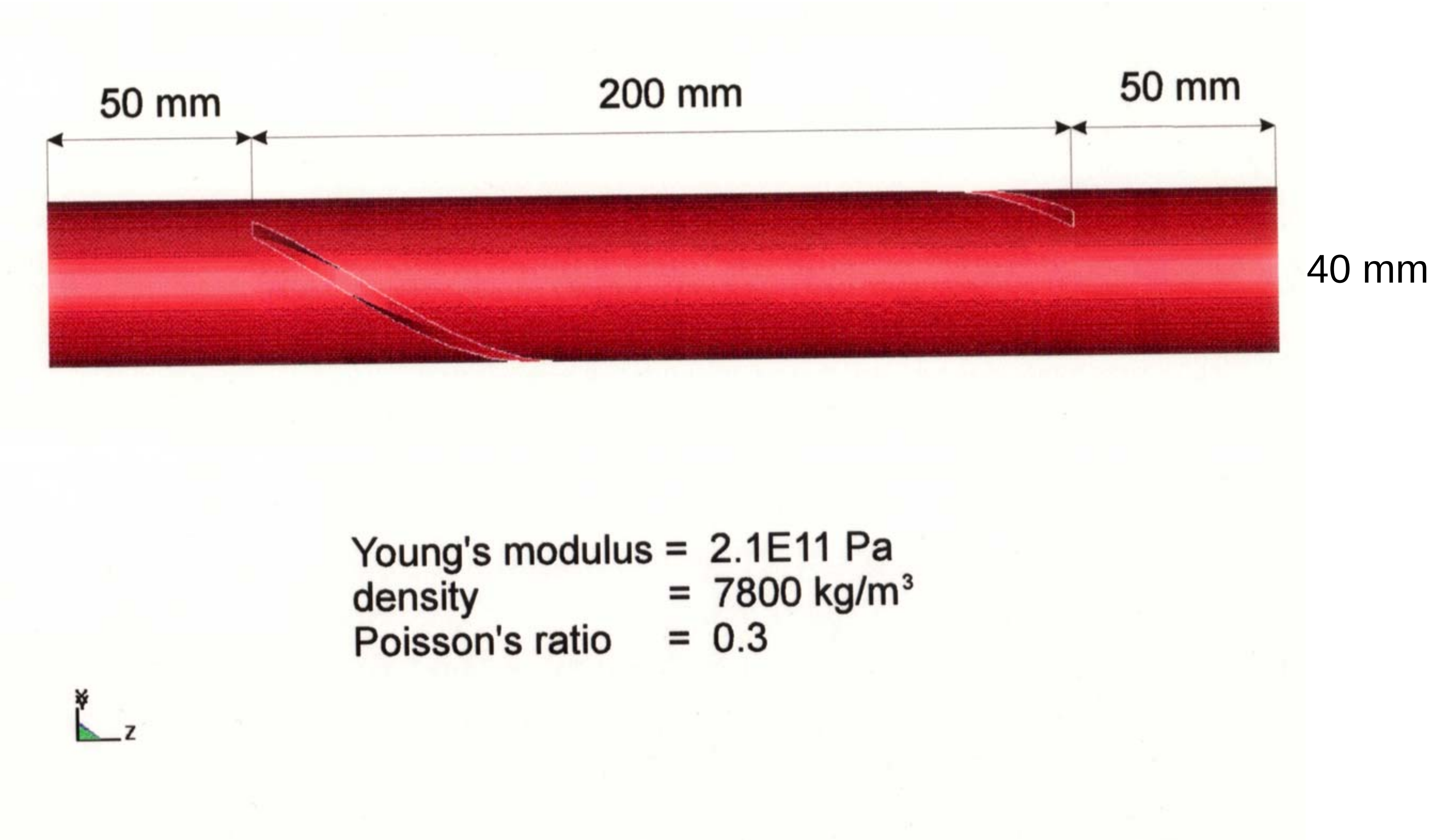
# Rod with a spiral groove

short rod, single right-hand spiral, 360 degrees



Young's modulus = 2.1E11 Pa
density = 7800 kg/m$^3$
Poisson's ratio = 0.3

# Motivation

- One intuitively feels that an axial impact loading on a rod with a spiral groove should invoke a torsional and bending wave effects, their ‘intensity’, however, is hard to predict.

- The presented study thus aims to describe transient phenomena in a longitudinal body having the form of a massive cylindrical rod whose middle part has a spiral groove on its surface. The rod is subjected to axial impact loading, expressed by a uniform pressure applied on the one face of the cylinder, while the other is completely fixed. The time dependence of the pulse is prescribed by a rectangular function.

# Two basic geometries

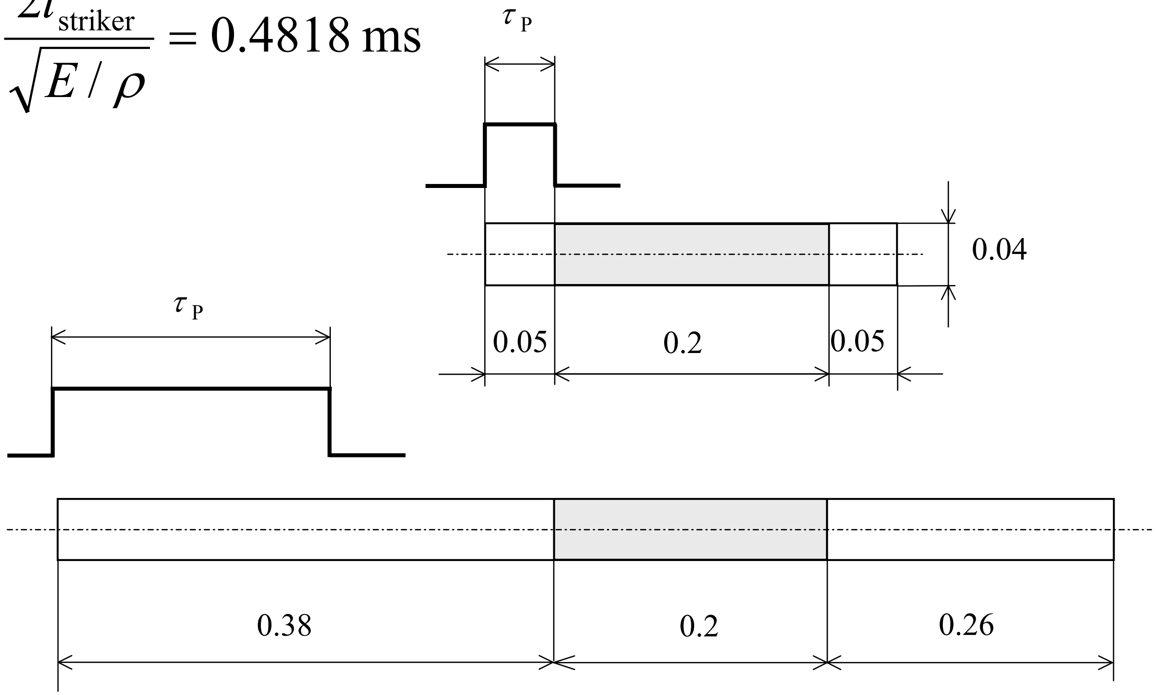
| Short and long rod [mm] | **Short rod** | | **Long rod** |
|---|---|---|---|
| diameter | | 40 | |
| the input cylindrical part | **50** | | **380** |
| groove part (full 360 degrees swing) | | 200 | |
| output part | **50** | | **260** |

# Short and long pulses and their comparison with 1D theory strikers

$$l_{\text{striker}} = 0.0025\,\text{m}; \quad \tau_{\text{P}} = \frac{2l_{\text{striker}}}{\sqrt{E/\rho}} = 0.4818\,\text{ms}$$

S

Equivalent striker 25mm



Equivalent striker 100 mm

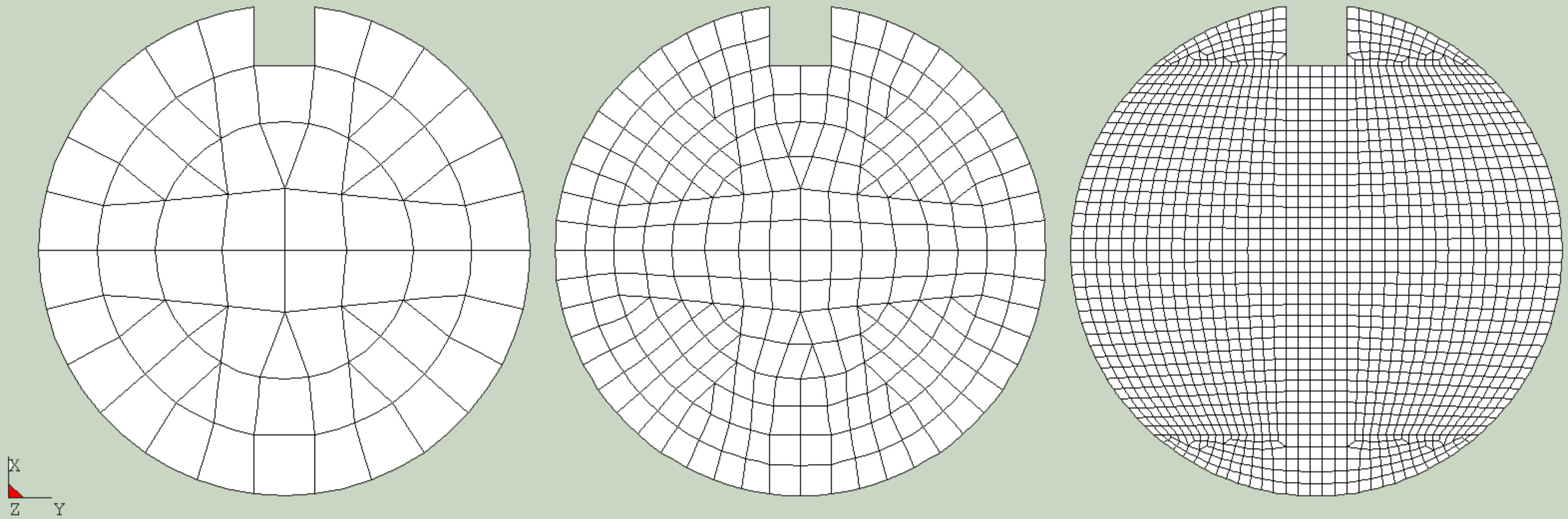
# Groove variants

- Single groove, right-hand pitch, 360 degrees
- Single groove, left-hand pitch, 360 degrees
- Single groove, right-hand pitch, 665 degrees
- Two grooves, right-hand pitch, 180 degrees
- Axial groove on surface in xz-plane
- No groove (‘smooth’ cylinder)

**Element types** (hexahedrons and pentahedrons) and **meshing**

L1 … linear elements, one element per groove
L2 … linear elements, two elements per groove
L5 … linear elements, five elements per groove
Q1 …quadratic elements (midsides), one element per groove
Q2…quadratic elements (midsides), two elements per groove

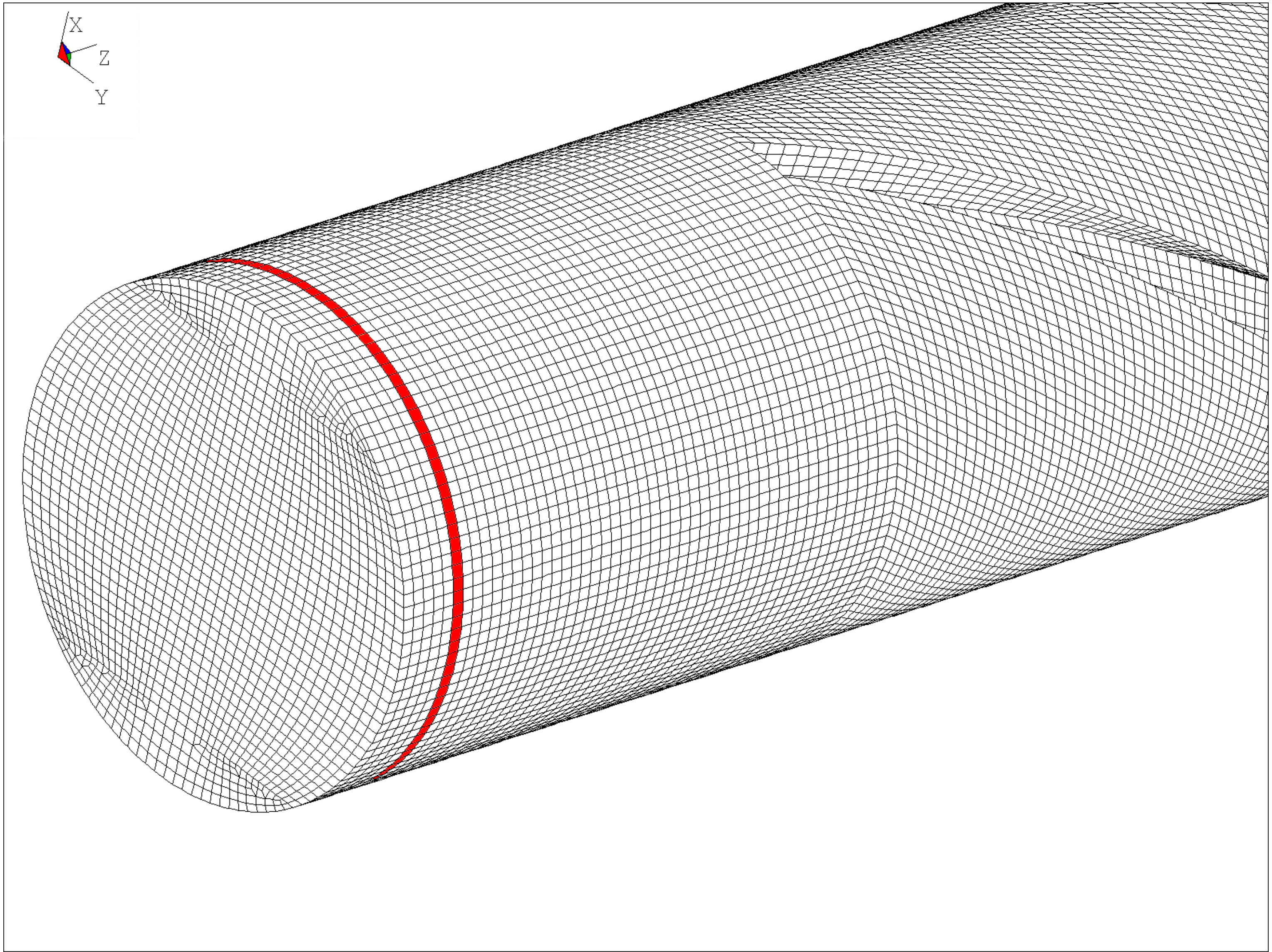
Full quadrature and consistent mass matrix



# The meshes are characterized by following input quantities

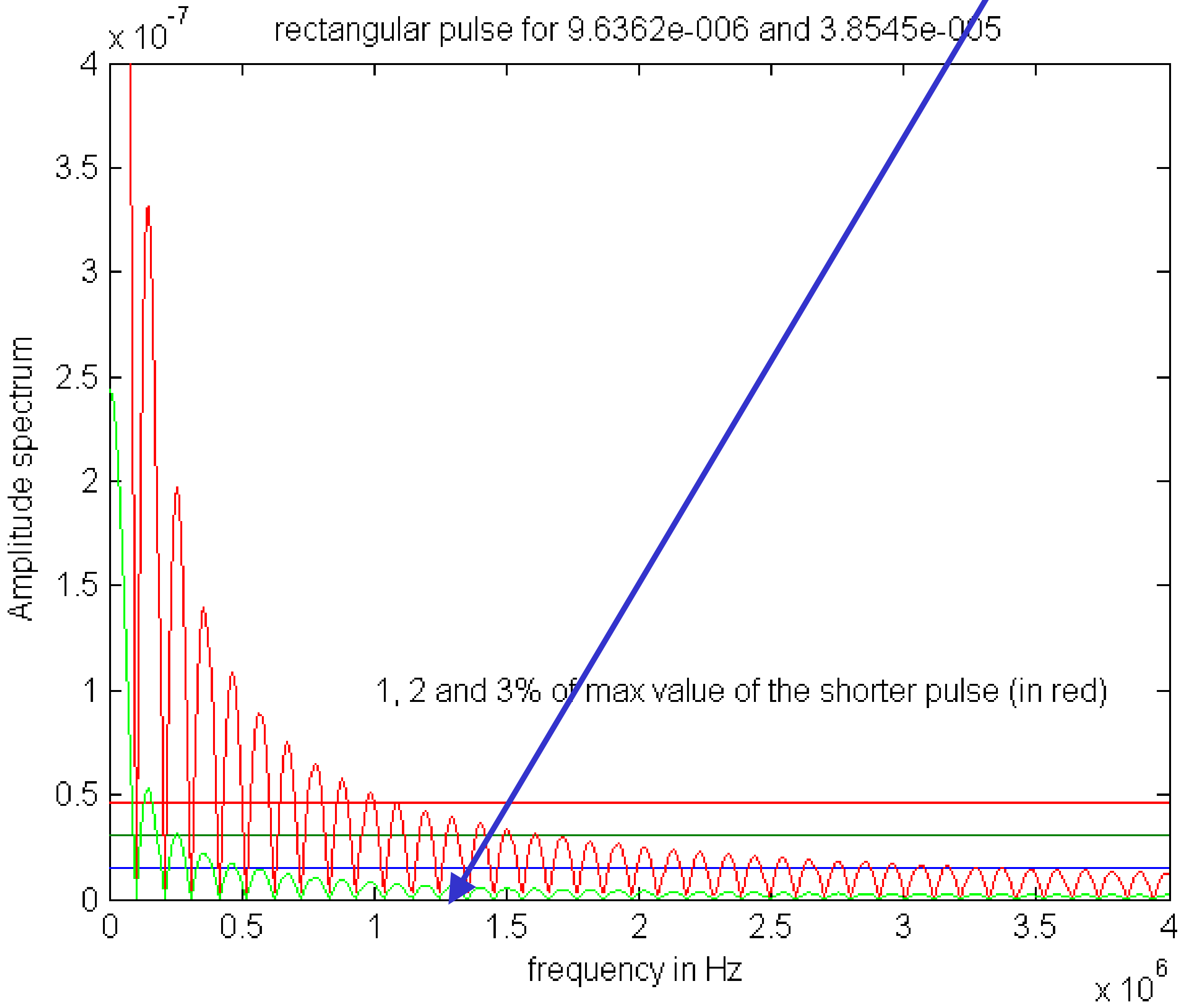
| | **L1** | **L2** | **Q1** | **Q2** | **L5** | **LX1** |
|---|---|---|---|---|---|---|
| **elements** | 4400 | 33040 | 4400 | 33040 | 483400 | 12392 |
| **nodes** | 4819 | 34569 | 18792 | 136619 | 505312 | 13351 |
| **dof’s** | 14457 | 103707 | 56376 | 409857 | **1515936** | 40053 |
| **front** | 291 | 975 | 852 | 2889 | 5232 | 291 |
| **elemsize[mm]** | 5 | 2.5 | 5 | 2.5 | 1 | 5 |

# Axonometric view and coordinate system L5 mesh, layer definition



# Maximum frequencies and minimum wavelenhths
# Fourier power spectrum of short and long pulse

| | **L1** | **L2** | **Q1** | **Q2** | **L5** | **LX1** |
|---|---|---|---|---|---|---|
| [MHz] | 0.2594 | 0.5189 | 0.5189 | 1.0378 | 1.2972 | 0.2594 |
| [mm] | 20 | 10 | 10 | 5 | 4 | 20 |



# Initial and Boundary conditions

- Initially the body is in rest, no initial stress
- Right-hand side of the rod is fully clamped

# Impact loading

is modelled by the pressure load applied
on the left-hand face of the rod

**Short pulse** … related to the length of cylindrical part of the short rod
**Long pulse** … related to the length of grooved part of the rod

**Dimensionless time** is defined as the ratio of

actual time
divided by
the time needed for 1D wave to pass through the length of the rod.

# Time discretization of

$$\mathbf{M}\ddot{\mathbf{q}}+\mathbf{K}\mathbf{q}=\mathbf{P}(t)$$

- By Newmark, no algorithmic damping

$$\Delta t = l_{\mathrm{min}} / (c_{\mathrm{L1}} * hmts) \quad c_{\mathrm{L1}} = \sqrt{E / \rho}$$

- **H**ow **m**any **t**ime **s**teps = 2

How many time steps are needed for the wave to pass through the ‘length’ of the shortest element

# Global quantities computation

$$\mathbf{H} = m\mathbf{v}, \quad \mathbf{MH} = \mathbf{r} \times m\mathbf{v}$$

Energy

$$E_{\mathrm{Pot}}\Big|_{\tau_k} = \frac{1}{2}\left(\mathbf{q}^{\mathrm{T}}\mathbf{K}\,\mathbf{q}\right)\Big|_{\tau_k} = \frac{1}{2}\sum_{\mathrm{e}=1}^{\mathrm{NELEM}}\left(\mathbf{q}_{\mathrm{e}}^{\mathrm{T}}\mathbf{K}_{\mathrm{e}}\,\mathbf{q}_{\mathrm{e}}\right)\Big|_{\tau_k};\; E_{\mathrm{Kin}}\Big|_{\tau_k} = \frac{1}{2}\left(\dot{\mathbf{q}}^{\mathrm{T}}\mathbf{M}\,\dot{\mathbf{q}}\right)\Big|_{\tau_k} = \frac{1}{2}\sum_{\mathrm{e}=1}^{\mathrm{NELEM}}\left(\dot{\mathbf{q}}_{\mathrm{e}}^{\mathrm{T}}\mathbf{M}_{\mathrm{e}}\,\dot{\mathbf{q}}_{\mathrm{e}}\right)\Big|_{\tau_k} \;,$$

Momentum

$$\mathbf{H}\Big|_{\tau_k} = \sum_{\mathrm{e}=1}^{\mathrm{NELEM}}\mathbf{H}_{\mathrm{e}}\Big|_{\tau_k} = \sum_{\mathrm{e}=1}^{\mathrm{NELEM}}\mathbf{T}_{\mathrm{e}}\left(\mathbf{M}_{\mathrm{e}}\,\dot{\mathbf{q}}_{\mathrm{e}}\right)\Big|_{\tau_k} \;; \qquad \mathbf{T}_{\mathrm{e}} = \left[\mathbf{E}_1, \ldots, \mathbf{E}_{\mathrm{INE}}, \ldots, \mathbf{E}_{\mathrm{NNODE}}\right]$$

Angular momentum

$$\mathbf{MH}\Big|_{\tau_k} = \sum_{\mathrm{e}=1}^{\mathrm{NELEM}}\mathbf{MH}_{\mathrm{e}}\Big|_{\tau_k} = \sum_{\mathrm{e}=1}^{\mathrm{NELEM}}\mathbf{X}_{\mathrm{e}}\left(\mathbf{M}_{\mathrm{e}}\,\dot{\mathbf{q}}_{\mathrm{e}}\right)\Big|_{\tau_k} \;; \qquad \mathbf{X}_{\mathrm{e}} = \left[\mathbf{X}_1, \ldots, \mathbf{X}_{\mathrm{INE}}, \ldots, \mathbf{X}_{\mathrm{NNODE}}\right]$$

$$\mathbf{E}_{\mathrm{INE}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{X}_{\mathrm{INE}} = \begin{bmatrix} 0 & -z_{\mathrm{INE}} & y_{\mathrm{INE}} \\ z_{\mathrm{INE}} & 0 & -x_{\mathrm{INE}} \\ -y_{\mathrm{INE}} & x_{\mathrm{INE}} & 0 \end{bmatrix}$$

# Force and moment computation

$$F_i(\tau_k) = \frac{H_i(\tau_k) - H_i(\tau_{k-1})}{\tau_k - \tau_{k-1}} = \frac{\mathit{\Delta} H_i(\tau_k)}{\mathit{\Delta}\tau_k} = \left.\frac{\mathit{\Delta} H_i(\tau)}{TSTEP}\right|_{\tau_k} \cong \left.\frac{\mathrm{d}H_i}{\mathrm{d}\tau}\right|_{\tau_k} \text{; } R_i(\tau_k) = F_i(\tau_k) \text{ for } \tau \geq \tau_{\mathrm{R}}$$

$$M_i(\tau_k) = \frac{MH_i(\tau_k) - MH_i(\tau_{k-1})}{\tau_k - \tau_{k-1}} = \left.\frac{\mathit{\Delta} MH_i(\tau)}{TSTEP}\right|_{\tau_k} \cong \left.\frac{\mathrm{d}MH_i}{\mathrm{d}\tau}\right|_{\tau_k} \text{; } MR_i(\tau_k) = M_i(\tau_k) \text{ for } \tau \geq \tau_{\mathrm{R}}$$

rate of momentum is equal to the impulse of external forces
rate of angular momentum … external moments

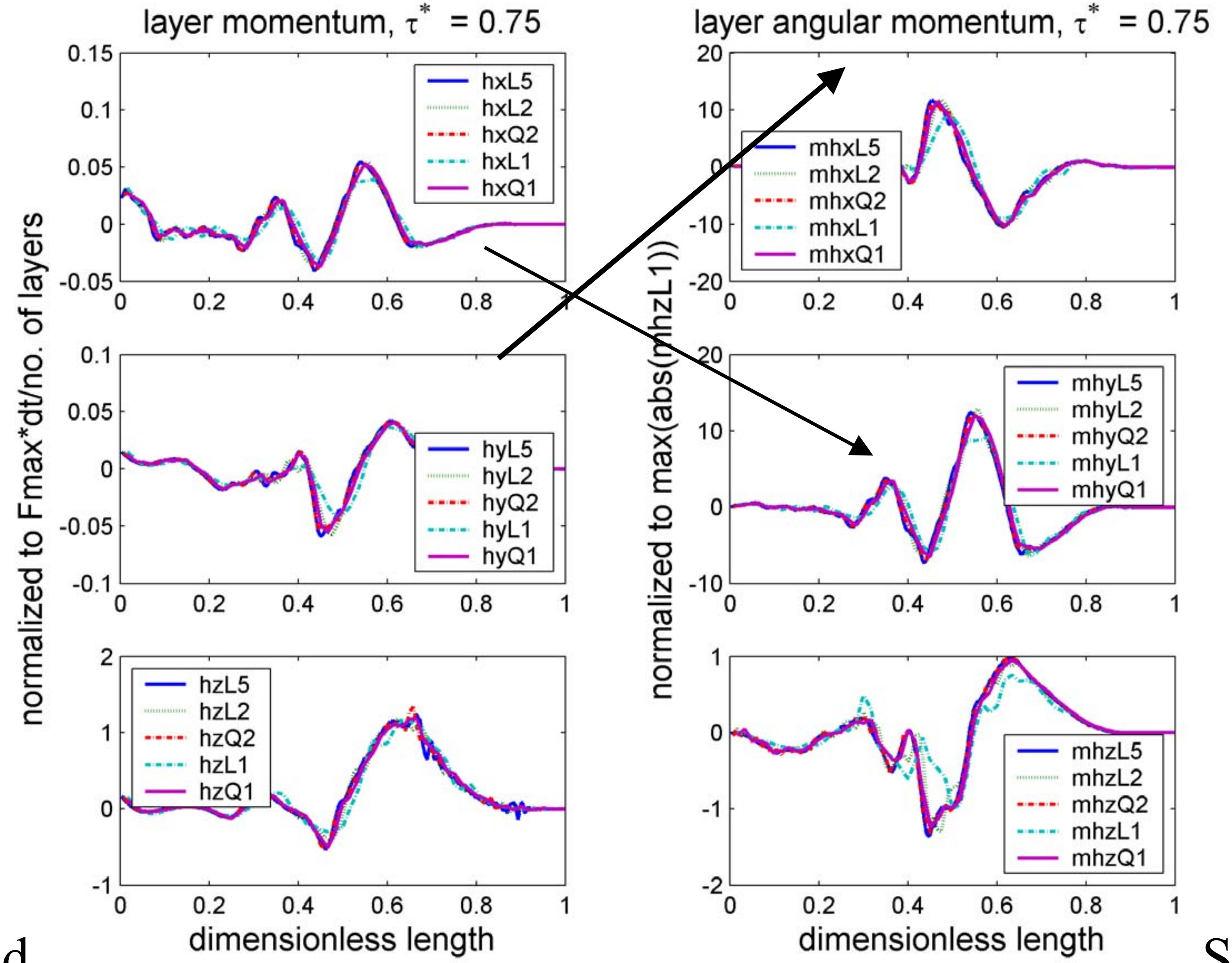
# Momentum and angular momentum.
# Layer quantities

$$\mathbf{h}\left(l_n\right)\Big|_{\tau_k} = \sum_{\mathrm{e}=\mathrm{IE_L}(l_n)}^{\mathrm{IE_U}(l_n)} \mathbf{H}_{\mathrm{e}}\Big|_{\tau_k} = \sum_{\mathrm{e}=\mathrm{IE_L}(l_n)}^{\mathrm{IE_U}(l_n)} \mathbf{T}_{\mathrm{e}} \left(\mathbf{M}_{\mathrm{e}}\, \dot{\mathbf{q}}_{\mathrm{e}}\right)\Big|_{\tau_k}$$

$$\mathbf{mh}\left(l_n\right)\Big|_{\tau_k} = \sum_{\mathrm{e}=\mathrm{IE_L}(l_n)}^{\mathrm{IE_U}(l_n)} \mathbf{MH}_{\mathrm{e}}\Big|_{\tau_k} = \sum_{\mathrm{e}=\mathrm{IE_L}(l_n)}^{\mathrm{IE_U}(l_n)} \mathbf{X}_{\mathrm{e}} \left(\mathbf{M}_{\mathrm{e}}\, \dot{\mathbf{q}}_{\mathrm{e}}\right)\Big|_{\tau_k}$$

# Layer momentum and angular momentum for a given time as functions of length

$$h_x \approx mh_y, \quad h_y \approx -mh_x$$

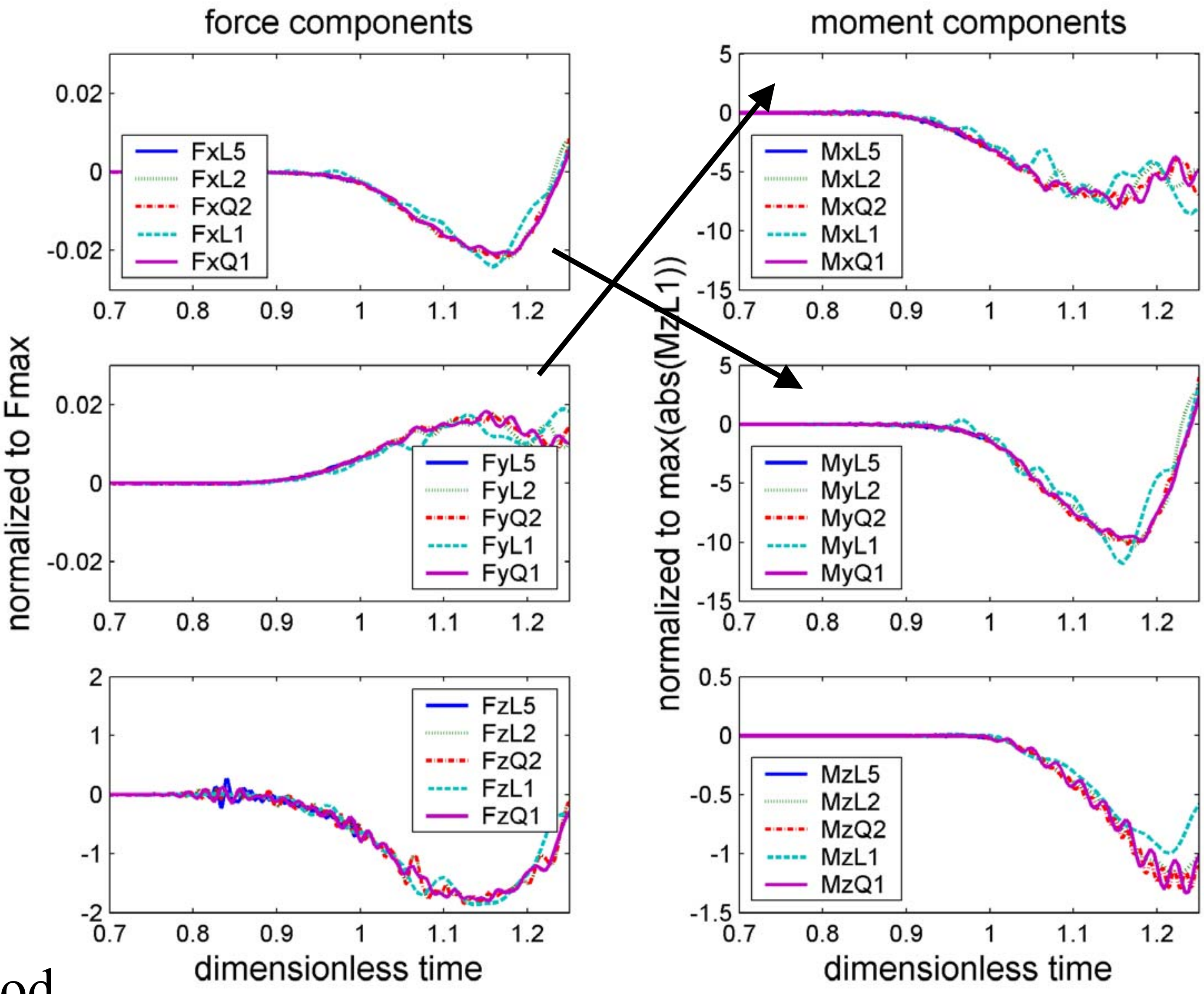


Short rod

Short pulse

# Reactions as a function of time

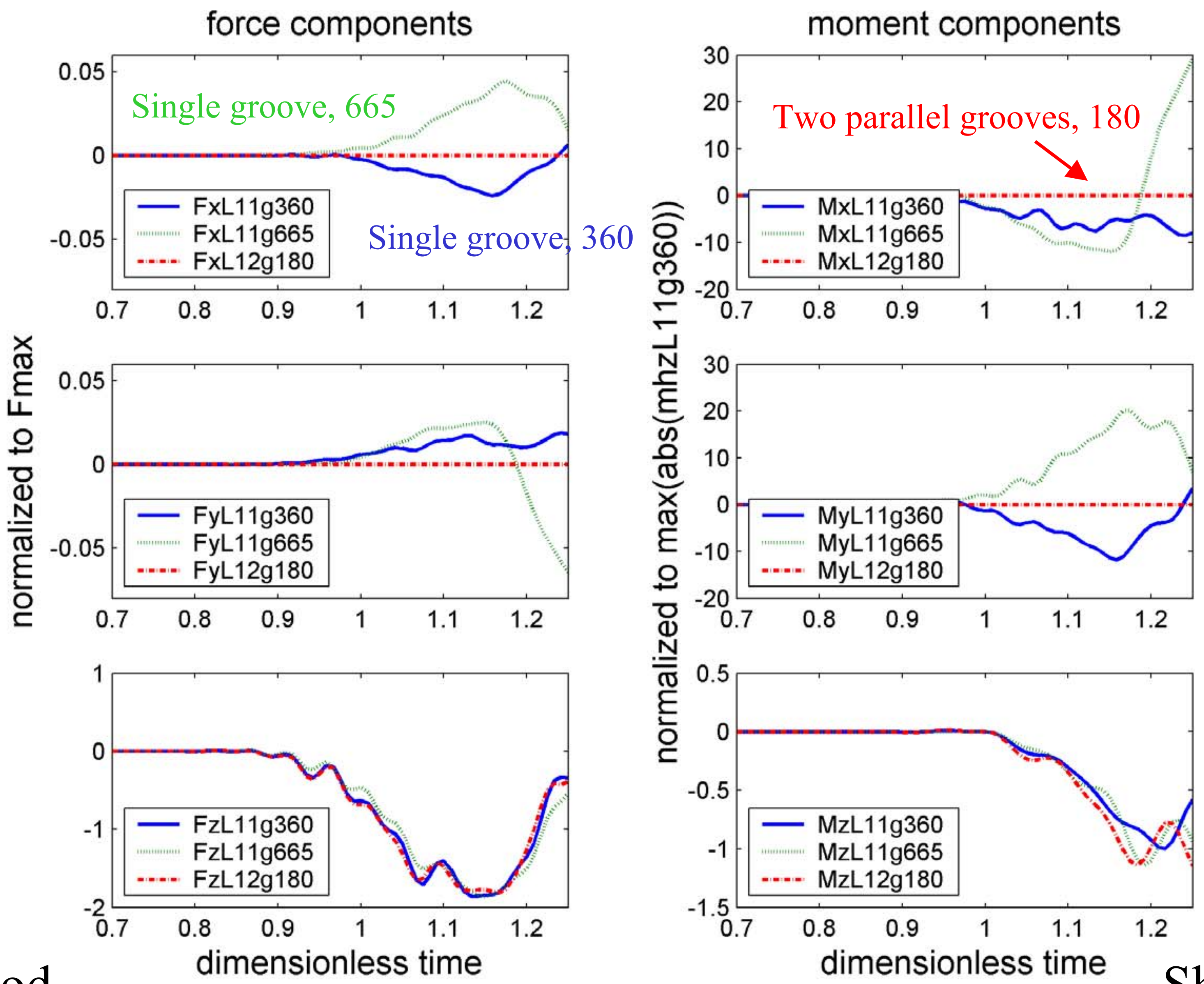
$$F_x \approx M_y, \quad F_y \approx -M_x$$



Short rod

Short pulse

# Reactions, 1 groove, 2 grooves, different pitch



Single groove, 665

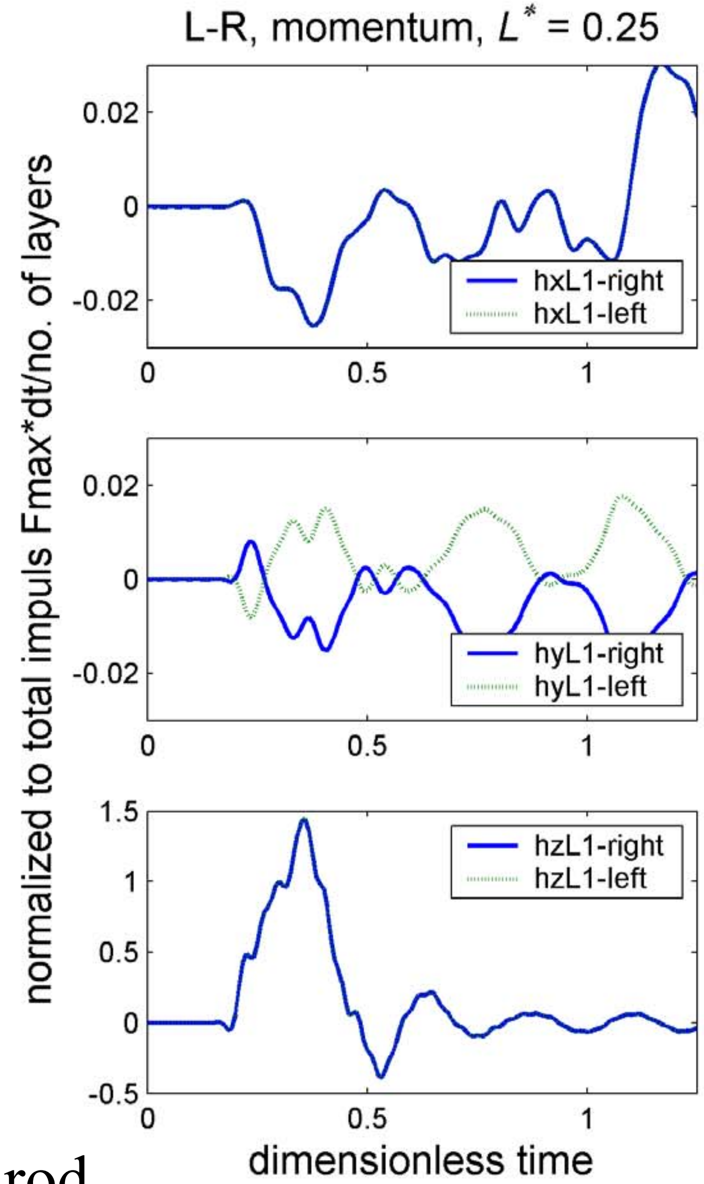
Single groove, 360

Two parallel grooves, 180

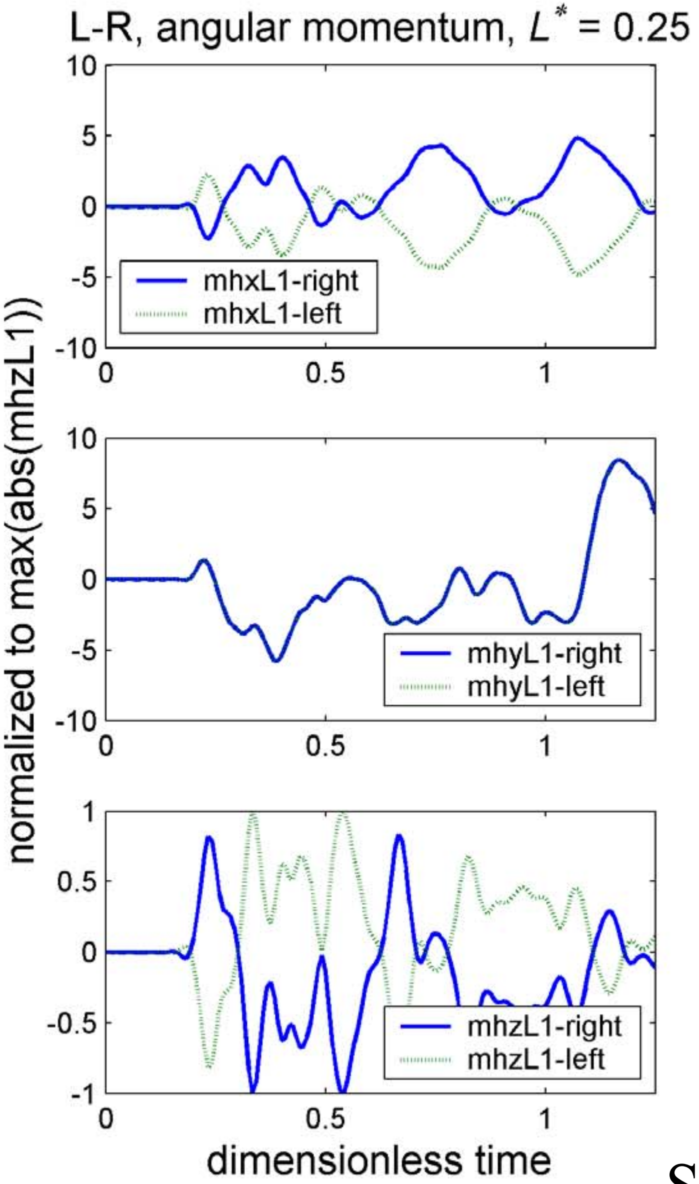
Short rod

Short pulse

# Reactions, Left- and right-hand thread



L-R, momentum, $L^* = 0.25$

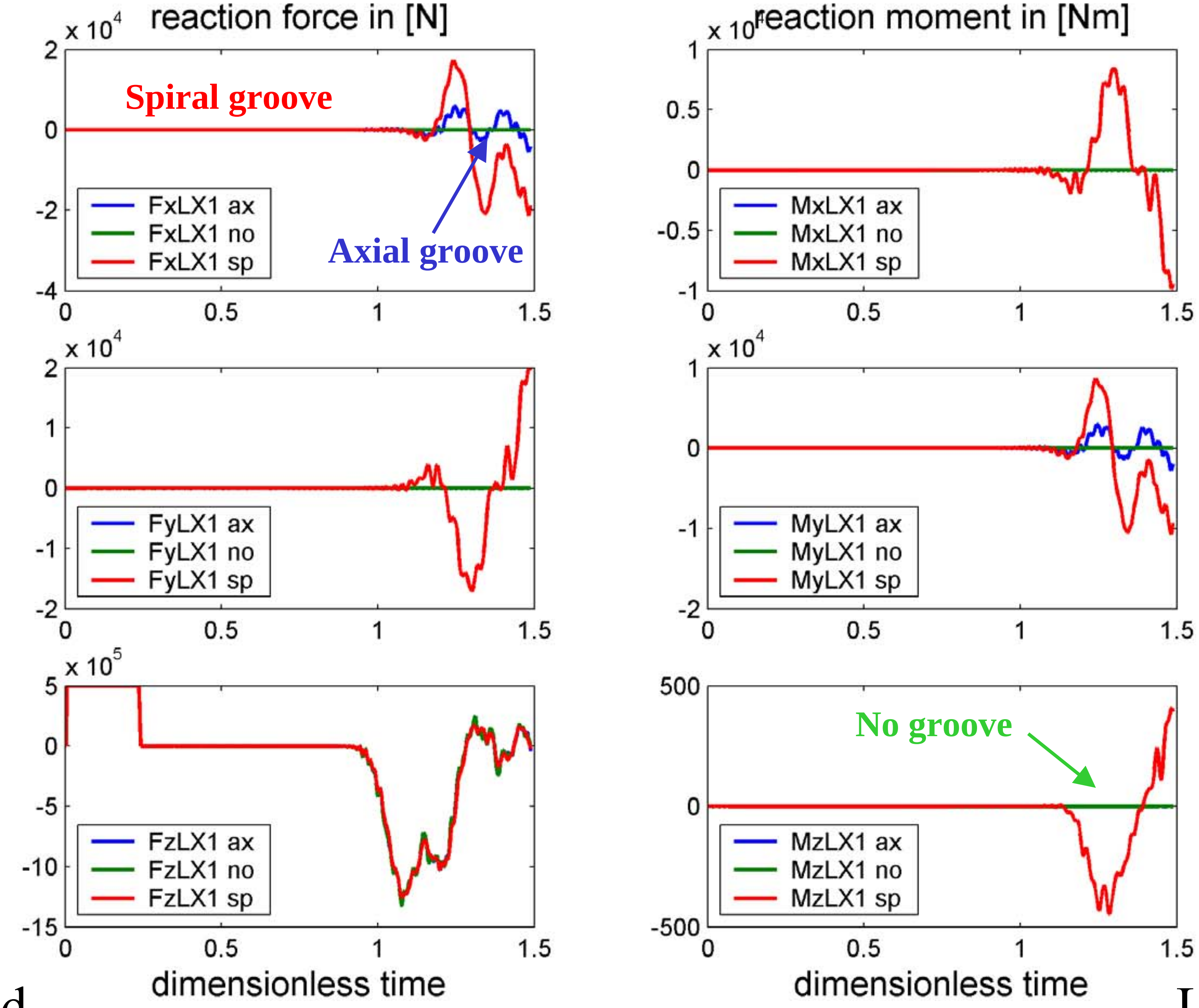


L-R, angular momentum, $L^* = 0.25$

Short rod

Short pulse

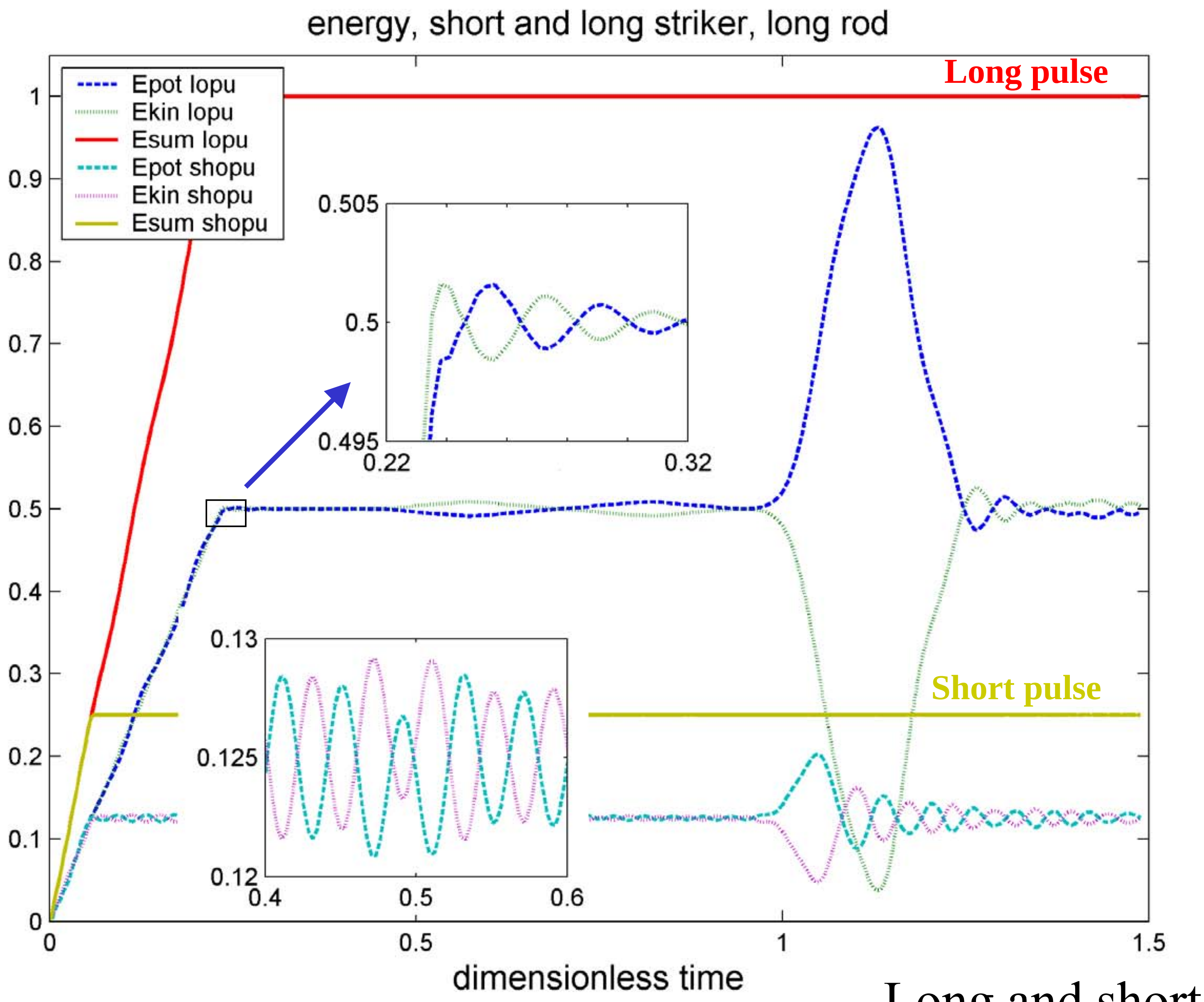
# Reactions, Axial, spiral and no groove



Long rod

Long pulse

# Long and short pulse
# 1D and 3D behaviour observed



Long rod

Long and short pulses

# Conclusions

Relations defining momentum and angular momentum defined above (slide 13 and 15) seems to be crucial for understanding the transient behaviour of the considered rod.

These relations, rewritten in components forms, explain why the distributions of layer momentum quantities as well as force resultants obey the following relations

$$h_x \approx m h_y, \quad h_y \approx -m h_x, \quad F_x \approx M_y, \quad F_y \approx -M_x$$

# Conclusions

You may notice that the **z-components of momentum and reaction force** are dominant.

**x- and y-components of momentum and angular momentum** are non-zero only in case of grooved rod.

Relations appearing in integrands defining layer momentum quantities are

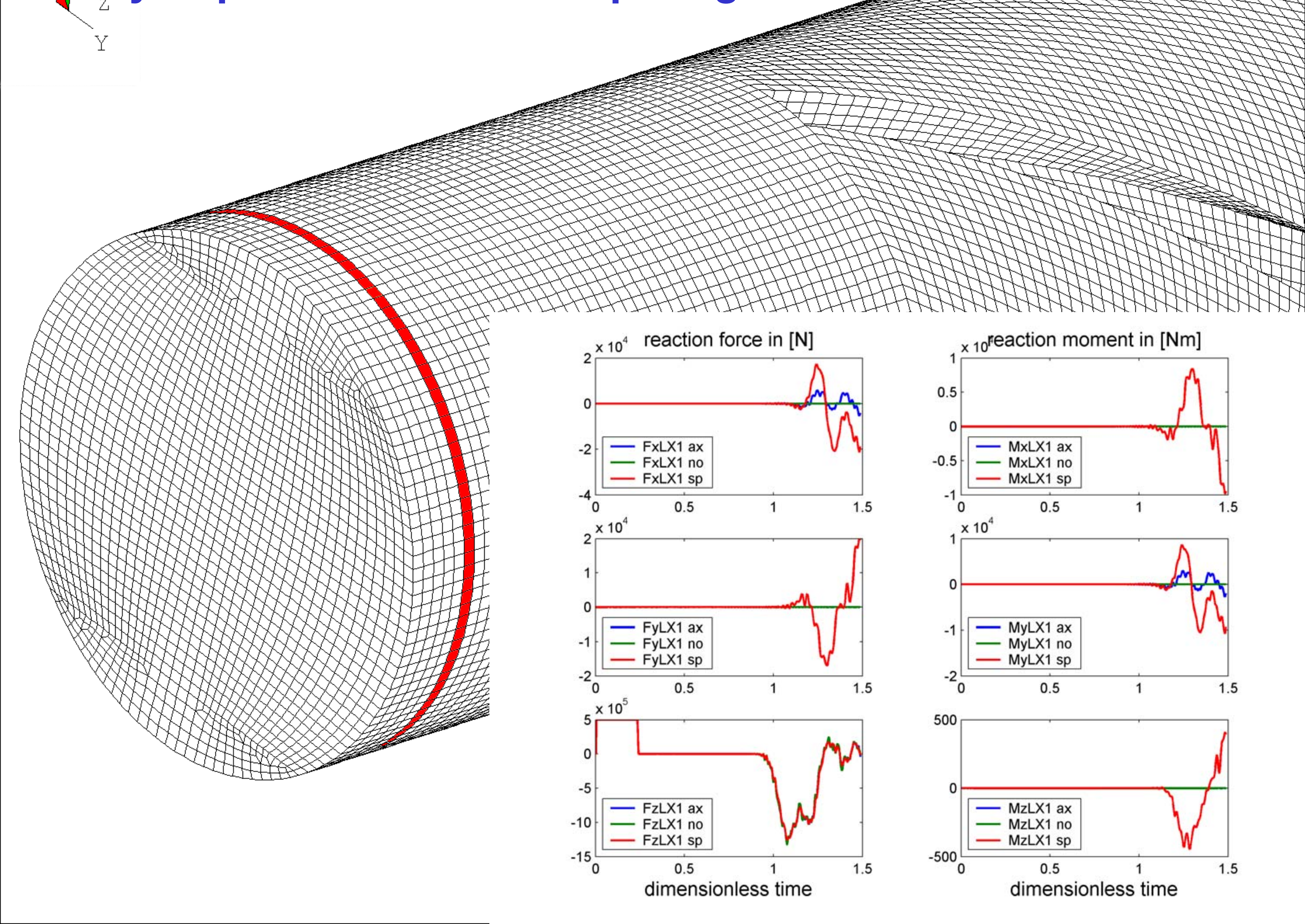
$$mh_x = \enclose{circle}{yh_z} - zh_y, \quad mh_y = zh_x - \enclose{circle}{xh_z}, \quad mh_z = \boxed{xh_y} - \boxed{yh_x}$$

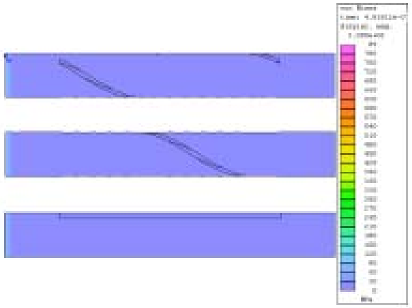
**dominant**

**less significant**

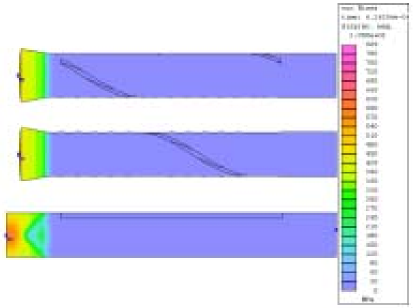
The dominancy of and  together with ‘longitudinalness’ of considered rods explains why the torsional effect induced due to the axial impact loading on a ‘thin’ rod with a spiral groove cannot be significant.

# Axially impacted rod with a spiral groove

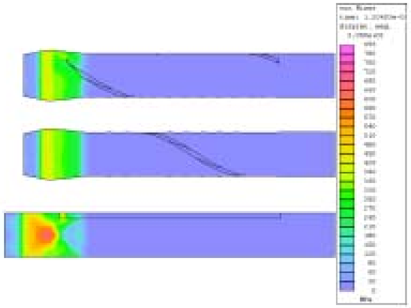




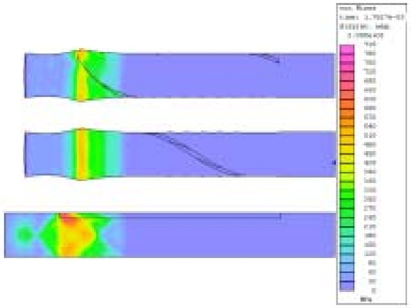
stress_L1_1g_001.tif



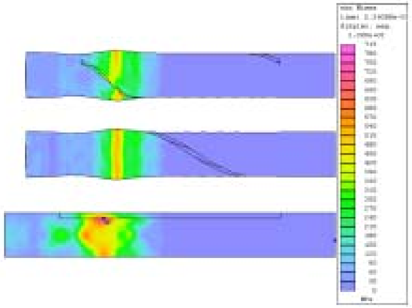
stress_L1_1g_013.tif



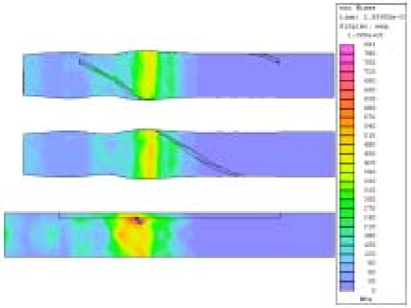
stress_L1_1g_025.tif



stress_L1_1g_037.tif



stress_L1_1g_049.tif



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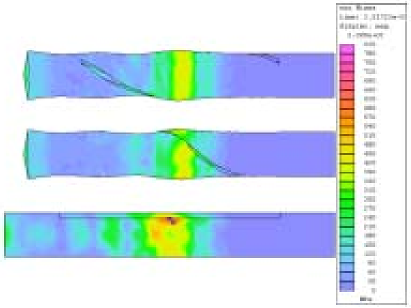

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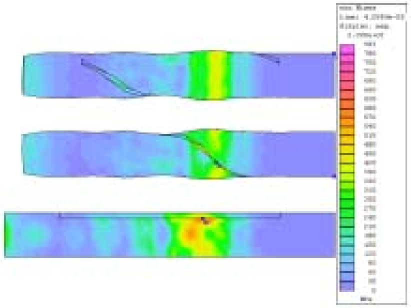

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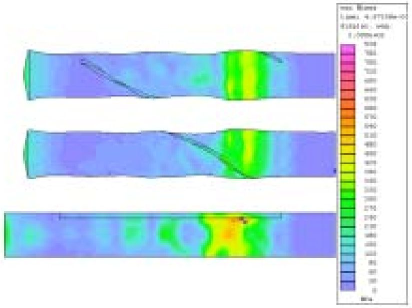

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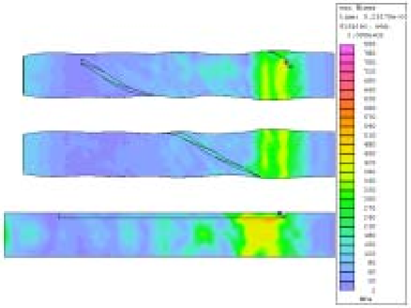

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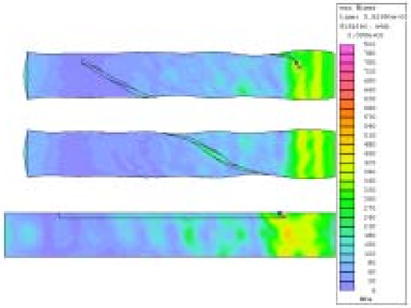

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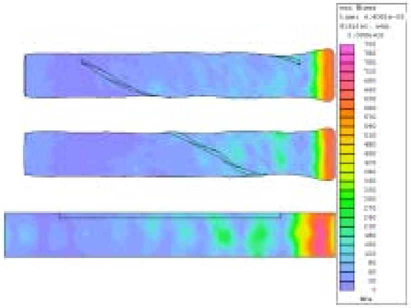

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