Spektrální rozklad

$U\ (3\times 3) \in R, \quad U^T = U$

$U\varphi = \lambda\varphi \qquad \lambda_k \in R, \ \varphi_k \in R^3$

$\phi = [\varphi_1\ \varphi_2\ \varphi_3] \qquad \varphi_i^T \varphi_j = \delta_{ij} \quad \Rightarrow \quad \phi^T\phi = I$

$\Lambda = \mathrm{diag}\,[\lambda_1\ \lambda_2\ \lambda_3]$

$U\phi = [U\varphi_1\ U\varphi_2\ U\varphi_3] = [\lambda_1\varphi_1\ \lambda_2\varphi_2\ \lambda_3\varphi_3] = \phi\Lambda$

$U\phi = \phi\Lambda \quad \Rightarrow \quad \boxed{U = \phi\Lambda\phi^T}$

Odmocnina matice

C sym. + def. $\quad \exists\, U$ sym + def. $\qquad C = UU$

$C = U^2 \qquad U = \sqrt{C}$

Výpočet odmocniny je jednoznačný.

Př

$$C = \begin{bmatrix} 9 & 0 \\ 0 & 4 \end{bmatrix} \quad U = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \quad U \neq \begin{bmatrix} -3 & 0 \\ 0 & 2 \end{bmatrix}$$

Předp. C pro existenci. Předp. U pro jednoznačnost.

Existence:

$$C = \Phi \Lambda^2 \Phi^T \quad \Lambda^2 = \mathrm{diag}\,[\mu_1\, \mu_2\, \mu_3] \quad \mu_k > 0$$

$$\lambda_k \stackrel{def.}{=} |\sqrt{\mu_k}| \quad \Lambda = \mathrm{diag}\,[\lambda_1\, \lambda_2\, \lambda_3] \quad \Lambda\Lambda = \Lambda^2$$

$$\boxed{U \stackrel{def}{=} \Phi \Lambda \Phi^T}$$

a) $U^2 = \Phi \Lambda \Phi^T \Phi \Lambda \Phi^T = \Phi \Lambda \Lambda \Phi^T = \Phi \Lambda^2 \Phi^T = C$

b) $U^T = (\Phi \Lambda \Phi^T)^T = \Phi \Lambda^T \Phi^T = \Phi \Lambda \Phi^T = U$ symetrie

c) + def.: $\forall x \neq 0 : x^T U x > 0$

$$x^T (\Phi \Lambda \Phi^T) x = \eta^T \Lambda \eta = \lambda_1 \eta_1^2 + \lambda_2 \eta_2^2 + \lambda_3 \eta_3^2 > 0$$

$$\Phi^T x = \eta \Rightarrow x = \Phi \eta \,, \quad \eta = 0 \Rightarrow x = 0$$

Jednoznačnost: (Stephenson, 1980)

$$U^2 = \bar{U}^2 = C \qquad (C - \mu I)\varphi = 0$$

$$(\bar{U}^2 - \mu I)\varphi = (\bar{U} + \lambda I)\underbrace{(\bar{U} - \lambda I)\varphi}_{x} = 0 \,, \quad \lambda = |\sqrt{\mu}| > 0$$

$$\bar{U} x = -\lambda x \,, \quad \bar{U} + \mathrm{def} \Rightarrow x = 0 \Rightarrow \bar{U}\varphi = \lambda \varphi$$

spektrální rozklad $\bar{U} = \Phi \Lambda \Phi^T = U$

Doplněk

$F\,(3\times 3)$ regulární

$$J = \det|F| = F_{11}\det\begin{vmatrix}F_{22} & F_{23}\\ F_{32} & F_{33}\end{vmatrix} - F_{12}\det\begin{vmatrix}F_{21} & F_{23}\\ F_{31} & F_{33}\end{vmatrix} + F_{13}\det\begin{vmatrix}F_{21} & F_{22}\\ F_{31} & F_{32}\end{vmatrix} =$$

$$= F_{11}\overline{F}_{11} + F_{12}\overline{F}_{12} + F_{13}\overline{F}_{13}$$ $\overline{F}_{ij}$ doplněk (kofaktor)

Pozn. $\overline{F}_{ij}$ neobsahuje prvky z i-tého řádku

$$F^{-1}_{ij} = \frac{1}{J}\overline{F}_{ji}$$ inverzní matice

$$\boxed{\overline{F}_{ij} = J F^{-1}_{ji}}$$

$$\frac{\partial J}{\partial F_{ij}} = \frac{\partial}{\partial F_{ij}}\left(F_{i1}\overline{F}_{i1} + F_{i2}\overline{F}_{i2} + F_{i3}\overline{F}_{i3}\right) = \overline{F}_{ij}$$

$$\boxed{\frac{\partial J}{\partial F_{ij}} = J F^{-1}_{ji}}$$

sym., antisym. část t. z. ř.

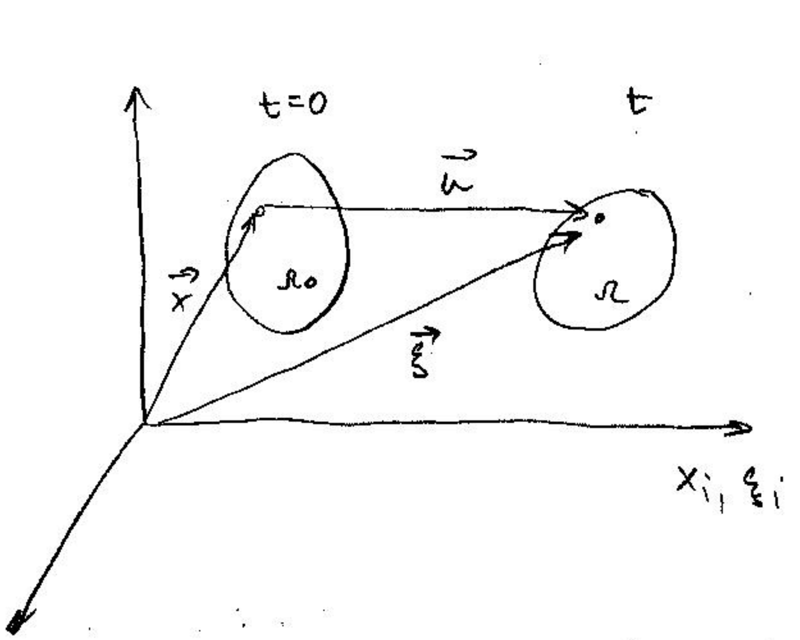
$$L = \underbrace{\tfrac{1}{2}(L + L^T)}_{D} + \underbrace{\tfrac{1}{2}(L - L^T)}_{W}$$ $\underset{\text{sym.}}{D^T = D}$, $\underset{\text{antisym.}}{W^T = \tfrac{1}{2}(L^T - L) = -W}$

$$\left.\begin{aligned} L &= \overline{D} + \overline{W} \\ L^T &= \overline{D} - \overline{W} \end{aligned}\right\} \quad L + L^T = 2\overline{D} \qquad L - L^T = 2\overline{W}$$

jednoznačnost $\left(X^T L X = X^T (D+W) X = X^T D X\right.$

$y = WX:\quad y^T X = X^T W^T X = -X^T W X = -X^T y = -y^T X = 0 \Rightarrow y \perp X$

KINEMATIKA DEFORMACE



$\vec{u}(\vec{x},t)$

$\vec{\xi} = \vec{x} + \vec{u}$

$\vec{\xi}(\vec{x},t)$

x_i ... materiálové s. (Lagrange)

ξ_i -- prostorové s. (Euler)

<u>Př</u> $\vec{u}(x_i,t)$ materiálové pole (Lagrangeovský popis)

$\vec{v}(\xi_i,t)$ prostorové pole (Eulerovský popis)

Lagrangeovský popis

grad posunutí — deformační grad

$$z_{ij} = \frac{\partial u_i}{\partial x_j} \qquad F_{ij} = \frac{\partial \xi_i}{\partial x_j}$$

$$F_{ij} = \frac{\partial}{\partial x_j}\left(x_i + u_i\right) = \delta_{ij} + z_{ij} \qquad \boxed{F = I + Z}$$

Deformační gradient

$$\xi_i(x_j, t) : \quad \Omega_0 \longrightarrow \Omega$$

Zobrazení je regulární ($\exists$ inverze)

$$J = \det\left|\frac{\partial \xi_i}{\partial x_j}\right| = \det|F| \neq 0 \quad \text{v } \Omega_0$$

vzhledem ke zjištění $J > 0$ nebo $J < 0$

$$V = \int_V dV = \int_{V_0} J \, dV_0$$

$V_0 \to 0: \quad \int_{V_0} J \, dV_0 \simeq J V_0 = V$ Jacobián je poměr objemů

$$V_0, V > 0 \implies \boxed{J > 0}$$

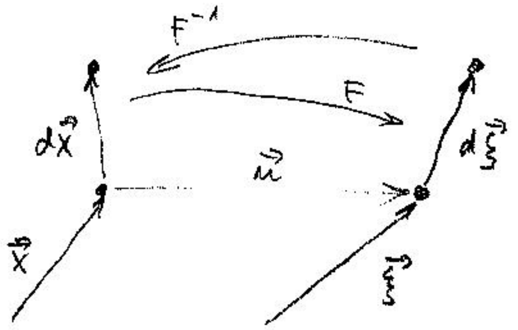
Pozn. $J < 0$ materiál se obrátil naruby

$J = 0$ singularita (nekonečná komprese)

Inverse: $\quad F^{-1}_{ij} = \dfrac{\partial x_i}{\partial \xi_j}$.

$$F_{ik} F^{-1}_{kj} = \frac{\partial \xi_i}{\partial x_k} \frac{\partial x_k}{\partial \xi_j} = \frac{\partial \xi_i}{\partial \xi_j} = \delta_{ij} \qquad F F^{-1} = I$$

Geometrický význam: $\quad d\xi_i = \dfrac{\partial \xi_i}{\partial x_j}\, dx_j = F_{ij}\, dx_j \qquad \boxed{d\xi = F dx}$



Green - Lagrange

$$\boxed{e \overset{\text{def}}{=} \tfrac{1}{2}\left(F^T F - I\right)} \qquad \text{symetrické}$$

$$(ds)^2 - (ds_0)^2 = d\xi^T d\xi - dx^T dx = dx^T F^T F dx - dx^T dx =$$

$$= dx^T \left(F^T F - I \right) dx = 2\, dx^T e\, dx$$

$$e = 0 \implies (ds)^2 - (ds_0)^2 = 0$$

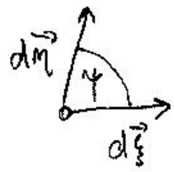
A: $x \to x'$ $\qquad x_i'$ - hlavní osy e $\qquad e' = \operatorname{diag}[e_1\ e_2\ e_3]$

$$dx^T e\, dx = dx^T A^T e' A\, dx = (dx')^T e' dx' = e_1 (dx_1')^2 + e_2 (dx_2')^2 + e_3 (dx_3')^2$$

$$\forall dx \neq 0: \quad dx^T e\, dx = 0 \implies e_1 = e_2 = e_3 = 0$$

$$(ds)^2 - (ds_0)^2 = 0 \implies e = 0$$

$\underline{e = 0}$



$$d\vec{x} \cdot d\vec{y} = \|d\vec{x}\| \cdot \|d\vec{y}\| \cos\varphi$$

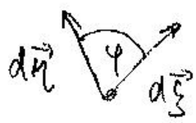
$$\underline{d\vec{\xi} \cdot d\vec{\eta}} = \|d\vec{\xi}\| \cdot \|d\vec{\eta}\| \cos\psi = \underline{\|d\vec{x}\| \cdot \|d\vec{y}\| \cos\psi}$$

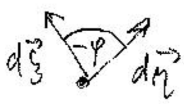
$$d\vec{\xi} \cdot d\vec{\eta} - d\vec{x} \cdot d\vec{y} = \|d\vec{x}\| \cdot \|d\vec{y}\| (\cos\psi - \cos\varphi)$$

$$d\xi^T d\eta - dx^T dy = dx^T F^T F dy - dx^T dy = dx^T (F^T F - I) dy =$$

$$= 2 dx^T e \, dy = 0$$

$$\boxed{\cos\psi = \cos\varphi} \quad \Rightarrow \quad |\psi| = |\varphi|$$





$J = +1$ $\qquad$ $J = -1$

<u>Pozn</u> Může být nepříjemné v numerice.

$$e = \tfrac{1}{2}(F^T F - I) = \tfrac{1}{2}\left[(Z+I)^T (Z+I) - I \right] = \tfrac{1}{2}(Z + Z^T + Z^T Z)$$

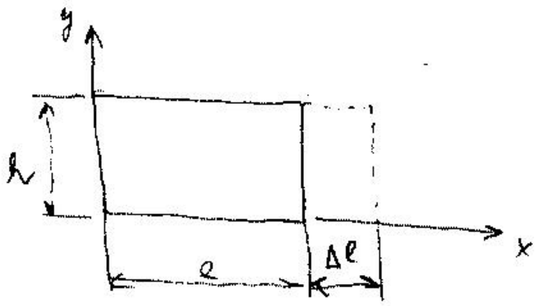
$$\varepsilon \overset{df}{=} \tfrac{1}{2}(Z + Z^T) \qquad \varepsilon_{ij} = \tfrac{1}{2}(Z_{ij} + Z_{ji}) = \tfrac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

$$e = \varepsilon + \tfrac{1}{2} Z^T Z \qquad Z \to 0: \quad e \simeq \varepsilon$$

Př:

$$u(x,y) = \frac{\Delta l}{l}\, x$$

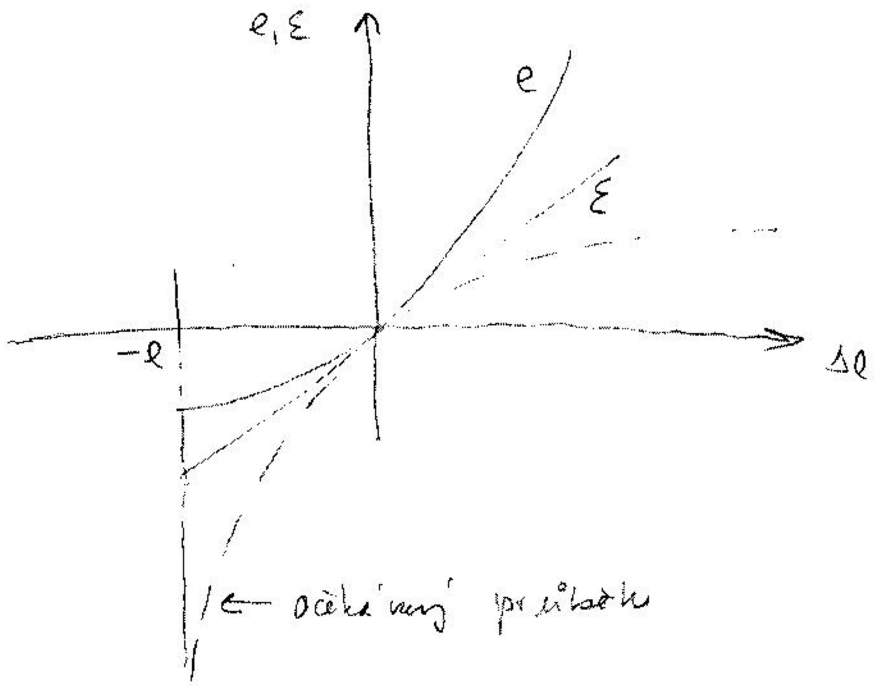
$$v(x,y) = 0$$



$$Z = \begin{bmatrix} \frac{\Delta l}{l} & 0 \\ 0 & 0 \end{bmatrix} \qquad F = \begin{bmatrix} 1+\frac{\Delta l}{l} & 0 \\ 0 & 1 \end{bmatrix}$$

$$J = \det|F| = 1+\frac{\Delta l}{l} = \frac{l+\Delta l}{l} = \frac{V}{V_0}$$

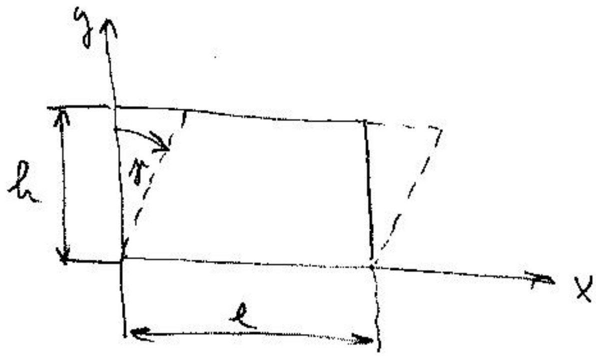
$$e = \begin{bmatrix} \frac{\Delta l}{l} + \frac{1}{2}\left(\frac{\Delta l}{l}\right)^2 & 0 \\ 0 & 0 \end{bmatrix} \qquad \varepsilon = \begin{bmatrix} \frac{\Delta l}{l} & 0 \\ 0 & 0 \end{bmatrix}$$



Př

$u(x,y) = y \, \mathrm{tg}\, \gamma$

$v(x,y) = 0$

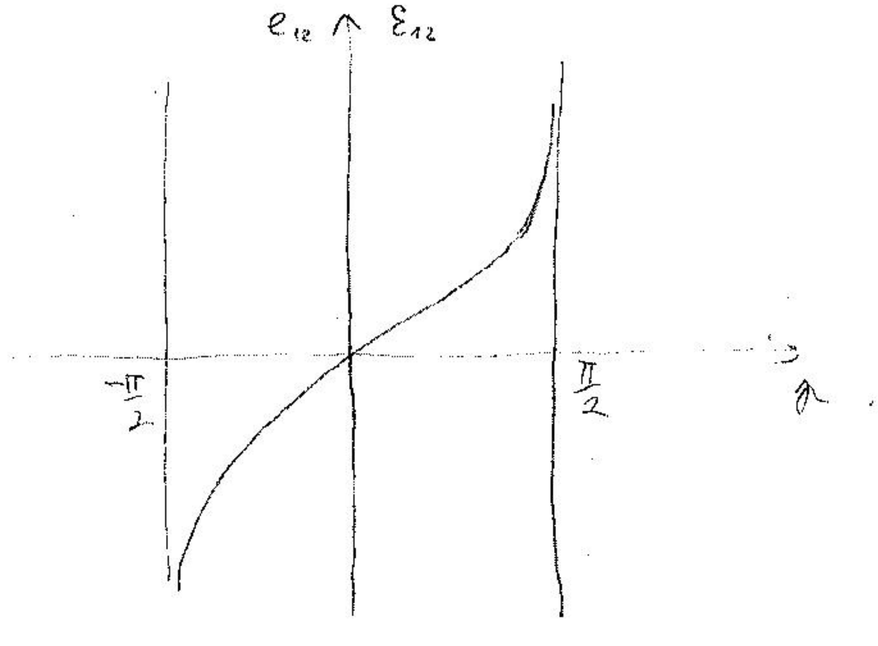


$$Z = \begin{bmatrix} 0 & \mathrm{tg}\,\gamma \\ 0 & 0 \end{bmatrix} \qquad F = \begin{bmatrix} 1 & \mathrm{tg}\,\gamma \\ 0 & 1 \end{bmatrix}$$

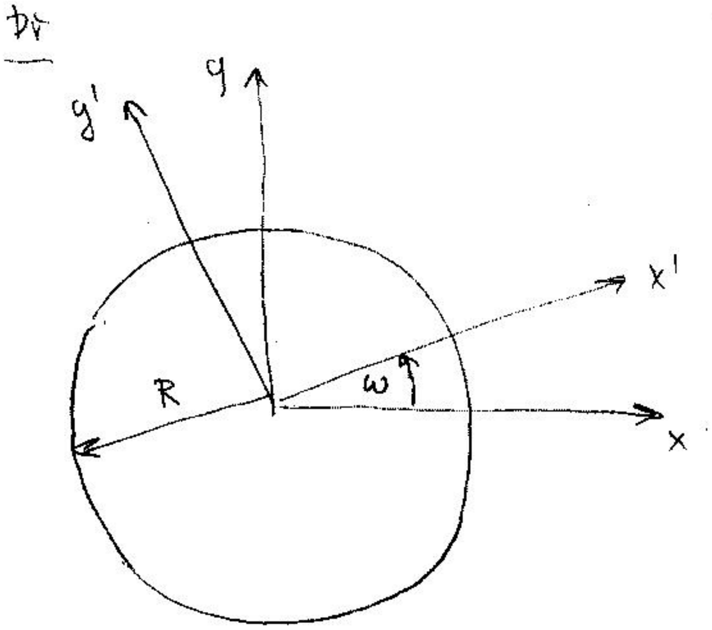
$J = \det |F| = 1$ izochorický děj

$$e = \frac{1}{2} \begin{bmatrix} 0 & \mathrm{tg}\,\gamma \\ \mathrm{tg}\,\gamma & \mathrm{tg}^2\gamma \end{bmatrix} \qquad \varepsilon = \frac{1}{2} \begin{bmatrix} 0 & \mathrm{tg}\,\gamma \\ \mathrm{tg}\,\gamma & 0 \end{bmatrix}$$

Pozn: $\gamma \to 0$, $\mathrm{tg}\,\gamma \to \gamma$



Př



$$\boxed{\xi' = x}$$

$$A = \begin{bmatrix} \cos\omega & \sin\omega \\ -\sin\omega & \cos\omega \end{bmatrix}$$

$$u = \xi - x = A^T \xi' - x = (A^T - I)\, x$$

$$u(x,y) = x(\cos\omega - 1) - y \sin\omega$$

$$\underline{v(x,y) = x \sin\omega + y(\cos\omega - 1)}$$

$$Z = \begin{bmatrix} \cos\omega - 1 & -\sin\omega \\ \sin\omega & \cos\omega - 1 \end{bmatrix} \qquad F = \begin{bmatrix} \cos\omega & -\sin\omega \\ \sin\omega & \cos\omega \end{bmatrix}$$

$$J = \det|F| = \cos^2\omega + \sin^2\omega = 1$$ isochorický děj

$$e = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad \varepsilon = \begin{bmatrix} \cos\omega - 1 & 0 \\ 0 & \cos\omega - 1 \end{bmatrix}$$

ε není invariantní vůči rotaci

Souhrn

1) Jednoosá deformace: $\varepsilon = 2$, $e = 3$

Průběh ε je lepší, avšak měl by jít $\to -\infty$ pro $\Delta l = -l$. Oba tensory teoreticky použitelné.

2) Prostý smyk: $\varepsilon = 2$, $e = 1$

$\operatorname{tg} \gamma$ je lepší míra smykové deformace než γ (z kos). Greenův tensor navíc popisuje prodloužení kolmé úsečky (efekty 2. řádu)

3) Rotace: $\varepsilon = 4$, $e = 1$.

Tensor malé deformace není invariantní vůči rotaci

Pozn: l, h, R může být nahrazeno dx, dy, dR

Pozn: Libovolné přetvoření lze vytvořit z těchto základních módů.

Pozn: Malé hodnoty ε neříkají zda byl tensor použit správně.

Polární rozklad F

$$F = RU = VR$$

$\det|F| = J > 0$

U sym. + def. pravostranný tenzor protažení

V sym. + def. levostranný —"—

$R^T R = I$, $\det|R| = 1$ matice rotace

Rozklad je jednoznačný

Algoritmus

1) $C \overset{\text{def}}{=} F^T F$ sym. + def. Cauchy – Green

$x^T C x = x^T F^T F x = y^T y > 0$ pro F regulární

2) $U \overset{\text{def}}{=} \sqrt{C} = \Phi \Lambda \Phi^T$ sym. + def.

3) $R \overset{\text{def}}{=} F U^{-1} = F \Phi \Lambda^{-1} \Phi^T$

$$R^T R = U^{-T} F^T F U^{-1} = U^{-1} C U^{-1} = U^{-1} U^2 U^{-1} = I$$

$\det|R| = \pm 1 \qquad \det|U| > 0$

$$\det|R| = \det|F U^{-1}| = \frac{J}{\det|U|} > 0 \Rightarrow \begin{cases} \det|R| = +1 \\ \det|U| = J \end{cases}$$

Pozn: $V \overset{\text{def}}{=} R U R^T \Rightarrow VR = RU = F$

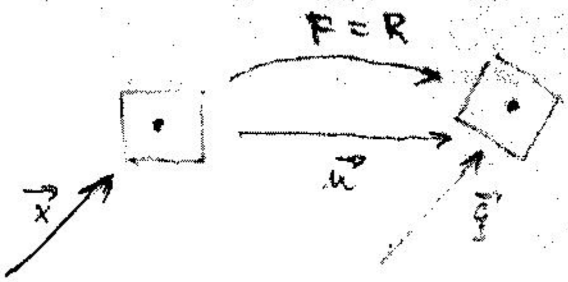
Jednoznačnost: $F = RU = \bar{R}\bar{U} \quad C = F^T F = \bar{U}^T \bar{R}^T \bar{R} \bar{U} = \bar{U}^2$

$\bar{U} = \sqrt{C} = U \qquad \bar{R} = F \bar{U}^{-1} = F U^{-1} = R$

Čistá rotace

$F = R \; , \quad U = I$

$$e = \frac{1}{2}\left(F^T F - I \right) = \frac{1}{2}\left(R^T R - I \right) = 0$$



Čistá deformace

$F = U \; ; \quad R = I$

$x_i' =$ hlavní osy U , $\quad U' = \mathrm{diag}\left[\lambda_1, \lambda_2, \lambda_3 \right]$

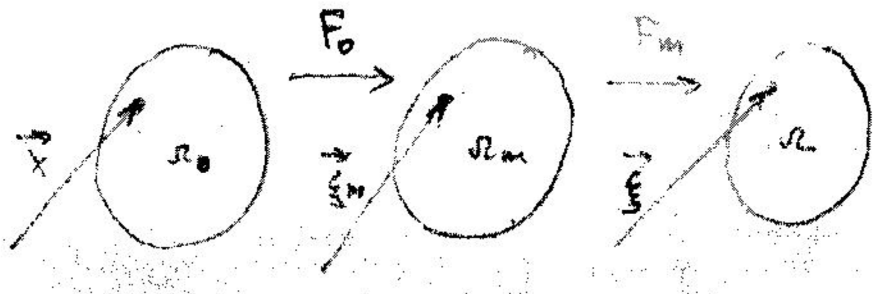
$d\xi = F\,dx = U\,dx \qquad d\xi' = U'\,dx'$

$d\xi_1' = \lambda_1\, dx_1'$ $\qquad \lambda_1, \lambda_2, \lambda_3 > 0$ hlavní protažení

$d\xi_2' = \lambda_2\, dx_2'$ $\qquad \lambda_k < 1$ tlak $\quad \lambda_k > 1$ tah

$d\xi_3' = \lambda_3\, dx_3'$

Multiplikativní rozklad F



$$F^0_{ij} = \frac{\partial \xi^m_i}{\partial x_j} \qquad F^m_{ij} = \frac{\partial \xi_i}{\partial \xi^m_j} \qquad F_{ij} = \frac{\partial \xi_i}{\partial x_j} = \frac{\partial \xi_i}{\partial \xi^m_k}\,\frac{\partial \xi^m_k}{\partial x_j} = F^m_{ik}\,F^0_{kj}$$

$$\boxed{F = F_m F_0}$$

$$d\xi = F_m\, d\xi_m = F_m F_0\, dx = F\, dx$$

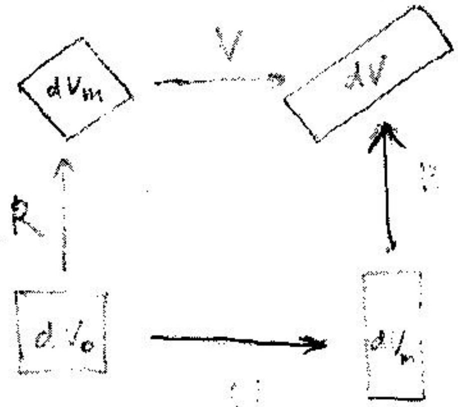
Pozn. Plasticita $F = F_e F_p$ multiplikativní

$$Z + I = Z_e + Z_p + Z_e Z_p + I$$

$$Z \approx Z_e + Z_p \Rightarrow E = E_e + E_p$$ aditivní rozklad

Polární rozklad

Eulerovské tenzory



Lagrangeovské tenzory

Lagrangeovské ~~tenzory~~ definice

zvažujme hlavní osy U:

$$E_k \overset{def}{=} f(\lambda_k)$$

1) $f(1) = 0$

2) $f(\lambda)$ = monotónně rostoucí

3) $f'(1) = 1$ redukce na E

$$E = \phi E' \phi^T \qquad E' = diag[E_1 E_2 E_3]$$

Hill

$$f(\lambda) = \frac{1}{m}(\lambda^m - 1) \qquad m \in R$$

$m = 0$: $\lim_{m\to 0} \frac{\lambda^m - 1}{m} \overset{LH}{=} \lim_{m\to 0} \frac{\lambda^m \ln \lambda}{1} = \ln \lambda$

$$E' = \frac{1}{m}(\Lambda^m - I) \qquad E = \frac{1}{m}(\phi \Lambda^m \phi^T - I)$$

m = celé číslo $\quad U^m = \phi \Lambda \phi^T \dots \phi \Lambda \phi^T = \phi \Lambda^m \phi^T$

m = reálné $\quad U^m \overset{def}{=} \phi \Lambda^m \phi^T$

$$\boxed{E = \frac{1}{m}(U^m - I)}$$

m = 2 Green-Lagrange

$$E^{(2)} = \frac{1}{2}(U^2 - I) = \frac{1}{2}(C - I) = \frac{1}{2}(F^T F - I) = e$$

m = 1 Biot

$$E^{(1)} = U - I \quad , \quad R = I \Rightarrow E^{(1)} = \varepsilon$$

m = 0 Hencky

$$E^{(0)} = \Phi(\ln \Lambda)\Phi^T \stackrel{def}{=} \ln U$$

m = materiálový parametr R.W. Ogden

Pozn. Nejlepší je E^0. Ogdenův model dává $m \sim 1-2$, což je způsobeno kalibrací.

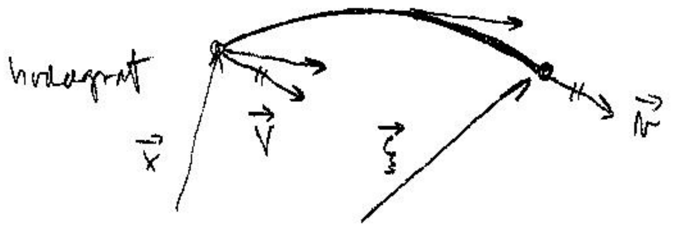
Pozn. GL je efektivní, protože se nepotřebují vl. čísla

Pozn.: Uplatnění GL pro geom. nelineární úlohy

Eulerovský popis

Rychlost

$$v_i(\xi_j,t) = v_i[\xi_j(x_k,t),t] = V_i(x_k,t)$$



různý vektor

$$\vec{V} \neq \vec{v}$$

$$V_i = v_i$$

trajektorie $\vec{\xi}(x_k,t)$ při pevném $\vec{X}$.

$$\vec{V}(x_k,t) = \frac{\partial\vec{\xi}}{\partial t} = \dot{\vec{\xi}} = \dot{\vec{u}}$$ materiálová derivace

paralelní přenos $\vec{X} \longmapsto \vec{\xi}$: $\vec{v}(\xi_j,t)$

$$V_i(x_k,t) = V_i[x_k(\xi_j,t),t] = v_i(\xi_j,t)$$

Př

$$\xi = x + at^2 \qquad x = \xi - at^2$$

$$\eta = y + bty + ct^2 \qquad y = \frac{\eta - ct^2}{1+bt}$$

$$V = \begin{bmatrix} 2at \\ by + 2c \end{bmatrix} \qquad v = \begin{bmatrix} 2at \\ \frac{b(\eta - ct^2)}{1+bt} + 2c \end{bmatrix}$$

$$x = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad t = 1\,s \quad \xi = \begin{bmatrix} a \\ c \end{bmatrix}$$

$$V(0,0,1) = \begin{bmatrix} 2a \\ 2c \end{bmatrix} \qquad v(a,c,1) = \begin{bmatrix} 2a \\ 2c \end{bmatrix}$$

Zrychlení

$\vec{A} = \dot{\vec{V}} = \ddot{\vec{\xi}} = \ddot{\vec{u}}$ materiálová derivace

paralelní přenos $\vec{X} \mapsto \vec{\xi}$: $\vec{a}(\xi_j, t)$

jiná veličina

$$A_i = \frac{\partial V_i}{\partial t} = \frac{\partial v_i}{\partial \xi_k}\dot{\xi}_k + \frac{\partial v_i}{\partial t} = \frac{\partial v_i}{\partial \xi_k} v_k + \frac{\partial v_i}{\partial t} = a_i(\xi_j, t)$$

$$\vec{a} = \frac{\partial \vec{v}}{\partial t} + \frac{\partial \vec{v}}{\partial \xi_k} v_k = \underbrace{\vec{v}_t}_{\text{lokální derivace}} + \underbrace{\vec{v} \cdot \operatorname{grad} \vec{v}}_{\text{konvektivní derivace}}$$

materiální derivace prostorového pole

Souhrn

$\vec{V}, \vec{v}$ $\qquad \dot{\vec{V}} = \frac{\partial \vec{V}}{\partial t}$ $\qquad \dot{\vec{v}} = \frac{\partial \vec{v}}{\partial t} + \frac{\partial \vec{v}}{\partial \xi_k} v_k$ mat. derivace

$\vec{A} = \dot{\vec{V}}$ $\qquad \vec{a} = \dot{\vec{v}}$ $\qquad \boxed{A_i = a_i}$

Rychlostní gradient

$$L_{ij} \stackrel{def}{=} \frac{\partial v_i}{\partial \xi_j} \qquad dv = L\, d\xi \qquad L = D + W$$

$$L_{ij} = \frac{\partial V_i}{\partial x_k}\frac{\partial x_k}{\partial \xi_j} = \frac{\partial \dot{\xi}_i}{\partial x_k} F^{-1}_{kj} = \dot{F}_{ik} F^{-1}_{kj} \qquad L = \dot{F} F^{-1}$$

symetrická část

$$D = \frac{1}{2}\left(L + L^T\right) \qquad D_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial \xi_j} + \frac{\partial v_j}{\partial \xi_i}\right)$$

Pozn $D_{ij} \neq \dot{\varepsilon}_{ij}$, $\int_0^t D\, d\tau$ závisí na integrační cestě

$$D_{ij} \neq \dot{E}^{(m)}_{ij} , \qquad D_{ij} = 0 \iff \dot{E}^{(m)}_{ij} = 0$$

$$\frac{d}{dt}(dS)^2 = \frac{d}{dt}\left(d\xi^T d\xi\right) = d\dot{\xi}^T d\xi + d\xi^T d\dot{\xi} = 2\, d\xi^T d\dot{\xi} =$$

$$= 2\, d\xi^T dv = 2\, d\xi^T L\, d\xi = 2\, d\xi^T D\, d\xi$$

podobné jako při analýze Greenova tensoru

$$D = 0 \iff \text{rychlost def.} = 0$$

Antisymetrická část

$$W = \frac{1}{2}\left(L - L^T\right) \qquad W_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial \xi_j} - \frac{\partial v_j}{\partial \xi_i}\right)$$

$W =$ spin (vírový tensor)

uvažujme lokální tuhý pohyb

$$d\vec{v} = \vec{\omega} \times d\vec{\xi} \qquad dv_i = \varepsilon_{ijk}\, \omega_j\, d\xi_k$$

$$W_{ik} \overset{def}{=} \varepsilon_{ijk}\, \omega_j \qquad dv_i = W_{ik}\, d\xi_k$$

$$dv = W\, d\xi \qquad W = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

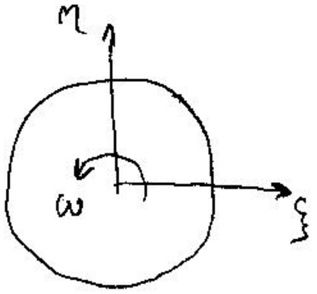
Pozn $\forall W$ lze přiřadit axiální vektor $\vec{\omega}$.

Př $\quad F = I, \quad L = \dot{F}F^{-1} = \dot{F}, \quad L_{ij} = \frac{\partial \dot{\xi}_i}{\partial x_j} = \frac{\partial \dot{u}_i}{\partial x_j}$

$$D_{ij} = \frac{1}{2}\left(\frac{\partial \dot{u}_i}{\partial x_j} + \frac{\partial \dot{u}_j}{\partial x_i}\right) = \dot{\varepsilon}_{ij} \qquad W_{ij} = \frac{1}{2}\left(\frac{\partial \dot{u}_i}{\partial x_j} - \frac{\partial \dot{u}_j}{\partial x_i}\right) = \dot{\omega}_{ij}$$

Lineární teorie je linearizací konzistentní.

Př



$$u_1(x,y,t) = x(\cos\omega t - 1) - y\sin\omega t$$

$$u_2(x,y,t) = x\sin\omega t + y(\cos\omega t - 1)$$

$$V_1 = -\omega x\sin\omega t - \omega y\cos\omega t = -\omega(u_2 + y) = -\omega\eta$$

$$V_2 = \omega x\cos\omega t - \omega y\sin\omega t = \omega(u_1 + x) = \omega\xi$$

$v_1 = -\omega\eta \qquad v_2 = \omega\xi \qquad$ prostorové pole

$$\left.\begin{aligned} a_1 &= \frac{\partial v_1}{\partial t} + \frac{\partial v_1}{\partial \xi_k} v_k = -\omega v_2 = -\omega^2\xi \\ a_2 &= \frac{\partial v_2}{\partial t} + \frac{\partial v_2}{\partial \xi_k} v_k = \omega v_1 = -\omega^2\eta \end{aligned}\right\} \quad \vec{a} = -\vec{r}\,\omega^2$$

$$L = \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} = W, \quad D = 0 \qquad \vec{w} = \begin{bmatrix} 0 \\ 0 \\ \omega_3 \end{bmatrix}$$