Probabilities as ratios of ranges in initial-state spaces

Foundations of Uncertainty:

Probability and Its Rivals

Prague, September 3rd, 2009

Objective interpretations of probability

- say that probability statements are made true or false by reality, and not by our state of mind or information.

- usually come in two varieties: frequency and propensity interpretations, which both face serious problems.

- Concerning the frequency interpretations, there are no non-probabilistic connections between probabilities and relative frequencies of events. Repetition of a random experiment can yield any possible outcome with any relative frequency in the short or long run.

Objective interpretations of probability

- Single-case propensity theories postulate fundamental normative entities in Nature that somehow constrain rational credence “just so”.

- But the propensity account can also be understood to say merely this: Instead of being identical to relative frequency, objective chance is that feature of an experimental set-up which explains (probabilistically!) the characteristic relative frequency with which an outcome typically occurs in the long run.

Objective interpretations of probability

- What could that feature be? In a deterministic context, we can look at initial states. Presumably, a probable outcome has “more” initial states leading to it than an improbable one.

- That gives rise to a further possibility for an objective interpretation: namely, probabilities as deriving from ranges in suitably structured spaces of initial states. The probability of an event is the proportion with which the event occurs in small regions of the initial state space. Call this the “range interpretation” of probability.

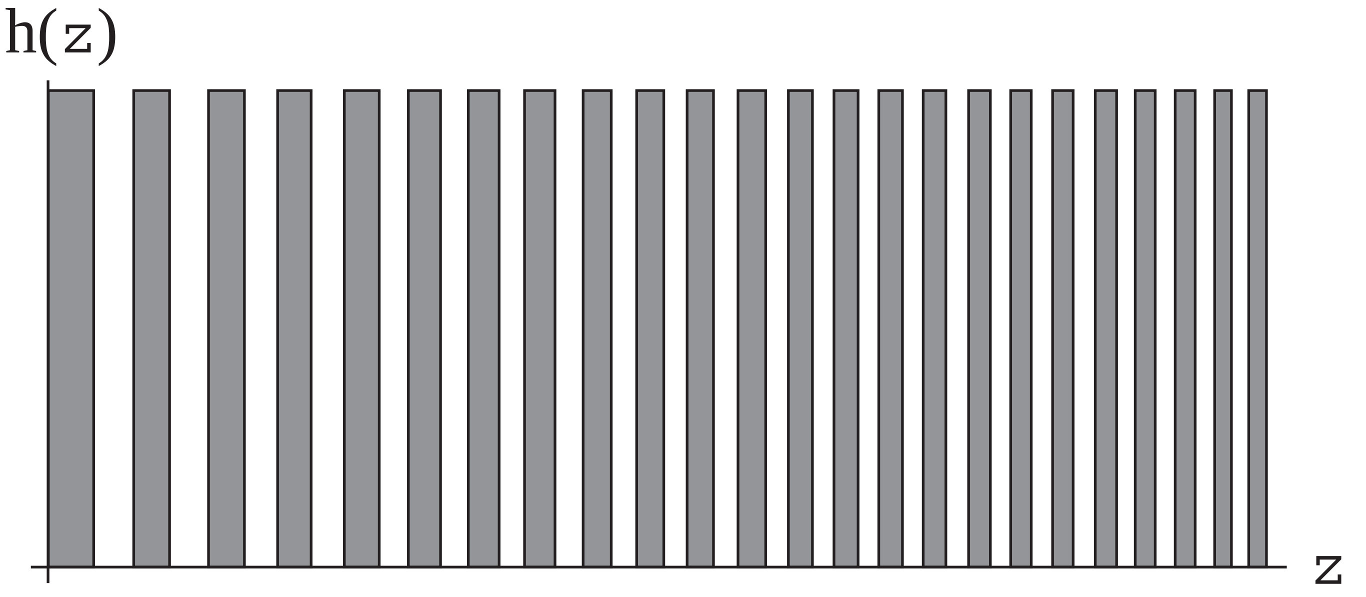
The range interpretation: 1st formulation

Let ***E*** be a random experiment and *A* a possible outcome. Let ***S*** be the initial state space attached to ***E***, and $\boldsymbol{S_A}$ the set of those initial states that lead to *A*. Let *μ* be the Lebesgue-measure. *A* occurs with probability *p* on a trial of ***E*** iff

for each interval ***I*** in ***S*** that is not too small we have:

$$\frac{\mu(\boldsymbol{I} \cap \boldsymbol{S_A})}{\mu(\boldsymbol{I})} \approx p$$

The wheel of fortune



From Strevens (2003), p. 50

The range interpretation: remarks

Let ***E*** be a random experiment and *A* a possible outcome. Let ***S*** be the initial state space attached to ***E***. If *A* is represented in each not too small interval in ***S*** with approximately the same proportion *p*, then there is an objective probability of *A* on a trial of ***E*** and its value is *p*.

- The “ranges” we are dealing with are physical or natural ranges. What one needs to apply the approach is a distinction between initial conditions and laws that rule the dynamics of the system. Moreover, the initial conditions have to be viewed as being continuously variable.

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- The outcome *A* is represented in each (not too) small segment of the initial-state space, but so are all other possible outcomes. Therefore, we cannot predict or control the outcome on a single trial of ***E***. On the other hand, *A* is represented in each such segment with approximately the same proportion. This explains the characteristic long-run frequency of *A*.

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- Of course, the actual frequency of *A* upon repetition of ***E*** may deviate as much as you like from *A'* s proportion within intervals of ***S***. What makes a probability statement true is not actual or hypothetical frequencies, but the physical circumstances that give rise to them.

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- The approach works only in a deterministic context, where the outcome of a chancy process is determined by initial conditions. This means that what is “random” or “chancy” depends on our epistemic and computational abilities.

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- *That* there are probabilities depends on us, but *what* they are depends on the world. The structure of the initial-state space and the proportion of *A* in each segment is a wholly objective matter. But that we call this proportion “the probability of *A* upon a trial of ***E***” has to do with our epistemic capacities.

The range interpretation: remarks

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- The ideal case would be the truly chaotic limiting case in which for *every* interval ***I*** in ***S*** the equation holds *exactly*. This never happens in reality, but it may be convenient to view and model chancy situations as if it were true. ("Near enough is good enough to apply the concept.")

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- We do not get single-case objective probabilities out of the range approach. Referring to a single case, the talk of “possible outcomes” has no objective sense, and neither is there an initial state space. The space is something attached to a *type* of experiment.

The range interpretation: remarks

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- This approach to probabilities was first proposed by von Kries (1886). He spoke of "Spielraum" (leeway, room to move, "play space"). The first mathematically rigorous investigations are Poincaré (1896) and Hopf (1934). The by far most comprehensive modern philosophical treatment is Strevens (2003), a thoroughgoing mathematically oriented treatise is Engel (1992).

The range interpretation: remarks

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- The critical question is if this approach is really suited for an *interpretation* of probability, in the sense of providing truth conditions for probability statements. Most writers consider it merely to give an *explanation* for the occurrence of probabilistic patterns in complex systems. (It is not well suited for assertibility conditions, as the requisite mathematics gets soon very complicated.)

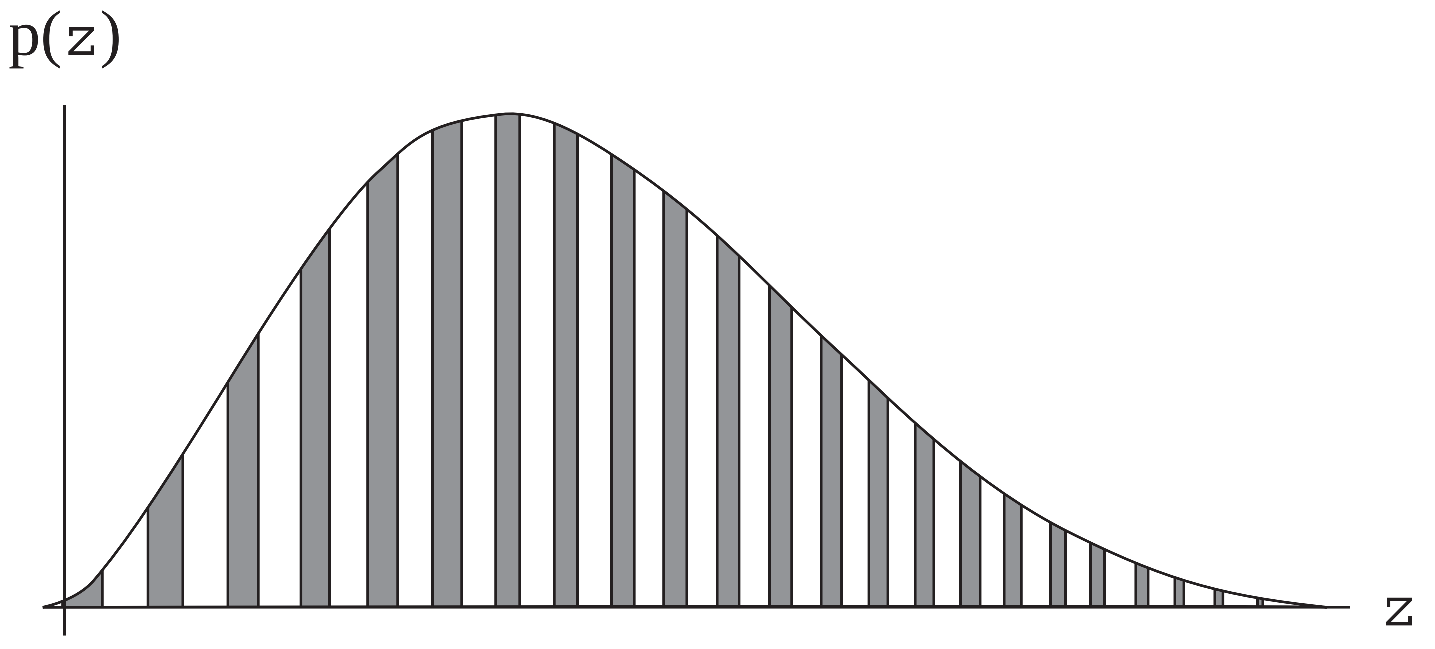
The method of arbitrary functions

A continuous function can be characterized as a function that is approximately constant on any sufficiently small interval. Therefore, we can restate the definition thus:

Let *A* be a possible outcome of a random experiment ***E***. Let ***S*** be the initial state space and $\boldsymbol{S_A}$ the set of those states that lead to *A*. The probability of *A* on a trial of ***E*** is *p* iff for any continuous density on ***S*** with appropriately bounded variation on small intervals we have:

$$\int_{\boldsymbol{S_A}} \delta(\mathbf{x})\,\mathrm{d}\mathbf{x} \approx p$$

The wheel of fortune revisited



from Strevens (2003), p. 51

Objection to the range approach

- Isn‘t the concept of probability in fact presupposed? To fix the probabilities according to the range approach we have to assume implicitly an approximately uniform probability distribution over the initial states in any small interval of ***S***.

- Or look at the method-of-arbitrary-functions-formulation: What are those density functions supposed to be? They are probability distributions, so the issue of interpreting probabilities is merely shifted back from the probabilities of outcomes to the probabilties of initial states.

- The approach concerns the „physics“, not the „metaphysics“, of probability (Strevens 2003).

Answers?

- When we say „The proportion of an outcome in any (not too) small interval of the initial state space is the probability of that outcome“, this is a perfectly objective answer.

- It would only be wrong if there was such a thing as the „true“ probability distribution over the initial state space, and if that distribution was very eccentric.

- But then we would conclude that we overlooked some nomologically relevant factor, i.e. that either we got the initial state space wrong or the space is not „primordial“.

Answers?

- The basic idea is this: The laws of nature determine the result, given initial conditions, but they leave open what those conditions are. As *they* do not care, and *we* can‘t control the initial conditions sufficiently, it can only be by accident if on repeated trials of ***E*** we approximate a very eccentric distribution over ***S***.

- *But*, if we somehow convince ourselves that there is something behind this very eccentric distribution, i.e. that it can be relied on for future predictions, then there must be some nomological factor we have overlooked, i.e. the laws of nature *do* care, contrary to what we thought. We would then re-model the experimental situation.

Answers?

- The problem of eccentric densities over the initial-state space is closely related to the problem of the choice of a measure on this space. „Eccentric“ measures have to be ruled out in order to get the probabilities right. As with the density functions, the class of „regular“ measures will be very large, but not contain all possible measures.

- Therefore, we have to presuppose that the „natural“ ways of measuring the distance between (vectors of) initial states are objectively distinguished and not merely a matter of convention.

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