Resting state network detection by graph partitioning using the Average Association criterion

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Resting state networks

Resting state networks (RSNs) can be rather loosely defined as regions exhibiting higher-than-average functional connectivity (FC) in the resting state. We propose a clustering approach to detect the functional structure of the human brain from measurements of spontaneous fluctuations using fMRI in single subjects.

The brain graph

- An undirected graph representing the functional connectivity in the brain is constructed.
  - vertices represent the voxels of the brain
  - edges between the vertices are weighted by the functional connectivity between the voxels
  - the sparsity depends on the connectivity but in general the graph is dense

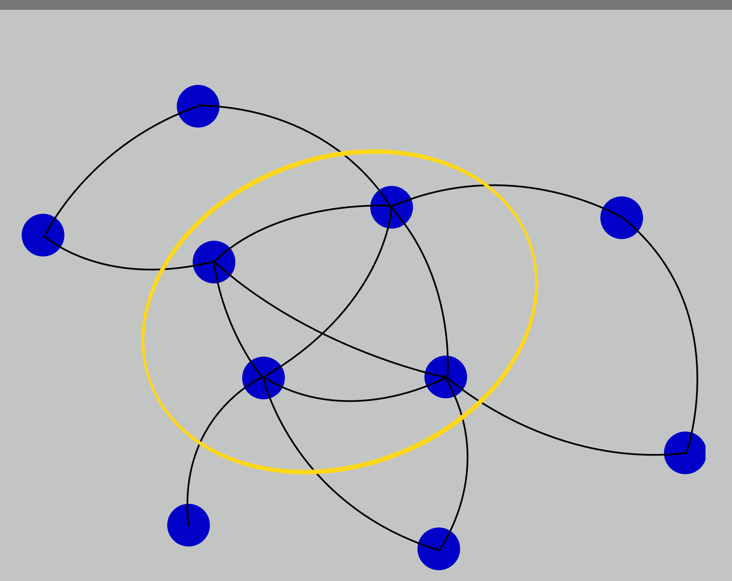


Figure: A schematic of the "brain graph". Circled region has a higher density of edges than other regions.

- Resting State Networks will correspond to sets of vertices in the graph which have a high density of strong edges among themselves.

Average Association

- Let a graph have vertices $p_i \in V$, $|V| = N$. Let the weight of the edge between vertices p_i and p_j be $w_{i,j}$ for all pairs $\{p_i, p_j\} \in V \times V$.
- The graph is to be split into K clusters V_k, $|V_k| = N_k$ and a residual set V_o, which will contain all the voxels that don't have a strong association with any cluster. The clusters V_k and the residual set V_o together form a partition of the set of vertices V.
- The **Average Association (AA)** objective [2] is defined as

$$J_K = \sum_{k=1}^{K} \sum_{p_i, p_j \in V_k} \frac{w_{i,j}}{N_k}. \qquad (1)$$

- The objective J_K is equal to the sum of all edges inside each cluster V_k divided by the number of its vertices.
- J_K must be optimized over all possible partitionings $\{V_1, V_2, ..., V_K, V_o\}$.
- The clustering is crisp, a voxel either belongs to a cluster or it does not.
- **Note** in general not all voxels are assigned to clusters, some may be in the residual set V_o. The residual set may also be empty, the method is unconstrained in this respect.

AA problem in matrix form

- The objective (1) is hard to optimize directly.
- A standard approach to solving the problem is a relaxation approach - we must reformulate the problem in matrix form.
- In matrix notation
  - $W = [w_{i,j}]_{NxN}$ is the connectivity matrix
  - cluster memberships given by indicator vectors $\mathbf{u_k} = \begin{cases} 1 \text{ if } p_i \in V_k \\ 0 \text{ otherwise} \end{cases}$
  - Since clusters are disjoint, $\mathbf{u_i}^T \mathbf{u_j} = 0$ if $i \neq j$
  - Gather indicator vectors into matrix $U = (\mathbf{u_1}, \mathbf{u_2}, ..., \mathbf{u_K})$, $U \in \mathbb{R}^{N \times K}$
  - The AA objective (1) becomes

$$J_K = tr\{(U^T U)^{-1} U^T W U\}$$

  - Substituting $Z = U(U^T U)^{-\frac{1}{2}}$ the optimization problem becomes

$$\max_{Z^T Z = I} Z^T W Z, \; \mathbf{z_k} = \begin{cases} \frac{1}{\sqrt{N_k}} \text{ if } p_i \in V_k \\ 0 \text{ otherwise,} \end{cases} \qquad (2)$$

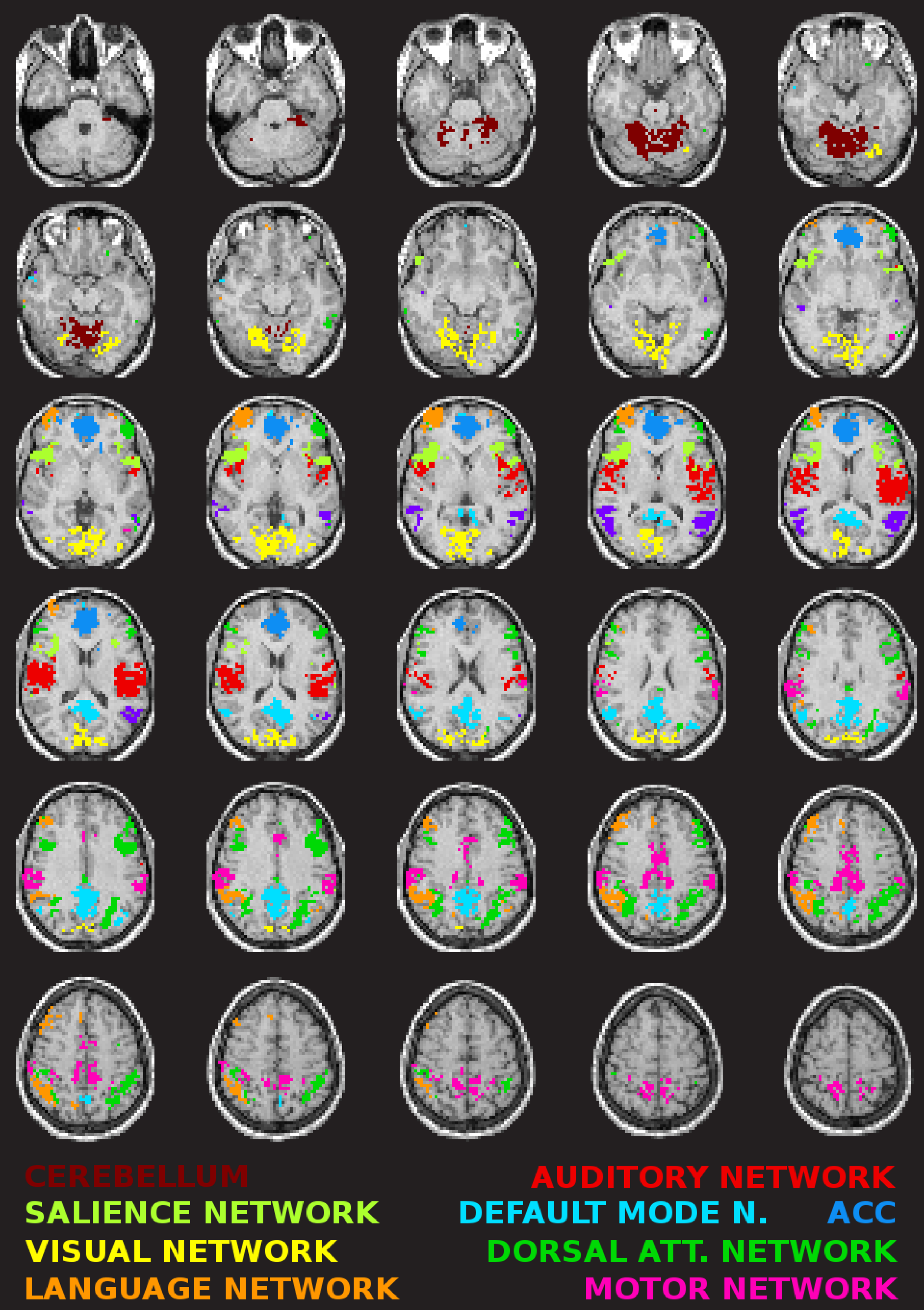
  where $\mathbf{z_k}$ is the k-th column vector of Z.

Solving the AA problem

- Relaxation: drop the requirements on the discrete form of $\mathbf{z_k}$ in (2).
- The relaxed solution $Z \in \mathbb{R}^{N \times K}$ is given in terms of eigenvectors of W
- The relaxed solution is not unique (for Normalized Cut see [3]). The space of relaxed solutions is parametrized by orthogonal transformations.
- This opens up the possibility of searching for better relaxed solutions.
- We have applied a classical procedure: the VARIMAX rotation [1].
- VARIMAX seeks an orthogonal $R_V \in \mathbb{R}^{K \times K}$ such that column vectors of the the matrix $Z_V = Z R_V$ have **simple structure**.
- Simple structure of an indicator vector implies a compact localization of each cluster V_k on the set of voxels V.
- Elements are assigned to the cluster where their loading is highest and the indicator vectors $[\mathbf{z_V}]_k$ are individually thresholded to obtain the final cluster memberships. Any remaining elements are gathered into the residual set.

Results

Selected clusters (out of 20) merged voxel-wise by majority vote from 6 subjects (except DAN: 2/5, Motor: 3/6 votes), 3600 ± 1200 voxels in the residual set. Measurement: 1.5T MRI, TR=2s, 10 min sessions, 27644 voxels $3mm^3$. Preprocessing: co-registration to Talairach atlas, spatial smoothing 6mm FWHM Gaussian, $0.009 - 0.08$Hz BP, orthogonalization w.r.t. head motion, {CSF, WM, whole-brain} signals. Analysis: AA clustering with FC estimated using correlation coefficients. Only positive CCs significant at $p < 0.004$ (uncorrected) were included.



References and Acknowledgments

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