Quantifying Interactions between Complex Oscillatory Systems: A Topic in Time Series Analysis

Thesis defense presentation

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Outline

1. Introduction
   - Overview
   - Weak interactions between pairs of systems
   - Focus of PhD thesis
2. Nonlinear time series analysis
   - Phase dynamics approach
   - Time series analysis methods
   - Original contributions
3. Summary

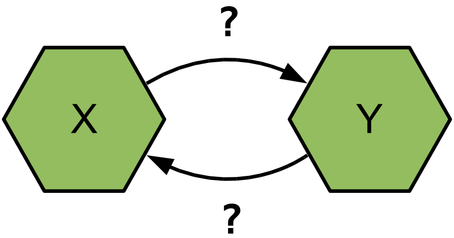
Motivation

- Complex systems are composed of subsystems
  Biological, chemical, meteorological, geophysical, ...
- Subsystems interact to create complicated behavior
  Neural interactions, mass/energy transport, ...
- Many systems have the character of nonlinear oscillators
  Stable amplitude, motion along limit cycle ...

Problem setting

- A suitable model of the entire system is not available
  Cannot parametrize the problem
- The nature of coupling between systems maybe unknown
  pulses/continuous, regular/irregular, persistence ...
- Systems are under external influences (noise, ...)
- Stationarity can be assumed for short time spans only
- Components can not be manipulated by experimenter
  human subject, past experiment, ...
- Available data: multivariate time series

Weak interactions between a pair of systems



- Coupling: none, unidirectional, bidirectional
- Synchronization effect (synchronized state)
- Challenge: strength of coupling

Focus of PhD thesis

- Understand state-of-the-art methods for
  - Directionality analysis based on Information Theory
  - Phase synchronization analysis
- Analyze the behavior of available methods
- Compare the methods against each other
  - Construct software infrastructure
  - Establish standardized tests
- Identify methods suitable for the analysis of real data for use in the BRACCIA EC FP6 programme

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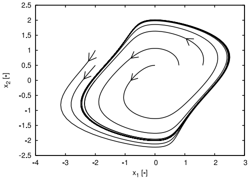
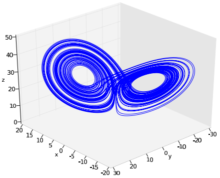
Phase dynamics approach

- Phase is a variable that describes the motion of oscillators
- Applicable to autonomous non-linear oscillators with stable amplitude
- Motion along limit cycle or strange attractor
- Applicable under weak coupling conditions
- Phase is obtained indirectly
- Phase considerably simplifies further processing



Van-der-Pol System



Lorenz System

Components of time series analysis methods

- Directionality detection
  - Directionality **index**
    Reacts to change of coupling from one to other system
  - Significance test
    Test with null hypothesis of independent systems
- Phase synchronization analysis
  - Phase synchronization **index**
    Reacts to strength of interaction between systems
  - Significance test
    Test with null hypothesis of **unsynchronized** systems

Directionality — Conditional mutual information

$$d_{x \to y} = \frac{1}{T} \sum_{\tau=1}^{T} I(X; Y_\tau | Y)$$

- Based on a generalized version of Granger causality
- X, Y are processes, Y_τ is shifted by time τ into the future
- Mutual information shared by X and Y_τ is *conditioned* on Y

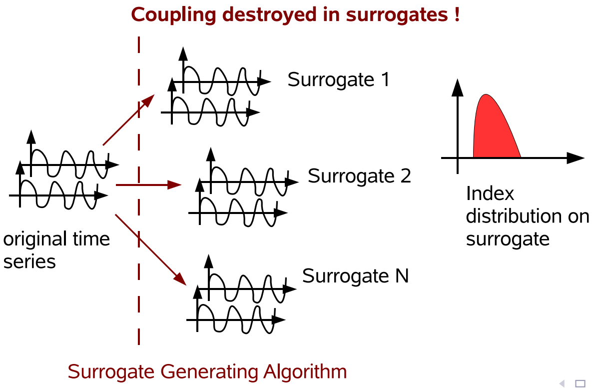
$$I(X; Y_\tau | Y) = H(Y_\tau | Y) - H(Y_\tau | Y, X)$$

- A description of the process state is needed
- Estimate of the pdf (pmf) of process state is required
- Many different estimators possible

  Kernel, Correlation integral, **k-NN**, B-spline, Histogram, ...

Significance testing

- Use of bootstrap-like strategy (surrogate time series)
- Ideally preserve all properties except tested (coupling)
- Multiple strategies: FT, AAFT, Permutation, Recurrence



Phase Synchronization

- Let ϕ_X, ϕ_Y — phases of X and Y
- Locking ratio *m*:*n*, e.g. 2:1, 1:1
- $\Delta\phi(t) = n\phi_1(t) - m\phi_2(t)$
- Indices measuring the 'spread' of distributions:
  - Mean phase coherence $\sqrt{\langle cos(\Delta\phi(t))\rangle^2 + \langle sin(\Delta\phi(t))\rangle^2}$
  - Mutual information $I(\phi_X; \phi_Y)$
  - Entropy of phase differences $H(\Delta\phi)$
- Same surrogates used as in directionality detection (!)

Original contributions — Directionality analysis

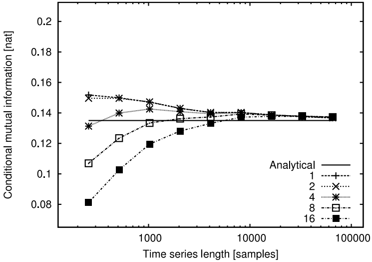
- Investigation of several pdf (pmf) estimation schemes
  Schindler–Hlaváčková et al., Phys. Rep. 441, 2007
- Adaptation of several schemes for estimating MI to CMI
- Extensive tests on model systems (vs. time series length)
  - Barnes ARMA model, bias analysis
  - Rössler systems, detection statistics
- Dir. detection between systems with higher relative frequency ratio is much more difficult
- Dir. index strongly interacts with surrogate data algorithms
- In practice algorithms based on discrete state space recommended

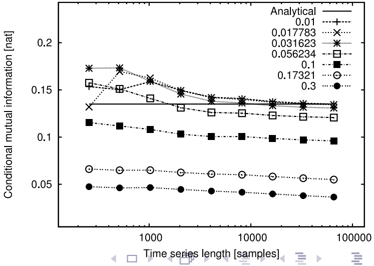
Original contributions — Directionality analysis II.

- Development of k-NN estimator of CMI

$$I(X;Y|Z) = \psi(k) - \langle \psi(n_{xz}(i)+1) + \psi(n_{yz}(i)+1) - \psi(n_z(i)+1) \rangle$$

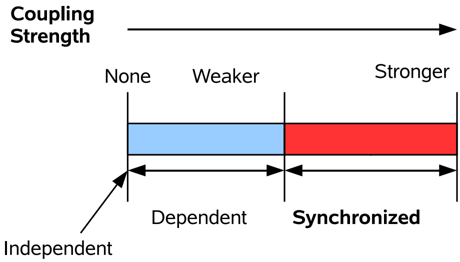
- Based on the works of Kozachenko & Leonenko (entropy) and Kraskov (mutual information)
- Published findings (Vejmelka & Paluš, Phys. Rev. E, 2008)





Original contributions — Phase sync. detection

- Tests of phase synchronization indices on model systems
- Numerically showed that standard surrogates not adequate
- Developed Bootstrap phase synchronization detector
- Manuscript in preparation (Vejmelka, Lee and Paluš)



Software construction

- Constructed software system (20k+ LOC C++)
- Scripting (Lua) for guiding experiments (15k+ LOC)
- Portable across many computing architectures
  POWER 4/5, Intel, SPARC
- ... and operating systems
  IBM AIX, Solaris, Windows, Linux/Unix
- Uses the MPI standard for launching/running processes
- Specialized subsystem for fixed-mass and fixed-radius queries (Vejmelka & Schindler-Hlaváčková, ICANNGA 2007)
- Now parts rewritten in Python

Human subject study

- Investigation of interactions between major oscillatory systems in human body
  - Heart (ECG)
  - Lungs (Respiratory effort)
  - Brain (Cortical oscillations)
- 46 patients analyzed from two measurement sites
  (Ulleval Hospital, Royal Lancaster Infirmary)
- Analysis of differences between waking state and general anesthesia
- Randomized Nimbex (curare) + artificial respiration group

Publications

- Vejmelka, Musílek, Paluš, Pelikán: K-means Clustering for Problems with Periodic Attributes. IJPRAI, to appear May/June 2009.
- Schindler-Hlaváčková, Paluš, Vejmelka, Bhattacharya: Causality detection based on information-theoretic approaches in time series analysis, Physics Reports 441, 2007.
- Paluš and Vejmelka: Directionality of coupling from bivariate time series: How to avoid false causalities and missed connections, Phys Rev E 75, 2007.
- Vejmelka and Paluš: Inferring the directionality of coupling with conditional mutual information, Phys Rev E 77, 2008.
- Vejmelka and Schindler-Hlaváčková: Mutual Information Estimation in Higher Dimensions: A Speed-Up of a k-Nearest Neighbor Based Estimator, ICANNGA 2007.
- Vejmelka, Fründ, Pillai: Traversing Scales: Large Scale Simulation of the Cat Cortex Using Single Neuron Models. Lectures in Supercomputational Neuroscience: Dynamics in Complex Brain Networks. Springer, 2008.

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Projects & Research visits

- EC FP6 BRACCIA (P.I. Milan Paluš) 2006-2008
  Analysis of ECG, Respiration and EEG time series from human subjects in resting state and under general anesthesia.
- EC FP7 BrainSync (P.I. Milan Paluš) 2008-2010
  Analysis of networks in the human brain in “resting state”. Data: MRI, MUA, EEG, MEG time series.
- Helmholtz Institute for Supercomp. Physics (Aug 2005)
  Modelling and analysis of large scale neural interactions.
- HPC-Europa Edinburgh Parallel Computing Center (Jan/Feb 2007)
  Use of supercomputers (HPCx, IBM Bluegene) to perform computationally intensive tests.

Summary

- Quick introduction to weak interaction analysis:
  - Phase dynamics
  - Directionality detection
  - Phase synchronization analysis
  - Significance testing
- Original contributions
  - Analysis of interactions between pair of systems
  - Mapping of behavior of available methods
  - K-NN based estimator for CMI
  - Bootstrap phase synchronization method
- Publications

Acknowledgments

- Milan Paluš

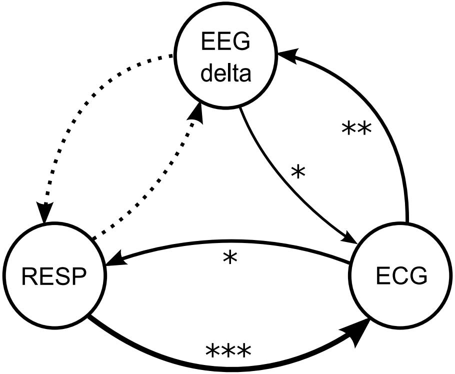
  for his guidance and time with which he has been generous.
- The reviewers for excellent feedback.

Thank you for your attention.

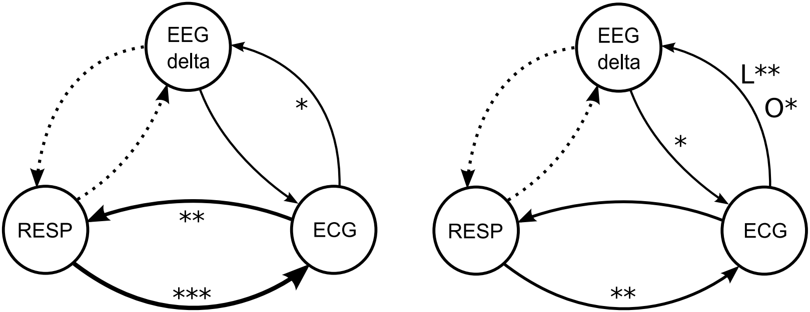
Questions

- General statements on lengths of time series ?
- Multiple interacting systems ?
- Direction of coupling vs. direction of mass flow ?
- Nonlinear time series analysis: art or science ?
- Neural interactions in BRACCIA study ?

BRACCIA Study results - Awake state



BRACCIA Study results - Anesthetised state



(a) non-Nimbex (b) Nimbex