

# ZÁKONY MECHANIKY KONTINUA

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## Zákon zachování hmotnosti

$$\boxed{m(V) = m(V_0)}$$

Lagrange

$$\left. \begin{aligned} m(V) &= \int_V \rho \, dV = \int_{V_0} J \rho \, dV_0 \\ m(V_0) &= \int_{V_0} \rho_0 \, dV \end{aligned} \right\} \forall V_0: \quad \boxed{J \rho = \rho_0}$$

Pi 
$$M = \int_{V_0} \rho_0 H^T H \, dV_0 = \int_{V_0} J \rho H^T H \, dV_0 = \int_V \rho H^T H \, dV$$

Euler

$$\dot{\rho}_0 = \dot{J} \rho + J \dot{\rho} = 0$$

$$\dot{J} = \frac{\partial J}{\partial F_{ij}} \dot{F}_{ij} = J F_{ji}^{-1} \dot{F}_{ij}$$

$$L = \dot{F} F^{-1} \Rightarrow L_{ik} = \dot{F}_{ij} F_{jk}^{-1} \Rightarrow L_{ii} = \dot{F}_{ij} F_{ji}^{-1}$$

$$\dot{J} = J L_{ii} \Rightarrow J L_{ii} \rho + J \dot{\rho} = 0, \quad J \neq 0$$

$$L_{ii} = \frac{\partial v_i}{\partial x_i} = \operatorname{div} \vec{v} \quad \text{Pom} \quad \operatorname{Div} \vec{V} = \frac{\partial V_i}{\partial x_i}$$

$$\boxed{\dot{\rho} + \rho \operatorname{div} \vec{v} = 0}$$

kvůli kontinuitě

Führen zählweise Lyapunov

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$$\vec{p} \stackrel{\text{def}}{=} \int_V \rho \vec{v} dV \quad \text{Lyapunov}$$

$$\boxed{\frac{d\vec{p}}{dt} = \int_V \vec{f} dV + \int_S \vec{T} dS}$$

$$\frac{dp_i}{dt} = \frac{d}{dt} \int_V \rho v_i dV = \frac{d}{dt} \int_{V_0} J \rho V_i dV_0 = \int_{V_0} \rho_0 \dot{V}_i dV_0 \quad \text{Lagrange}$$

$$= \int_{V_0} J \rho \dot{V}_i dV_0 = \int_V \rho \dot{v}_i dV \quad \text{Euler}$$

Euler

$$\int_S t_i dS = \int_S \sigma_{ij} n_j dS = \int_V \frac{\partial \sigma_{ij}}{\partial x_j} dV$$

$$\boxed{\frac{\partial \sigma_{ij}}{\partial x_j} + f_i = \rho a_i}$$

polymer variace ( $a_i = \dot{v}_i$ )

Lagrange

$$\int_S t_i dS = \int_{S_0} T_i dS_0 = \int_{S_0} \Sigma_{ij} N_j dS_0 = \int_{V_0} \frac{\partial \Sigma_{ij}}{\partial x_j} dV_0$$

$$\int_V f_i dV = \int_{V_0} J f_i dV_0 = \int_{V_0} F_i dV_0, \quad \vec{F} \stackrel{\text{def}}{=} J \vec{f}$$

$$\boxed{\frac{\partial \Sigma_{ij}}{\partial x_j} + F_i = \rho A_i}$$

( $A_i = \dot{V}_i$ )

# Balun podrobné momentu lpfaski

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$$\frac{d}{dt} \int_V \vec{\xi} \times \rho \vec{v} dV = \int_V \vec{\xi} \times \vec{f} dV + \int_S \vec{\xi} \times \vec{t} dS$$

$$(\vec{\xi} \times \vec{v})_i = \mu_{ijk} \xi_j v_k \quad \frac{\partial \sigma_{ij}}{\partial \xi_j} + f_i - \rho \ddot{u}_i = 0$$

Euler  $\int_V \mu_{ijk} \sigma_{jk} dV = 0$

$$\mu_{123} \sigma_{23} + \mu_{132} \sigma_{32} = 0 \Rightarrow \sigma_{23} - \sigma_{32} = 0$$

$$\boxed{\sigma_{ij} = \sigma_{ji}}$$

$$\boxed{\sigma^T = \sigma}$$

symetrie

## Lagrange

$$\Sigma = J \sigma F^{-T} \Rightarrow \sigma = \frac{1}{J} \Sigma F^T$$

$$\sigma^T = \sigma \Rightarrow \boxed{F \Sigma^T = \Sigma F^T} \text{ momenta' podmiata na APK}$$

Pom: Vektor momentu'ho napeti'  $\vec{T}$  tedy.

"vesplivaji" momentum rovnou'ho vektoru k  $\vec{x}$ .

# Zalium sadom'ui energije

(4)

$$\dot{W}(V) = \int_V \sigma_{ij} D_{ij} dV = \int_{V_0} s_{ij} \dot{e}_{ij} dV_0 = \int_{V_0} s_{ij}^{(m)} \dot{E}_{ij}^{(m)} dV_0$$

$$\dot{Q}(V) = \int_V r dV - \int_S \vec{h} \cdot \vec{n} dS = \int_V (r - \text{div} \vec{h}) dV$$

Laorange:

$$\int_V r dV = \int_{V_0} J r dV_0 = \int_{V_0} R dV_0$$

$$\boxed{R \stackrel{\text{def}}{=} J r}$$

lepely' fdraaj u  $R_0$

$$\int_S \vec{h} \cdot \vec{n} dS = \int_S h_i n_i dS = \int_{S_0} h_i J F_{ji}^{-1} N_j dS_0 =$$

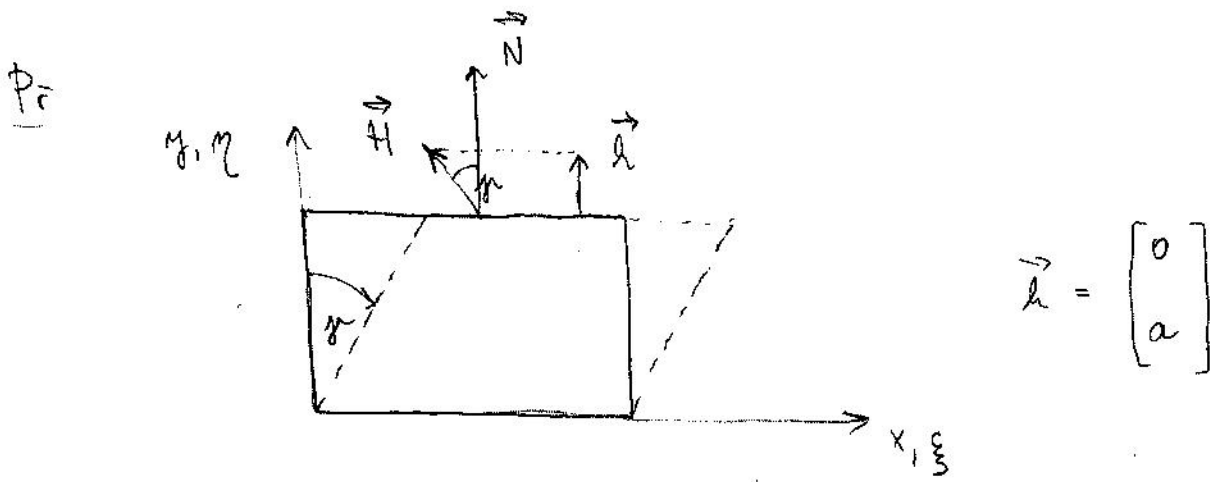
$$= \int_{S_0} H_j N_j dS_0 = \int_{S_0} \vec{H} \cdot \vec{N} dS_0$$

$$H_j \stackrel{\text{def}}{=} J F_{ji}^{-1} h_i$$

$$\boxed{H = J F^{-1} h}$$

Piola

$$\dot{Q}(V) = \int_V (r - \text{div} \vec{h}) dV = \int_{V_0} (R - \text{Div} \vec{H}) dV_0$$



$$F = \begin{bmatrix} 1 & \tan \alpha \\ 0 & 1 \end{bmatrix} \quad J = 1 \quad F^{-1} = \begin{bmatrix} 1 & -\tan \alpha \\ 0 & 1 \end{bmatrix}$$

$$\vec{H} = J F^{-1} \vec{h} = \begin{bmatrix} 1 & -\tan \alpha \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ a \end{bmatrix} = \begin{bmatrix} -a \tan \alpha \\ a \end{bmatrix}$$

$$\|\vec{H}\| = |a| \sqrt{1 + \tan^2 \alpha} = \|\vec{h}\| \sqrt{1 + \tan^2 \alpha}$$

Veľkosť  $\vec{H}$  zodpovedá vektora vici matematickej fyziky v danom bode.

(6)

Průběh energie bude klasičtější na jednodušší fyzice

$$U = \int_V \rho u dV = \int_{V_0} J \rho u dV_0 = \int_{V_0} \rho_0 u dV_0$$

$u$  [J/kg] vzhledem k  $\Omega_0$  i  $\Omega$

$$\dot{U} = \int_{V_0} \dot{\rho}_0 u dV_0 = \int_V \dot{\rho} u dV \quad \text{materiáloví derivace}$$

$$\dot{Q} + \dot{W} = \dot{U}$$

Euler

$$\rho - \operatorname{div} \vec{h}_i + \sigma_{ij} D_{ij} = \rho \ddot{u}_i$$

Lagrange

$$\rho - \operatorname{Div} \vec{H} + S_{ij}^{(m)} \dot{E}_{ij}^{(m)} = \rho_0 \ddot{u}_i$$

## Dondj' fallam termodinamika

(7)

$\exists \eta [J/KyK]$  huskaka entropie = skuzna veličina

$$CD: \quad \dot{S} \geq \int_V \frac{r}{T} dV + \int_S \frac{\vec{l} \cdot \vec{n}}{T} dS \quad \text{Euler}$$

$$\dot{S} \geq \int_{V_0} \frac{R}{T} dV_0 + \int_{S_0} \frac{\vec{H} \cdot \vec{N}}{T} dS \quad \text{Lagrange}$$

### Lokalen' forma

$$\rho \dot{\eta} \geq \frac{1}{T} (r - \text{div } \vec{h}) + \frac{1}{T^2} \vec{h} \cdot \text{grad } T$$

$$\rho_0 \dot{\eta} \geq \frac{1}{T} (R - \text{Div } \vec{H}) + \frac{1}{T^2} \vec{H} \cdot \text{Grad } T$$

### Dissipacni nerovnost

$$-\rho \eta \dot{T} + \sigma_{ij} D_{ij} - \frac{1}{T^2} \vec{h} \cdot \text{grad } T \geq \rho \dot{\eta}$$

$$-\rho_0 \eta \dot{T} + S_{ij}^{(m)} \dot{E}_{ij}^{(m)} - \frac{1}{T^2} \vec{H} \cdot \text{grad } T \geq \rho_0 \dot{\eta}$$

# PEVNĚ LÁTKY

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(1)  $\rho_0 = J\rho$  zákon zachování hmotnosti

(2)  $\frac{\partial \Sigma_{ij}}{\partial x_j} + F_i = \rho_0 \ddot{u}_i$  zákon zachování hybnosti

$\Sigma = JGF^T$ ,  $\vec{F} = J\vec{f}$ ,  $\ddot{u}_i = \frac{\partial^2 u_i}{\partial t^2}$  malá deformační

(3)  $\sigma = \sigma^T$ ,  $\Sigma F^T = F \Sigma^T$  zákon zachování momentů h.

(4)  $R - \text{Div } \vec{H} + S_{ij}^{(m)} \dot{E}_{ij}^{(m)} = \rho_0 \dot{w}$  z. zachování energie

$H = JF^{-1}h$ ,  $R = Jr$

(5)  $-\rho_0 m \dot{T} + S_{ij}^{(m)} \dot{E}_{ij}^{(m)} - \frac{1}{T} \vec{H} \cdot \text{Grad } T \geq \rho_0 \psi$

discipinace úlohou

7)  $S_{ij}^{(m)}(E_{kk}^{(m)}, T)$ ;  $T \rightarrow \vec{H}$  konstitutivní vztahy

nezávislé:  $\mu_1, \mu_2, \mu_3, T$  rovnice (2), (4)



## Kondični vrednosti termalnosti

⑨

$\epsilon_{ij}^{(m)}, T$ : ustrezni skrajni veličini

$$S_{ij}^{(m)} = \int_0^1 \frac{\partial \Psi}{\partial \epsilon_{ij}^{(m)}} \quad \eta = - \frac{\partial \Psi}{\partial T} \quad \vec{H} \cdot \text{Grad} T \leq 0$$

### Fonicevski neravnost

$$H_i = \int F_{ij}^{-1} h_j \quad [\text{Grad} T]_i = \frac{\partial T}{\partial x_i} = \frac{\partial T}{\partial \xi_k} \frac{\partial \xi_k}{\partial x_i} = \frac{\partial T}{\partial \xi_k} F_{ki}$$

$$\vec{H} \cdot \text{Grad} T = \int F_{ij}^{-1} h_j \frac{\partial T}{\partial \xi_k} F_{ki} = \int \delta_{kj} h_j \frac{\partial T}{\partial \xi_k} = \int \vec{h} \cdot \text{grad} T$$

$$\int \neq 0 \quad \boxed{\vec{h} \cdot \text{grad} T \leq 0}$$

### Variacijski formalizem

$\delta \mu_i \stackrel{\text{def}}{=} \tilde{\mu}_i - \mu_i$ ,  $\mu_i$  = lokalni potenciali,  $\tilde{\mu}_i$  = liberalizirani f.

osrednjen  $\mu_{i,j} = \frac{\partial \mu_i}{\partial x_j}$

$$(\delta \mu_i)_{,j} = (\tilde{\mu}_i - \mu_i)_{,j} = \tilde{\mu}_{i,j} - \mu_{i,j} = \delta(\mu_{i,j}) = \delta \mu_{i,j}$$

$$e = \frac{1}{2} (z + z^T + z^T z) \quad e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i} + u_{k,i} u_{k,j})$$

$$\dot{e}_{ij} = \frac{1}{2} (\dot{u}_{i,j} + \dot{u}_{j,i} + \dot{u}_{k,i} u_{k,j} + u_{k,i} \dot{u}_{k,j})$$

$$\delta e_{ij} \stackrel{def}{=} \frac{1}{2} (\delta u_{i,j} + \delta u_{j,i} + \delta u_{k,i} u_{k,j} + u_{k,i} \delta u_{k,j})$$

Prav:  $\delta e_{ij}$  není přímá úloha

Platí 
$$\int_{V_0} s_{ij} \dot{e}_{ij} = \int_V f_i \dot{u}_i dV + \int_S t_i \dot{u}_i dS$$

$$\Rightarrow \int_{V_0} s_{ij} \delta e_{ij} dV_0 = \int_V f_i \delta u_i dV + \int_S t_i \delta u_i dS$$
$$= \int_{V_0} F_i \delta u_i dV_0 + \int_{S_0} T_i \delta u_i dS_0$$

MKP

$$u = Hq \quad \delta e = B\delta q \quad (e \neq Bq)$$

$$\int_{V_0} B^T S dV_0 = R$$

$$R = \int_V H^T F dV + \int_S H^T T dS \quad \text{sledující zohlednění (H'ka)}$$

nebo

$$R = \int_{V_0} H^T F dV_0 + \int_{S_0} H^T T dS_0 \quad \text{konstantní zohlednění (H'ka)}$$

# Tecni metode

(11)

$$R = \int_{V_0} B^T S dV_0 \Rightarrow \delta q^T R = \int_{V_0} \delta e^T S dV_0 = \int_{V_0} \delta e_{ij} S_{ij} dV_0$$

$\mu_i = H_i q$   $\delta e_{ij} = \frac{1}{2} (H_{iij} + H_{jii} + H_{kii} \mu_{kij} + \mu_{kii} H_{kij}) \delta q$

$$S_{ij} \delta e_{ij} = (S_{ij} H_{iij} + S_{ij} H_{kii} \mu_{kij}) \delta q =$$

$$= \delta q^T (H_{iij}^T + H_{kii}^T \mu_{kij}) S_{ij}$$

$$\delta q^T R = \delta q^T \underbrace{\int_{V_0} (H_{iij}^T + H_{kii}^T \mu_{kij}) S_{ij} dV_0}_R$$

$$\dot{R} = \int_{V_0} (H_{iij}^T + H_{kii}^T \mu_{kij}) \dot{S}_{ij} dV_0 + \int_{V_0} H_{kii}^T \dot{\mu}_{kij} S_{ij} dV_0$$

$$\dot{S}_{ij} = C_{ijpq} \dot{e}_{pq} = C_{ijpq} (H_{piq} + H_{kip} \mu_{kij}) \dot{q}$$

$$\dot{\mu}_{kij} = H_{kij} \dot{q}$$

$$\dot{R} = \underbrace{\int_{V_0} (H_{iij}^T + H_{kii}^T \mu_{kij}) C_{ijpq} (H_{piq} + H_{kip} \mu_{kij}) dV_0}_{k_0 + k_L} \dot{q} +$$

$$+ \underbrace{\int_{V_0} H_{kii}^T S_{ij} H_{kij} dV_0}_{k_G} \dot{q} = k_T \dot{q}$$

$k_G$

## Stabilita

$$1) \quad \mu_{ij} \ll 1 \quad 2) \quad S \approx \sigma$$

$$\dot{R} = (K_0 + K_L) \dot{q} + K_\sigma \dot{q} \approx (K_0 + K_\sigma) \dot{q}$$

$$K_0 = \int_{V_0} H_{i,j}^T C_{ijpq} H_{pqr} dV_0$$

matice tuhosti  
& lineární průřeznosti

$$K_\sigma \approx \int_{V_0} H_{kij}^T \sigma_{ij} H_{kij} dV_0$$

matice průřeznosti  
kypění (geom. m.)

$$R(\lambda) = R_0 \lambda \Rightarrow K_\sigma = \lambda K_\sigma^0$$

$$K_T = (K_0 + K_\sigma^0 \lambda)$$

$$\det |K_T| = 0 \Rightarrow \text{problém ul. úzel.}$$

$$R_{\text{krit}} = \lambda_{\text{krit}} R_0$$

# TEKUTINY

(1)  $\rho + \rho \operatorname{div} \vec{v} = 0$  z. Eulerova rovnice

(2)  $\frac{\partial \sigma_{ij}}{\partial x_j} + f_i = \rho \dot{v}_i$  z. Eulerova rovnice

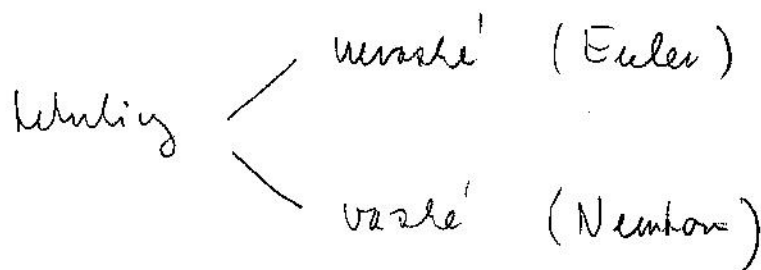
(3)  $\sigma^T = \sigma$  z. Eulerova rovnice

(4)  $\rho - \operatorname{div} \vec{h} + \sigma_{ij} D_{ij} = \rho \dot{u}$  z. Eulerova rovnice

(5)  $-\rho \dot{T} + \sigma_{ij} D_{ij} - \frac{1}{T} \vec{h} \cdot \operatorname{grad} T \geq \rho \dot{\psi}$  disipace

$\sigma_{ij}(\rho, T, D_{ke}), \quad T \rightarrow \vec{h}$

parametry:  $\rho_1, \rho_2, \rho_3, \rho, T \quad (1), (2), (4)$



# Nevale' ketuliy

$$\sigma_{ij} = -p \delta_{ij}$$

$p =$  dynamicly' tlak

$$\sigma_{ij} D_{ij} = -p \delta_{ij} D_{ij} = -p D_{ii} = -p \frac{\partial w_i}{\partial x_i} = -p \operatorname{div} \vec{v}$$

$\exists$  raice kvadraty  $\sigma_{ij} D_{ij} = \frac{p}{\rho} \dot{\rho}$

dissipatsii neravnost:  $-\rho \eta \dot{T} + \frac{p}{\rho} \dot{\rho} - \frac{1}{T} \vec{h} \cdot \operatorname{grad} T \geq \rho \dot{\psi}$

$\rho, T$ : nezavisle' stanna' velichiy

$$\dot{\psi} = \frac{\partial \psi}{\partial \rho} \dot{\rho} + \frac{\partial \psi}{\partial T} \dot{T}$$

$$-\underbrace{\rho \left( \eta + \frac{\partial \psi}{\partial T} \right)}_0 \dot{T} + \underbrace{\left( \frac{p}{\rho} - \rho \frac{\partial \psi}{\partial \rho} \right)}_0 \dot{\rho} - \frac{1}{T} \vec{h} \cdot \operatorname{grad} T \geq 0$$

$$\frac{p}{\rho} = \rho \frac{\partial \psi}{\partial \rho} \quad \eta = - \frac{\partial \psi}{\partial T} \quad \vec{h} \cdot \operatorname{grad} T \leq 0$$

$p(\rho, T)$  stanna' raice  $\Rightarrow \eta$   
(libonlaxi)

rishun'

(1)  $\frac{\partial p}{\partial t} + \frac{\partial p}{\partial \xi_k} v_k + p \frac{\partial v_i}{\partial \xi_i} = 0$

$\frac{\partial p}{\partial t} + \frac{\partial}{\partial \xi_k} (p v_k) = 0$

$\frac{\partial p}{\partial t} + \text{div}(p \vec{v}) = 0$

r. kontinuity

(2)  $\frac{\partial \sigma_{ij}}{\partial \xi_j} = - \frac{\partial p}{\partial \xi_j} \delta_{ij} = - \frac{\partial p}{\partial \xi_i}$

$-\text{grad } p + \vec{f} = p \vec{n}$

Euler

(4)  $r - \text{div } \vec{h} + \frac{p}{\rho} \vec{j} = p \vec{u}$

vedem' Laplas

(5)  $p(p, T) \Rightarrow \Psi, \eta \Rightarrow \mu(p, T) = \Psi + T\eta$

Prazn: Neshraniklena' khranilica  $p = p_0 = \text{konst.}$

(1)  $\text{div } \vec{v} = 0$  (2)  $-\text{grad } p + \vec{f} = p_0 \vec{n} \Rightarrow p_1 \vec{n}$

(4)  $r - \text{div } \vec{h} = p_0 \vec{u}$  upren' u mesh'ivole

Ideální plyn

vedení tepla:  $\vec{h} = -\lambda \text{grad} T$   $\lambda [W/mK] = \text{konst.}$ ,  $r = \tilde{r}$   
(omáček)

střeva práce:  $\frac{P}{\rho} = r T$   $r [J/kg K]$ .

$$\frac{P}{\rho} = \rho \frac{\partial \Psi}{\partial \rho} = r T \Rightarrow \Psi = r T \ln \frac{\rho}{\rho_0} + g(T)$$

$$\eta = -\frac{\partial \Psi}{\partial T} \quad \eta = -r \ln \frac{\rho}{\rho_0} - g'(T)$$

$$\mu = \Psi + T\eta = r T \ln \frac{\rho}{\rho_0} + g(T) - r T \ln \frac{\rho}{\rho_0} - Tg'(T)$$

$$\boxed{\mu = \varepsilon(T)}$$

$$(4) \quad \tilde{r} + \lambda \Delta T + \frac{P}{\rho} \dot{\rho} = \rho \frac{d\varepsilon}{dT} \dot{T}$$

$$C_v \stackrel{\text{def}}{=} \frac{d\varepsilon}{dT} [J/kg K]$$

$$\boxed{\tilde{r} + \lambda \Delta T + r T \dot{\rho} = \rho C_v \dot{T}}$$



Pr      Rozložení stabilního tlaku v atmosféře

(17)

$$(2) \quad -\frac{dp}{d\xi} - \rho g = 0 \quad g = \text{konst. rozložením}$$

$$(4) \quad \Delta T = 0$$

$$\frac{dT}{d\xi^2} = 0 \Rightarrow T(\xi) = T_0 - \alpha \xi \quad \alpha = \text{konst.}$$

$$\frac{dp}{d\xi} = -\rho g = -\frac{\rho p}{rT}$$

$$\ln \frac{p}{p_0} = -\frac{g}{r} \int_0^{\xi} \frac{d\xi}{T_0 - \alpha \xi} = \frac{g}{\alpha r} \ln \frac{T_0 - \alpha \xi}{T_0}$$

$$p(\xi) = p_0 \left( \frac{T}{T_0} \right)^{g/\alpha r}$$

Prův:       $\alpha \rightarrow 0$        $\lim_{\alpha \rightarrow 0} p(\xi) = p_0 \exp \left[ -\frac{g\xi}{rT_0} \right]$

což plyne z integrace       $\ln \frac{p}{p_0} = -\frac{g}{r} \int_0^{\xi} \frac{d\xi}{T_0}$

Pi:

Štítná' ra'zonn' mluvy  
(varimna' probléma' mluva)

$$N_1 (\xi_1 - ct)$$

$$n (\xi - ct)$$

$$N_2 = 0$$

$$f (\xi - ct)$$

$$N_3 = 0$$

$$p (\xi - ct)$$

$$\frac{\partial g}{\partial t} = -c \frac{\partial g}{\partial \xi}$$

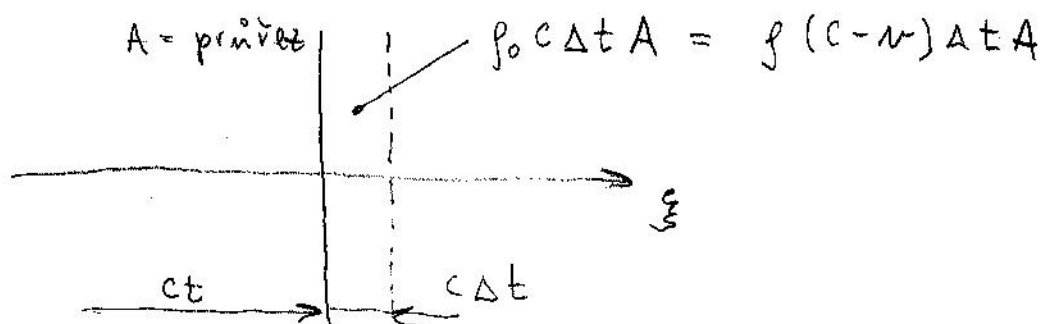
$$(1) \quad \frac{\partial p}{\partial t} + \frac{\partial p}{\partial \xi} n + f \frac{\partial n}{\partial \xi} = 0$$

$$-c \frac{\partial p}{\partial \xi} + \frac{\partial}{\partial \xi} (p n) = 0$$

$$-cp + pn = \alpha(T) = \text{konst.}$$

průhlední' pohlédnutí  $\xi > ct: -c p_0 + p = \alpha$

$$p(c-n) = p_0 c$$



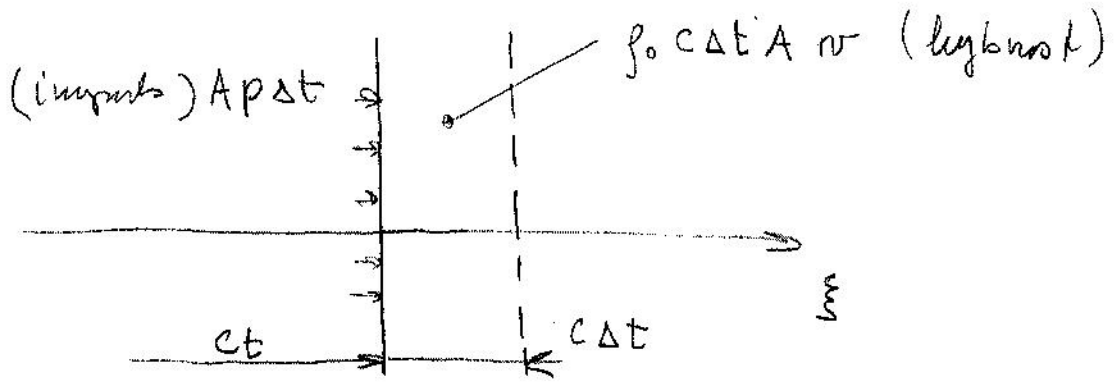
$$(2) \quad -\frac{\partial p}{\partial \xi} = f \left( \frac{\partial N}{\partial t} + \frac{\partial N}{\partial \xi} v \right) = f \frac{\partial N}{\partial \xi} (-c + v)$$

z. r. Kontinuität  $-\frac{\partial p}{\partial \xi} = -f_0 c \frac{\partial N}{\partial \xi}$

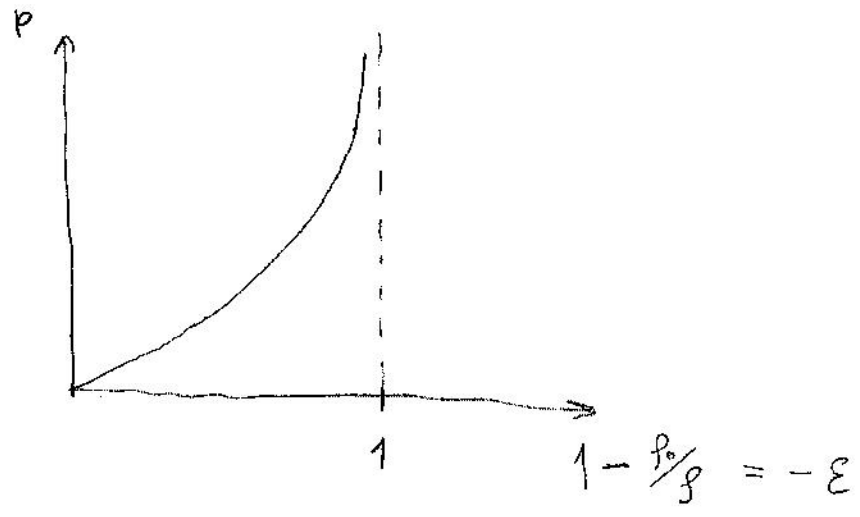
$$p = f_0 c N + \beta, \quad \beta = \text{konst.}$$

$\xi > ct$ :  $p = 0 \Rightarrow \beta = 0$

$$p = f_0 c v$$



adiabatisch d.h.  $p = p(\rho)$  Hugoniot



Trade liberalization

$$\sigma_{ij} = -p_{ij} + \tau_{ij}$$

Wicksell's cost

$$\tau_{ij} = c_{ij} - D_{ij} \quad (\text{Newton - status})$$

For optimal distribution we must be in equilibrium, also

$$A_{D_{ij}} : c_{ij} - D_{ij} \geq 0$$

Primum

$$c_{ij} - D_{ij} \geq 0 \Rightarrow + \text{def.}$$

From: Equilibrium price in  $\tau_{ij} = \frac{\partial \phi}{\partial D_{ij}}$ , where

$\phi = \text{dissemination product}$

Isoproduct technique:

$$\tau_{ij} = \lambda D_{ik} \sigma_{ij} + 2^m D_{ij}$$

$\lambda, \mu = \text{handwritten notation}$

$$\tau = \lambda \text{dim } N^2 I + 2^m D$$

$$\begin{aligned} \frac{\partial \tau_{ij}}{\partial \xi_j} &= \lambda \frac{\partial}{\partial \xi_j} (\operatorname{div} \vec{v}) \delta_{ij} + 2\mu \frac{\partial}{\partial \xi_j} D_{ij} = \\ &= \lambda [\operatorname{grad}(\operatorname{div} \vec{v})]_i + 2\mu \frac{1}{2} \left( \frac{\partial^2 v_i}{\partial \xi_j \partial \xi_j} + \frac{\partial^2 v_j}{\partial \xi_i \partial \xi_i} \right) = \\ &= \lambda [\operatorname{grad}(\operatorname{div} \vec{v})]_i + \mu [\Delta \vec{v}]_i + \mu [\operatorname{grad}(\operatorname{div} \vec{v})]_i \end{aligned}$$

$$-\operatorname{grad} p + (\lambda + \mu) \operatorname{grad}(\operatorname{div} \vec{v}) + \mu \Delta \vec{v} + \vec{f} = \rho \dot{\vec{v}}$$

Prm: konvektivná strúha  $\dot{\vec{v}}$  čím rýchlejšie nelineárna!

Reynoldsova čísla

$$\nu = \frac{\mu}{\rho} \quad [\text{m}^2/\text{s}] \quad \text{kinematická viskozita}$$

usklad. rovnica  $-\frac{1}{\rho} \operatorname{grad} p + \nu \Delta \vec{v} + \frac{1}{\rho} \vec{f} = \dot{\vec{v}}$

$\frac{\partial \vec{v}}{\partial \xi_i} v_i$	porovnáme s	$\nu \Delta \vec{v}$
$\frac{V^2}{L}$	$V_1 L =$ charakteristická veľkosť	$\nu \frac{V}{L^2}$
	rečivý	

$$Re = \frac{\text{konvekčná} \text{viskozita}}{\text{viskozita}} = \frac{VL}{\nu} \quad [\text{bez rozmier}]$$

$Re \rightarrow 0$  lineárna  $Re \rightarrow \infty$  Eulerova rovnica