# Mathematical methods for flow control

**Elfriede, Friedmann**
**Department of Applied Mathematics, Heidelberg University, Im Neuenheimer Feld 293, D-60120 Heidelberg, Germany**

**Abstract:** We are interested in drag reducing surfaces, i.e. in particular rough surfaces like riblets well known from shark skin and used in passive flow control. Their contribution to drag reduction depends on their size and on the flow type and velocity. Their maximal contribution to drag reduction known up to now is around 10% for turbulent flow for structures protruding the buffer layer. Smaller structures which stay inside the viscous sublayer are known to provide a much smaller drag reduction and were considered often as hydraulically smooth. Fundamental research in development of novel surface structures with higher drag reduction is continually carried out. First contributions were obtained for very small structures to be two to three times the highest value known until now. Performing mathematical modeling, analysis and numerical simulations, we will present recent drag issues and will discuss the flow physics, i.e. the drag reduction mechanism.

## Introduction

Rough surfaces like riblets are known as skin friction reducing surfaces since the oil crisis in the 1980s. Newest research [6] showed that very small grooves inside the viscous sublayer work best for turbulent flow. Their contribution to drag reduction is two to three times greater than known until now from higher structures protruding the buffer layer. Experimentally it was shown that these kind of microstructures work only in a small Reynolds number (Re) range [4]. Outside this range the drag reduction is small or even changes in drag increasing, which means that for a specific turbulent flow one must find the characteristic size of the optimal structures. This size seems to be approximately of the order of the Kolmogorov scale of the considered flow [4].

## Modeling

We consider flow which is described by the Navier-Stokes equation and cover with this a variety of applications. The majority of known theoretical results are related to laminar flow and viscous gas flow. This, together with our interest in skin friction reduction leads us to an access to turbulent flow via the boundary layer theory. In the viscous sublayer the flow equations can be described by the incompressible Navier-Stokes equations with no-slip boundary condition on the rough surfaces. In viscous gas dynamics we consider the compressible Navier-Stokes equations with the full Navier's slip condition on a rough body.

## Analysis

For the viscous sublayer model it was shown that there exists a unique solution if the size of the structures is small enough, restricted by the viscosity. Here, the homogenization theory was applied (by Jäger and Mikelić) and the drag calculated from a simplified so called effective model [2]. In viscous gas dynamics we were able to show the continuity of the drag in the low Mach limit for a rough body immersed in a channel and whose shape is allowed to change with the Mach number [1]. In the limit we obtained a smooth body immersed in an outer incompressible flow with no-slip boundary condition. This result can be interpreted as a justification of the non-physical no-slip boundary condition on perfectly smooth surfaces in incompressible flow.

**Numerical Simulations**

The numerical solution of the Navier-Stokes equation is known to be difficult at least for turbulent flow. The different mixing-length scales in this flow makes the computation time for Direct Numerical Simulations (DNS) infeasible especially if also the geometry (small grooves) requires already a very fine mesh. We perform DNS based on the finite element (FE) technique with a in-house software [5]. In [2] we simulated the instationary viscous sublayer model over riblets and pimpels. In [7] we validated reduced models, one with periodic boundary conditions and a homogenized one. We were also able to reproduce a turbulent flow over a smooth surface by coupling our FE simulation to time dependent boundary conditions coming from finite volume (FV) turbulent simulations. Our aim is to couple fine resolved boundary-layer models over rough structures (FE-DNS) with courser FV-DNS to reduce the computation costs for flow control problems.

**Drag reducing mechanism**

The drag reducing mechanism of riblets and grooves in turbulent flow can be explained with a shift in their mean velocity profile [2]. Due to the geometry the velocity profile can be decomposed in the main part (longitudinal flow) and the cross flow. The origin of the main flow lies beneath the one of the cross flow and the origin of the corresponding smooth surface lies in between [7]. The total amount of drag reduction depends then on the specific riblet or groove shape. On symmetric structures like pimpels there is no flow decomposition and no drag reduction. Riblets and grooves redirect the cross flow into the main flow field when immersed in the viscous sublayer and dampen the near-wall vortices if protruding in the buffer layer. Nevertheless, for very small structures another mechanism is coming into effect. Following Kolmogorov's hypothesis of local isotropy, that at sufficient high Re the small-scale turbulence is statistically isotropic. Small riblets and grooves increase thus the anisotropy of turbulence by forcing near wall turbulence to satisfy axisymmetry at large and small scales very close to the wall with invariance under rotation about the axis aligned with the main flow. For optimal structures, the limit state of one-component turbulence can be achieved, for which the turbulent dissipation vanishes and the total dissipation rate reaches its minimal value [3].

# Reference

[1] Feireisl, E., Friedmann, E., Continuity of drag and domain stability in the low Mach number limits, J. Math. Fluid. Mech, DOI 10.1007/s00021-012-0106-1, 2012.

[2] Friedmann, E., Richter, T., *Optimal microstructures. Drag reducing mechanism of riblets*, J. of Math. Fluid Mech., 13 (3), 429-447, 2010.

[3] Frohnapfel, B., Lammers, P., Jovanovic, J., Durst, F., Interpretation of the Mechanism Associated with Turbulent Drag Reduction in terms of Anisotropy Invariants, Journal of Fluid Mechanics 577:457-466, 2007.

[4] Frohnapfel, B., Jovanović, J. and Delgado, A.: Experimental investigation of turbulent drag reduction by surface-embedded grooves. J. Fluid Mech. 590, 107-166, 2007.

[5] GASCOIGNE, High Performance Adaptive Finite Element Toolkit, URL: `http://www.numerik.uni-kiel.de/~mabr/gascoigne/`.

[6] Jovanovic, J. and Frohnapfel, B. and Delgado, A., Viscous Drag Reduction with Surface Embedded Grooves, Notes on Numerical Fluid Mechanics and Multidisciplinary, Springer, 110:191-197, 2010.

[7] Maier M., Simulation von Grenzschichtströmungen über Ribletstrukturen, diploma thesis, Heidelberg University, 2011.