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Four Essays on Macroeconomic Aspects of the Transition Process

Jan Brůha

Dissertation

Prague, June 2013

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Abstract

This dissertation is a sample of my research on macroeconomic aspects of the transition process in the Central European countries. During my doctoral studies, I have been especially interested in two issues: (1) environmental regulation and policy and (2) macroeconomic convergence.

The first paper *The decomposition and econometric analysis of air emissions in a transition country: the case of the Czech Republic* is from the field of environmental economics and is empirically oriented. It analyzes air emissions of four classical pollutants in the Czech Republic during the transition period. The next three papers are macroeconomic papers based on theoretical modeling. The paper *Real Exchange Rate in Emerging Economies: The Role of Different Investment Margins* (written jointly with Jiří Podpiera) inquires about the mechanism of the strong pace of the real exchange rate appreciation observed in Central European transition economies. The paper introduces a quality investment margin and shows that the margin is needed for replicating the observed pace of the real exchange rate appreciation. The paper basically compares the steady state of the model for various levels of development of a converging country.

The next paper *The Dynamics of Economic Convergence: The Role of Alternative Investment Decisions* (written jointly with Jiří Podpiera) evaluates how various investment decisions explain the macroeconomic dynamics of European transition countries. Therefore, contrary to the previous paper, the transition dynamics is computed and compared to data for Central European economies. Finally, the last paper, *The convergence dynamics of a transition economy: The case of the Czech Republic* (written jointly with Jiří Podpiera and Stanislav Polák), aims at explaining the long-run trend in key macroeconomic variables in the Czech Republic using the framework introduced in the previous two papers.

The first paper has been rejected from Ecological Economics. Referees of Ecological Economics expressed similar concerns about the paper as the two referees of the thesis. I hope that I have learnt from these comments and a new submission to another journal is being planned.

The second paper was published in Economics of Transition, vol. 18 (3), July 2010, pp. 599–628. The third paper was published in Journal of Economic Dynamics and Control, vol. 35 (7), July 2011, pp. 1032-1044. The last paper appeared in Economic Modelling, vol. 27 (1), January 2010, pp. 116-124.

Abstrakt

Tato disertace je výběr z mého výzkumu, v němž jsem se zabýval makroekonomickými aspekty ekonomické transformace ve střední Evropě. V průběhu svého doktorského studia jsem se zabýval zejména těmito oblastmi: (1) environmentální regulací a (2) makroekonomickou konvergencí.

Disertace je složena ze čtyř článků. První článek The decomposition and econometric analysis of air emissions in a transition country: the case of the Czech Republic je z oblasti environmentální ekonomie a je empiricky orientován. Analyzuje emise čtyř klasických polutantů v české republice během ekonomické transformace. Další tři články se týkají makroekonomického modelování. článek Real Exchange Rate in Emerging Economies: The Role of Different Investment Margins (napsaný spolu s Jiřím Podpierou) zkoumá mechanismy reálné apreciace pozorované v konvergujících zemích střední Evropy. Tento článek považuje investice do kvality zboží za klíčový aspekt umožňující replikace pozorované reálné apreciace a porovnává ustálené hodnoty apreciace za různých modelových předpokladů. Další článek The Dynamics of Economic Convergence: The Role of Alternative Investment Decisions (napsaný spolu s Jiřím Podpierou) vyhodnocuje, jak různé modelové předpoklady o investicích pomáhají vysvětlit pozorovanou makroekonomickou dynamiku v transformujících se ekonomikách střední Evropy. Nakonec poslední článek The convergence dynamics of a transition economy: The case of the Czech Republic (napsaný spolu s Jiřím Podpierou a Stanislavem Polákem) vysvětluje dlouhodobé trendy v klíčových makroekonomických proměnných pomocí mechanismů navržených v předešlých dvou článcích.

První článek byl zamítnut v Ecological Economics. Recenzenti tohoto časopisu vyjádřili podobné pochybnosti jako recenzenti této disertace. Předpokládám, že se z připomínek obou skupin recenzentů poučím a zapracuji je do nové verze článku, kterou plánuji zaslat do jiného časopisu.

Druhý článek byl publikován v časopise Economics of Transition, vol. 18 (3), červenec 2010, s. 599–628. Třetí článek byl publikován v Journal of Economic Dynamics and Control, vol. 35 (7), červenec 2011, s. 1032-1044. Poslední článek vyšel v Economic Modelling, vol. 27 (1), leden 2010, s. 116-124.

Acknowledgments

I owe much to many people. Firstly, I want to acknowledge the support and the infinite patience of my supervisor Sergey Slobodyan, who provided me with many useful comments about my work during the various stages of my study. Secondly, I express my strong opinion about the stimulating environment at CERGE-EI. I have also benefited much from working with my co-authors. I also thank my two referees, Professor David Cobham and Assistant Professor Oleg Zamulin, for stimulating comments. Lastly, I am indebted to my wife Hana for her patience and forbearance during the periods of intensive work on the dissertation.

Czech Republic, Prague — Jan Brůha
June 2013

Introduction

This dissertation is a sample of my research on macroeconomic aspects of the transition process in the Central European countries. During my doctoral studies, I have been especially interested in two issues: (1) environmental regulation and policy and (2) macroeconomic convergence. The doctoral dissertation includes four papers, which witness my research.

The first paper is from the field of environmental economics and is empirically oriented. Its title is *The decomposition and econometric analysis of air emissions in a transition country: the case of the Czech Republic*. In the paper, I analyze air emissions of four classical pollutants in the Czech Republic during the transition period, i.e., up to the year 2004. The paper applies an index decomposition analysis to gauge the importance of the economic growth, the change in the structure of the national economy, and sector emission intensity on the air emissions of selected pollutants. Then, panel-data econometric techniques are applied to explain the change in emission intensities. I find that (i) during the first years of the transition, the structural effect was dominant, (ii) the intensity effect became more important in later years, and that (iii) the fall in emission intensity has been associated with the increase in labor productivity, and therefore the productivity growth has been environment-saving rather than environment-intensive. The paper is in a submission process.

The next papers are macroeconomic papers based on theoretical modeling. The paper *Real Exchange Rate in Emerging Economies: The Role of Different Investment Margins* (written jointly with Jiří Podpiera) asks about the mechanism of the strong pace of the real exchange rate appreciation observed in Central European transition economies. The paper introduces a quality investment margin and shows that the margin is needed for replicating the observed pace of the real exchange rate appreciation. The paper was published in Economics of Transition, vol. 18 (3), July 2010, pp. 599–628.

The paper *The Dynamics of Economic Convergence: The Role of Alternative Investment Decisions* (written jointly with Jiří Podpiera) evaluates how various investment decisions explain the macroeconomic dynamics of European transition countries. The model is successful in simultaneously addressing the following five facts, which dominate the picture of the macroeconomic development in Central and Eastern European transition countries:

Fact 1: The convergence in GDP per capita of an average Visegrad-4 country to the average of the EU15 attained 1 per cent a year on average over the decade.

Fact 2: The increase in export to GDP ratio on average over Visegrad-4 countries attained 2 per cent a year over the decade; the trade balance, after an initial deficit of around 5 per cent, reached a balanced position at the end of the decade.

Fact 3: The privatization and economic attractiveness of the region have resulted in a significant inflow of foreign direct investment; on average, the inflow in Visegrad-4 countries reached USD 5 billion a year over the decade.

Fact 4: Real exchange rates – *also in sub-index of tradable goods* – of Visegrad-4 currencies vis à vis the Euro have been appreciating by an average of about 3 per cent a year.

Fact 5: The proportion of medium-high and high-tech products in total exports has attained 1.5 to 2 per cent a year, see Fabrizio et al. (2007).

The major conceptual difference from the current literature stems from modeling the explicit decision about investment in quality in two countries that are unequally developed. The proposed extension is of crucial importance for consistent explanation of the macroeconomic developments in the Visegrad-4 region. Bringing the real exchange rate in line with the other macroeconomic variables (such as export performance) offers a reconciliation of the recent puzzle of the limited effect of the real exchange rate appreciation on external competitiveness in transition countries. The paper was published in *Journal of Economic Dynamics and Control 35*, pp. 1032-1044.

The last paper of my dissertation *The convergence dynamics of a transition economy: The case of the Czech Republic* (written jointly with Jiří Podpiera and Stanislav Polák) aims at explaining the long-run trend in key macroeconomic variables. Unlike a developed economy, which exhibits standard and settled characteristics for a sufficiently long period of time and for which long-run values (usually called equilibrium) can be obtained by averaging past observations, transition economies fall short in this respect. In order to find and assess these variables for a transition economy, one needs a specific model that delivers simultaneously determined long-term trajectories. To do that, the two-country model, introduced in the previous two papers, is used. The model, calibrated for the Czech economy and the EU15, shows that the symptoms of convergence can be explained by decreasing export and investment costs and by growing productivity and investment in quality in the transition country. The development of the economy is described by a quartet of endogenously determined trajectories for GDP, foreign direct investment, the trade balance, and the real exchange rate, which relate to their trends in the observed data. The long-run projections suggest that when the GDP per capita convergence is completed, the real exchange rate stabilizes at 45% of the base level in 1997. The paper has been published in *Economic Modeling 27*, January 2010, pp. 116-124.

The three papers on macroeconomic convergence present variants of a model which is an extension to the framework of the two-country model used by many authors in past years in international macroeconomics. Therefore, it is worth emphasizing how the three papers differ. The first paper simply asks whether the pace of the real exchange rate appreciation observed in Central European economies in the past ten years can be explained without the proposed extension. The answer is that the proposed extension is vital for explanation of the observed dynamics. The second paper goes a step further and compares the dynamics of the main variables (output, consumption, exports, real exchange rate, trade balance, and debt) implied by the extended model with data for the Visegrad-4 countries. The paper concludes that the model is successful on this front. The last paper then applies the model to the Czech Republic.

The two-country framework has been used as the main trading partner for these countries is the Euro Area. Moreover, since the explanation of the observed dynamics is based on the endogenous decision of firms about exporting and about product quality, the small open economy framework would be deficient as the decisions in the advanced country would be exogenous. These two considerations led to the choice of the two-country framework.

Other papers

I have written other papers devoted to the fascinating topic of economic transition as well. These papers are not included in this thesis, but they may still be of interest to the reader.

Environmental papers

I have written a series of papers jointly with H. Brůhová-Foltýnová on environmental regulation in the transport sector applied to the Czech Republic.

In the paper *Assessment of Fiscal Measures on Atmospheric Pollution from Transport in Urban Areas*, we formulate and estimate a microeconometric model of household transport demand, which allows for latent separability of fuel consumption between urban and rural areas. The micro-simulation model assesses impacts of changes in passenger transport prices on fuel consumption, external effects of air pollution, household welfare, and public finance in a transition country. An exemplar calibration is taken for the five largest cities in the Czech Republic (Prague, Ostrava, Brno, Pilsen, and Olomouc). We show that ignoring the different price elasticities of fuel consumption in the urban and rural areas can lead to serious errors. The results suggest that the Mohring effect is not strong enough to warrant further subsidies to public transport, that a reduction in public transport fares is not a good policy option (because of public-fund costs), and that an increase in motor-fuel taxation may be a good policy option. The paper was published as Chapter 14 in Chalifour, N., Milne, E.J., Ashiabor, H., Deketelaere, K., Kreiser, L. (eds): Critical Issues in Environmental Taxation. International and Comparative Perspectives: Volume V. Oxford University Press, pp. 335 – 350, 2008, ISBN-13: 978-0-19-954218-5.

The paper *Evaluation of regulation of external cost of air pollution from transport: An inquiry using global sensitivity analysis* takes seriously the uncertainty in parameter values in transport studies and demonstrates how to apply sensitivity analysis techniques to deal with it. These techniques are exemplified on the microsimulation model introduced in the previous paper. The paper assesses the importance of various parameters for the model outcome. The results show that a significant portion of the variation in model results can be explained by a small number of model parameters. These results can bring useful information to policymakers and other stakeholders concerning the relative merits of directing research. The paper has been published as Chapter 15 in H. Geerlings, Y. Shiftan and D. Stead (Eds.) ‘Transition towards Sustainable Mobility: the Role of

Instruments, Individuals and Institutions', pp. 289-306. ISBN: 978-1-4094-2469-7.

Finally, the paper *An international comparison of factors influencing modal split: implications for environmental taxation* is an empirical paper which analyzes the factors influencing modal split in European cities using the Urban Audit database. The paper was published as Chapter 32 in J. Cottrell, J.E. Milne, H. Ashiabor, L. Kreiser, K. Deketelaere (eds): Critical Issues in Environmental Taxation. International and Comparative Perspectives: Volume VI., Oxford University Press, 2009, ISBN: ISBN 978-0-19-956648-8.

Other international macroeconomic papers

The paper *Transition Economy Convergence in a Two-Country Model: Implications for Monetary Integration* uses the model presented in the third paper of this thesis to project the real exchange rate in four transition countries. The exchange rate projections bear important policy implications, which are illustrated on the collision between the price and nominal exchange rate criterion for the European Monetary Union. The paper appeared as a European Central Bank working paper no. 740.

The paper *Macroeconomic Factors and the Balanced Value of the Czech Koruna/Eur Exchange Rate* (written jointly with Alexis Derviz) studies the dependence of the Czech koruna's exchange rate to the euro on risk factors that cannot be reduced to standard macroeconomic fundamentals. For this purpose, an international asset-pricing model in which the exchange rate is co-determined by a risk factor imperfectly correlated with other priced risks in the economy is introduced. The model embeds the standard no-arbitrage setup. It also contains an additional equation that links the autarchic currency price with the foreign-exchange order flow. In the state-space form, the unobserved variables that determine the dynamics of the asset markets, the autarchic exchange rate, and the FX order flow span a number of macroeconomic and latent risk factors. The model for the Czech koruna/euro exchange rate uses Kalman filter techniques. The results indicate the existence of a 'non-fundamental' source of systematic divergence between the observed and the autarchic (i.e. fundamental) FX returns. The paper was published in the Czech Journal of Economics and Finance, vol. 56(7-8), pp. 318-343, 2006.

The paper *The origins of global imbalances* (jointly with Jiří Podpiera) studies the endogenous response of unequally developed regions to a drop in investment and trade costs in a general equilibrium model. The response is characterized by a rise in foreign direct investment in the underdeveloped region and increased consumption in the developed one, leading to trade imbalances between the regions. The paper hereby proposes that declining investment and trade costs could have caused this century's global imbalances. The paper has appeared as CNB Working paper 2008/07.

The decomposition and econometric analysis of air emissions in a transition country: the case of the Czech Republic

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Abstract

The purpose of this paper is to analyze air emissions of selected pollutants in a transition country – the Czech Republic. The paper applies an index decomposition analysis to gauge the importance of economic growth, the change in the structure of the national economy, and emission intensity of the air emissions of selected pollutants by sectors. A novel feature of the decomposition exercise is that two approaches to the decomposition, coined economic and engineering, are defined and compared. Then, the paper applies the panel-data econometric techniques to explain the change in the emission intensities. The paper concludes that (i) during the first years of the transition, the structural effect was dominant, (ii) the intensity effect became more important in later years, and (iii) the fall in the emission intensity has been associated with an increase in labor productivity, and therefore the productivity growth has been environment-saving rather than environment-intensive.

Key words: Decomposition analysis, Index numbers, Air Emissions, Transition country

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All errors remaining in this text are the responsibility of the author.

1 Introduction

The Czech Republic, together with other Central European transition economies, has undergone crucial changes when transforming from a centrally planned economic system towards a market system since 1990. These changes have been not only economic or political, but environmental as well. Although the transition economies are still material and energy intensive (relative to the old EU countries[1]), significant structural economic changes have occurred and the quality of the environment has improved in many areas. This also applies to the air quality, which has been bettered by a significant reduction in air emissions[2].

This paper asks how this happened. There are multiple possible explanations of the change in environmental stress; the most important ones include: changes in economic structure, environmental investments, regulation, and technological change. This paper aims at gauging the role of these determinants with the emphasis on recent changes in air emissions in the Czech Republic.

The index decomposition analysis (hereafter, IDA) is a popular tool to determine the impact of economic activity, the structure of a national economy, technology, and possibly of other factors on a chosen environmental indicator; Ang (1999). Nevertheless, alternative approaches, such as econometric decomposition analysis; Stern (2002); or structural decomposition analysis; Hoekstra, van den Bergh (2003); have been proposed too. Each approach has different data requirements and provides different pieces of information.

This paper attempts at reaping benefits of the various approaches. First, it uses the IDA to isolate the impacts of changes in economic activity, economic structure, and emission intensity. Then, panel-data econometrics is applied to explain the relation between the changes in emission intensities and factor accumulation in the manufacturing sector. By this, it is possible to combine the advantages of IDA with the advantages of the econometric approach to the production function suggested by Stern (2002).

There are several papers investigating the change in environmental burden in

[1] The energy intensity, measured by the energy used per unit of GDP, was about 50% higher in the former Czechoslovakia than in the EU countries in 1990. Although, the energy intensity has fallen considerably, it is still about 30% higher today in the Czech Republic than in the old EU countries.

[2] For example, the emissions of NO_x fell after the first 15 years of transition to about 50% of the 1990 level in the Czech Republic, 45% level in the Slovak Republic, 65% in Poland, and 85% in Hungary. Similar figures apply for the emissions of SO_2 and particulate matter.

transition countries. Viguier (1999) compares the emissions of SO_2, NO_X, and CO_2 in three transition countries (Hungary, Poland, Russia) to three high-income OECD countries (France, the UK, the US) for the time period 1971-1994. Based on the IDA, he concludes that high emission intensities (measured by emissions per unit of GDP) are due to high energy intensities. Cherp et al. (2003) study the environmental performance in Russia and conclude that environmental improvements in the 1990s were caused mainly by the economic decline during the transition process. Markandya et al. (2006) analyze energy consumption in transition countries and find that the fall in their energy intensity is counterbalanced by increase in their economic activity.

There have been several studies focused on the Czech Republic. Earnhart, Lízal (2006) analyze the effects of ownership structure on corporate environmental performance and examine the relationship between financial performance and environmental performance on a panel of Czech corporations. Earnhart, Lízal (2007) study the effect of corporate environmental performance on financial performance: they assess the extent and direction of whether good environmental performance affects revenues, costs, or both.

Brůha, Ščasný (2006) is closest to the present paper and investigate the air emissions in the Czech Republic between 1995 and 2003. The present paper is a significant methodological improvement over Brůha, Ščasný (2006) in several dimensions: it applies several IDA formulas to robustify results; it introduces and compares economic versus engineering approaches to the emission intensity change; it improves the econometric exercise and makes it more robust using a larger set of models.

The rest of the chapter is organized as follows: Section 2 describes the main stylized facts for the Czech Republic and explains the data used in the analysis. Section 3 describes the IDA approach to gauge the factors behind the change in emissions of selected air pollutants. Section 4 then applies panel-data techniques to explain the change in emission intensities in the manufacturing sector. Section 5 concludes.

2 Data and stylized facts

The first years of the economic transition in the Czech Republic (1990-1992) witnessed a 12% drop in the real GDP. After the year 1993, the Czech economy started recovering and the economic growth has become positive since then (with the exception of the mild recession during the years 1997-1999). The transition was also accompanied by strong changes in the economic structure: the agriculture, the mining industry and the energy sector have lost their shares, while the sectors of manufacturing goods, market services (including construction and transportation), and public services have gained (see Figure 1 for the Czech GDP from 1995 to 2004).

The bad state of the environment at the beginning of the transition called for public interventions. The most important interventions related to the air emissions were the following:

Emission limits applied to the large power plants, required to be fulfilled by the year 1998.

Gasification subsidies provided from the State Environmental Fund to municipalities and households, preferentially in heavily polluted regions in the period 1993-1997. The aim of the subsidies was to substitute brown coal for natural gas used for heating.

Subsidies for installation of electric heating systems from the State Environmental Fund to households and small private and public bodies under the Air Quality Program in 1993-1997.

An increase in the VAT rate on energies: from the lower rate (5%) to the standard rate (23%) in 1998, which impacted on the consumer prices of energies (coal, gas, electricity).

A change in energy market regulation: a gradual abolishment of electricity-price cross-subsidy from industry to households; the effect being a continual decrease in the electricity price for industry.

As a response to the public environmental interventions, and to the increased environmental concern in general, production sectors spent significant amounts of environmental investments in the Czech Republic. Figure 2 displays the environmental investments in air protection by sectors. The figure reveals that environmental investments for in protection were significant only in a limited set of sectors, which included chemical and metallurgical industry, the energy sector, transportation and public services. In absolute figures, the investments were most significant in the sectors of energy and public services. The energy sector is also prominent in relative numbers: the investments amounted about 15% of the value added. This relative performance is followed by the chemical industry (about 4% of the value added). The metallurgical sector and public services had environmental investments in air protection at about 2% of the value added, and the ratio of the investment on the value added in the rest of the sectors was less than 0.5%. The time pattern is interesting as well: the environmental investments in air protection attained highest values (both absolutely and relatively to the value added) in the years 1994-2000. Since the year 2002, they are less than 1% of the value added in almost all sectors.

The levels of emissions in the Czech Republic in 1995-2004 are summarized in Figures 3 - 6 below. I will discuss them in more detail.

The emissions of sulphur dioxide (SO_2) were reduced in all sectors, but the most significant reduction occurred in the energy sector and manufacturing (both in the absolute terms and relative to the 1990 sectoral level). A significant relative decline also occurred in the sectors of market and public services. The transportation and mining sectors witnessed the lowest relative declines (by about 40%). Overall, at the end of the 1990s, the level of SO_2 emissions was less than 20% of the 1990 level. Since the beginning of the 2000s, the levels of the emissions have remained relatively unchanged.

The fall in the level of nitrogen oxides (NO_x) emissions has not been as dramatic as for SO_2 emissions. Moreover, there are sectors which have increased their emissions: these are the public services, transportation sector, and construction. The most significant absolute increase occurred in the transportation sector. Nevertheless, the total level of the NO_X emissions is now about 50% of their 1990 level due to the decrease in the energy, agriculture, and manufacturing sectors.

The level of carbon monoxide (CO) emissions has slightly decreased during the 15 years of the transition, the decrease being about 30%. The decline in the manufacturing sector has been partially offset by the increase in the transportation sector. Also, construction has increased its CO emissions.

The particulate matter (PM) emissions have decreased to less than 20% of the 1990 level. The decrease was most significant during the 1990s and occurred especially in the energy and manufacturing sectors. With the exception of the transportation sector, which has increased its emissions by about 40%, all other sectors have decreased their emissions. Since 2000, the level and structure of the PM emissions have been relatively stable.

3 The index decomposition analysis

3.1 General theory

The goal of the index decomposition analysis is to understand historical changes in an environmental or other socio-economic indicator, and to gauge the driving forces or determinants that underlie these changes.

The application of the IDA to environmental and energy indicators has been used especially to assess the influence of economic growth, sectoral shifts, and technology changes, which correspond to the following three effects. The **scale effect** measures the effect of total output change on the indicator. The **composition effect** assesses the effect of a shift in the value-added shares of sectors in the economy. The **intensity effect** describes the effect of changes in the sector level use of the indicator per unit of value added. The decomposition to these three effect is a standard exercise in energy and environmental economics and can be motivated by sustainability concerns. As shown by Brock and Taylor (2005), sustainable development with a non-decreasing level of economic activity is impossible without the intensity effect [3]. Moreover, as transition countries underwent significant changes both in the structure of their GDP as well as in technology used, it is worth distinguishing these two effects in historical data.

This paper performs this decomposition and analyzes the case of four 'classical' pollutants: SO_2, NO_X, CO, and particulate matter (PM).

Let us consider the environmental indicator Φ, which is given as

$$\Phi_t = \Upsilon_t \sum_i \phi_{1it} \dots \phi_{Mit}, \tag{1}$$

where Υ_t is the real GDP, and the summation runs over sector shares, relative energy carriers, or another interesting dimension. The goal is to decompose the change in the indicator into a number of determinants.

If observations were available in continuous time, the decomposition would be straightforward: the percentage change in the indicator $\dot{\Phi}_t/\Phi_t$ could be written as follows:

$$\frac{\dot{\Phi}_t}{\Phi_t} = \frac{\dot{\Upsilon}_t}{\Upsilon_t} + \frac{\sum_i \frac{\dot{\phi}_{1it}}{\phi_{1it}} \phi_{1it} \dots \phi_{Mit}}{\sum_i \phi_{1it} \dots \phi_{Mit}} + \dots + \frac{\sum_i \frac{\dot{\phi}_{Mit}}{\phi_{Mit}} \phi_{1it} \dots \phi_{Mit}}{\sum_i \phi_{1it} \dots \phi_{Mit}},$$

where $\frac{\dot{\Upsilon}_t}{\Upsilon_t}$ is the real GDP growth, and the expression $\frac{\sum_i \frac{\dot{\phi}_{mit}}{\phi_{mit}} \phi_{1it} \cdots \phi_{Mit}}{\sum_i \phi_{1it} \cdots \phi_{Mit}}$ could be interpreted as a weighted percentage change in the factors ϕ_{mit}. The problem is that observations are not available in continuous time, and therefore discrete-time approximations should be used.

A discrete-time decomposition approximation can adopt an additive or a multiplicative mathematical form. The additive form decomposes the difference in the indicator Φ between times t_1 and t_2 into the sum of determinants D_i and a residual term R:

$$\Phi_{t_2} - \Phi_{t_1} = D_1 + D_2 + \dots + D_N + R.$$

[3] The intuition why the composition effect cannot solve sustainability problems in the long run is clear. Even if the least polluting sector were dominating the economy, its environmental intensity must decrease for achieving sustainable development.

The multiplicative form decomposes the relative growth of the indicator into the product of determinant effects:

$$\frac{\Phi_{t_2}}{\Phi_{t_1}} = D_1 \times D_2 \times \ldots \times D_N \times R.$$

It is also possible to consider a hybrid additive-multiplicative form:

$$\frac{\Phi_{t_2}}{\Phi_{t_1}} = \frac{\Upsilon_{t_2}}{\Upsilon_{t_1}} \left(D_1 + \ldots + D_N + R\right),$$

where the scale effect $\Upsilon_{t_2}/\Upsilon_{t_1}$, i.e.; the growth in the real GDP Υ, enters in a multiplicative way, while other effects enter in an additive way; such a formula has been used e.g. by Brůha, Ščasný (2006).

A number of mathematical forms for the additive as well as multiplicative decomposition forms has been proposed. Ang (1999, 2004) provide useful overviews of mathematical forms and their useful properties. The following four properties are particularly relevant to the index decomposition analysis:

Exactness: an exact decomposition has no residual; in the additive case this means that the residual equals 0, while it equals 1 in the multiplicative case.

Time reversal: the decomposition satisfies this property whenever the decomposition yields the reciprocal results after the reversal of the time periods.

Factor reversal: concerns the invariance with respect to the permutation of determinants.

Robustness: a decomposition is robust if it does not fail when it comes across zero (or even negative) values in the dataset.

3.2 Economic versus engineering approach to decomposition

As mentioned above, the IDA is applied to decompose environmental indicators into the three effects: the scale effect, the intensity effect and the composition effect. The scale effect is related to the change in the real GDP and its definition is uncontroversial. However, the definition of the two other effects – composition and intensity – are not as straightforward: alternative definitions of sector shares and emission intensities are possible. The complication is that not only the overall price level changes, but also relative prices of sector products change from one year to another. Therefore, one should be careful about the appropriate definition of the sector shares in the decomposition formula. In general, they can be based on nominal sectoral prices, or on real sectoral prices (with respect to the base year). I will propose two approaches, which

I call 'economic' and 'engineering', because of their different interpretation of results.

The *economic approach* answers the question how many emissions are used to produce a unit of the value added in a given year. The *engineering* approach is more relevant for the understanding of emission intensities of physical amounts (rather than monetary valuations), and therefore is directly related to technological issues.

To describe the two approaches formally, I need to introduce the following notation: Ω_t denotes the environmental indicator (total emissions), Y_t denotes the nominal GDP, Υ_t is the real GDP, and Ω_{it}, Y_{it} and Υ_{it} are sector-wide counterparts[4]. Let d_t be the GDP deflator, and d_{it} is the deflator relevant for sector i, i.e., $Y_t = (1 + d_t)\Upsilon_t$, and $Y_{it} = (1 + d_{it})\Upsilon_{it}$. Define the nominal and real shares as follows: $s_{it} = Y_{it}/Y_t$, and $\sigma_{it} = \Upsilon_{it}/\Upsilon_t$. Define environmental intensities as: $a_{it} = \frac{\Omega_{it}}{Y_{it}/(1+d_t)}$ and $\alpha_{it} = \frac{\Omega_{it}}{\Upsilon_{it}}$. The two intensities are linked by the identity $\alpha_{it} = a_{it}\frac{1+d_{it}}{1+d_t}$. Note that I use Latin characters for nominal variables, while Greek letters are reserved for real variables.

It can be easily checked that total emissions of the year t can be decomposed either as:

$$\Omega_t = \Upsilon_t \sum_i a_{it} s_{it}, \tag{2}$$

or as:

$$\Omega_t = \Upsilon_t \sum_i \alpha_{it} \sigma_{it}. \tag{3}$$

The economic approach is based on Decomposition (2): the shares should be defined using the nominal structure of the GDP: s_{it}. The reason is that the nominal structure of the GDP informs us about the distribution of the value added across sectors. The intensity and the composition effects then also encompass the change in the relative valuation of sectors. To illustrate the point, consider a sector which does not change its technology, but the market increases the valuation of the sector products relative to the other sectors. Then, the emission intensity of this sector (according to the economic approach) will fall, only because the value added produced in the sector is now better valued. This makes a perfect sense from the economic point of view: it answers how the production of the real GDP (value added) is environment-intensive.

[4] Thus, Ω_{it} is emissions by a sector i in the year t, Y_{it} and Υ_{it} are its nominal and real products respectively.

The engineering approach is based on a different notion of emission intensities. The sectoral shares are based on σ_{it}, i.e., the composition effect ignores relative-price movements. This means that the emission intensity of a sector will change only if the technology changes, not because of a change in relative prices. Therefore, Equation (3) is relevant here.

These two approaches can be linked as follows:

$$\Omega_t = \Upsilon_t \sum_i \alpha_{it} \varpi_{it} s_{it}, \tag{4}$$

where $\varpi_{it} \equiv \frac{1+d_{it}}{1+d_t}$. Now, Equation (4) decomposes the emissions into four effects:

Scale effect is related to the real economic growth, i.e., a change in Υ_t.
Pure technology effect is related to changes in α_{it}.
Pure composition effect is related to changes in the structure of the current-year GDP, s_{it}.
Relative-price effect: is due to changes in relative prices of sector outputs, i.e., changes in ϖ_{it}.

Economic studies usually consider the scale effect, the pure composition effect (s_{it}) and the intensity effect, which is given as a combination of the pure technology effect and the relative-price effect ($a_{it} = \alpha_{it}\varpi_{it}$). From the engineering point of view, it could be more appropriate to isolate the pure technology effect, and to base the decomposition on (3) rather than on (2). Note that if the relative prices were constant, the two approaches would be equivalent.

A mathematical form of the IDA, which passes the factor reversal test, will have the following nice property: the decomposition results based on (4) can yield directly the results for decompositions based on (2) and (3) by a direct summation (the additive formulas) or multiplication (the multiplicative formulas) of the relevant factors.

3.3 Log mean Divisia index

This paper applies the log mean Divisia index (henceforth *LMDI*), suggested by Ang (2004) as the preferred method under a wide range of circumstances: the LMDI satisfies the four requirements mentioned above. The LMDI has both a multiplicative and an additive form. The multiplicative form will be used in the subsequent analysis.

The multiplicative log mean Divisia index is defined as follows:

$$D_j^{t_2,t_1} \equiv \exp\left(\sum_i \frac{\mathcal{L}(\Phi_{it_2},\Phi_{it_1})}{\mathcal{L}(\Phi_{t_2},\Phi_{t_1})}\log\left(\frac{\phi_{jit_2}}{\phi_{jit_1}}\right)\right), \qquad (5)$$

where $\Phi_{it} \equiv \prod_{j=1}^{m} \phi_{jit}$ and $\mathcal{L}$ is so-called logarithmic average:

$$\mathcal{L}(x_1,x_2) \equiv \begin{cases} \frac{x_1-x_2}{\log x_1 - \log x_2} & \text{if } x_1 \neq x_2 \\ x_1 & \text{otherwise.} \end{cases}$$

The residual term satisfies $R = 1$, since the *LMDI* is an exact approach.

Thus, when applied to (2), the *intensity effect* is given as:

$$D_a^{t_2,t_1} = \exp\left(\sum_i \frac{\mathcal{L}(a_{it_2}s_{it_2}, a_{it_1}s_{it_1})}{\mathcal{L}(\sum_j a_{jt_2}s_{jt_2}, \sum_j a_{jt_1}s_{jt_1})}\log\left(\frac{a_{it_2}}{a_{it_1}}\right)\right),$$

and the *structure* effect is given as follows:

$$D_s^{t_2,t_1} = \exp\left(\sum_i \frac{\mathcal{L}(a_{it_2}s_{it_2}, a_{it_1}s_{it_1})}{\mathcal{L}(\sum_j a_{jt_2}s_{jt_2}, \sum_j a_{jt_1}s_{jt_1})}\log\left(\frac{s_{it_2}}{s_{it_1}}\right)\right).$$

Similar formulas can easily be derived for (3) and (4).

Note that the LMDI formula satisfies the factor reversal test and therefore the decomposition results $D_\alpha^{t_2,t_1}$, $D_s^{t_2,t_1}$, and $D_\varpi^{t_2,t_1}$ based on (4) immediately yield the decomposition in (2) and (3). In particular, $D_a^{t_2,t_1} = D_\alpha^{t_2,t_1}D_\varpi^{t_2,t_1}$ does so for the economic decomposition, while $D_\sigma^{t_2,t_1} = D_s^{t_2,t_1}D_\varpi^{t_2,t_1}$ for the engineering decomposition. This can also easily be checked using Formula (5).

3.4 Empirical results

Figure 7 displays the decomposition of the air emissions of the four pollutants based on the LMDI. The decomposition is based on Equation (4) for the ten-year period of 1995-2004.

For presentation purposes, I present the results of the multiplicative decomposition in logs: the multiplicative formula can be re-cast in the 'percentage' decomposition as follows (using the notorious approximation $\log(X_2/X_1) \cong$

$X_2/X_1 - 1$):

$$\frac{\Phi_{t_2} - \Phi_{t_1}}{\Phi_{t_1}} = \log(D_1) + \ldots + \log(D_N) + \tilde{R},$$

where now, the percentage change in the indicator Φ is decomposed in determinants, interpreted as contribution to the percentage change. Now, the residual term $\tilde{R}$ is not just the logarithm of the original residual term R, but it also contains the approximation error $\log\left(\frac{\Phi_{t_2}}{\Phi_{t_1}}\right) = \frac{\Phi_{t_2}-\Phi_{t_1}}{\Phi_{t_1}}$.

The results of the decomposition are as follows. The SO_2 emissions were falling in almost all the years. The most significant drop occurred in the years 1995 to 1999, and it is explained by the pure technology effect. This change in 'pure' technology coefficients came especially from the energy sector: by the year 1999, the power plants were obliged to comply with the stringent regulation (emission limits). During the 1990s, also the change in the economic structure helped the emission decrease, driven mainly by the pure composition effect. The relative prices fluctuated so that they do not seem to contribute in a clear way to the emission intensity of the GDP. Since the year 2001, there has been little change in the SO_2 emissions and the determinants have change their signs from year to year.

The NO_x emissions decreased mainly during the 1990s. Since the year 2001, they have fluctuated around the value of 2.75 million tonnes per year. Moreover, they even increased during the last years unlike the other pollutants, which stay almost constant. Contrary to the case of SO_2 emissions, the main determinant of the decrease during the 1990s was the pure composition effect.

The CO emissions were decreasing during the 1990s and they have been almost constant since the year 2001. The decrease during the 1990s was due to the pure technology effect. The contribution of the two effects has fluctuated from positive to negative values since the year 2000.

The amount of the particulate matter emissions has decreased in almost all the years in the sample (with the exception of the slight growth from 2000 to 2001). The main determinant was the pure technology effect. From the year 1996 to the year 2000, the average annual decrease in the manufacturing sector was almost 50%.

To check whether the results are sensitive to the chosen mathematical form of the decomposition (the LMDI formula), I also apply an alternative approach: the ideal Fisher decomposition, introduced by Siegel (1945), and further investigated by Ang et al (2004). The results of the Fisher decomposition are reported in Figure 8. The reader can easily check that both the approaches yield virtually identical results.

4 Econometric analysis

The decomposition analysis in Section 3 decomposes the total emissions into the scale effect, the pure composition effect, the relative price effect, and the pure technology effect. This is important, but the driving forces behind these effects are worth further analysis. The composition effect has been probably driven by the convergence of the transition economy to the structure of the more developed OECD economies. Therefore, this paper seeks to explain the intensity effect [5].

There is another reason to study the intensity effect: the composition effect alone is not a long-run solution for environmental improvements. Either the level or the intensity (technology) effects are needed if the level of the environmental quality has to be sustained or increased; Brock and Taylor (2005).

To study the intensity effect, I use panel-data techniques to regress changes in the emission intensity (measured by the amount of emissions on the value added) of sectors in the manufacturing industries on a number of potential determinants. The analyzed sectors are as follows:

- food products, beverages and tobacco,
- textiles and textile products,
- wood and wood products,
- pulp, paper and paper products; publishing and printing,
- chemicals, chemical products and man-made fibres,
- rubber and plastic products,
- other non- metallic mineral products,
- basic metals and fabricated metal products,
- machinery and equipment,
- electrical and optical equipment,
- transport equipment,
- other manufacturing.

The following two manufacturing sectors are excluded: the sector of leather and leather products, and of coke, refined petroleum products. The reason is that these two sectors had zero (or even negative) value added in some years during the sample period and therefore it would be impossible to define emission intensity.

[5] Here, I explain the intensity effect from the economic point of view, i.e., the effect which is the combination of the improvement in technology and movements in relative prices. It would be hard to distinguish these two effects using the national account data only.

The following regressors are considered among the explanatory variables:

- change in labor productivity (labor productivity measured by the ratio of the value added to wages);
- various investments;
- the share of environmental investments in air protection on total investments;
- time trend and a dummy for years before 2000.

The reason why I include the change in labor productivity is to infer whether the increase in the productivity was associated with a better environmental performance, i.e., whether the increase in labor productivity was resource-saving or rather resource-using. On the one hand, if transition countries were used as the 'pollution haven' (because of relatively less stringent environmental legislation at the beginning of the transition), then one may expect that the increase in labor productivity would increase emission intensity. On the other hand, if technological processes were highly inefficient comparing to the ones used in advanced countries, then one may expect that the technological spill-overs (either through FDI or through imitation) would help environmental performance despite any comparative advantages of transition countries in lax environmental standards.

I considered various types of investments, such as investments in buildings (relevant especially for SO_2 and PM emissions since they are related to heating), investments in transport equipment (relevant especially for NO_x and PM emissions), and investments in software, which I consider as a proxy for high-tech investments. All investments are measured as a ratio of the value of investment to the value added. By considering the investment, I want to inquire whether the new capital is better from the environmental point of view than the old one. For example, the investment in transport equipment may increase the emission intensity if firms use more transport services in production, but on the other hand, if new transport equipment is more environmentally effective, then the emission intensity may indeed fall as a result of these investments.

I consider the time trend and the time dummy to control for huge changes in emission intensities in earlier years of the transition, which may reflect an autonomous technological change (i.e., imitations of foreign technologies).

Because dozens of variables can enter the regression, there is a variable selection problem. The statistical literature has proposed a bulk of methods; see Burnham, Anderson (2002) for a survey. In this paper, I apply the cross-validation method suggested by Shao (1993) and further discussed by Shao (1997). I evaluate a large set of candidate regression models. As a model selection tool, I use the cross-validated approximation to the model mean-square error. As an alternative, I use the bootstrapped approximation to the population mean-square error, as suggested by Shao (1997). Both approaches yield exactly the same 'winning' model for all pollutants, which makes me feel fine about the results.

The winning model was then compared with the null benchmark, consisting of fixed-effect constants only. The winning model and the benchmark were then compared based on the mean-square error criterion. Since the winning model was obtained essentially by a data-mining approach, the usual statistics can give misleading results as of whether the winning model is really better than the benchmark[6]. To overcome this difficulty, I follow White (2000) and implement the 'Bootstrap Reality Check' (henceforth BRC) to inquire whether the 'winning' model is indeed better than the benchmark.

Moreover, to inquire the robustness of the winning regression, I also report four alternative regressions. It seems that the coefficients of the winning regressions are robust to the inclusion of other regressors. In all cases, I apply the fixed-effect panel data model to control for potential correlations of regressors with unobserved heterogeneity of sectors. The random-effect models were either rejected (Hausman test), or in some cases the random-effect estimators had troubles converging. Therefore, I report the fixed-effect models only.

Table 1 reports the results for the change in SO_2 intensity in manufacturing. The following regressors appear in the 'winning' model: change in the labor productivity, investments in buildings, and dummy for years 1995 to 1999. The change in labor productivity appears to be the most influential regressor and the sign is negative. This suggests that the increase in the labor productivity is associated with the decrease in the emission intensity. The coefficient of investments in buildings is insignificant and negative, which may be interpreted as weak evidence that new buildings are more energy-saving (the coefficient becomes significant if the time dummy is removed from regression). The time dummy is significant and negative, which implies that the most important improvements occurred before the year 2000. When environmental investments are included, they have the correct sign, but the coefficient is small. The inclusion of the proxy for high-tech investments (software investments) does not reveal anything interesting: the coefficient is small and insignificant. The BRC rejects the hypothesis that the 'winning' model is better than the null benchmark by chance only.

Since the dummy variable for the years 1995-1999 explains a large portion of the fall in emissions, I check the robustness of the regression results. Basically,

[6] This is discussed by White (2000): '*Data snooping occurs when a given set of data is used more than once for purposes of inference or model selection. When such data reuse occurs, there is always the possibility that any satisfactory results obtained may simply be due to chance rather than to any merit inherent in the method yielding the results.*'

I re-run the regressions on the sample since the year 2000 (and obviously the time dummy is removed). The results are displayed in Table 2. The rise in labor productivity and investments in building remain significant and the sign of the two coefficients does not change. This indicates that the growth in labor productivity was environment-saving even after the year 2000. Environmental investments in air protection ceases to be significant, which is consistent with the notion that these investment had the major effect in the earlier phase of the transition.

Table 3 reports the results for the change in NO_x intensity. The following regressors appear in the 'winning' model: the change in labor productivity, the time trend, and the total investments. Nevertheless, the model is poor (the R^2 is very low: 8%) and moreover, the BRC accepts the hypothesis that the winning model is better than the null benchmark by chance. The only significant variable is the change in labor productivity with the negative sign (which means that the productivity improvement was associated with the fall in emission intensities). Overall, the model is unable to offer a satisfactory explanation for the NO_x emissions at the sub-sector level.

Table 4 displays the regression results for the chosen models when the sample is restricted to the post-2000 period. The growth in labor productivity is again the significant variable (and its negative sign is preserved). On the other hand, the overall performance of the regressions improves (e.g., the R^2 measure increases to about 25%).

Table 5 reports the results for the change in CO intensity in manufacturing. Five regressors appear in the 'winning' model, but only two of them are significant at the conventional 5% level: the change in the labor productivity and investments in buildings. The signs of these coefficients are the same as in the case of SO_2 intensity: the increase in labor productivity is associated with the decrease in emission intensity, as are investments in buildings, which may confirm the interpretation of the SO_2 results that new buildings are more energy-efficient. Also, similarly to the SO_2 case, the time dummy has the negative sign, which means that the most important improvements occurred before the year 2000, but now the coefficient is on the edge of significance. Environmental investments and software investments do not prove to be significant. The BRC rejects the hypothesis that the 'winning' model is better than the null benchmark only by chance.

Table 6 displays the regression results for regression models when the sample is restricted to the post-2000 period. As in the case of SO_2 emissions, the rise in labor productivity and investments in buildings are associated with the fall in emissions, even after the year 2000.

Table 7 reports the results for the change in PM intensity. The winning model contains three regressors: environmental investments, the change in labor productivity, and the time dummy. The environmental-investment coefficient has the expected sign (negative), but is not significant. The coefficients for the change in labor productivity and the time dummy are significant and have the same signs as in the case of SO_2 and CO intensities, with the similar interpretation. Other possible regressors (investments in buildings or transport equipment, or the time trend) are insignificant when included in the model. Also for the PM emissions, the BRC rejects the hypothesis that the 'winning' model is better than the null benchmark only by chance.

Lastly, Table 8 checks the robustness of the regressions for PM emissions on the restricted sample. The results suggest that the results for the full sample are robust in this respect.

To summarize, in all four cases, the increase in labor productivity is associated with the fall in emission intensity, which means that the productivity growth was environment-saving rather than environment-intensive. With the exception of the NO_X emissions, the data-snooping procedure of Shao (1993) succeeds in finding a model which explains about one fourth of the change in the emission intensities (and more than one third for the SO_2 emissions), which is not bad at the sector level data. The bootstrapped version of the Reality Check (suggested by White, 2000) reveals that the winning regression is better than a simplistic benchmark, which consists of sector intercepts only, for all emissions except of NO_X. I was unable to find a satisfactory model for the latter case. If data are restricted to the post-2000 period, the econometric model can explain about 25% of the change in NO_X emissions and the model with labor productivity beats the null benchmark. The regression results for all pollutants seem to be robust both with respect to the inclusion of other variables.

5 Conclusion

This paper attempts at explaining a decrease in the levels of selected air pollutants in the Czech Republic during the first years of its economic and political transition. First, an index decomposition exercise is used to assess the relative importance of the level, sectoral composition, relative price, and technological changes to air pollution emissions during the transition. I find that — although the composition effect was important — a reduction of in certain pollutants (SO_2 and particulate matter) was caused mainly by a significant drop in emission intensities. This drop was caused by environmental regulation especially in the energy sector.

Then, an econometric exercise is used to explain the change in the technology effect in manufacturing. This exercise reveals that environmental investments had only a limited impact in the manufacturing sector. On the other hand, the most significant reductions in emission intensities have been associated with increases in labor productivity, which suggests that the productivity increase in the Czech Republic has been environment-saving rather than environment-intensive.

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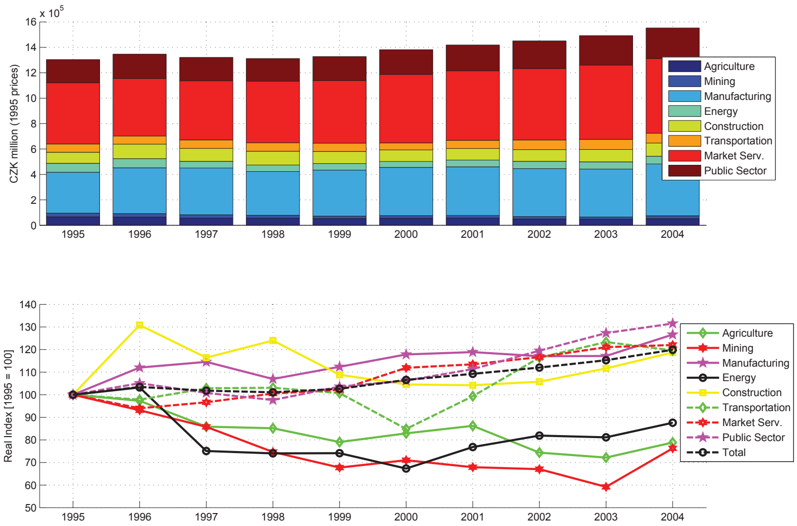
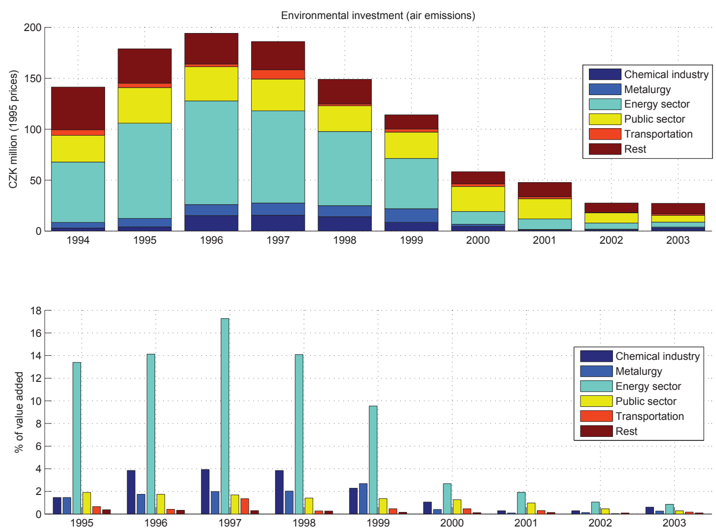
Fig. 1. Real GDP in the Czech Republic in 1995-2004



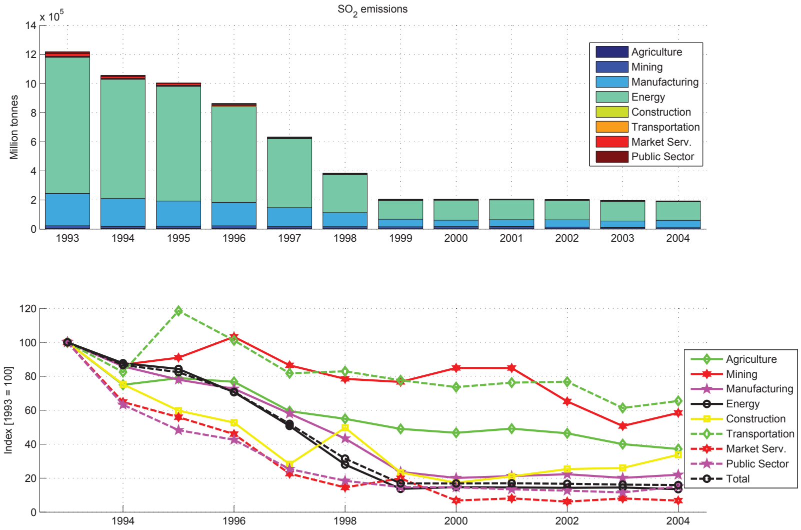
Source: Czech Statistical Office

Fig. 2. Environmental investment in air protection in the Czech Republic



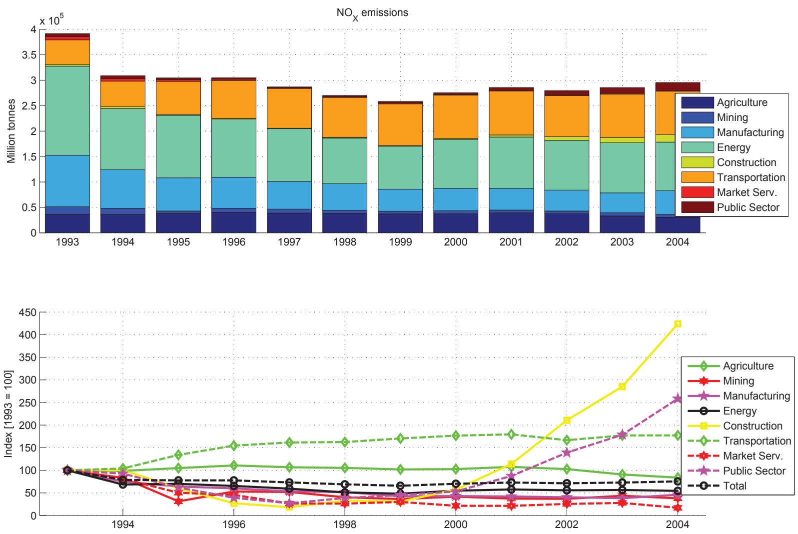
Source: Czech Statistical Office, own calculation

Fig. 3. Sulphur Dioxide (SO_2) emissions in the Czech Republic



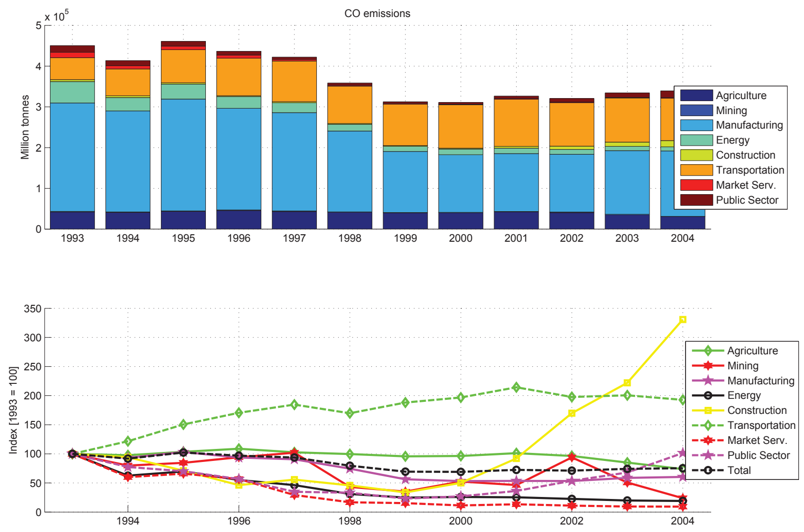
Source: Czech Hydrometeorlogical Institute, Czech Statistical Office, own calculation

Fig. 4. Nitrogen Oxides (NO_X) emissions in the Czech Republic



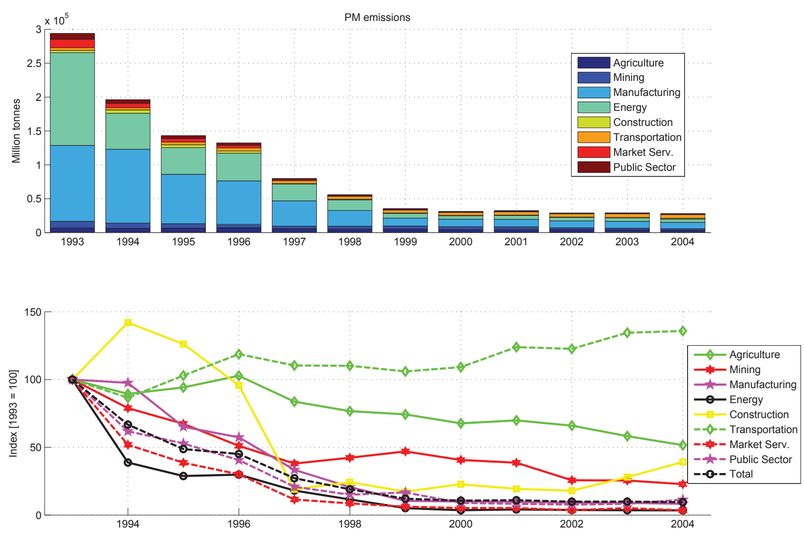
Source: Czech Hydrometeorological Institute, Czech Statistical Office, own calculation

Fig. 5. Carbon monoxide (CO) emissions in the Czech Republic



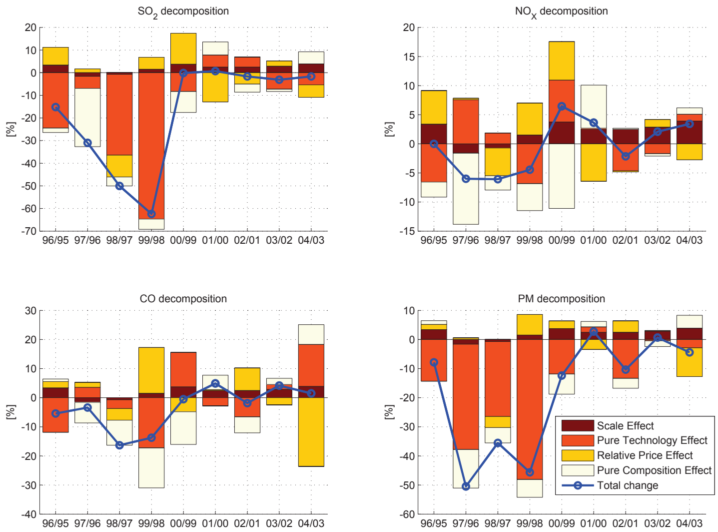
Source: Czech Hydrometeorlogical Institute, Czech Statistical Office, own calculation

Fig. 6. Particulate matter (PM) emissions in the Czech Republic



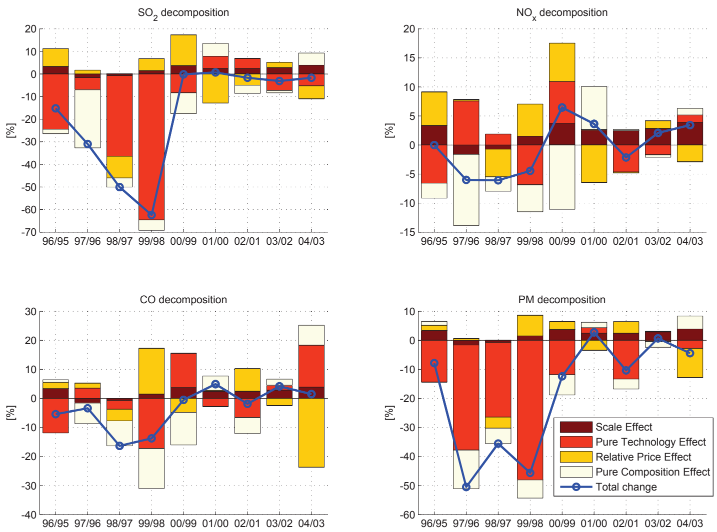
Source: Czech Hydrometeorlogical Institute, Czech Statistical Office, own calculation

Fig. 7. Log mean Divisia index decomposition of air emissions in the Czech Republic



Source: Own computation

Fig. 8. Ideal Fisher index decomposition of air emissions in the Czech Republic



Source: Own computation

Table 1
Explanation of the change in SO_2 intensity in manufacturing (full sample)

Dependent variable: change in SO_2 intensity					
Method: fixed-effect model					
Variable	Winning model	Alt. model 1	Alt. model 2	Alt. model 3	Alt.model 4
Change in labor productivity	-0.683 0.000	-0.719 0.000	- 0.727 0.000	-0.716 0.000	-0.742 0.000
Investments in buildings	-0.007 0.250	-0.008 0.245	-0.009 0.183		-0.013 0.034
Dummy (95 - 99)	-0.144 0.003			-0.161 0.000	
Trend		0.018 0.085	0.018 0.089		
Investments in air protection		-0.000 0.093	-0.000 0.087		
Software investments			0.040 0.392		
R^2	0.379	0.388	0.392	0.372	0.330
$\overline{R}^2$	0.368	0.371	0.369	0.366	0.324
signf.	0.056	0.056	0.056	0.056	0.060

The point estimates of the coefficients are typed using normal-size script, the p-values are typed using the tiny script.

Table 2

Explanation of the change in SO_2 intensity in manufacturing (sample since 2000)

Dependent variable: change in SO_2 intensity				
Method: fixed-effect model				
Variable	Winning model	Alt. model 1	Alt. model 2	Alt. model 3
Change in labor productivity	-0.583 0.000	-0.589 0.000	- 0.597 0.000	-0.636 0.000
Investments in buildings	-0.019 0.018	-0.018 0.045	-0.018 0.045	
Trend		-0.000 0.968	-0.000 0.966	
Investments in air protection		-0.000 0.207	-0.000 0.214	
Software investments			0.033 0.835	
R^2	0.275	0.297	0.298	0.208
$\overline{R}^2$	0.264	0.263	0.251	0.208
signf.	0.069	0.069	0.070	0.074

The point estimates of the coefficients are typed using normal-size script, the p-values are typed using the tiny script.

Table 3
Explanation of the change in NO_X intensity in manufacturing (full sample)

Dependent variable: change in NO_X intensity					
Method: fixed-effect model					
Variable	Winning model	Alt. model 1	Alt. model 2	Alt. model 3	Alt. model 4
Trend	0.024	0.020	0.020	0.025	0.024
	0.286	0.479	0.494	0.267	0.299
Change in labor productivity	-0.771	-0.765	- 0.775	-0.802	
	0.048	0.054	0.053	0.006	
Total investments	-0.004	-0.006	-0.004		-0.046
	0.903	0.839	0.912		0.060
Investments in air protection		-0.000	-0.000		
		0.885	0.876		
Investments in transp. equip.		0.015	0.016		
		0.719	0.702		
Software Investment			-0.040		
			0.751		
R^2	0.080	0.082	0.083	0.080	0.049
$\overline{R}^2$	0.065	0.049	0.042	0.072	0.041
signf.	0.378	0.384	0.387	0.374	0.388

The point estimates of the coefficients are typed using normal-size script, the p-values are typed using the tiny script.

Table 4

Explanation of the change in NO_X intensity in manufacturing (sample since 2000)

Dependent variable: change in NO_X intensity					
Method: fixed-effect model					
Variable	Winning model	Alt. model 1	Alt. model 2	Alt. model 3	Alt. model 4
Trend	0.019 0.400	0.017 0.525	0.017 0504	0.017 0.389	0.002 0.916
Change in labor productivity	-0.930 0.000	-0.926 0.001	-0.939 0.000	-0.870 0.000	
Total investments	-0.007 0.741	0.005 0.827	0.002 0.923		-0.040 0.013
Investments in air protection		0.000 0.968	-0.000 0.941		
Investments in transp. equip.		0.009 0.795	-0.002 0.946		
Software investments			0.215 0.360		
R^2	0.245	0.246	0.256	0.244	0.101
$\overline{R}^2$	0.221	0.196	0.194	0.232	0.087
signf.	0.122	0.126	0.126	0.120	0.1423

The point estimates of the coefficients are typed using normal-size script, the p-values are typed using the tiny script.

Table 5
Explanation of the change in CO intensity in manufacturing (full sample)

Dependent variable: change in CO intensity					
Method: fixed-effect model					
Variable	Winning model	Alt. model 1	Alt. model 2	Alt. model 3	Alt. model 4
Investments in air protection	-0.000		-0.000		
	0.647		0.585		
Change in labor productivity	-0.326	-0.328	-0.305	-0.306	-0.315
	0.012	0.012	0.017	0.017	0.014
Investments in buildings	-0.017	-0.017	-0.018	-0.018	-0.019
	0.013	0.015	0.015	0.016	0.16
Investments in transp. equip.	0.013	0.014			
	0.093	0.366			
Dummy (95 - 99)	-0.102	-0.113	-0.116	-0.130	-0.133
	0.093	0.040	0.047	0.012	0.011
Software investments					0.042
					0.397
R^2	0.253	0.251	0.249	0.247	0.242
$\overline{R}^2$	0.227	0.232	0.229	0.233	0.216
signf.	0.065	0.064	0.064	0.064	0.066

The point estimates of the coefficients are typed using normal-size script, the p-values are typed using the tiny script.

Table 6
Explanation of the change in CO intensity in manufacturing (sample since 2000)

Dependent variable: change in CO intensity					
Method: fixed-effect model					
Variable	Winning model	Alt. model 1	Alt. model 2	Alt. model 3	Alt. model 4
Investments in air protection	-0.000 0.768		-0.000 0.618		
Change in labor productivity	-0.252 0.108	-0.257 0.097	-0.244 0.106	-0.241 0.107	-0.267 0.857
Investment in buildings	-0.021 0.012	-0.020 0.013	-0.021 0.008	-0.022 0.007	-0.022 0.067
Investments in transp. equip.	0.005 0.831	0.009 0.655			
Software investments					0.104 0.505
R^2	0.162	0.162	0.162	0.159	0.165
$\overline{R}^2$	0.121	0.134	0.134	0.145	0.138
signf.	0.069	0.068	0.068	0.066	0.067

The point estimates of the coefficients are typed using normal-size script, the p-values are typed using the tiny script.

Table 7
Explanation of the change in PM intensity in manufacturing (full sample)

Dependent variable: change in PM intensity					
Method: fixed-effect model					
Variable	Winning model	Alt. model 1	Alt. model 2	Alt. model 3	Alt. model 4
Investments in air protection	-0.000 0.165	-0.000 0.159	-0.000 0.168	-0.000 0.159	-0.000 0.153
Change in labor productivity	-0.581 0.000	-0.569 0.000	-0.590 0.000	-0.574 0.000	-0.564 0.000
Dummy (95 - 99)	-0.185 0.010	-0.113 0.003	-0.116 0.010	-0.130 0.021	-0.133 0.056
Investment in buildings		-0.003 0.755		-0.004 0.688	
Investments in transp. equip.			-0.021 0.299	-0.022 0.278	
Trend					-0.015 0.536
R^2	0.246	0.246	0.252	0.254	0.248
$\overline{R}^2$	0.232	0.226	0.232	0.227	0.228
signf.	0.109	0.111	0.116	0.111	0.110

The point estimates of the coefficients are typed using normal-size script, the p-values are typed using the tiny script.

Table 8
Explanation of the change in PM intensity in manufacturing (sample since 2000)

Dependent variable: change in PM intensity					
Method: fixed-effect model					
Variable	Winning model	Alt. model 1	Alt. model 2	Alt. model 3	Alt. model 4
Investments in air protection	-0.000 0.376	-0.000 0.539	-0.000 0.060	-0.000 0.048	-0.000 0.106
Change in labor productivity	-0.522 0.010	-0.476 0.018	-0.511 0.014	-0.442 0.034	-0.518 0.011
Investment in buildings		-0.016 0.124		-0.018 0.010	
Investment in transp. equip.			-0.008 0.795	-0.022 0.505	
Trend					0.012 0.594
R^2	0.151	0.181	0.151	0.188	0.154
$\overline{R}^2$	0.137	0.156	0.124	0.148	0.127
signf.	0.122	0.112	0.124	0.121	0.128

The point estimates of the coefficients are typed using normal-size script, the p-values are typed using the tiny script.

Real Exchange Rate in Emerging Economies: The Role of Different Investment Margins

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Abstract

In this paper we analyze the role of various investment margins in explaining the real exchange rate appreciation recorded in European transition countries. We present a model that introduces a quality investment margin and show that the margin is needed for replicating the observed pace of the real exchange rate appreciation.

Key words: Real Exchange Rate, Emerging Economies, Two-Country Modeling
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1 Introduction

The real exchange rates of the transition economies in Central and Eastern Europe have been continuously and significantly appreciating, in a period of rapid economic growth and gains in international competitiveness. For the years 1995-2005 the average annual pace of the real exchange rate appreciation in the Visegrad-4 countries (i.e. the Czech Republic, Hungary, Poland, and Slovakia) ranged from 3.0 to 4.0% (see Figure 1). We ask the question how this happened.

The prime concept explaining a trend real exchange rate appreciation of fast growing economies is the so called Harrod (1933) – Balassa (1964) – Samuelson (1964) effect. This effect appears as a consequence of unbalanced growth in productivity in favor of tradable versus non-tradable sectors, which is stronger for fast growing countries. Since the Visegrad-4 countries converge to the developed Western EU countries, they could be prone to such an effect. However, the recent empirical evidence, surveyed in Égert et al. (2006) and Mihaljek and Klau (2006), shows that this effect (0.3 – 1.8 p.p. per annum during 1997-2005) accounts only for a fraction (approximately 30%) of the overall real exchange rate appreciation in the Visegrad-4. This is likely caused by small differences in productivity dynamics in tradable (manufacturing) and non-tradable sectors (services), which ranges from 3 to 9% on average over 1995-2005. Moreover, the extent of the effect is further lowered by a relatively small share of non-tradables in the price indices, which in the same decade accounted for 18–23%. Thus, this concept can rationalize the real exchange rate appreciation only to an insufficient extent.

Since the majority of the real exchange rate appreciation in the Visegrad-4 countries remains to be explained, we consider two alternative strands of literature, namely the extensive margin proposed by Krugman (1981) and vertical product differentiation by Flam and Helpman (1987), featuring the quality margin.

In the literature, the extensive margin is used in dynamic general equilibrium two country models to explain an endogenous productivity bias towards exporting firms, i.e., an endogenous Harrod-Balassa-Samuelson effect (Ghironi and Melitz, 2005, Bergin et al., 2006). Therefore, even a relatively faster uniform productivity growth in the converging country can generate real exchange rate appreciation. Since the expansion of the number of available varieties is faster in a converging economy compared to an advanced counterpart, the real exchange rate based on consumer price indices (as opposed to welfare-theoretic indices) will appreciate for the converging economy.

We use a model with an extensive margin to analyze its potential for explaining the real exchange rate appreciation in transition countries. Our simulations show that for a reasonable range of parameters, the extensive margin can explain only between 0.2-1.5 p.p. annual appreciation, which corresponds to the empirical findings on the Harrod-Balassa-Samuelson effect reported above. Therefore, a large portion of the mean real exchange rate appreciation in the Visegrad-4 countries remains to be explained.

Therefore, we turn to the quality margin. An increase in product quality enables exporting at higher prices, which can be compatible with real exchange rate appreciation. Statisticians adjust price indices for quality incompletely, since hedonic indices are rarely used (see a survey by Ahnert and Kenny, 2004). It is therefore very likely that due to the quality bias in price indices, there appears the well established correlation between price level and economic development, documented and explored by Kravis and Lipsey (1988).

The available evidence speaks for quality change in the Visegrad-4 region. Hallak and Schott (2006) estimate cross-country differences in product quality using data on U.S. trading partners. They find that although the quality of products in Hungary and Poland stood at only 40% and 30%, respectively, compared to the Swiss benchmark, it has significantly increased during 1989-2003 by an annual 4% and 2.3%, respectively, relative to the benchmark. Also Cincibuch and Podpiera (2006) assess quality changes in tradable goods during 1997-2003 and find an annual improvement of the Czech Republic and Slovakia relative to Germany by 2.8 and 3.6%, respectively. And finally, Fabrizio et al. (2007) use the unit value ratio of export (the unit value of a country's export divided by the unit value of the world export) to show how the Visegrad-4 region gained competitiveness through improvements in quality. The yearly average improvement in the unit value ratio of exports amounted to 5% during 1994-2004. In addition they report that the share of high quality exports has increased quite dramatically in the Visegrad-4 countries, from 45% to 65% over the decade 1995-2005. Therefore, we see a potential for quality improvements to explain the real exchange rate appreciation in the Visegrad-4 countries.[1]

We ask whether a two-country model with extensive margin, extended by the endogenous quality investment, can generate a realistic range of real exchange rate appreciation. The simulations show that the extended model generates between 0.5 - 4.5 p.p. annual real exchange rate appreciation. We conclude that the quality margin has the potential to explain the real exchange rate appreciation (two-thirds of the entire appreciation) that remains to be explained after accounting for the well-established Harrod-Balassa-Samuleson

[1] Therefore, our research could be also considered as a complement to current thinking on the relation between quality, export volumes, and export prices in the international trade literature, see e.g., Hummels and Klenow (2005), Hallak and Schott (2008), or Feenstra et al (2007).

effect.

The rest of the paper is organized as follows. In Section 2, we describe the proposed model extension and in Section 3 we report the results of simulation. Section 4 concludes. The steady state of the model under a particular functional form is described in the Appendix.

2 Description of the Model

This section presents the proposed model. Since the model aims at explaining long-run issues (the real exchange rate in transition countries) rather than short-run fluctuations around a steady state, the model is formulated as a deterministic dynamic perfect-foresight model. The usage of such a modeling framework is a standard choice in the international trade literature for investigating long-run issues.[2] Thus, contrary to the usual practice of applied DSGE models, which attempt to characterize the short-run fluctuations around a steady state or around an exogenously given development trajectory, the proposed model investigates a long-run convergence trajectory of ex-ante asymmetric countries.

The two countries are modeled in discrete time that runs from zero to infinity. The home country is populated by a representative competitive household that has recursive preferences over discounted streams of momentary utilities. The momentary utility is derived from consumption. A similar household inhabits the foreign country. Production takes place in heterogenous production entities called firms.

2.1 Firms

In the domestic country, there is a large number of firms that are owned by the domestic household. In each period there is an unbounded mass of potential, ex-ante identical, entrants. Each entrant has to pay the fixed entry costs c; the cost is paid in terms of the aggregate consumption bundle. The actual number of entrants is determined by the zero-profit condition.

Firms' ex-post entry differ by an idiosyncratic variation of the total factor productivity: when a firm enters, it draws a shock z from a distribution $G(z)$. At the end of each period, there is an exogenous probability that a firm is hit

[2] See Baldwin and Forslid (2000), Melitz (2003), Bergin et al. (2006), and Melitz and Ottaviano (2008) for examples of perfect-foresight models that are deterministic at the aggregate level.

by an exit shock. This probability is δ and is assumed to be independent on aggregate as well as individual states. Hit firms shut down.

The production of a firm is characterized by two features: physical quantity x and product quality h. If firm j wishes to produce its product with quality level h_j, the firm has to pay the fixed quality investment at the level h_j. Similar to entry costs, quality investments take the form of an aggregate consumption bundle. The quality choice is a once-and-for-all decision undertaken at the entry time (but after the idiosyncratic productivity is revealed).

The production of physical quantities, given as $x_{jt} = z_j A_t \ell(l_{jt}, h_j)$, requires a variable input, labor l. The production function ℓ is strictly increasing in the variable input (labor), but strictly decreasing in the second argument (quality level).[3] This implies that the chosen quality increases the labor inputs needed to produce physical quantities. Thus, quality investment is costly for two reasons: first, it requires fixed input h_j, and second, more variable input is required to produce better goods.

The production of physical quantities is increasing in the level of firm total factor productivity $A_t z_j$, which has two components: (a) the idiosyncratic component z_j, which is i.i.d. across firms and which follows the distribution $G(z)$ introduced above, and (b) the common component A_t. Domestic firms enjoy at time t common productivity A_t, while foreign firms enjoy common productivity A_t^*.

We assume that the final output of the firm is given by a product of quality and quantity as follows: $q_{jt} = h_j x_{jt}$. The final quality-quantity bundle is what is sold at the market. This assumption follows the standard approach of growth theoreticians, for example Young (1998). Thus, the production of the final bundle can be described as $q_{jt} = z_j A_t f(h_j, l_{jt})$, where f is given as $f(h_j, l_{jt}) \equiv h_j \ell(l_{jt}, h_j)$. We assume that the final bundle production function is increasing in both arguments and is homogeneous of degree one. We explicitly distinguish the quality-quantity bundle from the physical quantity since the explanation for the observed real exchange rate appreciation is based on a dichotomy between quality-adjusted and -unadjusted prices.

Firms may export only if special fixed costs are invested. If a firm at the entry time decides to invest the fixed export costs, then it becomes eligible to export in all subsequent periods, otherwise it is in all periods not eligible to export.[4] Therefore, we call such firms exporters, while the other firms are

[3] We require that the function ℓ is strictly decreasing in invested quality. Otherwise the model would imply endogenous growth, as in Young (1998) and Baldwin and Forslid (2000).

[4] Under the Constant-elasticity-of-substitution (CES) market structure assumed in this paper, all firms that paid the fixed export-eligibility costs will find it profitable

non-exporters. Unit iceberg exporting costs ς represent transportation costs and policy barriers such as tariffs, while the fixed costs may represent expenditures associated with acquiring necessary expertise such as legal, business, or accounting issues in foreign markets. The fixed export costs are again paid in terms of the aggregate consumption bundle and are denoted as $c^x > 0$. This assumption implies, as in Melitz (2003), that in the equilibrium there is a cut-off productivity value $\overline{z}$, such that firms with lower idiosyncratic productivity $z_j < \overline{z}$ will not invest to become eligible to export, while firms with a sufficiently high productivity level $z_j \geq \overline{z}$ will.

To make reading the paper easier, we introduce the following convention. The domestic country's variables are without a *, while the foreign country's variables have a *. The good produced by the firm located in the destination market is denoted by the d superscript, while goods imported are denoted by the m superscript. Thus $p^d_{j\tau t}$ will denote the time t price of a good produced by a vintage τ firm j located in the domestic country and sold to the domestic market; $p^m_{j\tau t}$ is the time t price of a good j imported to the domestic market from the foreign country; while $p^{m*}_{j\tau t}$ would be the price of a good exported from the domestic country to the foreign household. We further assume that prices are denominated in the currency of the market, where the good is sold.

The quality investment is a sunk factor as well as the fixed export cost if borne, set at the time of entry, while labor can be freely adjusted. Given a realization of the productivity shock z_j, the probability of the exit shock δ, and a chosen production plan, the value of a firm is determined by the expected present value of the stream of profits.

Let $\mathbb{O}^e_{j\tau t}$ denote the t-period *real* ***o****perating profit* of a domestic exporter of vintage τ enjoying idiosyncratic productivity z_j, and be given as follows:

$$\mathbb{O}^e_{j\tau t} = \left[\kappa_t \frac{p^d_{j\tau t}}{P_t} + (1-\kappa_t)\frac{\eta_t}{1+\varsigma}\frac{p^{m*}_{j\tau t}}{P^*_t}\right] A_t z_j f(h_{j\tau}, l_{j\tau t}) - \mathbb{W}_t l_{j\tau t},$$

where $0 \leq \kappa_t \leq 1$ is the share of the product sold on the domestic markets,[5] P_t is the domestic price level, P^*_t is the foreign price level, η_t is the *real exchange rate*, which is linked to the nominal exchange rate s_t as $\eta_t = s_t P^*_t / P_t$, $\varsigma \geq 0$ represents unit iceberg exporting costs, $\mathbb{W}_t$ is the *real wage*, and $l_{j\tau t}$ is the labor demand by the firm. Similarly, the *real operating profit* of a domestic

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to export in all periods. This is proven in Lemma 2 in the Appendix.

[5] We show in the Appendix (Lemma 2) that in the equilibrium, all domestic exporters export at a particular date t the same share of production to the foreign market, regardless of their vintage τ or productivity j. Therefore, we shall simply write κ_t instead of $\kappa_{j\tau t}$. The vintage and productivity only determine whether a particular firm is an exporter or not.

non-exporter is given as follows:

$$\mathbb{O}^{n}_{j\tau t} = \frac{p^{d}_{j\tau t}}{P_t} A_t z_j f(h_{j\tau}, l_{j\tau t}) - \mathbb{W}_t l_{j\tau t}.$$

Analogous definitions apply to the foreign firm as well.

The products of firms of different vintages have different quality levels (since incentives to invest in quality differ as macroeconomic conditions change), and that is why $\mathbb{O}^{e}_{j\tau t}$, $\mathbb{O}^{n}_{j\tau t}$, $p^{d}_{j\tau t}$, and $q^{d}_{j\tau t}$ will be in general different.

We assume that the firm's manager maximizes the expected discounted stream of profits. Thus, the value of the profit stream of the domestic firm of vintage τ, enjoying the idiosyncratic productivity level z_j is (in real terms):

$$V_\tau(z_j) = \max_{\mathbf{1}^{x}_{j}, h_{j\tau}, \{l_\tau\},} \sum_{t=\tau}^{\infty} (1-\delta)^{t-\tau} \mu^{t}_{\tau} \left[\mathbf{1}^{x}_{j\tau} \mathbb{O}^{e}_{j\tau t} + (1 - \mathbf{1}^{x}_{j\tau}) \mathbb{O}^{n}_{j\tau t} \right] - (c + \mathbf{1}^{x}_{j\tau} c^{x} + h_{j\tau}), \tag{1}$$

where $\mathbf{1}^{x}_{j}$ is the indicator of exporters (i.e. $\mathbf{1}^{x}_{j} = 1$, if firm j is an exporter and $\mathbf{1}^{x}_{j} = 0$ for non-exporters), and the effective discount factor is given as $(1-\delta)^{\tau - t} \mu^{t}_{\tau}$, where μ^{t}_{τ} is the marginal rate of intertemporal substitution between dates τ and t. The rate of the intertemporal substitution is defined in subsection 2.2. The value of the firm owned by the foreign household is defined analogously.

Note that prices such as $p^{d}_{j\tau t}$ are prices of the final quantity-quality bundles and therefore derived indexes P_t, P^{*}_{t}, and η_t are related to aggregations of these final bundles. The prices related to physical quantities are then given by $\wp^{d}_{j\tau t} \equiv h_{j\tau} p^{d}_{j\tau t}$. The discussion about the distinct role of prices per quality-quantity bundle and that of prices defined on physical quantities is left to subsection 2.3.

To summarize the sequencing, the timing proceeds first with the entry of prospecting entrants. Then, each new entrant draws a productivity level from the distribution G and it decides the quality of its production $h_{j\tau}$ and whether to invest for export eligibility c^{x}. Then, labor demand and production (of both entrants and incumbents) take place. At the end of the period, some firms experience the exit shock and shut down.

2.1.1 Market Structure

The final good Q in the domestic country is composed of a continuum of quality-quantity bundles (goods), some of which are produced in the domestic country and some are imported. There is an imperfect substitution among these goods, which is modeled using the standard constant-elasticity-of-substitution (CES) function with the parameter $\theta > 1$. The aggregate good in the domestic country is defined as:

$$Q_t = \left(\sum_{\tau \le t} (1-\delta)^{t-\tau} \left[n_\tau \int q^{d \, \frac{\theta-1}{\theta}}_{j\tau t} \, \mathrm{d}G(j) + n^*_\tau \int \mathbf{1}^{x*}_{j\tau} q^{m \, \frac{\theta-1}{\theta}}_{j\tau t} \, \mathrm{d}G(j) \right] \right)^{\frac{\theta}{\theta-1}},$$

where n_τ is the number of domestic entrants, who enter the market at time τ. At time t, only $(1-\delta)^{t-\tau} n_\tau$ of such entrants survive. The final good in the foreign country is defined analogously. The market structure implies the following definition of the aggregate price index:

$$P_t = \left(\sum_{\tau \le t} (1-\delta)^{t-\tau} \left[n_\tau \int p^{d \, 1-\theta}_{j\tau t} \, \mathrm{d}G(j) + n^*_\tau \int \mathbf{1}^{x*}_{j\tau} p^{m \, 1-\theta}_{j\tau t} \, \mathrm{d}G(j) \right] \right)^{\frac{1}{1-\theta}}, \tag{2}$$

where $p_{j\tau t}$ is the time t price of products of vintage τ of firm j. The pricing decisions of firms are described by the subsequent equations below. Note that the final good Q_t represents both physical quantities as well as qualities and that the price indexes P_t, and P^*_t aggregate both available quantities and qualities. In that sense, these are quality-adjusted price indexes. See subsection 2.3 for more discussion.

The CES market structure implies that the demand for the domestic firm j product is given as:

$$p^d_{j\tau t} = \left(\frac{q^d_{j\tau t}}{Q_t} \right)^{-\frac{1}{\theta}} P_t, \tag{3}$$

and

$$p^m_{j\tau t} = \left(\frac{q^m_{j\tau t}}{Q_t} \right)^{-\frac{1}{\theta}} P_t$$

for importers. Analogous formulae apply to the demand for products in the foreign market.

2.1.2 Optimal Production Plans

We derive optimal production and investment plans using backward induction for a general neoclassical production function. The parametric example of model equations for the Cobb-Douglas production function is given in the Appendix. We present the derivation for the domestic firm, which is easily generalized for the foreign firm.

Let us assume the problem of maximizing the value of a domestic firm. Since there are no labor adjustment costs, labor decisions are made on a period-by-period basis. The standard results of monopolistically competitive pricing suggest that prices are set as a mark-up over marginal costs. Simultaneously with prices, firms also decide the share of the product sold in the domestic market κ_t.

Now, let us take the perspective of a non-exporter of vintage τ and common productivity level A_t. Its real operating profit $\mathbb{O}^n_{j\tau t}$ in period t is given – conditional on non-exporter status, common productivity, and idiosyncratic productivity z_j – as a solution to the following program:

$$\mathbb{O}^n_{j\tau t} = \max_{l_{jt}} \left\{ \frac{p^d_{jt}}{P_t} A_t z_j f(h_{j\tau}, l_{jt}) - \mathbb{W}_t l_{jt} \right\} = \max_{l_{jt}} \left\{ \left[A_t z_j f(h_{j\tau}, l_{jt})\right]^{\frac{\theta-1}{\theta}} Q_t^{\frac{1}{\theta}} - \mathbb{W}_t l_{jt} \right\}. \tag{4}$$

The second equality in (4) follows from the CES market structure.[6] Similarly, the real operating profit of an export firm $\mathbb{O}^e_{j\tau t}$ of vintage τ in a period t is given by:

$$\mathbb{O}^e_{j\tau t} = \max_{l_{jt}} \left\{ \left(\kappa_t \frac{p_{j\tau t}}{P_t} + (1-\kappa_t) \frac{\eta_t}{1+\varsigma} \frac{p^*_{j\tau t}}{P^*_t} \right) A_t z_j f(h_{j\tau}, l_{j\tau t}) - \mathbb{W}_t l_{j\tau t} \right\} = \tag{5}$$

$$= \max_{l_{jt}} \left\{ \left(\kappa_t Q_t^{\frac{1}{\theta}} + (1-\kappa_t) \frac{\eta_t}{1+\varsigma} Q_t^{*\frac{1}{\theta}} \right) \left[A_t z_j f(h_{j\tau}, l_{j\tau t})\right]^{\frac{\theta-1}{\theta}} - \mathbb{W}_t l_{j\tau t} \right\}.$$

The expected present value of the stream of operating profits is given as follows:

$$\mathbb{O}^\xi_{j\tau} = \sum_{t=\tau}^{\infty} \mu^t_\tau (1-\delta)^{t-\tau} \mathbb{O}^\xi_{j\tau t}$$

with $\xi \in \{n, e\}$. The expected present values depend on idiosyncratic productivity z_j, quality investment $h_{j\tau}$, and the future path of productivities, real wages and demands. The following proposition will be useful:

Proposition 1

The net present value of the stream of exporters' real operating profits $\mathbb{O}^e_{j\tau}$ is increasing in z_j, and similarly for non-exporters. Moreover, for any z_j and τ, $\mathbb{O}^e_{j\tau} > \mathbb{O}^n_{j\tau}$.

Proof

[6] The equality is obtained as follows: the real turnover is $\frac{p^d_{j\tau t}}{P_t} q^d_{j\tau t} = \left(q^d_{j\tau t}\right)^{\frac{-1}{\theta}} Q_t^{\frac{1}{\theta}} q^d_{j\tau t} = \left(q^d_{j\tau t}\right)^{\frac{\theta-1}{\theta}} Q_t^{\frac{1}{\theta}}$ by Equation (3). Substituting the production function $A_t z_j f(h_{j\tau}, l_{j\tau t})$ for $q^d_{j\tau t}$ yields the result.

The first part of the claim is a direct application of the envelope theorem. Indeed, the envelope theorem ensures that $\frac{d\mathbb{O}^e_{j\tau t}}{dz_j} = \frac{\partial \mathbb{O}^e_{j\tau t}}{\partial z_j}$. By (4) one obtains that $\frac{\partial \mathbb{O}^e_{j\tau t}}{\partial z_j} = \frac{\theta-1}{\theta z_j}\left[A_t z_j f(h_{j\tau}, l_{jt})\right]^{\frac{\theta-1}{\theta}} Q_t^{\frac{1}{\theta}}$, which is clearly positive for any finite z_j, A_t, and Q_t. Therefore $\frac{d\mathbb{O}^e_{j\tau}}{dz_j} = \sum_{t=\tau}^{\infty} \mu^t_\tau (1-\delta)^{t-\tau} \frac{d\mathbb{O}^e_{j\tau t}}{dz_j} = \sum_{t=\tau}^{\infty} \mu^t_\tau (1-\delta)^{t-\tau} \frac{\partial \mathbb{O}^e_{j\tau t}}{\partial z_j} > 0$. The exactly analogous reasoning applies for non-exporters. This proves the first part of the Proposition. To prove the second part of the Proposition, observe that the exporter can secure at least as high a profit as the non-exporter by choosing $\kappa \equiv 1$, and by choosing the same level of quality investment $h_{j\tau}$. Therefore $\mathbb{O}^e_{j\tau} \geq \mathbb{O}^n_{j\tau}$. The strict inequality follows from the fact that $0 < \kappa_t < 1$ by Lemma 2 in the Appendix.

The optimal investment decision of a firm that enjoys a productivity level z_j maximizes the value of the firm given as $\mathbf{V}^\xi_\tau(h_{j\tau}|z_j) = \mathbb{O}^{d\xi}_{j\tau} - \left(c + \mathbf{1}^x_{j\tau} c^x + h_j\right)$, for $\xi \in \{n, e\}$. The maximization of $\mathbf{V}^e_\tau(h_{j\tau}|z_j)$ (resp. $\mathbf{V}^n_\tau(h_{j\tau}|z_j)$) yields the optimal demand for quality investment for exporters (resp. non-exporters), and the value of the firm is:

$$V^\xi_\tau(z_j) = \max_{h_{j\tau} \geq 0} \mathbf{V}^\xi_\tau(h_{j\tau}|z_j),$$

where $\xi \in \{e, n\}$. Value functions $V^n_\tau(z_j)$, $V^e_\tau(z_j)$ implicitly define the cut-off value $\overline{z}$, which is the lowest idiosyncratic shock, which makes the export-eligibility investment profitable. Thus it is defined as[7]

$$\overline{z}_\tau = \min_{z_j}(V^e_\tau(z_j) \geq V^n_\tau(z_j)).$$

The value of a firm is given by

$$V_\tau(z_j) = \max_{\xi \in \{n,e\}} V^\xi_\tau(z_j) = \begin{cases} V^e_\tau(z_j) & \text{if } z_j \geq \overline{z}_\tau \\ V^n_\tau(z_j) & \text{if } z_j < \overline{z}_\tau \end{cases},$$

and the expected value of a new entrant $\mathcal{V}_\tau$ is

$$\mathcal{V}_\tau = \int_{z_L}^{z_u} V_\tau(z)\, G(\mathrm{d}z). \tag{6}$$

This completes the backward induction.

[7] One of the referees points out that this definition of the cut-off value is correct only if the expected present value of the profit is increasing in z_j. Proposition 1 demonstrates that this is indeed the case.

The optimal production plan induces a measure over firms. Denote $\widetilde{\mathbb{O}}_{\tau t}$ the t-time expected[8] real operating profit of a domestically-owned firm, which enters in time τ, $\widetilde{\mathbb{O}}_{\tau t} = \int_{z_L}^{z_u} \mathbb{O}_{j\tau t}\, G(\mathrm{d}z)$, and $\widetilde{c}_\tau$ the expected real investment costs under such a measure. Then:

$$\mathcal{V}_\tau = \sum_{\sigma \geq 0} \mu_\tau^{\tau+\sigma}(1-\delta)^\sigma \widetilde{\mathbb{O}}_{\tau,\tau+\sigma} - \widetilde{c}_\tau,$$

where the expected real investment costs consist of three terms:

$$\widetilde{c}_\tau = c + c^x(1 - G(\overline{z}_\tau)) + \widetilde{h},$$

where the first term is the fixed entry cost c paid by all entrants prior to entry, the second term $c^x(1-G(\overline{z}_\tau^d))$ is the expectation of the export-eligibility costs (recall that only firms with $z_j \geq \overline{z}_\tau$ pay these costs), and the final term $\widetilde{h}$ is the expected quality investment, given by:

$$\widetilde{h} = \int_{z_L}^{\overline{z}_\tau} h_{j\tau}^{opt,n}\, G(\mathrm{d}z) + \int_{\overline{z}_\tau}^{z_U} h_{j\tau}^{opt,e}\, G(\mathrm{d}z).$$

2.2 Households

The home country is populated by a representative competitive household who has recursive preferences over discounted stochastic streams of period utilities. The period utilities are derived from the consumption of the aggregate good. Leisure does not enter the utility, so labor is supplied inelastically. The aggregate labor supply in the domestic country is $\mathcal{L}$, while $\mathcal{L}^*$ is the aggregate labor supply in the foreign country. Households can trade bonds denominated in the foreign currency.

The domestic household maximizes

$$\max U = \sum_{t=0}^{\infty} \beta^t u(C_t),$$

subject to

$$B_t = (1 + r_{t-1}^*)B_{t-1} + \frac{-1}{\eta_t}(C_t - \mathbb{W}_t\mathcal{L}) + \frac{1}{\eta_t}(\Xi_t - \widetilde{c}_t n_t) - \frac{\Psi_B}{2}B_t^2 + \mathcal{T}_t, \quad (7)$$

where B_t is the real bond holding of the domestic household, C_t is consumption, r_{t-1}^* is the real interest rate of the internationally traded bond, Ψ_B represents portfolio adjustment costs, as in Schmitt-Grohe and Uribe (2003) to

[8] This expectation is taken with respect to the measure given by the optimal production plan.

stabilize the model,[9] and $\mathcal{T}_t$ is the rebate of these costs in a lump-sum fashion to the household. The flow of real operating profits from all domestic firms is denoted as Ξ_t and is given by

$$\Xi_t = \sum_{s \leq t} (1-\delta)^{t-s} n_s \widetilde{\mathbb{O}}_{s,t}.$$

Because of the law of large numbers and of perfect foresight, the *ex-ante* expected values of the key variables for household decisions (such as investment costs or profit flows) coincide with *ex-post* realizations.

The first-order conditions for the domestic household are standard:

$$(1 + \Psi_B B_t) = \frac{\eta_{t+1}}{\eta_t}(1 + r_t^*)\mu_t^{t+1}, \tag{8}$$

$$\lim_{t \to \infty} B_{t+1} = 0, \tag{9}$$

$$\widetilde{c}_t = \sum_{v \geq 0} (1-\delta)^v \mu_t^{t+v} \widetilde{\mathbb{O}}_{t,t+v}, \tag{10}$$

where the marginal rate of substitution is defined as usual as:

$$\mu_{t_1}^{t_2} \equiv \beta^{t_2 - t_1} \frac{u'(C_{t_2})}{u'(C_{t_1})}.$$

Equation (8) determines the bond holding, equation (9) is the standard transversality condition, and equation (10) is the expected zero-profit condition, which determines the number of new domestic entrants n_t.

It is worth noting that although there is an idiosyncratic variance at the firm level, the model is deterministic at the aggregate level, thus the dynasty problem is deterministic, too. Therefore the marginal rate of substitution does not involve the expectation operator. The household problem in the foreign country is defined symmetrically.

Bonds are denominated in the foreign currency and since the model is deterministic, this is a completely innocent assumption. The international bond market equilibrium requires that $B_t + B_t^* = 0$.

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[9] In a strict sense, the model is stable even without portfolio adjustment costs (i.e., under $\Psi_B = 0$). The model is deterministic and therefore it would not exhibit unit-root behavior even under $\Psi_B = 0$. On the other hand, if $\Psi_B = 0$, then the model would exhibit steady state dependence on the initial asset holding. Therefore we use nontrivial adjustment costs $\Psi_B > 0$ to give up the dependence of the steady state on the initial asset holding.

As mentioned above, prices $p_{j\tau t}$ and the corresponding price indexes P_t, and P_t^* are quality-adjusted prices. Therefore, the real wages $\mathbb{W}_t$ and $\mathbb{W}_t^*$ and the real exchange rate η_t are measured in the terms of qualities. These measures correspond to real-world price indexes only if the latter are quality-adjusted using a hedonic approach, which is rarely the case for transition countries (see Ahnert and Kenny, 2004, p. 28). To get indexes closer to real-world measures, we have to define aggregate indexes over $\wp_{j\tau t}$. We denote such indexes as $\mathcal{P}_t$ and $\mathcal{P}_t^*$.

The quality-unadjusted price index should satisfy the aggregation consistency, i.e., the aggregate expenditure (measured in quality-unadjusted prices) $\mathcal{P}_t Q_t$ should be equal to the aggregation of the individual (quality-unadjusted) prices. Therefore, the quality-unadjusted price index should be defined as follows:

$$\mathcal{P}_t = \frac{\sum_{\tau \leq t}(1-\delta)^{t-\tau}\left[n_\tau \int_{z_L}^{z_U} q_{j\tau t}^d \wp_{j\tau t}^d \, \mathrm{d}G(j) + n_\tau^* \int_{\tilde{z}_\tau}^{z_U} q_{j\tau t}^m \wp_{j\tau t}^m \, \mathrm{d}G(j)\right]}{Q_t}.$$

The algebraic form of the the quality-unadjusted price index (in terms of productivities A_t, A_t^* and aggregates Q_t, Q_t^*, P_t, P_t^*, $\mathbb{W}_t$, $\mathbb{W}_t^*$) is given in the Appendix.

Nevertheless, $\mathcal{P}_t$ might differ from the CPI-based real-world indexes by one more term. The market structure based on the CES aggregation implies the *love-for-variety* effect, which means that the welfare-theoretical price index differs from the 'average' price (CPI-based) index by the term $\nu^{\frac{1}{\theta-1}}$, where ν is the number of available varieties and θ is the parameter of substitution in the CES function (see Melitz, 2003 for definition and derivation). Therefore, we distinguish the following definitions of the real exchange rate:

Quality-adjusted theoretically-consistent RER η_t is the real exchange rate, which enters the decisions of agents in the model.

Quality-unadjusted theoretically-consistent RER is the real exchange rate defined over physical quantities and is related to the quality-adjusted theoretically-consistent RER as $\frac{\mathcal{P}_t^*/P_t^*}{\mathcal{P}_t/P_t}\eta_t$.

Quality-adjusted CPI-based RER is related to its theoretically consistent counterpart as $\left(\frac{\nu_t^*}{\nu_t}\right)^{\frac{1}{\theta-1}}\eta_t$, where ν_t and ν_t^* is the number of varieties available at time t in the domestic and foreign country, respectively.

Quality-unadjusted CPI-based RER is probably the correct counterpart of the *measured real exchange rate* and is defined as $\left(\frac{\nu_t^*}{\nu_t}\right)^{\frac{1}{\theta-1}}\frac{\mathcal{P}_t^*/P_t^*}{\mathcal{P}_t/P_t}\eta_t$.

The quality-adjusted theoretically consistent real exchange rate η_t depreciates for the transition country during the convergence and the reason is the downward-sloping demand curve. On the other hand, the three remaining indexes may appreciate under some conditions; see Section 3 for discussion and intuition.

The number of available varieties in the domestic country can be written as:

$$v_t = \sum_{\tau \leq t} (1-\delta)^{t-\tau} n_\tau + \sum_{\tau \leq t} (1-\delta)^{t-\tau} \left(1 - G(\bar{z}^*_\tau)\right) n^*_\tau,$$

where the first term is the number of domestic firms of different vintages existing at time t, while the second term is the number of exporters in the foreign country existing at time t. The analogous formula holds also for the number of varieties in the foreign country.

2.4 General Equilibrium

As usual, the general equilibrium is defined as a time profile of prices such that all households optimize and all markets clear. Since there are no price rigidities, only the relative prices matter. The general equilibrium requires that the market-clearing conditions hold.

The aggregate resource constraint is given as follows:

$$C_t + n_t \widetilde{c}_t = Q_t. \tag{11}$$

The labor market equilibrium requires

$$\sum_{\tau \leq t} (1-\delta)^{t-\tau} n_\tau \int_{z_L}^{z_U} l_{j\tau t} \, \mathrm{d}G(j) = \mathcal{L}, \tag{12}$$

where $l_{j\tau t}$ is the labor demand by individual firms, and $\mathcal{L}$ is the aggregate, inelastic, labor supply. Analogous market clearing conditions hold in the foreign country.

The international bond market equilibrium requires that

$$B_t + B^*_t = 0. \tag{13}$$

The last equilibrium condition is the balance-of-payment equilibrium, which requires that

$$B_{t+1} = (1 + r^*_t) B_t + X_t, \tag{14}$$

where X_t is the value of *net* real exports of the domestic country expressed in the foreign currency.

The Appendix summarizes *steady-state* model equations. The reader is referred to Appendix A.2 in Brůha and Podpiera (2007a) for a description of the recursive form of a variant of the model, which can be used for dynamic simulations. Papers by Brůha and Podpiera (2007b) and Brůha, Podpiera and Polák (2009) applied the dynamic solutions to policy questions (European economic integration and convergence in a small open economy, respectively).

3 Quantitative Analysis

We inquire whether the 'endogenous' Harrod-Balassa-Samuelson effect can be a sufficient explanation for the real exchange rate appreciation in European transition countries. This is doable because the steady state of the model outlined in Section 2 encompasses the steady state of the model by Ghironi and Melitz (2005) as a special case under the assumption of homogenous products' quality (no quality investment) and the linear production function with labor as the only input: $f(l) = l$. We contrast the outcome of this model with that from the model with explicit quality investment, where we opt for the Cobb-Douglas production function: $f(l) = h^{\alpha} l^{1-\alpha}$ (see the Appendix for a derivation of the model under this specific functional form). In the immediate next subsection we present the models' comparison for benchmark calibration. Subsequently, we provide a comprehensive sensitivity analysis.

3.1 The Models' Comparison for Benchmark Calibration

We employ the calibration used by Ghironi and Melitz (2005) to carry out the following comparison of steady states. We set two productivity levels in the converging country such that the converging economy attains 60 and 75% of GDP per capita of the advanced economy, which corresponds to the situations of Visegrad-4 countries in 1995 and 2005, respectively. Then, we compute the change in the (CPI-based) real exchange rate in the two steady states and establish the implied yearly appreciation between 1995 and 2005. It is worth noting that the two steady states differ by the level of domestic productivity A_t only, i.e., we hold the rest of the parameters (including the distribution $G(z)$) fixed.

In their calibration, Ghironi and Melitz (2005) set the value of the common productivity parameter to one: $A^* = 1$; similarly the entry cost is normalized to one as well: $c = 1$. The exit rate δ is set at 10%. The size of the intertemporal rate of substitution, for yearly frequency, is $\beta = 0.95$. Their choice of the value 3.8 for the parameter of the intratemporal substitution θ is based on empirically found mark-ups for the U.S. by Bernard et al. (2003). The iceberg cost ς equals 0.3 and the present value of the fixed export costs c^x is 23.5% higher than the entry cost in annualized terms. The distribution $G(z)$ for the idiosyncratic productivity shocks takes the Pareto distribution with the parameter $k = 3.4$. For the steady state comparison, it is not necessary to specify the parametrization of the momentary utility function u. It is sufficient to assume the usual properties of u, i.e. that it is increasing and concave.

The just-described calibration of parameters is used to simulate the model without explicit investment in quality. The implied yearly CPI-based real exchange rate appreciation equals 0.2%. The model with investment in quality contains one additional parameter (the proportion of quality in the production of the quality-quantity bundle), which we set at $\alpha = 0.3$. The steady state comparison for the model with investment in quality for the benchmark calibration yields the average yearly CPI-based real exchange rate appreciation of 2%. It follows that the investment in quality is a significantly more important (ten times) driving force for the CPI-based real exchange rate appreciation than the endogenous Harrod-Balassa-Samuelson effect.

Hence, we show that the Harrod-Balassa-Samuelson effect is an insufficient explanation tool for the observed pace of the real exchange rate appreciation in European transition economies. The conclusion might be, however, subject to the particular calibration of the parameters chosen for the benchmark calibration. It may be possible that there is a reasonable combination of the parameter values that would yield the required appreciation even for the model without investment in quality. In order to provide a sensitivity analysis of the results to the choice of parameters and establish firmly the findings from the benchmark calibration, we carry out a sensitivity analysis in the next section.

3.2 Sensitivity Analysis

We assess the size of the implied CPI-based real exchange rate appreciation for various combinations of the model's parameter values.

The numerical ranges for the parameters are predominantly motivated by relevant empirical micro and macroeconomic evidence. We specify a large interval for the parameter θ, since Ghironi and Melitz (2005) choose 3.8, while the standard DSGE models (Rotemberg and Woodford, 1992) opt for $\theta = 6$. Our range [3.25 7.5] includes both values. The iceberg cost ς is considered to be between 0.05 and 0.30. The reason is that a realistic calibration of the export share of a Visegrad-4 country on GDP would require a number of about 0.1, which is lower than the value 0.30 used by Ghironi and Melitz (2005). The range for parameter δ is chosen from a half of to double the value suggested by Ghironi and Melitz (2005), i.e. from 0.05 to 0.20. Similarly, we choose the range for exporting costs c^x between a half of and double the value used by Ghironi and Melitz (2005).

The rate of the intertemporal rate of substitution[10] is $\beta = 0.95$. The final steady state value of the common productivity parameter is fixed at the conventional value[11] $A^* = 1$. The distribution of idiosyncratic productivity shocks $G(z)$ takes a Pareto distribution with the parameter k. The parameter k is sampled from a uniform distribution on [2 6.5] subject to the restriction $k > \theta - 1$. For the set of simulations with the model with investment in quality, we extend the parameter space by $\alpha \in [0.05, 0.30]$. The set of parameter values from which we draw is a multidimensional cube as summarized in Table 1. As a sampling scheme, we use the Halton sequences and we sample 10,000 parameter combinations for each.

Figure 2 shows histograms of the predicted average yearly CPI-based real exchange rate appreciation from the two model simulations. As we can see, the median exchange rate appreciation for the model without investment in quality is the range between 0 and 0.5 p.p. per annum and the maximum falls in the range 1-1.5 p.p. This contrasts with the histogram for the model with investment in quality, where the median is in the range between 2 and 3 p.p. and the maximum attained appreciation falls into the interval 4-5 p.p.

The relative importance of different parameters with respect to the implied exchange rate appreciation is very similar for both models (with and without investment in quality). First, an increase in the exit rate δ to 0.25 does not change the implied yearly appreciation. Second, we report that the higher values[12] of the parameter of intratemporal substitution θ decrease the exchange rate appreciation (the value $\theta = 6$ would imply appreciation less than 1% in the model without quality investment). Third, a decreasing value of the iceberg cost ς is associated with a slight increase in the implied real exchange rate appreciation. For instance, a decrease in iceberg cost to $\varsigma = 0.1$ implies an increase in the yearly exchange rate appreciation by 0.02 p.p.

The last sensitivity concerns the distribution G. High values of the shape parameter k increases the predicted appreciation. For a value of k as large as 6, the implied average annual appreciation is 0.8% and 3.5% for the model

[10] In the steady state, the parameters β and δ are individually unimportant. They matter through the product $1 - \beta(1 - \delta)$ and thus it makes sense to fix one and to let vary the other.
[11] The values for A^* and c are normalized, irrelevant for the comparison of the steady state real exchange rate.
[12] However, when experimenting with high values of the parameter of intratemporal substitution θ, we have to increase the parameter k to ensure that the value of a new entrant is bounded (we increase the parameter k to 6). As in Ghironi and Melitz (2005), the model requires that $k > \theta - 1$.

without and with investment in quality, respectively. It is worth noting that the similar number results if the Pareto distribution is replaced with the exponential distribution.

As follows from the results for the sensitivity analysis, the model without investment in quality can rarely attain an appreciation greater than 1.25 p.p. per annum. However, even these are obtained for the extreme values of the parameters. On the other hand, the model with investment in quality does readily generate a sufficient (3.0-4.0%) speed of real exchange rate appreciation to justify the recent evidence of the real exchange rate appreciation in Visegrad-4 countries. The results basically confirm the findings from the model's benchmark calibration.

4 Concluding remarks

We suggest that an upgrade in product quality is an important factor explaining the CPI-based real exchange rate appreciation in the Visegrad-4 countries (the Czech Republic, Hungary, Poland, and Slovakia). We first use a dynamic general equilibrium two-country model without quality investment margin and compare two steady states. One steady state corresponds to a situation in which the converging country attains 60% of the productivity of the advanced counterpart. In the other steady state the converging economy reaches 75% of the productivity of the advanced country. We find that the 15 p.p. increase in productivity of the converging country implies between 0.2 - 1.25% annual CPI-based real exchange rate appreciation of the converging economy (which corresponds to an endogenous Harrod-Balassa-Samuelson effect). Subsequently, we repeat this exercise for the model with investment in quality and find the implied annual CPI-based real exchange rate appreciation equals between 0.5 and 4.5%. It follows that for matching the empirical data for the average annual CPI-based real exchange rate appreciation in the Visegrad-4 region (averaging 3.5% during 1995-2005), one ought to take into account the changes in product quality besides the Harrod-Balassa-Samuelson effect.

A The detailed derivation of the model

This part of the paper derives in details the optimal decision of firms under the CES market structure and a particular production function. As a benchmark calibration, we use the iso-elastic production function $\ell(l,h) \equiv \left(\frac{l}{h}\right)^{1-\alpha}$ for production of physical quantities. This formulation implies the Cobb-Douglas production function $f(h,l) = h^{\alpha} l^{1-\alpha}$ for the production of the quality-quantity bundle. The curvature of the momentary utility function u does not matter for the steady-state properties as long as u is strictly increasing and concave. Note that for $\alpha = 0$ (and taking the relevant limits where necessary), the production function is linear in a single input – labor, which corresponds to the parametrization used by Ghironi and Melitz (2005).

A.1 The detailed derivation of firm behavior

The short-run cost function associated with the Cobb-Douglas production function is given as follows:

$$\mathbb{C}(q, \mathbb{W}_t, A_t, z_j, h_{j\tau}) = \mathbb{W}_t \left[\frac{q}{A_t z_j h_{j\tau}^{\alpha}}\right]^{\frac{1}{1-\alpha}}.$$

First, we derive the maximizing behavior of non-exporters.[13] The period t supply decision of a vintage τ non-exporter, who enjoys the productivity z_j and who has invested in the product quality $h_{j\tau}$, solves the following program[14]

$$\max_{q^d_{j\tau t}} \left\{ \left[q^d_{j\tau t}\right]^{\frac{\theta-1}{\theta}} Q_t^{\frac{1}{\theta}} - \mathbb{C}(q^d_{j\tau t}, \mathbb{W}_t, A_t, z_j, h_{j\tau}) \right\}.$$

A simple algebra yields the optimal supply

$$q^d_{j\tau t} = \left(\left[\frac{\theta-1}{\theta} (1-\alpha) \mathbb{W}_t^{-1} \left[A_t z_j h_{j\tau}^{\alpha}\right]^{\frac{1}{1-\alpha}} \right]^{\theta} Q_t \right)^{\frac{(1-\alpha)}{\alpha(\theta-1)+1}},$$

[13] We derive expressions only for domestic firms. The expressions for foreign firms are derived analogously.

[14] Note that this program is equivalent to the program (4). The reason is that the quality level $h_{j\tau}$ and the export-eligibility status has been already decided. Therefore the problem of the choice of the output $q_{j\tau t}$ is perfectly equivalent to the choice of the only variable input – labor $l_{j\tau t}$.

and the optimal labor demand

$$l_{j\tau t}=\left[\frac{q_{j\tau t}^{d}}{A_t z_j h_{j\tau}^{\alpha}}\right]^{\frac{1}{1-\alpha}}=\left(\left[\frac{\theta-1}{\theta}(1-\alpha)\mathbb{W}_t^{-1}\right]^{\theta}\left[A_t z_j h_{j\tau}^{\alpha}\right]^{\theta-1}Q_t\right)^{\frac{1}{\alpha(\theta-1)+1}}. \tag{A.1}$$

Now, using the CES market demand (3), we derive the real turnover

$$\frac{p_{jt}^{d}}{P_t}q_{jt}^{d}=z_j^{\frac{\theta-1}{\alpha(\theta-1)+1}}h_{j\tau}^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}}\left[\frac{\theta-1}{\theta}(1-\alpha)\mathbb{W}_t^{-1}A_t^{\frac{1}{1-\alpha}}\right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}}Q_t^{\frac{1}{\alpha(\theta-1)+1}}, \tag{A.2}$$

and the real operating profit [15]

$$\mathbb{O}_{j\tau t}=\frac{p_{jt}^{d}}{P_t}q_{jt}^{d}-\mathbb{C}(q_{jt}^{d},\mathbb{W}_t,A_t,z_j,h_{j\tau})=\mathcal{W}_1 z_j^{\frac{\theta-1}{\alpha(\theta-1)+1}}h_{j\tau}^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}}\mathbb{W}_t^{\frac{-(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}}A_t^{\frac{(\theta-1)}{\alpha(\theta-1)+1}}Q_t^{\frac{1}{\alpha(\theta-1)+1}}.$$

Second, we derive the optimal production decisions of exporters. The problem can be characterized as follows (with the definition $q_{j\tau t}=\kappa_{j\tau t}q_{j\tau t}^{d}+(1-\kappa_{j\tau t})q_{j\tau t}^{m*}$)

$$\max_{q_{j\tau t}^{d},q_{j\tau t}^{m*}}\left\{\left(q_{j\tau t}^{d}\right)^{\frac{\theta-1}{\theta}}Q_t^{\frac{1}{\theta}}+\left(\frac{\eta_t}{1+\varsigma}\right)Q_t^{*\frac{1}{\theta}}\left(q_{j\tau t}^{m*}\right)^{\frac{\theta-1}{\theta}}-\mathbb{C}(q_{j\tau t},\mathbb{W}_t,A_t,z_j,h_{j\tau})\right\}.$$

The solution yields $q_{j\tau t}^{d}=\left[\frac{\theta-1}{\theta}\left(\frac{\partial\mathbb{C}}{\partial q_{j\tau t}}\right)^{-1}\right]^{\theta}Q_t$, and $q_{j\tau t}^{*m}=\left[\frac{\theta-1}{\theta}\frac{\eta_t}{1+\varsigma}\left(\frac{\partial\mathbb{C}}{\partial q_{j\tau t}}\right)^{-1}\right]^{\theta}Q_t^{*}$. Some simple, but tedious, algebraic manipulations yield

$$\kappa_{j\tau t}q_{j\tau t}\equiv q_{j\tau t}^{d}=\left[\frac{\theta-1}{\theta}(1-\alpha)\mathbb{W}_t^{-1}\left(A_t z_j h_{j\tau}^{\alpha}\right)^{\frac{1}{1-\alpha}}\right]^{\theta}\frac{Q_t}{q_{jt}^{\frac{\alpha\theta}{1-\alpha}}},$$

and

$$(1-\kappa_{j\tau t})q_{j\tau t}\equiv q_{j\tau t}^{m*}=\left[\frac{\theta-1}{\theta}(1-\alpha)\frac{\eta_t}{1+\varsigma}\mathbb{W}_t^{-1}\left(A_t z_j h_{j\tau}^{\alpha}\right)^{\frac{1}{1-\alpha}}\right]^{\theta}\frac{Q_t^{*}}{q_{j\tau t}^{\frac{\alpha\theta}{1-\alpha}}}.$$

It implies that $\kappa_{j\tau t}=\frac{Q_t}{Q_t+Q_t^{*}\left(\frac{\eta_t}{1+\varsigma}\right)^{\theta}}$. It follows that $\kappa_{j\tau t}$ does not depend on individual characteristics of firms, such as z_j and $h_{j\tau}$, it depends only on relative tightness of both markets and on the real exchange rate corrected for transport costs $\mathbf{t}$. Therefore, we proved the following Lemma:

[15] We define $\mathcal{W}_1\equiv\left[\frac{\theta-1}{\theta}(1-\alpha)\right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}}-\left[\frac{\theta-1}{\theta}(1-\alpha)\right]^{\frac{\theta}{\alpha(\theta-1)+1}}=\frac{\alpha(\theta-1)+1}{(\theta-1)(1-\alpha)}\left[\frac{\theta-1}{\theta}(1-\alpha)\right]^{\frac{\theta}{\alpha(\theta-1)+1}}$.

Lemma 2: It is never optimal for exporters to export all production and not to export in a given period. Moreover, the optimal exporting share κ_t depends only on the current macroeconomic conditions and given the exporting status of a firm, it does not depend on vintage or productivity.

Thus, we hereafter write κ_t instead of $\kappa_{j\tau t}$. Define $\xi_t \equiv Q_t + Q_t^* \left(\frac{\eta_t}{1+\varsigma}\right)^\theta = \frac{Q_t}{\kappa_t}$. The total production of eligible firms can be written as follows

$$q_{j\tau t} = \left(z_j^\theta h_{j\tau}^{\alpha\theta}\right)^{\frac{1}{\alpha(\theta-1)+1}} \left\{\left[\frac{\theta-1}{\theta}(1-\alpha)\mathbb{W}_t^{-1} A_t^{\frac{1}{1-\alpha}}\right]^\theta \xi_t\right\}^{\frac{(1-\alpha)}{\alpha(\theta-1)+1}},$$

and the optimal labor demand as

$$l_{j\tau t} = \left[\frac{q_{j\tau t}}{A_t z_j h_{j\tau}^\alpha}\right]^{\frac{1}{1-\alpha}} = \left(\left[\frac{\theta-1}{\theta}(1-\alpha)\mathbb{W}_t^{-1}\right]^\theta \left[A_t z_j h_{j\tau}^\alpha\right]^{\theta-1} \xi_t\right)^{\frac{1}{\alpha(\theta-1)+1}}. \tag{A.3}$$

The firms' real turnover on the domestic and the foreign markets, respectively, are given by

$$\frac{p_{j\tau t}^d}{P_t} q_{j\tau t}^d = z_j^{\frac{\theta-1}{\alpha(\theta-1)+1}} h_{j\tau}^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}} \kappa_t^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}} \left[\frac{\theta-1}{\theta}(1-\alpha)\mathbb{W}_t^{-1} A_t^{\frac{1}{1-\alpha}}\right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} Q_t^{\frac{1}{\alpha(\theta-1)+1}}, \tag{A.4}$$

$$\left(\frac{\eta_t}{1+t}\right) \frac{p_{j\tau t}^{m*}}{P_t^*} q_{jt}^{m*} = z_j^{\frac{\theta-1}{\alpha(\theta-1)+1}} h_{j\tau}^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}} (1-\kappa_t)^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}} \left(\frac{\eta_t}{1+\varsigma}\right)^{\frac{\theta}{\alpha(\theta-1)+1}} \times \tag{A.5}$$
$$\times \left[\frac{\theta-1}{\theta}(1-\alpha)\mathbb{W}_t^{-1} A_t^{\frac{1}{1-\alpha}}\right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} Q_t^{*\frac{1}{\alpha(\theta-1)+1}}.$$

The real production costs of exporters read as follows

$$\mathbb{C}_{jt} = z_j^{\frac{\theta-1}{\alpha(\theta-1)+1}} h_{j\tau}^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}} A_t^{\frac{(\theta-1)}{\alpha(\theta-1)+1}} \mathbb{W}_t^{\frac{-(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} \left\{\left[\frac{\theta-1}{\theta}(1-\alpha)\right]^\theta \xi_t\right\}^{\frac{1}{\alpha(\theta-1)+1}},$$

thus, the real operating profit in a period t is given as

$$\mathbb{O}_{j\tau t} = \mathcal{W}_1 z_j^{\frac{\theta-1}{\alpha(\theta-1)+1}} h_{j\tau}^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}} A_t^{\frac{(\theta-1)}{\alpha(\theta-1)+1}} \mathbb{W}_t^{\frac{-(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} \xi_t^{\frac{1}{\alpha(\theta-1)+1}}.$$

Now, we are able to derive the expected present value of a profit stream. We start with an exporter $\mathbb{O}^e_{j\tau}$, whose expected present value satisfies

$$\mathbb{O}^e_{j\tau} = z_j^{\frac{\theta-1}{\alpha(\theta-1)+1}} h_j^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}} \underbrace{\mathcal{W}_1 \sum_{t=\tau}^{\infty} (1-\delta)^{t-\tau} \mu^t_\tau A_t^{\frac{(\theta-1)}{\alpha(\theta-1)+1}} \mathbb{W}_t^{\frac{-(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} \xi_t^{\frac{1}{\alpha(\theta-1)+1}}}_{\varpi^e_\tau}, \tag{A.6}$$

while the expected present value of a non-exporter $\mathbb{O}^n_{j\tau}$ satisfies

$$\mathbb{O}^n_{j\tau} = z_j^{\frac{\theta-1}{\alpha(\theta-1)+1}} h_{j\tau}^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}} \underbrace{\mathcal{W}_1 \sum_{t=\tau}^{\infty} (1-\delta)^{t-\tau} \mu^t_\tau A_t^{\frac{(\theta-1)}{\alpha(\theta-1)+1}} \mathbb{W}_t^{\frac{-(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} Q_t^{\frac{1}{\alpha(\theta-1)+1}}}_{\varpi^n_\tau}. \tag{A.7}$$

The value of an exporter, who enjoys a productivity level z_j, is determined by quality investment

$$\mathbf{V}^e_\tau(h_{j\tau}|z_j) = \mathbb{O}^e_{j\tau} - (c + c^x + h_{j\tau}) \equiv z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} h_{j\tau}^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \varpi^e_\tau - (c + c^x + h_{j\tau})\,;$$

and similarly for a non-exporter:

$$\mathbf{V}^n_\tau(h_{j\tau}|z_j) = \mathbb{O}^n_{j\tau} - (c + h_{j\tau}) = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} h_{j\tau}^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \varpi^n_\tau - (c + h_{j\tau})\,.$$

If a firm's manager is maximizing the value of firm then chooses the following quality level

$$h^{opt,e}_{j\tau} = z_j^{\theta-1} \left[\frac{\alpha(\theta-1)\varpi^e_\tau}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)+1}, \tag{A.8}$$

and the value of an exporting firm is[16]

$$V^e_\tau(z_j) = \max_{h\geq 0} \mathbf{V}^e_\tau(h|z_j) = z_j^{(\theta-1)} \left[\varpi^e_\tau\right]^{\alpha(\theta-1)+1} \mathcal{G} - (c + c^x), \tag{A.9}$$

similarly, the value of a non-exporting firm is

$$V^{dn}_\tau(z_j) = \max_{h\geq 0} \mathbf{V}^{dn}_\tau(h|z_j) = z_j^{\theta-1} \left[\varpi^n_\tau\right]^{\alpha(\theta-1)+1} \mathcal{G} - c, \tag{A.10}$$

and the optimal capital investment to quality is

$$h^{opt,n}_{j\tau} = z_j^{\theta-1} \left[\frac{\alpha(\theta-1)\varpi^n_\tau}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)+1}. \tag{A.11}$$

[16] Define $\mathcal{G} \equiv \left[\left(\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}\right)^{\alpha(\theta-1)} - \left(\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}\right)^{\alpha(\theta-1)+1}\right]$, which can be simplified to $\mathcal{G} = \frac{1}{\alpha(\theta-1)+1}\left(\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}\right)^{\alpha(\theta-1)}$.

Expressions (A.9) and (A.10) confirm the Proposition 1 that a firm's value is indeed increasing in z_j.

Value functions $V_\tau^n(z_j)$ and $V_\tau^e(z_j)$ implicitly define the cut-off value $\overline{z}$, which is the least idiosyncratic shock, which makes the export-eligibility investment profitable, i.e. $\overline{z}_\tau = \min_{z_j}(V_\tau^e(z_j) \geq V_\tau^n(z_j))$, which for the particular parametrization is given as follows

$$\overline{z}_\tau = \left(\frac{c^x}{\mathcal{G}[[\varpi_\tau^e]^{\alpha(\theta-1)+1} - [\varpi_\tau^n]^{\alpha(\theta-1)+1}]} \right)^{\frac{1}{\theta-1}}.$$

A.2 The Steady State

Since the paper effectively compares two steady states, we characterize the steady state.[17] The steady state is a long-run equilibrium and it is obtained when exogenous parameters (particulary productivity parameters A and A^*) are constant for a sufficiently long period of time. The steady state is characterized by a number of features. Most importantly

- zero bond holding $B_{ss} = 0$, which is due to adjustment costs $\psi_B > 0$; the steady-state zero bond holding implies that steady-state net exports equal zero.
- constant endogenous quantities and prices;
- the marginal rate of the intertemporal substitution $\mu_{t_1}^{t_2} = \beta^{t_2-t_1}$;
- the steady-state effective discount rate $\mathcal{R} \equiv \sum_{t\geq 0}(1-\delta)^t \mu_\tau^{\tau+t}$ reads as $\mathcal{R} = \frac{1}{1-\beta(1-\delta)}$ and the steady-state interest rate $r_{ss} = \beta^{-1} - 1$;
- the constant number of entrants n, which implies that the number of firms in a country is given by n/δ.

Since we deal with the steady state, we omit the time subscripts. Therefore, it is possible to write

$$\varpi^e = \mathcal{W}_1 \mathcal{R} A^{\frac{(\theta-1)}{(1-\alpha)+\alpha\theta}} \mathbb{W}^{\frac{-(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} \xi^{\frac{1}{(1-\alpha)+\alpha\theta}}, \tag{A.12}$$

$$\varpi^n = \mathcal{W}_1 \mathcal{R} A^{\frac{(\theta-1)}{(1-\alpha)+\alpha\theta}} \mathbb{W}^{\frac{-(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} Q^{\frac{1}{(1-\alpha)+\alpha\theta}}, \tag{A.13}$$

[17] We do not give details how to solve the transition dynamics here. The solution of the transition dynamics requires a trick: the model is a vintage-type model and therefore hard-to-solve. Nevertheless, Brůha and Podpiera (2007a) show that the model can be rewritten into a recursive (first-order) form, which makes the application of standard numerical techniques for solving perfect-foresight models routine. Since the computation of the transition dynamics is not relevant here, we derive only equations, which characterize the steady state. The reader is referred to the Appendix in Brůha and Podpiera (2007a) for details on transition dynamics.

and the steady-state productivity cut-off satisfies $\overline{z}^d = \left(\frac{c^x}{\mathcal{G}[\mathcal{W}_1\mathcal{R}]^{\alpha(\theta-1)+1}(\xi-Q)}\right)^{\frac{1}{\theta-1}} \frac{\mathbb{W}^{1-\alpha}}{A} = \left(\frac{c^x}{\mathcal{G}[\mathcal{W}_1\mathcal{R}]^{\alpha(\theta-1)+1}Q^*\left(\frac{\eta}{1+\varsigma}\right)^{\theta}}\right)^{\frac{1}{\theta-1}} \frac{\mathbb{W}^{1-\alpha}}{A}$.

A.3 Steady state derivation of the quality-unadjusted price index

Using the results of the preceding section, it is possible to characterize the quality-unadjusted price index $\mathcal{P}$ in more details. This is done in several steps. First, we will consider the expression $q^d_{j\tau t}\wp^d_{j\tau t}$ for the domestic non-exporter. It obviously holds that $q^d_{j\tau t}\wp^d_{j\tau t} = q^d_{j\tau t}p^d_{j\tau t}h_{j\tau} = P_t q^d_{j\tau t}\frac{p^d_{j\tau t}}{P_t}h_{j\tau}$. The expression $q^d_{j\tau t}\frac{p^d_{j\tau t}}{P_t}$ is the real turnover of the domestic non-exporter and it has been already derived in (A.2). Therefore, it is possible to write

$$q^d_{j\tau t}\wp^d_{j\tau t} = P_t z_j^{\frac{\theta-1}{\alpha(\theta-1)+1}} h_{j\tau}^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}+1} \left[\frac{\theta-1}{\theta}(1-\alpha)\,\mathbb{W}_t^{-1}A_t^{\frac{1}{1-\alpha}}\right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} Q_t^{\frac{1}{\alpha(\theta-1)+1}}.$$

Now, this can be combined with the expression for the optimal quality investment for non-exporters (A.11), to get the following expression

$$q^d_{j\tau t}\wp^d_{j\tau t} = \mathcal{W}_P P_t z_j^{2(\theta-1)} \varpi_\tau^{n^{2\alpha(\theta-1)+1}} \mathbb{W}_t^{-\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} A_t^{\frac{(\theta-1)}{\alpha(\theta-1)+1}} Q_t^{\frac{1}{\alpha(\theta-1)+1}},$$

where we define $\mathcal{W}_P = \left[\frac{\theta-1}{\theta}(1-\alpha)\right]_j^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}\theta-1} \left[\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}\right]^{2\alpha(\theta-1)+1}$. In the steady state, we can use (A.13), which would lead to the following steady-state expression

$$q^d_j\wp^d_j = \mathcal{W}_P\,[\mathcal{W}_1\mathcal{R}]^{2\alpha(\theta-1)+1}\,P_t z_j^{2(\theta-1)}\mathbb{W}^{-2(\theta-1)(1-\alpha)}A^{2(\theta-1)}Q^2.$$

Second, consider the case of domestic exporters. It is possible to derive the desired expression completely analogously, i.e. using the exporter's turnover on the domestic market (A.4) and substituting the expression for the optimal quality investment in (A.8). The algebraic manipulations yields the following formula

$$q^d_{j\tau t}\wp^d_{j\tau t} = \mathcal{W}_P P_t z_j^{\frac{\theta-1}{\alpha(\theta-1)+1}} \varpi_\tau^{e^{2\alpha(\theta-1)+1}} \kappa_t^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}} \mathbb{W}_t^{-\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} A_t^{\frac{(\theta-1)}{\alpha(\theta-1)+1}} Q_t^{\frac{1}{\alpha(\theta-1)+1}},$$

which can be in the steady state, using (A.12), simplified as follows

$$q^d_j\wp^d_j = \mathcal{W}_P\,[\mathcal{W}_1\mathcal{R}]^{2\alpha(\theta-1)+1}\,P z_j^{2(\theta-1)}\mathbb{W}^{-2(\theta-1)(1-\alpha)}A^{2(\theta-1)}Q\xi.$$

Finally, we should consider the foreign exporters to derive $q^m_{j\tau t}\wp^m_{j\tau t}$. The expression can be derived as $q^m_{j\tau t}\wp^m_{j\tau t} = q^m_{j\tau t}p^m_{j\tau t}h^*_{j\tau} = P_t q^m_{j\tau t}\frac{p^m_{j\tau t}}{P_t}h^*_{j\tau}$ after a suitable modification of (A.5), namely this expression should be taken from the perspective of the foreign firm, but expressed in the domestic currency.

Combining the three results, it is possible to derive the final steady-state formula:

$$\begin{aligned}\frac{\mathcal{P}_t}{P_t} =& \mathcal{W}_P\left[\mathcal{W}_1\mathcal{R}\right]^{2\alpha(\theta-1)+1}\left(\frac{A}{\mathbb{W}^{(1-\alpha)}}\right)^{2(\theta-1)}\left[Q\int_{z_L}^{\overline{z}} z_j^{2(\theta-1)}\,\mathrm{d}G(z_j) + \xi\int_{\overline{z}}^{z_U} z_j^{2(\theta-1)}\,\mathrm{d}G(z_j)\right] + \\ &+\mathcal{W}_P\left[\mathcal{W}_1\mathcal{R}\right]^{2\alpha(\theta-1)+1}\left(\frac{A}{\mathbb{W}^{(1-\alpha)}}\right)^{2(\theta-1)}\left(\frac{\eta_t^{-1}}{1+\varsigma}\right)\xi^*\int_{\overline{z}^*}^{z_U} z_j^{2(\theta-1)}\,\mathrm{d}G(z_j).\end{aligned}$$

Note that the final expression for $\frac{\mathcal{P}_t}{P_t}$ may be unbounded because the integral $\int_{\overline{z}}^{z_U} z_j^{2(\theta-1)}\,\mathrm{d}G(z_j)$ may diverge if the domain of G is unbounded, i.e., if $z_U = \infty$. This can happen, for instance, when using the usual choice of Pareto distribution for G for $k \leq 2(\theta-1)$. However, even in this case, the expression $\frac{\mathcal{P}^*_t/P^*_t}{\mathcal{P}_t/P_t}$ can be defined as a limit, when the distribution G is successively approximated by a sequence of right-truncated versions[18] $G_n(x) = \frac{\mathbf{1}_{x\in[z_L,n)}G(x)}{\int_{z_L}^{n}\mathrm{d}G(s)}$ of G. It follows that in this way defined limit of $\eta\frac{\mathcal{P}^*_t/P^*_t}{\mathcal{P}_t/P_t}$ would converge to the following expression $\eta\left(\frac{\frac{A^*}{A}}{\left[\frac{\mathbb{W}^*}{\mathbb{W}}\right]^{1-\alpha}}\right)^{2(\theta-1)}\frac{\frac{Q^*}{Q}\left[\frac{Q^*}{Q}+\left(\frac{\eta^{-1}}{1+\varsigma}\right)^{\theta}+\kappa\left(\frac{\eta}{1+\varsigma}\right)^{\theta}\right]}{\left[1+\frac{Q^*}{Q}\kappa^*\left(\frac{\eta^{-1}}{1+\varsigma}\right)^{\theta}+\frac{Q^*}{Q}\kappa\left(\frac{\eta}{1+\varsigma}\right)^{\theta}\right]}$.

A.4 Steady-state Market-clearing conditions

The steady-state market clearing conditions are the following:

(1) Zero-profit condition in equilibrium;
(2) Goods-market clearing in both countries;
(3) Labor-market clearing in both countries;
(4) Balance of Payment

Zero expected profits in equilibrium The condition of zero expected profits in equilibrium implies that the expected value of a domestic-country

[18] Melitz and Ottaviano (2008) use the truncated Pareto distribution in their model.

entrant be zero. $\int_{z_L}^{\overline{z}^d} V_\tau^n(z_j)\,\mathrm{d}G(z_j) + \int_{\overline{z}^d}^{z_U} V_\tau^e(z_j)\,\mathrm{d}G(z_j) = 0$. This can be reexpressed as

$$\mathcal{G}\left\{[\varpi^n]^{\alpha(\theta-1)+1}\int_{z_L}^{\overline{z}} z_j^{\theta-1}\,\mathrm{d}G(z) + [\varpi^e]^{\alpha(\theta-1)+1}\int_{\overline{z}}^{z_U} z_j^{\theta-1}\,\mathrm{d}G(z)\right\} = c + c^x\left[1-G(\overline{z})\right].$$

In the steady state, the expression can be rewritten, using (A.12) and (A.13), explicitly as

$$\left(\frac{A}{\mathbb{W}^{1-\alpha}}\right)^{\theta-1}\left\{Q\int_{z_L}^{\overline{z}} z_j^{\theta-1}\,\mathrm{d}G(z) + \xi\int_{\overline{z}}^{z_U} z_j^{\theta-1}\,\mathrm{d}G(z)\right\} = \frac{c+c^x\left[1-G(\overline{z})\right]}{\mathcal{G}(\mathcal{W}_1\mathcal{R})^{\alpha(\theta-1)+1}}.$$

The analogous condition holds for the foreign-country entrants.

Goods-market clearing The goods-market clearing condition is the GDP identity. The condition in the steady state stipulates (because net exports are zero) that $Q = \underbrace{(\mathcal{L}\mathbb{W} + \Xi - n\widetilde{c})}_{consumption} + \underbrace{n\widetilde{c}}_{investnment} = \mathcal{L}\mathbb{W} + \Xi$ and analogously $Q^* = \mathcal{L}^*\mathbb{W}^* + \Xi^*$. The formula for the steady-state labor demand is provided below and the steady-state profit flow satisfies

$$\begin{aligned}\Xi &= \frac{n}{\delta}\widetilde{\mathbb{O}} = \\ &= \frac{n(\mathcal{W}_1\mathcal{R})^{\alpha(\theta-1)+1}}{\delta\mathcal{R}}\left[\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)}\left(\frac{A}{\mathbb{W}^{1-\alpha}}\right)^{\theta-1}\left\{Q\int_{z_L}^{\overline{z}} z_j^{\theta-1}\,\mathrm{d}G(z) + \xi\int_{\overline{z}}^{z_U} z_j^{\theta-1}\,\mathrm{d}G(z)\right\}.\end{aligned}$$

The second equality follows from a substitution of (A.8) and (A.11) into (A.6) and (A.7).

Labor-market clearing The labor demand by individual domestic firms can be obtained by using expressions (A.1) and (A.3). First, we plug in the expressions for the optimal quality investment and then we integrate them to obtain the aggregate labor demand as follows

$$\frac{n}{\delta}\left[\mathcal{W}_1\mathcal{R}\right]^{\alpha(\theta-1)}\left[\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)}\frac{A^{(\theta-1)}}{\mathbb{W}^{(\theta-1)(1-\alpha)+1}}\left[Q\int_{z_L}^{\overline{z}} z_j^{\theta-1}\,\mathrm{d}G(z) + \xi\int_{\overline{z}}^{z_U} z_j^{\theta-1}\,\mathrm{d}G(z)\right],$$

which should be equal to $\mathcal{L}$. The labor-market condition determines the total number of firms in the steady state. The analogous formula holds for the foreign country.

The Balance of Payment The balance of payment equilibrium condition can be written as

$$\eta^{2\theta-1} \frac{\int_{\overline{z}^*}^{z_U} z^{\theta-1}\, \mathrm{d}G(z)}{\int_{\overline{z}}^{z_U} z^{\theta-1}\, \mathrm{d}G(z)} = \left(\frac{n^*}{n}\right)\left(\frac{Q}{Q^*}\right)\left(\frac{A^*}{A}\right)^{\theta-1}\left(\frac{\mathbb{W}}{\mathbb{W}^*}\right)^{(\theta-1)(1-\alpha)}.$$

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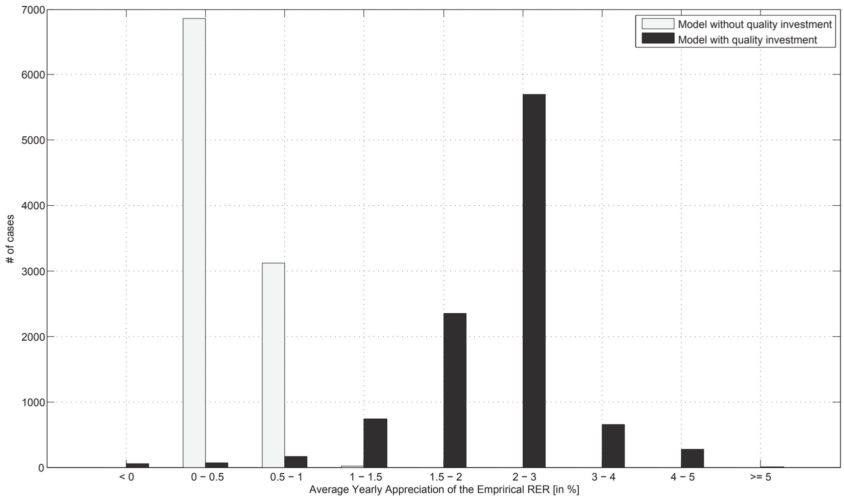
Table 1: Sampling scheme

Parameter	Lower bound	Upper bound
δ	0.05	0.20
θ	3.25	7.50
ς	0.05	0.30
c^x	0.12	0.50
α	0.05	0.50

Fig. A.1. Real exchange rate appreciation in Visegrad-4 countries

Source: Eurostat, Real exchange rate vis-à-vis the Euro Area.

Fig. A.2. The simulation results



The Dynamics of Economic Convergence: The Role of Alternative Investment Decisions

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Abstract

In this paper we evaluate how various investment decisions explain the macroeconomic dynamics of European transition countries. We introduce quality investment decisions into a model with other two standard investment margins assumed in the advanced trade literature – investment in new varieties and in export eligibility. We show that the standard investment margins are not sufficient to simultaneously match the dynamics in the macroeconomic variables, especially the export performance and the real exchange rate. In contrast, the extended model with quality investment provides reconciliation.

Key words: Two-country Modeling, Convergence, Real Exchange Rate
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1 Introduction

Macroeconomic dynamics of Central and Eastern European transition countries[1] (henceforth CEE countries) is puzzling from the perspective of standard dynamic general equilibrium models. The purpose of this paper is to introduce an extension to the existing two-country dynamic general equilibrium models with advanced trade features for understanding the convergence process of emerging market economies.

During a transition decade 1995-2005 (i.e., after the basic institutional foundations of a market economy have been created, Roland, 2004), the following five facts (see also Figure 1) dominate the picture of the economic development in CEE countries:

Fact 1: The convergence in GDP per capita of an average Visegrad-4 country to the average of the EU15 attained 1 per cent a year on average over the decade.

Fact 2: An increase in export to GDP ratio on average over Visegrad-4 countries attained 2 per cent a year over the decade, trade balance, after initial deficit around 5 per cent, has reached balanced position at the end of the decade.

Fact 3: The privatization and economic attractiveness of the region have resulted in a significant inflow of the foreign direct investment; on average the inflow in Visegrad-4 countries reached 5 bln. USD a year over the decade.

Fact 4: Real exchange rates – *also in sub-index of tradable goods* – of Visegrad-4 currencies vis à vis the Euro have been appreciating by an average about 3 per cent a year[2].

Fact 5: The proportion of medium-high and high tech products in total exports has gained 1.5 to 2 per cent a year, see Fabrizio et al. (2007).

We investigate the implications of different modeling assumptions on investment decisions for capturing the macroeconomic dynamics of the transition economies. The modeling benchmarks are recent macroeconomic models with

[1] These are so-called 4-Visegrad countries, i.e. the Czech Republic, Hungary, Poland, and Slovakia.

[2] The real exchange rate appreciation could be a consequence of unbalanced growth in productivities in favor of tradable versus non-tradable sectors which is stronger for fast growing countries. Since the Visegrad-4 countries converge to the developed Western EU countries, they could be prone to such an effect. However, Égert et al (2006) presents a survey of 14 studies estimating the Harrod-Balassa-Samuelson effect in Visegrad-4 countries using data for productivity growth in tradable as well as non-tradable sectors. The average annual effect is 0.7 percent, which is roughly one third of the observed average close to 3 percent annual real exchange rate appreciation in the region during 1995-2005.

advanced trade features; such as Ghironi and Melitz (2005). This kind of models usually works with two investment margins: horizontal investments in new varieties and investment in export eligibility.

However, Dury and Oomen (2007) and Brůha and Podpiera (2008, 2009) suggest that the concurrently observed Fact 2 and Fact 4 calls for an extension of the available framework by introducing an exogenous quality shocks (the former study) and decisions on investment in quality [3] (the latter study). Both studies basically associate the deficit of the two-country dynamic general-equilibrium models for explanation of the experience of converging countries in an implicit assumption that along the transition path products of both countries have comparable qualities. Indeed, since relatively more goods (Fact 2) are sold for relatively higher prices (Fact 4), the trend development in Visegrad-4 countries can be only reconciled by a steady improvement in quality of products (Fact 5).

Therefore, we present a model, which treats various investment decisions endogenously and, on aggregate, the decisions influence the real exchange rate and convergence dynamics. The contribution is that (i) we show how to formalize various kinds of investments in a rigorous framework of the dynamic general equilibrium and (ii) we explain their role for macroeconomic dynamics.

The comprehensive two-country modeling framework is formulated with the purpose of capturing long-run trends in main macroeconomic variables of a converging economy. Thus, contrary to the usual practice of applied DSGE models, which attempt to characterize the short-run fluctuations around a steady state or around an exogenously given development trajectory, the proposed model yields a long-run trajectory of convergence of asymmetric countries. Since the stress is on the long-run trends, the model is formulated as a dynamic, perfect-foresight model.

The rest of the paper is organized as follows: Section 2 describes the model, Section 3 highlights features of the proposed model by contrasting them with an alternative setup. Section 4 concludes. Appendix A contains detailed derivation of the model. Appendix B discusses the numerical methods for model simulation, while Appendix C describes the model extension by the elastic labor supply.

[3] An increase in product quality enables to export products at higher prices, which is compatible with the structural (equilibrium) real exchange rate appreciation. Hallack and Schott (2008), Cincibuch and Podpiera (2006), and Fabrizio et al. (2007) document on average 4 percent annual increase in relative product quality in the CEE countries compared to the various benchmarks during 1989-2004. This is consistent with the increasing proportion of high-tech production in export of Visegrad-4; approx. 2 percent annually between 1995-2005.

2 Description of the Model

There are two countries that are modeled in discrete time that runs from zero to infinity. Each country is populated by a representative competitive household who has recursive preferences over discounted streams of momentary utilities. The momentary utility is derived from consumption. Production takes place in heterogenous production entities called firms.

2.1 Firms

In the domestic country, there is a large number of firms, which are owned by the domestic household. In each period there is an unbounded mass of potential, ex-ante identical, entrants. Each entrant has to pay the fixed entry cost c; the cost is paid in terms of the aggregate consumption bundle. The actual number of entrants is determined by the zero-profit condition.

Firms differ ex-post entry by an idiosyncratic variation of total factor productivity: when a firm enters, it draws a shock z from a distribution $G(z)$. At the end of each period, there is an exogenous probability that a firm is hit by an exit shock. This probability is δ and is assumed to be independent of aggregate as well as individual states. Hit firms shut down.

The production of a firm is characterized by two features: physical quantity x and the product quality h. If the firm j wishes to produce its product with the quality level h_j, it has to pay the fixed quality investment at the level h_j. Similarly to the entry cost, the quality investments take the form of the aggregate consumption bundle. The quality choice is a once-and-for-all decision undertaken at the entry time (but after the idiosyncratic productivity is revealed).

In addition to the quality input h, the production requires a variable input –labor l. The production of the final bundle q_{jt} can be described using the neoclassical production function f and the firm's total factor productivity $A_t z_j$, $q_{jt} = z_j A_t f(h_j, l_{jt})$. The quality of the final bundle is h_j, and therefore the physical quantity is given simply as $x_{jt} = q_{jt}/h_j$. Such a distinction between the final bundle (quality included) and the physical quantity is standard in the literature; e.g., Young (1998). We explicitly distinguish the quality-quantity bundle from the physical quantity, since the explanation for the observed real exchange rate appreciation is based on a dichotomy between quality-adjusted and -unadjusted prices. This feature conciliates increasing external competitiveness with an appreciation real exchange rate[4].

[4] The quality investment is an endogenous decision and is in our model driven by

The production is affected by the level of firm's total factor productivity $A_t z_j$, which has two components: (a) an idiosyncratic component z_j, which is i.i.d. across firms and which follows the distribution $G(z)$ introduced above, and (b) a common component A_t. Domestic firms enjoy A_t, while foreign firms enjoy A_t^*.

Firms may export only if special fixed cost is invested. If a firm at the entry time decides to invest the fixed export costs, then it becomes eligible to export in all subsequent periods, otherwise it is never eligible to export. Therefore, we call such firms exporters, while the other firms non-exporters. Unit iceberg exporting cost ς represents transportation cost, policy barriers such as tariffs, while the fixed cost may represent expenditures associated with acquiring necessary expertise such as legal, business, or accounting issues of the foreign market. The fixed export cost is again paid in terms of the aggregate consumption bundle and is denoted as $c^x > 0$. This assumption implies, as in Melitz (2003), that in equilibrium there is a cut-off productivity value $\overline{z}$ such that firms with lower idiosyncratic productivity $z_j < \overline{z}$ will not invest to become eligible to export, while firms with a sufficiently high productivity level $z_j \geq \overline{z}$ will do so.

To make easier reading the paper, we introduce the following convention. The countries are distinguished by the * superscript: domestic country's variables are without *, while foreign country's variables do have one. The good produced by the firm located in the destination market is denoted by the superscript d, while goods imported are denoted by the superscript m. Thus $p^d_{j\tau t}$ will denote the time t price of a good produced by a vintage τ firm j located in the domestic country and sold to the domestic market, $p^m_{j\tau t}$ is the time t price of a good j imported to the domestic market from the foreign country, while $p^{m*}_{j\tau t}$ would be a price of a good exported from the domestic country to the foreign household. We further assume that prices are denominated in the currency of the market, where the good is sold.

The quality investment is a sunk factor as well as the fixed export cost if borne, set at the time of entry, while labor can be freely adjusted. Given a realization of the productivity shock z_j, the probability of the exit shock δ, and a chosen production plan, the value of a firm is determined by expected present value of the stream of profits.

Let $\mathbb{P}^e_{j\tau t}$ denote the t-period *real operating profit* of a domestic exporter of

demand factors. First, as the converging economy becomes richer, its consumers demand higher-quality goods and second, exporters need to invest into quality to compete with firms in the advanced country.

vintage τ enjoying the idiosyncratic productivity z_j, and be given as follows:

$$\mathbb{P}^e_{j\tau t} = \left[\kappa_t \frac{p^d_{j\tau t}}{P_t} + (1-\kappa_t)\,\frac{\eta_t}{1+\varsigma}\frac{p^{m*}_{j\tau t}}{P^*_t}\right] A_t z_j f(h_{j\tau}, l_{j\tau t}) - \mathbb{W}_t l_{j\tau t},$$

where $0 \leq \kappa_t \leq 1$ is the output share sold in the domestic market[5], P_t is the domestic price level, P^*_t is the foreign price level, η_t is the *real exchange rate*, which is linked to the nominal exchange rate s_t as $\eta_t = s_t P^*_t / P_t$, $\varsigma \geq 0$ represents the unit iceberg exporting costs, $\mathbb{W}_t$ is the *real wage*, and $l_{j\tau t}$ is the labor demand of the firm. Similarly, the *real operating profit* of a domestic non-exporter is given as follows:

$$\mathbb{P}^n_{j\tau t} = \frac{p^d_{j\tau t}}{P_t} A_t z_j f(h_{j\tau}, l_{j\tau t}) - \mathbb{W}_t l_{j\tau t}.$$

Analogous definitions apply to the foreign firms as well.

Products of firms of different vintages have different quality levels (since incentives to invest in quality differ as macroeconomic conditions change), and that is why $\mathbb{P}^e_{j\tau t}$, $\mathbb{P}^n_{j\tau t}$, $p^d_{j\tau t}$, and $q^d_{j\tau t}$ will be, in general, different.

We assume that firm's managers maximize the expected discounted stream of profits. Thus, the value of the profit stream of the domestic firm of vintage τ, enjoying the idiosyncratic productivity level z_j is (in real terms):

$$V_\tau(z_j) = \max_{\mathbf{1}^x_j, h_{j\tau}, \{l_\tau\},} \sum_{t=\tau}^{\infty} (1-\delta)^{t-\tau} \mu^t_\tau \left[\mathbf{1}^x_{j\tau} \mathbb{P}^e_{j\tau t} + (1-\mathbf{1}^x_{j\tau}) \mathbb{P}^n_{j\tau t}\right] - (c + \mathbf{1}^x_{j\tau} c^x + h_{j\tau}), \tag{1}$$

where $\mathbf{1}^x_j$ is the indicator of exporters (i.e. $\mathbf{1}^x_j = 1$, if the firm j is an exporter and $\mathbf{1}^x_j = 0$ for non-exporters), and the effective discount factor is given as $(1-\delta)^{\tau - t} \mu^t_\tau$, where μ^t_τ is the marginal rate of intertemporal substitution between dates τ and t . The rate of the intertemporal substitution is defined in Subsection 2.2. The value of the foreign firm is defined analogously.

Note that prices such as $p^d_{j\tau t}$ are prices of the final quantity-quality bundles and therefore derived indexes P_t, P^*_t, and η_t are related to aggregations of these final bundles. The prices related to physical quantities are then given by $\wp^d_{j\tau t} \equiv h_{j\tau} p^d_{j\tau t}$. The discussion about distinct role of prices per quality-quantity bundle and that of prices defined on physical quantities is left to Subsection 2.3.

[5] We show in Appendix A (Lemma 1) that in the equilibrium, all domestic exporters export at a particular date t the same share of its production to the foreign market, regardless their vintage τ or productivity j. Therefore, we shall simply write κ_t instead of $\kappa_{j\tau t}$. The vintage and productivity only determine whether a particular firm is an exporter or not.

2.1.1 Market Structure

The final good Q in the domestic country is composed of a continuum of quality-quantity bundles (goods), some of which are produced in the domestic country and some are imported. There is an imperfect substitution among these goods, which is modeled using the standard constant-elasticity-of-substitution (CES) function with the parameter $\theta > 1$. The aggregate good in the domestic country is defined as:

$$Q_t = \left(\sum_{\tau \le t}(1-\delta)^{t-\tau}\left[n_\tau \int q^{d\frac{\theta-1}{\theta}}_{j\tau t}\, \mathrm{d}G(j) + n^*_\tau \int \mathbf{1}^{x*}_{j\tau} q^{m\frac{\theta-1}{\theta}}_{j\tau t}\, \mathrm{d}G(j)\right]\right)^{\frac{\theta}{\theta-1}},$$

where n_τ is the number of domestic entrants, who enter the market at time τ. At time t, only $(1-\delta)^{t-\tau} n_\tau$ of such entrants survive. The final good in the foreign country is defined analogously. The market structure implies the following definition of the aggregate price index:

$$P_t = \left(\sum_{\tau \le t}(1-\delta)^{t-\tau}\left[n_\tau \int p^{d^{1-\theta}}_{j\tau t}\, \mathrm{d}G(j) + n^*_\tau \int \mathbf{1}^{x*}_{j\tau} p^{m^{1-\theta}}_{j\tau t}\, \mathrm{d}G(j)\right]\right)^{\frac{1}{1-\theta}}, \qquad (2)$$

where $p_{j\tau t}$ is the time t price of products of the vintage τ of the firm j. The pricing decisions of firms are described by the subsequent equations below. Note that the final good Q_t represents both physical quantities as well as qualities and that the price indexes P_t, and P^*_t aggregate both: available quantities and qualities. In that sense, these are quality-adjusted price indexes. See section 2.3 for more discussion.

2.1.2 Optimal Production Plans

We derive optimal production and investment plans using backward induction for general neoclassical production function. The parametric example of model equations for the Cobb-Douglas production function is given in Appendix A. We present the derivation for a domestic firm, which is easily generalized for a foreign firm.

Let us assume the problem of maximizing the value of a domestic firm. Since there are no labor adjustment costs, labor decisions are made on a period-by-period basis. Standard results of monopolistically competitive pricing suggest that prices are set as a mark-up over marginal costs. Simultaneously with prices, firms decide κ_t.

Now, let us take the perspective of a non-exporter of vintage τ and common productivity level A_t. Its real operating profit $\mathbb{P}^n_{j\tau t}$ in a period t is given – conditional on non-exporter status, common productivity, and idiosyncratic productivity z_j, – as a solution to the following program:

$$\mathbb{P}^n_{j\tau t} = \max_{l_{jt}} \left\{ \frac{p^d_{j\tau t}}{P_t} A_t z_j f(h_{j\tau}, l_{jt}) - \mathbb{W}_t l_{jt} \right\}. \tag{3}$$

Similarly, the real operating profit of an exporter $\mathbb{P}^e_{j\tau t}$ of vintage τ in a period t is given by:

$$\mathbb{P}^e_{j\tau t} = \max_{l_{jt}} \left\{ \left(\kappa_t \frac{p^d_{j\tau t}}{P_t} + (1-\kappa_t) \frac{\eta_t}{1+\varsigma} \frac{p^{m*}_{j\tau t}}{P^*_t} \right) A_t z_j f(h_{j\tau}, l_{j\tau t}) - \mathbb{W}_t l_{j\tau t} \right\}. \tag{4}$$

The expected present value of the stream of operating profits is given as follows: $\mathbb{P}^\xi_{j\tau} = \sum\limits_{t=\tau}^{\infty} \mu^t_\tau (1-\delta)^{t-\tau} \mathbb{P}^\xi_{j\tau t}$, with $\xi \in \{n, e\}$. The expected present values depend on idiosyncratic productivity z_j, quality investment $h_{j\tau}$, and the future path of productivities, real wages, and demands. The optimal investment decision of a firm, which enjoys a productivity level z_j, maximizes the value of the firm is given as $\mathbf{V}^\xi_\tau(h_{j\tau}|z_j) = \mathbb{P}^\xi_{j\tau} - \left(c + \mathbf{1}^x_{j\tau} c^x + h_j\right)$, for $\xi \in \{n, e\}$. The maximization of $\mathbf{V}^e_\tau(h_{j\tau}|z_j)$ (resp. $\mathbf{V}^n_\tau(h_{j\tau}|z_j)$) yields the optimal demand for quality investment for exporters (resp. non-exporters), and the value of the firm is:

$$V^\xi_\tau(z_j) = \max_{h_{j\tau} \geq 0} \mathbf{V}^\xi_\tau(h_{j\tau}|z_j),$$

where $\xi \in \{e, n\}$. The value functions $V^n_\tau(z_j)$, $V^e_\tau(z_j)$ implicitly define the cut-off value $\overline{z}$, which is the lowest idiosyncratic shock, which makes the export-eligibility investment profitable. Thus, it is defined as

$$\overline{z}_\tau = \arg\min_{z_j} (V^e_\tau(z_j) \geq V^n_\tau(z_j)).$$

The value of a firm is given by

$$V_\tau(z_j) = \max_{\xi \in \{n,e\}} V^\xi_\tau(z_j) = \begin{cases} V^e_\tau(z_j) & \text{if } z_j \geq \overline{z}_\tau \\ V^n_\tau(z_j) & \text{if } z_j < \overline{z}_\tau \end{cases},$$

and the expected value of a new entrant $\mathcal{V}_\tau$ is:

$$\mathcal{V}_\tau = \int_{z_L}^{z_u} V_\tau(z)\, G(\mathrm{d}z), \tag{5}$$

This completes the backward induction.

The optimal production plan derived above induces a measure over firms.

Denote by $\widetilde{\mathbb{P}}_{\tau t}$ the t-time expected[6] real operating profit of a domestic firm, which enters in time τ, $\widetilde{\mathbb{P}}_{\tau t} = \int_{z_L}^{z_u} \mathbb{P}_{j\tau t}\, G(\mathrm{d}z)$, and $\widetilde{c}_\tau$ represents the expected real investment cost under such measure. Then:

$$\mathcal{V}_\tau = \sum_{\sigma \geq 0} \mu_\tau^{\tau+\sigma}(1-\delta)^\sigma \widetilde{\mathbb{P}}_{\tau,\tau+\sigma} - \widetilde{c}_\tau,$$

where the expected real investment cost consists of three terms:

$$\widetilde{c}_\tau = c + c^x(1 - G(\overline{z}_\tau)) + \widetilde{h}.$$

The first term is the fixed entry cost c paid by all entrants prior the entry; the second term $c^x(1 - G(\overline{z}_\tau))$ is the expected export-eligibility cost (recall that only firms with $z_j \geq \overline{z}_\tau$ pay the cost). And the final term $\widetilde{h}$ is the expected quality investments, given by: $\widetilde{h} = \int_{z_L}^{\overline{z}_\tau} h_{j\tau}^{opt,n}\, G(\mathrm{d}z) + \int_{\overline{z}_\tau}^{z_U} h_{j\tau}^{opt,e}\, G(\mathrm{d}z)$.

2.2 Households

The domestic as well as foreign country is populated by a representative competitive household who has recursive preferences over discounted stochastic streams of period utilities. The period utilities are derived from consumption of the aggregate good. Leisure does not enter the utility, so labor is supplied inelastically[7]. The aggregate labor supply in the domestic country is $\mathcal{L}$, while $\mathcal{L}^*$ is the aggregate labor supply in the foreign country. Households can trade bonds denominated in the foreign currency.

The domestic household maximizes

$$\max U = \sum_{t=0}^{\infty} \beta^t u(C_t),$$

subject to

$$B_t = (1 + r^*_{t-1})B_{t-1} + \frac{-1}{\eta_t}(C_t - \mathbb{W}_t\mathcal{L}) + \frac{1}{\eta_t}(\Xi_t - \widetilde{c}_t n_t) - \frac{\Psi_B}{2}B_t^2 + \mathcal{T}_t, \quad (6)$$

where B_t is the real bond holding of the domestic household, C_t is consumption, r^*_{t-1} is the real interest rate on the internationally traded bond, Ψ_B represents portfolio adjustment costs, as in Schmitt-Grohe, Uribe (2003) to

[6] This expectation is taken with respect to the measure given by the optimal production plan.

[7] We consider the model extension with elastic labor supply in Appendix C.

stabilize the model[8], and $\mathcal{T}_t$ is the rebate of these costs in a lump-sum fashion to the household. The flow of real operating profits from all domestic firms is denoted as Ξ_t and is given by

$$\Xi_t = \sum_{s \leq t} (1-\delta)^{t-s} n_s \widetilde{\mathbb{P}}_{s,t}.$$

Because of the law of large numbers and of perfect foresight, the *ex-ante* expected values of the key variables for household decisions (such as investment cost or profit flows) coincide with *ex-post* realizations.

The first-order conditions for the domestic household are standard ones:

$$(1 + \Psi_B B_t) = \frac{\eta_{t+1}}{\eta_t}(1 + r_t^*)\mu_t^{t+1}, \tag{7}$$

$$\widetilde{c}_t = \sum_{v \geq 0} (1-\delta)^v \mu_t^{t+v} \widetilde{\mathbb{P}}_{t,t+v}, \tag{8}$$

along with the transversality condition $\lim_{t\to\infty} B_{t+1} = 0$, and where the marginal rate of substitution is defined as usual by :

$$\mu_{t_1}^{t_2} \equiv \beta^{t_2 - t_1} \frac{u'(C_{t_2})}{u'(C_{t_1})}.$$

Equation (7) determines the bond holding, and equation (8) is the expected zero-profit condition, which determines the number of new domestic entrants n_t.

It is worth noting that although there is an idiosyncratic variance at the firm level, the model is deterministic at the aggregate level, thus the dynasty problem is deterministic too. Therefore the marginal rate of substitution does not involve the expectation operator. The household problem in the foreign country is defined symmetrically.

Bonds are denominated in the foreign currency[9] and since the model is deterministic, this is a completely innocent assumption. The international bond market equilibrium requires that $B_t + B_t^* = 0$.

To summarize, the timing proceeds first with the entry of prospecting entrants in both countries. Then, each new entrant draws a productivity level from the

[8] In a strict sense, the model is stable even without portfolio adjustment costs (i.e., under $\Psi_B = 0$). The model is deterministic and therefore it would not exhibit unit-root behavior even under $\Psi_B = 0$. Nevertheless, if $\Psi_B = 0$, then the model would exhibit steady state dependence on the initial asset holding. Therefore we use nontrivial adjustment costs $\Psi_B > 0$ to give up the dependence of the steady state on the initial asset holding.

[9] The bond is real, which also means that the unit of foreign currency is equivalent to the unit of the foreign consumption bundle.

distribution G and it decides the quality of its production $h_{j\tau}$ and whether to invest for export eligibility. Then, labor demand and production (of both entrants and incumbents) take place. At the end of the period, some firms experience the exit shock and shut down.

2.3 Notes on Price Indexes

The prices $p_{j\tau t}$ and the corresponding price indexes P_t, and P_t^* are quality-adjusted. Therefore, the real wages $\mathbb{W}_t$ and $\mathbb{W}_t^*$ and the real exchange rate η_t are measured in the terms of quality-quantities bundles. These measures correspond to real-world price indexes only if the latter are quality-adjusted perhaps using a hedonic approach, which is rarely the case for transition countries (see Ahnert and Kenny, 2004, p. 28). To get indexes closer to real-world measures, we have to define aggregate indexes over $\wp_{j\tau t}$. We denote such indexes as $\mathcal{P}_t$ and $\mathcal{P}_t^*$.

The quality-unadjusted price index should satisfy the aggregation consistency, i.e., the aggregate expenditure (measured in quality-unadjusted prices) $\mathcal{P}_t Q_t$ should be equal to the aggregation of the individual (quality-unadjusted) prices. Therefore, the quality-unadjusted price index should be defined as follows:

$$\mathcal{P}_t = \frac{\sum_{\tau \le t}(1-\delta)^{t-\tau}\left[n_\tau \int_{z_L}^{z_U} q_{j\tau t}^d \wp_{j\tau t}^d \,\mathrm{d}G(j) + n_\tau^* \int_{\bar{z}_\tau}^{z_U} q_{j\tau t}^m \wp_{j\tau t}^m \,\mathrm{d}G(j)\right]}{Q_t}.$$

The algebraic form of the quality-unadjusted price index (in terms of productivities A_t, A_t^*, and aggregates Q_t, Q_t^*, P_t, P_t^*, $\mathbb{W}_t$, $\mathbb{W}_t^*$) is given in Appendix B.

Nevertheless, $\mathcal{P}_t$ might differ from the CPI-based real-world indexes by one more term. The market structure based on the CES aggregation implies the *love-for-variety* effect, which means that the welfare-theoretical price index differs from the 'average' price (CPI-based) index by the term $\nu^{\frac{1}{\theta-1}}$, where ν is the number of available varieties and θ is the parameter of substitution in the CES function (see Melitz, 2003 for definition and derivation). Therefore, we distinguish the following two definitions of the real exchange rate:

Quality-adjusted theoretically-consistent RER η_t is the real exchange rate, which enters the decisions of agents in the model.

CPI-based (quality-unadjusted) RER is the closest counterpart of the *measured real exchange rate* and is defined as $\left(\frac{\nu_t^*}{\nu_t}\right)^{\frac{1}{\theta-1}} \frac{\mathcal{P}_t^*/P_t^*}{\mathcal{P}_t/P_t}\eta_t$.

The quality-adjusted theoretically consistent real exchange rate η_t depreciates for the transition country during the convergence due to the downward-sloping demand curve. On the other hand, the CPI-based RER index may appreciate under some conditions, see Section 3 for discussion and intuition.

The number of available varieties in the domestic country can be written as:

$$\nu_t = \sum_{\tau \le t} (1-\delta)^{t-\tau} n_\tau + \sum_{\tau \le t} (1-\delta)^{t-\tau} \left(1 - G(\overline{z}_\tau^*)\right) n_\tau^*,$$

where the first term is the number of domestic firms of different vintages existing at time t, while the second term is the number of exporters in the foreign country existing at time t. The analogous formula holds also for the number of varieties in the foreign country.

2.4 General Equilibrium

The general equilibrium is defined as a time profile of prices such that all households optimize and all markets clear.

The aggregate resource constraint is given as follows:

$$C_t + n_t \tilde{c}_t = Q_t, \tag{9}$$

the labor market equilibrium requires:

$$\sum_{\tau \le t} (1-\delta)^{t-\tau} n_\tau \int_{z_L}^{z_U} l_{j\tau t} \, \mathrm{d}G(j) = \mathcal{L}, \tag{10}$$

where $l_{j\tau t}$ is the labor demand by individual firms, and $\mathcal{L}$ is the aggregate, inelastic, labor supply. Analogous market clearing conditions hold in the foreign country.

The international bond market equilibrium requires that

$$B_t + B_t^* = 0. \tag{11}$$

The last equilibrium condition is the balance-of-payment equilibrium, which requires that:

$$B_{t+1} = (1 + r_t^*) B_t + X_t, \tag{12}$$

where X_t is the value of *net* real exports of the domestic country expressed in the foreign currency.

A more involved task is to simulate the transition dynamics, because the model is effectively a vintage type model. However, if we rewrite the model in the recursive form (the full set of equations of the model in the first-order form is available in Appendix B), then the variety of efficient methods can be used to simulate the model.

2.5 Steady state

The steady state is the long-run equilibrium and it is obtained when exogenous parameters (particularly A and A^*) are constant for a sufficiently long period of time. The speed of convergence to the steady state is influenced mainly by parameters β and δ.

The steady state is characterized by a number of features. The most important (and intuitive) ones include:

- Zero bond holding $B_{ss} = 0$, which is due to adjustment costs Ψ_B.
- Constant quantities and prices.
- The steady-state effective discount rate reads as $\frac{1}{1-\beta(1-\delta)}$ and the steady-state interest rate $r_{ss} = \beta^{-1} - 1$.
- The zero net foreign asset positions implies that the net exports are zero as well.

3 Inquiry on Model Dynamics

We make use of the model introduced in the preceding sections to inquire whether vertical investment margin is a necessary component for a consistent explanation of the key stylized facts of converging economies (represented by the real exchange rate, GDP per capita, trade balance development, and the external debt). The vertical investment margin is alternatively switched on and off by alternative model's calibration and taking limits to the expressions, where necessary. The model with active vertical investment margin is labeled as *Benchmark*, while the model without vertical margin is labeled as *Alternative*. The comparison of the two model simulations yields our argument.

3.1 Calibrating the Model

We assume that the small and less developed economy experiences an exogenous convergence of the domestic total factor productivity to the level of its large and developed counterpart: $A_t \rightarrow A^*$. After the convergence is reached $A_t = A^*$, both economies converge to the steady state; the speed of convergence to the steady state is determined mainly by parameters β and δ. The majority of parameters are calibrated in accordance with the choice by Ghironi and Melitz (2005). The calibration for the *Benchmark* and the *Alternative* differs in the vertical investment margin calibration.

The simulation experiments are carried out under the Cobb-Douglas production function for production of the quality-quantity basket $f(k, l) = k^{\alpha} l^{1-\alpha}$, the constant-relative-risk-aversion momentary utility function u with the parameter of the intertemporal rate of substitution ϵ, and the uniform distribution [10] for $G(z)$. More details about functional forms and their implications are given in the Appendix A.

In the parametrization we assume two countries that have liberalized current and financial account of the balance of payments: free debt securities trading on which is levied a portfolio adjustment cost of $\Psi_B = 0.025$ (a value similar to Ghironi and Melitz, 2005). The trade liberalization is represented by a low value of transaction costs ($\varsigma = 0.05$), and the calibration of the export eligibility cost $c^x/c = 1.235$ is similar to that of Ghironi and Melitz (2005). The values of c is calibrated to reflect the consumption-to-absorbtion and investment-to-absorbtion ratio observed in data for the CEE countries (the Czech Republic, Slovakia, Hungary, and Poland). These ratios (both in data [11] and in the model) are about 70% and 30%, respectively.

In both countries there is an average mark-up over marginal cost of 28 percent, which falls into the conventional calibration range in the literature. The standard macroeconomic models such as Rotemberg and Woodford (1992) use $\theta = 6$, while Ghironi amd Melitz (2005) opt for a value of 3.8 (based on empirically found mark-ups for the U.S. by Bernard et al, 2003). Since the difference in the two mentioned models is in the presence or absence of entry cost, the interpretation of the average vs. marginal costs is crucial. While the mark-up over average and over marginal cost are equal in the model without entry cost, the model with entry cost has different mark-up over marginal and over average cost. Consequently, a model with entry cost and lower θ would correspond to the same mark-up over average cost in a model with higher θ and without entry cost. Based on the evidence of mark-up over average cost in the Czech Republic, provided by Podpiera and Raková (2008), in the range of 15-20 percent, we set the elasticity of substitution at 4.5.

The calibration of the extent to which quality investment influences the pro-

[10] Microeconomists usually use other distributions than uniform for modelling the distribution of productivities across firms. The usual choice is the Pareto distribution. This practise is followed by Ghironi and Melitz (2005). The problem with the Pareto distribution is that it restricts the parameter θ, since for large values of θ, the value of a new entrant may not be bounded (due to the shape of the Pareto distribution). That is why we use a distribution with the bounded support (i.e. uniform). Moreover, the uniform distribution shares some useful properties with the Pareto distribution (the both distributions are preserved under truncation).

[11] Note that when dealing with the absorption in data, we divide the government consumption into consumption and investments. This is necessary for comparison of the model with data, since the model abstracts from the public sector.

duction of quality-quantity basket, i.e. the parameter α, is set to 0.35 for the *Benchmark* and zero for the *Alternative model*. The former value is based on the calibration experiments with regard to the pace of the real exchange rate development in the CEE countries during 1995-2005. The choice of the elasticity of intertemporal substitution ϵ and the discount factor β are based on conventional calibration in the literature, i.e. 2 and 95 percent, respectively, which is an annual equivalent to the quarterly calibration in Ghironi and Melitz (2005). The annual exit rate for companies δ is 10 percent, which is the number used by Ghironi and Melitz (2005).

The simulations run from 1995 to 2100. It is assumed that by 2040 the convergence is completed, i.e. by 2040 the difference in total factor productivity is negligible. Years beyond 2040 are simulated in order to settle the model in the steady state. Table 1 provides an overview of calibrated parameters.

The TFP of the converging economy A_t growths according to the logistic curve $A_t = A^* \frac{1+m\exp(-(t-1995)/\iota)}{1+n\exp(-(t-1995)/\iota)}$, with the following numerical values: $m = 7.5$ (for the *Benchmark model*), $m = 6.5$ (for the *Alternative model*), $n = 11$, $\iota = 5$. These values imply that the initial total factor productivity of the converging economy reaches a slightly more than 60% of the value of the advanced country. This is motivated by the initial position of a typical transition country from the CEE. Note that the parameter m differs in the two model versions; the reason for this calibration is to obtain the identical initial conditions for the output ratio.

3.2 Simulation results

The output of the simulations for both the *Benchmark* and the *Alternative model* is displayed in Figure 2 and is represented by a set of five variables: the ratio of *per capita* GDP in the less developed country over that in the developed counterpart, an index of the welfare-theoretical real exchange rate η_t, the empirical real exchange rate (the index of the quality-unadjusted CPI-based real exchange rate), trade balance (as a percentage of the converging country GDP), and external debt of the converging country, i.e. its international bold holding position (expressed in a percentage of the converging country GDP).

In the *Benchmark model*, the convergence of the less developed country is characterized by halving the gap between GDP per capita within 15 years, empirical exchange rate appreciation by 40% by the end of convergence, the welfare-theoretic real exchange rate depreciation by 2.5%. The initial trade balance deficit reaching the lowest level of 5.7% is turning subsequently into surplus of roughly 5% in 15 years. And finally, the temporary accumulation of debt to GDP ratio towards the size of 60% is gradually reduced later.

In a comparison of the *Alternative* to the *Benchmark*, the absence of the quality investment in the *Alternative* causes a slightly faster closure of the convergence gap; the half of the gap is reached in roughly 13 years. However, the empirical real exchange rate appreciates very negligibly (which suggests that the effect of the new varieties is rather small) and the real exchange rate depreciates by roughly 2.5%. The major effect comes again from the consumption smoothing mechanism during the convergence in perfect foresight models. In particular, the dynamics of the trade balance exhibits similar pattern as in the *Benchmark*.

The simulation results for the *Benchmark* and the *Alternative* reveal that the introduction of the vertical investment does not change significantly the dynamics of other variables, but implies a significant appreciation of the empirical real exchange rate for the converging economy. It follows that in order to explain concurrently observed Fact 1, 2, and 4, stated in the Introduction, one needs to extend the standard framework with the vertical investment margin (documented in Fact 5).

3.3 Sensitivity analysis

We start with the parameter Ψ_B, which does not influence the steady state (and therefore the long-run exchange rate appreciation), but significantly affects the transitory dynamics. Its larger value than chosen would reduce consumption-smoothing and therefore the debt accumulation and trade imbalances would be lower during the convergence. The transition dynamics is influenced also by the parameter of the intertemporal substitution ε (lower values than chosen mean lower incentives to smooth consumption), β (higher values than chosen increase patience and therefore reduce the debt accumulation by the converging economy), and δ (higher values speed up the convergence process since old vintages are being rapidly replaced by new vintages).

The 'openness parameters' (fixed costs c^x and iceberg costs ς) impact the real exchange rate appreciation by reducing the extent of both 'endogenous' Harrod-Balassa-Samuelson effect as well as the quality channel. In fact, if the converging economy is more open at the beginning of the transition, then its real exchange rate is relatively stronger compared to the situation of initially less open transition economy. Therefore, there is lower scope for the real exchange rate adjustment for an initially more open converging economy.

And finally, the parameter of intratemporal substitution θ affects the exchange rate dynamics as follows: low values of θ (i.e. low substitution) increases the love-for-variety effect of the CES utility function, which means that the empirical real exchange rate appreciation would be somewhat stronger even for the model without quality investment (*Alternative model*). Still, even an extreme calibration, such as $\theta = 3$, would not make the *love-for-variety effect* on the empirical real exchange rate strong enough to replicate the real exchange rate appreciation observed in Visegrad-4 countries.

4 Conclusion

In this paper we describe an extension of two-country models in the literature that stems in quality investment decision and show that such an extension is necessary for a consistent explanation of key macroeconomic variables in transition converging economies. The major conceptual difference from the current literature stems from modeling the explicit decision about investment in quality in two countries that are unequally developed. From the technical point of view, the difference in the modeling approach compared to the literature is in the use of dynamic simulations for solving the model.

The paper presents simulations for two alternative models. First, a rather standard two-country model, in which only investment in varieties and export eligibility is considered. Such a model is shown to generate dynamics in many macroeconomic variables that matches the data quite well. Nevertheless, the dynamics in the real exchange rate is very subdued. Consequently, it by far misses the dynamics observed in Visegrad-4 countries. The second model, that embraces the endogenous decision about investment in quality is shown to perform equally well in matching macroeconomic variables but in addition also matches well the real exchange rate development.

As it follows from the results, the proposed extension is of crucial importance for consistent explanation of the macroeconomic developments in the Visegrad-4 region. Bringing the real exchange rate in line with the other macroeconomic variables (such as export performance) offers the reconciliation of the recent puzzle of the limited effect of the real exchange rate appreciation on external competitiveness in transition countries.

A Detailed Derivation of the Model

In this part of the paper, we derive the main model equations for particular functional forms of the production function, utility function, and investment cost functions. In particular, we use the Cobb-Douglas production function $f(k,l) = k^{\alpha} l^{1-\alpha}$ for the production of the quality-quantity bundle. The momentary utility function is parameterized using the common constant-relative-risk-aversion form $u(C) = (1-\varepsilon)^{-1} C^{1-\varepsilon}$, with the parameter of intertemporal substitution ε.

The short-run cost function associated with the Cobb-Douglas production function is given as follows:

$$\mathbb{C}(q, \mathbb{W}_t, A_t, z_j, h_{j\tau}) = \mathbb{W}_t \left[\frac{q}{A_t z_j h_{j\tau}^{\alpha}} \right]^{\frac{1}{1-\alpha}} .$$

First, we derive the maximizing behavior of non-exporters [12]. The period t supply decision of a vintage τ non-exporter, who enjoys the productivity z_j and who has invested in the product quality $h_{j\tau}$, is a solution to the following program [13]:

$$\max_{q_{j\tau t}^d} \left\{ \left[q_{j\tau t}^d \right]^{\frac{\theta-1}{\theta}} Q_t^{\frac{1}{\theta}} - \mathbb{C}(q_{j\tau t}^d, \mathbb{W}_t, A_t, z_j, h_{j\tau}) \right\} .$$

A simple algebra yields the optimal supply:

$$q_{j\tau t}^d = \left(\left[\frac{\theta-1}{\theta} (1-\alpha)\, \mathbb{W}_t^{-1} \left[A_t z_j h_{j\tau}^{\alpha} \right]^{\frac{1}{1-\alpha}} \right]^{\theta} Q_t \right)^{\frac{(1-\alpha)}{\alpha(\theta-1)+1}} ,$$

[12] We derive expressions only for domestic firms. The expression for foreign firms are easily derived analogously.

[13] Note that this program is equivalent to the program (3). The reason is that the quality level $h_{j\tau}$ and the export-eligibility status has been already decided. Therefore the problem of the output choice $q_{j\tau t}$ is perfectly equivalent to the choice of the only variable input (labor) $l_{j\tau t}$. In the derivation, we use the properties of the CES market structure: the real turnover is $\frac{p_{j\tau t}^d}{P_t} q_{j\tau t}^d = \left(q_{j\tau t}^d \right)^{\frac{-1}{\theta}} Q_t^{\frac{1}{\theta}} q_{j\tau t}^d = \left(q_{j\tau t}^d \right)^{\frac{\theta-1}{\theta}} Q_t^{\frac{1}{\theta}}$ by the residual demand function: $\frac{p_{j\tau t}^d}{P_t} = \left(\frac{q_{j\tau t}^d}{Q_t} \right)^{\frac{-1}{\theta}}$.

and the optimal labor demand:

$$l_{j\tau t}=\left[\frac{q^d_{j\tau t}}{A_t z_j h^\alpha_{j\tau}}\right]^{\frac{1}{1-\alpha}}=\left(\left[\frac{\theta-1}{\theta}(1-\alpha)\mathbb{W}_t^{-1}\right]^\theta\left[A_t z_j h^\alpha_{j\tau}\right]^{\theta-1}Q_t\right)^{\frac{1}{\alpha(\theta-1)+1}}. \tag{A.1}$$

Now, using the CES market structure, it is easy to derive the real turnover:

$$\frac{p^d_{j\tau t}}{P_t}q^d_{j\tau t}=z_j^{\frac{\theta-1}{\alpha(\theta-1)+1}}h_{j\tau}^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}}\left[\frac{\theta-1}{\theta}(1-\alpha)\mathbb{W}_t^{-1}A_t^{\frac{1}{1-\alpha}}\right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}}Q_t^{\frac{1}{\alpha(\theta-1)+1}}, \tag{A.2}$$

and the real operating profit [14] :

$$\mathbb{P}_{j\tau t}=\frac{p^d_{j\tau t}}{P_t}q^d_{j\tau t}-\mathbb{C}(q^d_{jt},\mathbb{W}_t,A_t,z_j,h_{j\tau})=\mathcal{W}_1 z_j^{\frac{\theta-1}{\alpha(\theta-1)+1}}h_{j\tau}^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}}\mathbb{W}_t^{\frac{-(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}}A_t^{\frac{(\theta-1)}{\alpha(\theta-1)+1}}Q_t^{\frac{1}{\alpha(\theta-1)+1}}.$$

Second, the optimal production decisions of exporters is derived. The problem can be characterized as follows (with the definition $q_{j\tau t}=\kappa_{j\tau t}q^d_{j\tau t}+(1-\kappa_{j\tau t})q^{m*}_{j\tau t}$):

$$\max_{q^d_{j\tau t},q^{m*}_{j\tau t}}\left\{\left(q^d_{j\tau t}\right)^{\frac{\theta-1}{\theta}}Q_t^{\frac{1}{\theta}}+\left(\frac{\eta_t}{1+\varsigma}\right)Q_t^{*\frac{1}{\theta}}\left(q^{m*}_{j\tau t}\right)^{\frac{\theta-1}{\theta}}-\mathbb{C}(q_{j\tau t},\mathbb{W}_t,A_t,z_j,h_{j\tau})\right\}.$$

The solution yields that $q^d_{j\tau t}=\left[\frac{\theta-1}{\theta}\left(\frac{\partial\mathbb{C}}{\partial q_{j\tau t}}\right)^{-1}\right]^\theta Q_t$, and $q^{*m}_{j\tau t}=\left[\frac{\theta-1}{\theta}\frac{\eta_t}{1+\varsigma}\left(\frac{\partial\mathbb{C}}{\partial q_{j\tau t}}\right)^{-1}\right]^\theta Q^*_t$. Some simple, but tedious, algebraic manipulations yield:

$$\kappa_{j\tau t}q_{j\tau t}\equiv q^d_{j\tau t}=\left[\frac{\theta-1}{\theta}(1-\alpha)\mathbb{W}_t^{-1}\left(A_t z_j h^\alpha_{j\tau}\right)^{\frac{1}{1-\alpha}}\right]^\theta\frac{Q_t}{q_{jt}^{\frac{\alpha\theta}{1-\alpha}}},$$

and

$$(1-\kappa_{j\tau t})q_{j\tau t}\equiv q^{m*}_{j\tau t}=\left[\frac{\theta-1}{\theta}(1-\alpha)\frac{\eta_t}{1+\varsigma}\mathbb{W}_t^{-1}\left(A_t z_j h^\alpha_{j\tau}\right)^{\frac{1}{1-\alpha}}\right]^\theta\frac{Q^*_t}{q_{j\tau t}^{\frac{\alpha\theta}{1-\alpha}}}.$$

This implies that $\kappa_{j\tau t}=\frac{Q_t}{Q_t+Q^*_t\left(\frac{\eta_t}{1+\varsigma}\right)^\theta}$. Observe that $\kappa_{j\tau t}$ does not depend on individual characteristics of firms: z_j and $h_{j\tau}$; it depends only on relative tightness of both markets and on the real exchange rate corrected for transport costs ς. Therefore, we just proved the following Lemma:

[14] We define $\mathcal{W}_1\equiv\left[\frac{\theta-1}{\theta}(1-\alpha)\right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}}-\left[\frac{\theta-1}{\theta}(1-\alpha)\right]^{\frac{\theta}{\alpha(\theta-1)+1}}=\frac{\alpha(\theta-1)+1}{(\theta-1)(1-\alpha)}\left[\frac{\theta-1}{\theta}(1-\alpha)\right]^{\frac{\theta}{\alpha(\theta-1)+1}}$.

Lemma 1 *It is never optimal for exporters to export all production and not to export in a given period. Moreover, the optimal exporting share κ_t depends only on the current macroeconomic conditions, and – given the exporting status of a firm – it does not depend on its vintage or on its productivity.*

Thus, we will simply write κ_t instead of $\kappa_{j\tau t}$. Define $\xi_t \equiv Q_t + Q_t^* \left(\frac{\eta_t}{1+\varsigma}\right)^\theta = \frac{Q_t}{\kappa_t}$. The total production of eligible firms can be written as follows:

$$q_{j\tau t} = \left(z_j^\theta h_{j\tau}^{\alpha\theta}\right)^{\frac{1}{\alpha(\theta-1)+1}} \left\{ \left[\frac{\theta-1}{\theta}(1-\alpha)\,\mathbb{W}_t^{-1} A_t^{\frac{1}{1-\alpha}}\right]^\theta \xi_t \right\}^{\frac{(1-\alpha)}{\alpha(\theta-1)+1}},$$

and the optimal labor demand:

$$l_{j\tau t} = \left[\frac{q_{j\tau t}}{A_t z_j h_{j\tau}^\alpha}\right]^{\frac{1}{1-\alpha}} = \left(\left[\frac{\theta-1}{\theta}(1-\alpha)\,\mathbb{W}_t^{-1}\right]^\theta \left[A_t z_j h_{j\tau}^\alpha\right]^{\theta-1} \xi_t\right)^{\frac{1}{\alpha(\theta-1)+1}}. \tag{A.3}$$

The firms' real turnovers on the domestic and the foreign markets, respectively are given by:

$$\frac{p_{j\tau t}^d}{P_t} q_{j\tau t}^d = z_j^{\frac{\theta-1}{\alpha(\theta-1)+1}} h_{j\tau}^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}} \kappa_t^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}} \left[\frac{\theta-1}{\theta}(1-\alpha)\,\mathbb{W}_t^{-1} A_t^{\frac{1}{1-\alpha}}\right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} Q_t^{\frac{1}{\alpha(\theta-1)+1}}, \tag{A.4}$$

$$\begin{aligned}\left(\frac{\eta_t}{1+\varsigma}\right)\frac{p_{j\tau t}^{m*}}{P_t^*} q_{jt}^{m*} &= z_j^{\frac{\theta-1}{\alpha(\theta-1)+1}} h_{j\tau}^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}} (1-\kappa_t)^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}} \left(\frac{\eta_t}{1+\varsigma}\right)^{\frac{\theta}{\alpha(\theta-1)+1}} \times \\ &\times \left[\frac{\theta-1}{\theta}(1-\alpha)\,\mathbb{W}_t^{-1} A_t^{\frac{1}{1-\alpha}}\right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} Q_t^{*\frac{1}{\alpha(\theta-1)+1}}.\end{aligned} \tag{A.5}$$

Real production costs of exporters read as follows:

$$\mathbb{C}_{jt} = z_j^{\frac{\theta-1}{\alpha(\theta-1)+1}} h_{j\tau}^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}} A_t^{\frac{(\theta-1)}{\alpha(\theta-1)+1}} \mathbb{W}_t^{\frac{-(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} \left\{\left[\frac{\theta-1}{\theta}(1-\alpha)\right]^\theta \xi_t\right\}^{\frac{1}{\alpha(\theta-1)+1}},$$

thus, the real operating profit in a period t is given as:

$$\mathbb{P}_{j\tau t}^e = \mathcal{W}_1 z_j^{\frac{\theta-1}{\alpha(\theta-1)+1}} h_{j\tau}^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}} A_t^{\frac{(\theta-1)}{\alpha(\theta-1)+1}} \mathbb{W}_t^{\frac{-(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} \xi_t^{\frac{1}{\alpha(\theta-1)+1}}.$$

Now, we are able to derive the expected present value of profit stream. We start with an exporter $\mathbb{P}^e_{j\tau}$, whose expected present value satisfies:

$$\mathbb{P}^e_{j\tau} = z_j^{\frac{\theta-1}{\alpha(\theta-1)+1}} h_{j\tau}^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}} \underbrace{\mathcal{W}_1 \sum_{t=\tau}^{\infty} (1-\delta)^{t-\tau} \mu^t_\tau A_t^{\frac{(\theta-1)}{\alpha(\theta-1)+1}} \mathbb{W}_t^{\frac{-(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} \xi_t^{\frac{1}{\alpha(\theta-1)+1}}}_{\varpi^e_\tau}, \tag{A.6}$$

while the expected present value of a non-exporter $\mathbb{P}^n_{j\tau}$ satisfies:

$$\mathbb{P}^n_{j\tau} = z_j^{\frac{\theta-1}{\alpha(\theta-1)+1}} h_{j\tau}^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}} \underbrace{\mathcal{W}_1 \sum_{t=\tau}^{\infty} (1-\delta)^{t-\tau} \mu^t_\tau A_t^{\frac{(\theta-1)}{\alpha(\theta-1)+1}} \mathbb{W}_t^{\frac{-(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} Q_t^{\frac{1}{\alpha(\theta-1)+1}}}_{\varpi^n_\tau}. \tag{A.7}$$

The value of an exporter, who enjoys a productivity level z_j, is determined by quality investment:

$$\mathbf{V}^e_\tau(h_{j\tau}|z_j) = \mathbb{P}^e_{j\tau} - (c + c^x + h_{j\tau}) \equiv z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} h_{j\tau}^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \varpi^e_\tau - (c + c^x + h_{j\tau});$$

and similarly for a non-exporter:

$$\mathbf{V}^n_\tau(h_{j\tau}|z_j) = \mathbb{P}^n_{j\tau} - (c + h_{j\tau}) = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} h_{j\tau}^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \varpi^n_\tau - (c + h_{j\tau}).$$

If firms' manager maximizing the value of the firm chooses the following quality level:

$$h^{opt,e}_{j\tau} = z_j^{\theta-1} \left[\frac{\alpha(\theta-1)\varpi^e_\tau}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)+1}, \tag{A.8}$$

and the value of an exporting firm is[15]:

$$V^e_\tau(z_j) = \max_{h\geq 0} \mathbf{V}^e_\tau(h|z_j) = z_j^{(\theta-1)} \left[\varpi^e_\tau\right]^{\alpha(\theta-1)+1} \mathcal{G} - (c + c^x), \tag{A.9}$$

similarly, the value of a non-exporting firm is

$$V^{dn}_\tau(z_j) = \max_{h\geq 0} \mathbf{V}^{dn}_\tau(h|z_j) = z_j^{\theta-1} \left[\varpi^n_\tau\right]^{\alpha(\theta-1)+1} \mathcal{G} - c, \tag{A.10}$$

and the optimal investment to quality is:

$$h^{opt,n}_{j\tau} = z_j^{\theta-1} \left[\frac{\alpha(\theta-1)\varpi^n_\tau}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)+1}. \tag{A.11}$$

[15] Define $\mathcal{G} \equiv \left[\left(\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}\right)^{\alpha(\theta-1)} - \left(\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}\right)^{\alpha(\theta-1)+1} \right]$, which can be simplified to $\mathcal{G} = \frac{1}{\alpha(\theta-1)+1} \left(\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}\right)^{\alpha(\theta-1)}$.

The value functions $V^n_\tau(z_j)$, and $V^e_\tau(z_j)$ implicitly define the cut-off value $\overline{z}$, which is the least idiosyncratic shock, which makes the export-eligibility investment profitable, i.e. $\overline{z}_\tau = \arg\min_{z_j}(V^e_\tau(z_j) \geq V^n_\tau(z_j))$, which for the particular parametrization is given as follows:

$$\overline{z}_\tau = \left(\frac{c^x}{\mathcal{G}[[\varpi^e_\tau]^{\alpha(\theta-1)+1} - [\varpi^n_\tau]^{\alpha(\theta-1)+1}]} \right)^{\frac{1}{\theta-1}}. \tag{A.12}$$

Note that the definition of the cut-off value is correct, only if the expected present value of profit is increasing in z_j. The proposition below demonstrates that this is indeed the case:

Lemma 2 *: The net present value of the stream of exporter's real operating profits $\mathbb{P}^e_{j\tau}$ is increasing in z_j, and similarly for non-exporters. Moreover, for any z_j and τ: $\mathbb{P}^e_{j\tau} > \mathbb{P}^n_{j\tau}$.*

PROOF. The first part of the claim is a direct application of the envelope theorem. Indeed, the envelope theorem ensures that $\frac{d\mathbb{P}^e_{j\tau t}}{dz_j} = \frac{\partial \mathbb{P}^e_{j\tau t}}{\partial z_j}$. By (3) one obtains that $\frac{\partial \mathbb{P}^e_{j\tau t}}{\partial z_j} = \frac{\theta-1}{\theta z_j} [A_t z_j f(h_{j\tau}, l_{jt})]^{\frac{\theta-1}{\theta}} Q_t^{\frac{1}{\theta}}$, which is clearly positive for any finite z_j, A_t, and Q_t. Therefore $\frac{d\mathbb{P}^e_{j\tau}}{dz_j} = \sum_{t=\tau}^{\infty} \mu^t_\tau (1-\delta)^{t-\tau} \frac{d\mathbb{P}^e_{j\tau t}}{dz_j} = \sum_{t=\tau}^{\infty} \mu^t_\tau (1-\delta)^{t-\tau} \frac{\partial \mathbb{P}^e_{j\tau t}}{\partial z_j} > 0$. The exactly analogous reasoning applies for exporters. This proves the first part of Lemma. To prove the second part of Lemma, observe that the exporter can secure at least as high profit as the non-exporter by choosing $\kappa \equiv 1$, and by choosing the same level of the quality investment $h_{j\tau}$. Therefore $\mathbb{P}^e_{j\tau} \geq \mathbb{P}^n_{j\tau}$. The strict inequality follows from the fact that $0 < \kappa_t < 1$ by Lemma 1.

B Numerical solution

This section describes numerical techniques to solve the model. The model in its original formulation is a vintage-type model, which is hard to solve, since it is necessary to keep track of each of the vintages of the productivity distribution. This is demanding from the computational point of view (demanding on memory and computing time). A recursive formulation is much more appropriate for numerical techniques, since the standard, widely understood, techniques, such as the projection method (Judd, 2002); or the L-B-J technique (Juillard et al, 1998) can be applied. Therefore, in Part B.1, we discuss how to rewrite the model in a recursive (first-order) form. Part B.2

describes how to recursively compute the quality-unadjusted price index $\mathcal{P}$, and the final part B.3 contains some details on numerical solution.

B.1 Model in the recursive form

In this part, we transform the model into the recursive form, which is suitable for efficient application of numerical methods. The recursive form consists of dynamic (first-order difference) equations and static (algebraic) equations. We first describe dynamic equations, and then the static equations. We use the same parametrization as in the paper, i.e., the CES market structure, the CRRA utility function $u = \frac{c^{1-\varepsilon}}{1-\varepsilon}$, and the Cobb-Douglas production function $f(k,l) = k^{\alpha} l^{1-\alpha}$.

We list equations for variables related to domestic agents (firms and household) only. The corresponding equations for foreign agents can be derived analogously.

B.1.1 Dynamic Equations

Intertemporal Marginal Rate of Substitution

$$\mu_t^{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{\varepsilon}, \tag{B.1}$$

Profit Flows

$$\begin{aligned} \varpi_t^n &= \mathcal{W}_1 \left(A_t^{\theta-1} \mathbb{W}_t^{-(\theta-1)(1-\alpha)} Q_t \right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{t+1} \varpi_{t+1}^n, \\ \varpi_t^e &= \mathcal{W}_1 \left(A_t^{\theta-1} \mathbb{W}_t^{-(\theta-1)(1-\alpha)} \xi_t \right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{t+1} \varpi_{t+1}^e. \end{aligned} \tag{B.2}$$

The expected present value of the stream of future profits (from a new entrant) can be easily expressed as follows:

$$\Omega_t = \left[\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)} \left(\varpi_{t+1}^{n^{\alpha(\theta-1)+1}} \int_{z_L}^{\overline{z}_{t+1}} z^{\theta-1}\, G(\mathrm{d}z) + \varpi_{t+1}^{e^{\alpha(\theta-1)+1}} \int_{\overline{z}_{t+1}}^{z_U} z^{\theta-1}\, G(\mathrm{d}z) \right), \tag{B.3}$$

where the cut-off values are given by (B.9).

To get the representation of realized profit flows Ξ_t, the value of exports, the labor demand, and of the CPI-based real exchange rate, it is necessary to define the 'weighted' numbers of exporters and non-exporters, where the weights are based on firms' size. The weighted number of exporters $\widehat{n}_t^e$ obeys the following recursive relation:

$$\widehat{n}_{t+1}^e = (1-\delta)\widehat{n}_t^e + n_{t+1}^e \left[\frac{\alpha(\theta-1)\varpi_{t+1}^e}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)} \int_{\overline{z}_{t+1}}^{z_U} z^{\theta-1}\, G(\mathrm{d}z), \tag{B.4}$$

while a similar recursive equation holds for non-exporting firms:

$$\widehat{n}_{t+1}^n = (1-\delta)\widehat{n}_t^n + n_{t+1}^n \left[\frac{\alpha(\theta-1)\varpi_{t+1}^n}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)} \int_{z_L}^{\overline{z}_{t+1}} z^{\theta-1}\, G(\mathrm{d}z). \tag{B.5}$$

To get the recursive representation for actual realized profits Ξ_t, we have to split it into two parts (according to the export-status): $\Xi_t = \Xi_t^e + \Xi_t^n$. Then:

$$\begin{aligned}\Xi_t^e &= \mathcal{W}_1 \widehat{n}_t^e \left(A_t^{\theta-1}\mathbb{W}_t^{-(\theta-1)(1-\alpha)}\xi_t\right)^{\frac{1}{\alpha(\theta-1)+1}}, \\ \Xi_t^n &= \mathcal{W}_1 \widehat{n}_t^n \left(A_t^{\theta-1}\mathbb{W}_t^{-(\theta-1)(1-\alpha)}Q_t\right)^{\frac{1}{\alpha(\theta-1)+1}}.\end{aligned} \tag{B.6}$$

The value of net exports used in the equilibrium condition (12) is defined as the difference between the value of gross domestic exports minus the value of gross domestic imports. The value of gross domestic exports (expressed in the domestic currency) X_t^d satisfies the following equation:

$$X_t^d = \widehat{n}_t^e (1-\kappa_t)^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}} \left(\frac{\eta_t}{1+\varsigma}\right)^{\frac{\theta}{\alpha(\theta-1)+1}} \left[\frac{\theta-1}{\theta}(1-\alpha)\mathbb{W}_t^{-1}A_t^{\frac{1}{1-\alpha}}\right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} Q_t^{*\frac{1}{\alpha(\theta-1)+1}}.$$

The analogical formula applies to the value of gross exports of the foreign country.

The rest of the model dynamic equations are *balance-of-payment equation* (12), household's *budget constraint* (6), household's *Euler equation* (7), and their foreign counterparts. These equations are already in the form of first-order difference equations.

B.1.2 Static Equations

The static equations are mainly market clearing conditions and definitions. The market clearing conditions include the clearing of the goods markets (9), international bond market clearing (11), and labor market clearing conditions. The labor market clearing conditions can be restated into the recursive form as follows: define $\eth_t$ as

$$\eth_t^n = \left(\left[\frac{\theta-1}{\theta}(1-\alpha)\right]^\theta A_t^{\theta-1}\mathbb{W}_t^{-\theta}Q_t\right)^{\frac{1}{\alpha(\theta-1)+1}}, \tag{B.7}$$

$$\eth_t^e = \left(\left[\frac{\theta-1}{\theta}(1-\alpha)\right]^\theta A_t^{\theta-1}\mathbb{W}_t^{-\theta}\xi_t\right)^{\frac{1}{\alpha(\theta-1)+1}}.$$

Then the domestic labor demand is given as $\mathcal{L}_t = \sum_{\xi\in\{e,n\}} \eth_t^\xi \widehat{n}_t^\xi$. The labor demands should be equal to inelastic labor supply.

The *zero-profit equations* can be easily converted to the algebraic form:

$$\widetilde{c}_t = \Omega_t,$$

where expected investment costs obey:

$$\begin{aligned}\widetilde{c}_t &= c + (1-G(\overline{z}_t))c^x + \dots \qquad (B.8)\\ &+ \left[\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)+1}\left(\varpi_t^{n\,\alpha(\theta-1)+1}\int_{z_L}^{\overline{z}_{t+1}} z^{\theta-1}\,G(\mathrm{d}z) + \varpi_t^{e\,\alpha(\theta-1)+1}\int_{\overline{z}_{t+1}}^{z_U} z^{\theta-1}\,G(\mathrm{d}z)\right).\end{aligned}$$

It remains to find the representation of the cut-off value $\overline{z}$, which is the least idiosyncratic shock, which makes the export-eligibility investment profitable, i.e. $\overline{z}_\tau = \arg\min_{z_j}(V_\tau^e(z_j) \geq V_\tau^n(z_j))$. The cut-off value satisfies:

$$\overline{z}_\tau = \left(\frac{c^x}{\mathcal{G}\left[[\varpi_\tau^e]^{\alpha(\theta-1)+1} - [\varpi_\tau^n]^{\alpha(\theta-1)+1}\right]}\right)^{\frac{1}{\theta-1}}. \tag{B.9}$$

B.2 Derivation of the quality-unadjusted price index

Model simulations require the characterization of the quality-unadjusted price index $\mathcal{P}$ in the recursive form too. Recall that the price index is given as:

$$\mathcal{P}_t = \frac{\sum_{\tau\leq t}(1-\delta)^{t-\tau}\left[n_\tau \int_{z_L}^{z_U} q_{j\tau t}^d \wp_{j\tau t}^d\,\mathrm{d}G(j) + n_\tau^* \int_{\overline{z}_\tau}^{z_U} q_{j\tau t}^m \wp_{j\tau t}^m\,\mathrm{d}G(j)\right]}{Q_t}.$$

The recursive representation of the key formula $\sum_{\tau\leq t}(1-\delta)^{t-\tau}\left[n_\tau \int_{z_L}^{z_U} q_{j\tau t}^d \wp_{j\tau t}^d\,\mathrm{d}G(j) + n_\tau^* \int_{\overline{z}_\tau}^{z_U} q_{j\tau t}^m \wp_{j\tau t}^m\right.$ can be outlined in three steps.

First, we will consider the $\mathcal{P}_t^n \mathrel{\mathring{=}} \sum_{\tau\leq t}(1-\delta)^{t-\tau} n_\tau \int_{z_L}^{\overline{z}_\tau} q_{j\tau t}^d \wp_{j\tau t}^d\,\mathrm{d}G(j)$, i.e., the part related to domestic non-exporters. It obviously holds that $q_{j\tau t}^d \wp_{j\tau t}^d =$

$q^d_{j\tau t}p^d_{j\tau t}h_{j\tau} = P_t q^d_{j\tau t}\frac{p^d_{j\tau t}}{P_t}h_{j\tau}$. The expression $q^d_{j\tau t}\frac{p^d_{j\tau t}}{P_t}$ is the real turnover of the domestic non-exporter and it has been already derived in (A.2). Therefore, it is possible to write:

$$q^d_{j\tau t}\wp^d_{j\tau t} = P_t z_j^{\frac{\theta-1}{\alpha(\theta-1)+1}} h_{j\tau}^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}+1}\left[\frac{\theta-1}{\theta}(1-\alpha)\mathbb{W}_t^{-1}A_t^{\frac{1}{1-\alpha}}\right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} Q_t^{\frac{1}{\alpha(\theta-1)+1}}.$$

Now, this can be combined with the expression for the optimal quality investment for non-exporters (A.11), to get the following expression:

$$q^d_{j\tau t}\wp^d_{j\tau t} = \mathcal{W}_P P_t z_j^{2(\theta-1)}\varpi_\tau^{n2\alpha(\theta-1)+1}\mathbb{W}_t^{-\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} A_t^{\frac{(\theta-1)}{\alpha(\theta-1)+1}} Q_t^{\frac{1}{\alpha(\theta-1)+1}},$$

where we define $\mathcal{W}_P = \left[\frac{\theta-1}{\theta}(1-\alpha)\right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}}\left[\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}\right]^{2\alpha(\theta-1)+1}$. Therefore:

$$\frac{\mathcal{P}^n_t}{P_t} = \widehat{\widehat{n}}^n_t\left[\frac{\theta-1}{\theta}(1-\alpha)\mathbb{W}_t^{-1}A_t^{\frac{1}{1-\alpha}}\right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} Q_t^{\frac{1}{\alpha(\theta-1)+1}},$$

where the weighted number[16] $\widehat{\widehat{n}}^n_t$ of domestic non-exporters satisfies:

$$\widehat{\widehat{n}}^n_{t+1} = (1-\delta)\widehat{\widehat{n}}^n_t + n_{t+1}\left[\frac{\alpha(\theta-1)\varpi^n_{t+1}}{\alpha(\theta-1)+1}\right]^{2\alpha(\theta-1)+1}\int_{z_L}^{\overline{z}_{t+1}} z^{2(\theta-1)}\,G(\mathrm{d}z). \quad \text{(B.10)}$$

Second, consider the case of domestic exporters $\mathcal{P}^e_t \triangleq \sum_{\tau\le t}(1-\delta)^{t-\tau}n_\tau\int_{\overline{z}_\tau}^{z_U} q^d_{j\tau t}\wp^d_{j\tau t}\,\mathrm{d}G(j),$. The derivation of the desired expression is completely analogous: use the exporter's turnover on the domestic market (A.4) and substituting the expression for the optimal quality investment in (A.8). The algebraic manipulations yield the following formula:

$$\frac{\mathcal{P}^e_t}{P_t} = \widehat{\widehat{n}}^e_t\left[\frac{\theta-1}{\theta}(1-\alpha)\mathbb{W}_t^{-1}A_t^{\frac{1}{1-\alpha}}\right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} Q_t^{\frac{1}{\alpha(\theta-1)+1}},$$

where the weighted number $\widehat{\widehat{n}}^e_t$ of domestic exporters satisfies:

$$\widehat{\widehat{n}}^e_{t+1} = (1-\delta)\widehat{\widehat{n}}^e_t + n_{t+1}\left[\frac{\alpha(\theta-1)\varpi^e_{t+1}}{\alpha(\theta-1)+1}\right]^{2\alpha(\theta-1)+1}\int_{\overline{z}_{t+1}}^{z_U} z^{2(\theta-1)}\,G(\mathrm{d}z).$$

Finally, the recursive formula for: $\mathcal{P}^m_t \triangleq \sum_{\tau\le t}(1-\delta)^{t-\tau}n^*_\tau\int_{\overline{z}_\tau}^{z_U} q^m_{j\tau t}\wp^m_{j\tau t}\,\mathrm{d}G(j)$ is given by the suitable modification of the formula for $\mathcal{P}^e_t/P_t$. In sum, the

[16] Note that there is a difference between $\widehat{\widehat{n}}^n_t$ and $\widehat{n}^n_t$, which can be inferred from the integrands in (B.5) and in (B.10). The analogous comment applies to $\widehat{\widehat{n}}^e_t$ and $\widehat{n}^e_t$.

desired expression for $\mathcal{P}_t$ is given as:

$$\mathcal{P}_t = \frac{P_t}{Q_t}\left[\frac{\mathcal{P}_t^n}{P_t} + \frac{\mathcal{P}_t^e}{P_t} + \frac{\mathcal{P}_t^m}{P_t}\right] = Q_t^{-1}\left[\mathcal{P}_t^n + \mathcal{P}_t^e + \mathcal{P}_t^m\right].$$

B.3 Numerical methods

We propose the following implementation of numerical method to the recursive representation of the model. Given a guess of the time profiles of the following seven variables: domestic output $\{Q_t\}_{t=0}^{\infty}$, domestic real wage $\{\mathbb{W}_t\}_{t=0}^{\infty}$, domestic consumption $\{C_t\}_{t=0}^{\infty}$, their foreign counterparts: $\{Q_t^*\}_{t=0}^{\infty}$, $\{\mathbb{W}_t^*\}_{t=0}^{\infty}$, $\{C_t^*\}_{t=0}^{\infty}$ and the real exchange rate $\{\eta_t\}_{t=0}^{\infty}$, it is straightforward to compute the time profile of all other endogenous variables (given exogenous and policy variables). The algorithm is following:

(1) Given $\{C_t\}_{t=0}^{\infty}$, $\{C_t^*\}_{t=0}^{\infty}$ compute the marginal rate of substitutions $\left\{\mu_t^{t+1}\right\}_{t=0}^{\infty}$, $\left\{\mu_t^{*t+1}\right\}_{t=0}^{\infty}$ using (B.1).
(2) Given $\{Q_t\}_{t=0}^{\infty}$, $\{\mathbb{W}_t\}_{t=0}^{\infty}$, $\{Q_t^*\}_{t=0}^{\infty}$, $\{\mathbb{W}_t^*\}_{t=0}^{\infty}$ and $\left\{\mu_t^{t+1}\right\}_{t=0}^{\infty}$, $\left\{\mu_t^{*t+1}\right\}_{t=0}^{\infty}$, it is possible to solve for[17] $\{\varpi_t^{\circ}\}_{t=0}^{\infty}$, and therefore for $\{\overline{z}_t^{\circ}\}_{t=0}^{\infty}$, and $\{\Omega_t^{\circ}\}_{t=0}^{\infty}$. Use (B.2), (B.9), and (B.3).
(3) Then, compute the expected investment costs $\{\widetilde{c}_t\}_{t=0}^{\infty}$ and the GDP identity to find the numbers of new entrants.
(4) Then, use the forward difference equations (B.4) and (B.5) to solve for profits flows $\left\{\Xi_{t+1}^{\circ}\right\}_{t=0}^{\infty}$ and to find the labor demand.
(5) One can use households' Euler equations to derive the optimal bond holding and from the international-bond market clearing condition to derive the interest rate $\{r_t\}_{t=0}^{\infty}$;

The idea is therefore to guess the time profiles of the seven variables and verify the guess. The guess should be verified as follows:

(1) Budget constraints for both households have to be satisfied.
(2) Labor markets in both countries have to be cleared.
(3) The zero-profit condition must hold for entrants in both countries.
(4) The balance-of-payment condition has to be satisfied.

Denote the guess of the seven variables as

$$\overrightarrow{\mathcal{H}} = \left\{\{Q_t\}_{t=0}^{\infty}, \{\mathbb{W}_t\}_{t=0}^{\infty}, \{C_t\}_{t=0}^{\infty}, \{Q_t^*\}_{t=0}^{\infty}, \{\mathbb{W}_t^*\}_{t=0}^{\infty}, \{C_t^*\}_{t=0}^{\infty}, \{\eta_t\}_{t=0}^{\infty}\right\},$$

[17] The circle $\circ$ in the superscript is used as a formal argument distinguishing types of firms.

and the seven equilibrium conditions as $\left\{\hbar_t(\overrightarrow{\mathcal{H}})\right\}_{t=0}^{\infty}$, where we interpret $\hbar_t(\overrightarrow{\mathcal{H}^0}) = 0$ as the fulfilment of these conditions at time t for a guess $\overrightarrow{\mathcal{H}^0}$ [18].

The equilibrium candidate $\overrightarrow{\mathcal{H}}$ is an infinite-dimensional object and for practical simulations, we have to approximate it by a finite-dimensional representation. There are two ways of doing that:

(1) the domain-truncation approach;
(2) projection methods.

The *domain-truncation approach* reduces dimensionality of $\overrightarrow{\mathcal{H}}$ by setting $\{Q_t\}_{t=0}^{\infty} \approx \widehat{Q} = \{Q_1, ..., Q_N, Q_+, Q_+, ..., Q_+\}$, where Q_+ is the steady state of the variable Q_t (and similarly for other variables too). Setting

$$\widehat{\mathcal{H}} = \left\{\hat{Q}, \widehat{\mathbb{W}}, \hat{C}, \widehat{Q^*}, \widehat{\mathbb{W}^*}, \widehat{C^*}, \hat{\eta}\right\},$$

and solving the system

$$\begin{aligned} \hbar_1(\widehat{\mathcal{H}}) &= 0 \\ \hbar_2(\widehat{\mathcal{H}}) &= 0 \\ &\vdots \\ \hbar_M(\widehat{\mathcal{H}}) &= 0 \end{aligned} \tag{B.11}$$

for $M \gg N$. This is a system of $7M$ unknowns. Lafargue (1990) proposed this approach, and Boucekkine (1995) and Juillard et al. (1998) have further elaborated it. Hence, the approach is called the L-B-J approach.

The *projection method* parameterizes the time profile of variables in $\overrightarrow{\mathcal{H}}$ using a finite set of parameters. Various choices of parameterizations are possible. Our experimentation suggests that cubic splines are particularly suitable and that the spline-based projection can improve over the L-B-J approach (the projection based on cubic splines is able to reduce the computation time to one-third comparing to the L-B-J).

[18] Note that the fulfillment of equilibrium condition at time t, $\hbar_t = 0$ does not depend on the value of the seven variables at time t only: it depends on their whole time profiles. It depends on future values because of expectations of profits, e.g. today's investment decisions depend on the expected present value of future profit streams, and the it depends on past values because of predetermined variables in budget constraints.

C Elastic labor supply extension

The modeling of firms does not interfere with an assumption about labor supply, and therefore, we will only summarize the changes due to elastic labor supply in households' behavior modeling. We follow the approach used in Bilbiie et al. (2008), which relies heavily on Ghironi and Melitz (2005). The representative household maximizes:

$$\max U = \sum_{t=0}^{\infty} \beta^t \left[u(C_t) - \chi \frac{L_t^{1+1/\varphi}}{1+1/\varphi} \right],$$

subject to

$$B_t = (1 + r_{t-1}^*) B_{t-1} + \frac{-1}{\eta_t} (C_t - \mathbb{W}_t L_t) + \frac{1}{\eta_t} (\Xi_t - \tilde{c}_t n_t) - \frac{\Psi_B}{2} B_t^2 + \mathcal{T}_t,$$

where φ is the Frisch elasticity of labor supply, the parameter χ is greater than 0, and L_t is labor supply in time t. For the rest of symbols, we refer to the main paper.

The first-order optimality conditions for household optimization are extended by the labor supply equation, which takes the familiar form:

$$u'(C_t) W_t = \chi L_t^{1/\varphi}.$$

The rest of the first-order conditions remains the same as in the main paper, i.e. Equations (7), (8), plus the transversality condition. The final important change is that the labor-market clearing condition (10) now changes to

$$\sum_{\tau \le t} (1-\delta)^{t-\tau} \, n_\tau \int_{z_L}^{z_U} l_{j\tau t} \, \mathrm{d}G(j) = \left[\frac{u'(C_t)\mathbb{W}_t}{\chi} \right]^{\varphi}.$$

The reader is referred to Appendix B for the recursive representation of the left-hand side of this equation.

Our baseline calibration follows Bilbiie et al. (2008), who use the consensual number $\varphi = 2$ (see also King and Rebello, 1999). The parameter χ is calibrated so that the steady state labor supply is the same as in the model with inelastic labor supply. This means that for the benchmark model, we set $\chi = 1.25$, while for the alternative model we set $\chi = 1.35$.

We also recalibrate the productivity path for the converging economy A_t so that the simulations for the elastic and inelastic labor have identical initial conditions for output. The productivity path is still calibrated using the logistic function $A_t = A^* \frac{1+m\exp(-(t-1995)/\iota)}{1+n\exp(-(t-1995)/\iota)}$, but contrary to the calibration in

Table 1 (in the main paper), we set $m = 6$ (benchmark model) and $m = 5$ (alternative model). The parameters n and ι preserve their values. Note that this recalibration means that the initial level of the converging economy productivity is lower. This is due to the fact that as the economy becomes richer, the marginal valuation of leisure increases and the labor supply decreases. This channel partly offsets the productivity increase. In other words, if we kept the productivity path the same as for the economy with inelastic labor supply, the initial level of output in the converging economy would be smaller.

Simulation results for the economies with elastic labor supply are reported in Figure C.1. This figure is organized similarly to the figure in the main paper. We add results on the real wage in the converging economy.

The main finding is the following. First, the dynamics of the real exchange rate is almost unchanged from the model with fixed labor supply. Second, the models with elastic labor supply imply a more significant increase in the converging-economy real wage. This is due to the additional channel missing from the models with inelastic labor supply: the marginal valuation of leisure increases as the converging economy becomes richer and therefore firms have to pay higher wages to attract workers. Third, the initial accumulation of the debt by the converging economy and the trade balance dynamics is stronger under the elastic labor supply economy. The explanation is based on the fact that due to the 'leisure channel' (as was explained above) the initial gap in productivities must be larger, which boosts incentives to smooth the consumption.

The overall conclusion is that the elastic labor supply assumption affects dynamics of some variables (real wage, debt, trade balance), but keeps the main point of the paper unchanged, mainly that the converging economy can experience a fast exchange rate appreciation simultaneously with the trade-balance improvements by investing in product quality.

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Fig. 1. Stylized facts

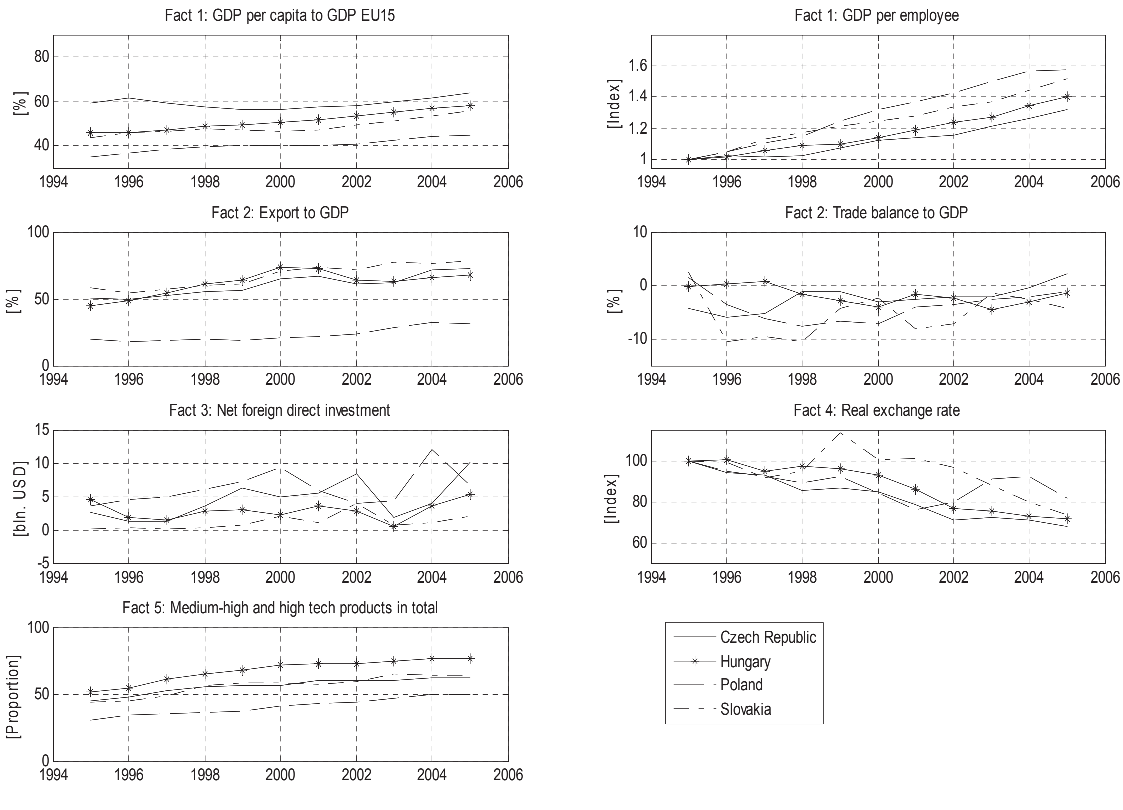


Table 1
Model's Parametrization

Parameter	*Benchmark Model*	*Alternative Model*
	Model with quality investment	*Model without quality investment*
α	0.35	0
θ	4.50	4.50
β	0.95	0.95
δ	0.10	0.10
ς	0.05	0.05
ε	2	2
c^x	0.25	0.25
c	1.0	1.0
Ψ_B	0.025	0.025
A^*	1	1

Fig. 2. Simulation results

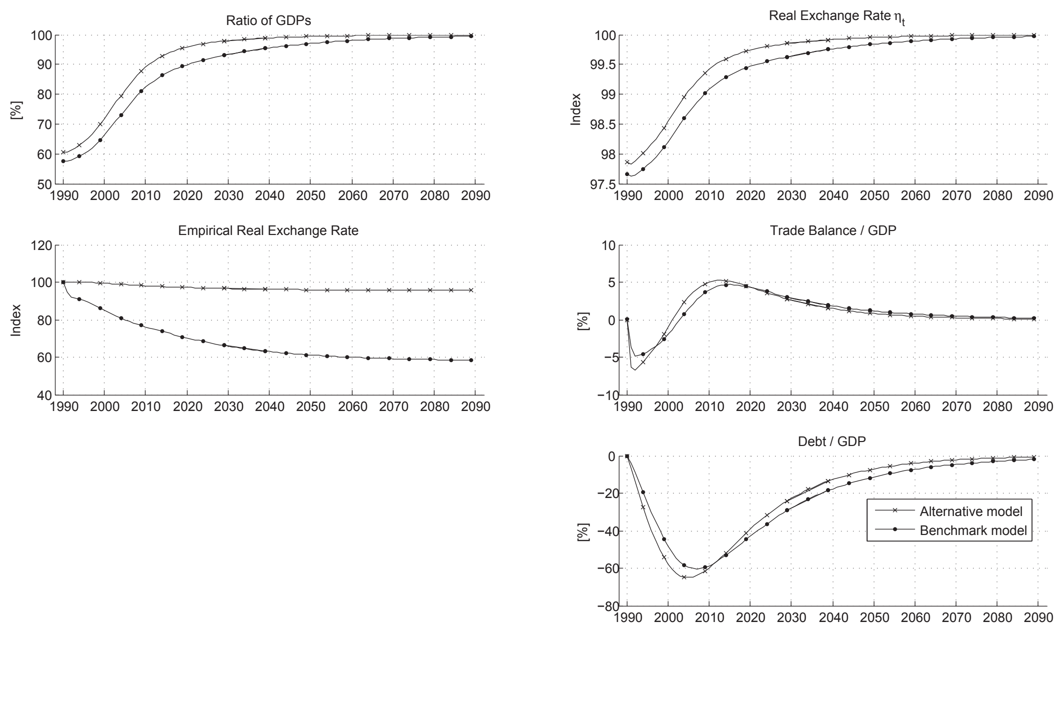
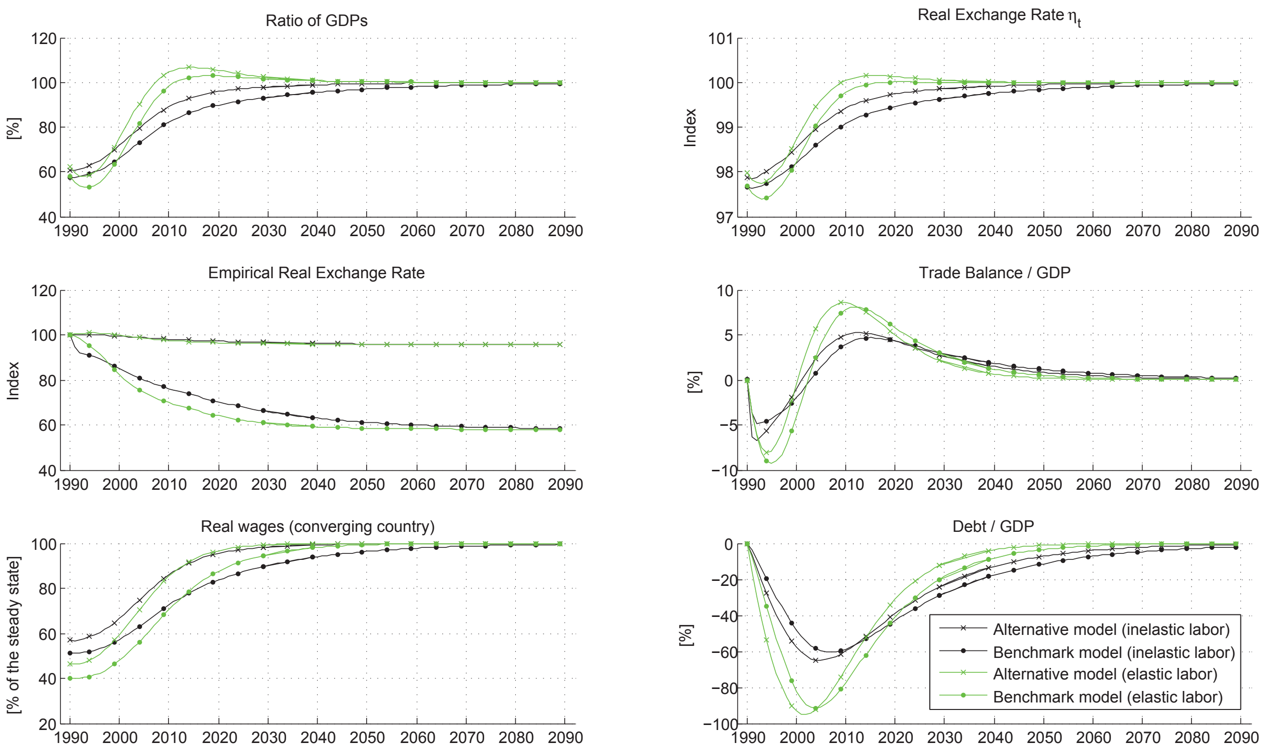


Fig. C.1. Simulation with the elastic-labor



The Convergence Dynamics of a Transition Economy: The Case of the Czech Republic

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Abstract

In this paper we develop a two-country dynamic general equilibrium model by means of which we seek to explain the long-run path of a transition economy. The model's novel feature is the inclusion of quality investment in the standard framework of applied general equilibrium two-country models. This feature is necessary to explain the trend in the real exchange rate. We present an application to the Czech economy.

Key words: Two-country modeling, convergence
J.E.L. Classification: F12, F41, F43.

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1 Introduction

In this paper we explain the macroeconomic dynamics in the Czech Republic, one of the transition economies in Central and Eastern Europe (CEE). In particular, we suggest a reconciliation of the real exchange rate dynamics with the rest of the macroeconomy, which is so far unaddressed in the literature. We provide a theoretical framework and simulations with a calibrated model for the Czech Republic during 1995–2005.

Such a reconciliation is especially needed for policymakers, since they usually make inferences about the simultaneous development of the entire economy. And since the real exchange rate plays an important role in many policy-related decisions, establishing jointly the long-term trajectories of all the key policy relevant variables is crucial for measured policy decision making.

Following the establishment of the Czech Republic in 1993, the Czech economy recorded solid economic growth, which was interrupted by a slowdown in 1996 and a subsequent two-year recession. GDP fell by 0.7% and 0.8% in the recession years, mainly because of falling investment and consumption (in the second year). Subsequently, first consumption and later investment resumed and absorbed significant imports. The improvement in the supply side of the economy was only gradual; net exports contributed to economic growth negatively until 2004. However, in 2005 the economic growth of 6.3% was dominantly driven by the net exports (with a contribution of 5.1%).

Sizeable foreign direct investment directed into both the tradable and non-tradable sectors gradually boosted efficiency in the entire economy. An emphasis on the production of higher-value sophisticated tradable products (through technology transfer via direct investment) steadily improved export performance and the trade balance. Consequently, domestic product accelerated and the Czech koruna real exchange rate continuously strengthened.

The average net inflow of foreign direct investment in the Czech Republic during 1995–2005 amounted to USD 2.65 billion, which is 7.5% of GDP. In regional relative terms, this is 21.5% of the average net direct investment inflow in the entire CEE region (Bulgaria, the Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Romania, Slovakia, and Slovenia). The prime factor for the high inflow of direct investment was large-scale privatization of existing, often run-down, production facilities. Additional factors, according to Meyer (1998), included market penetration opportunity and low labor costs.

Roughly half of the total net foreign direct investment inflow was directed into non-tradable sectors, such as water supply, telecommunications, real estate, and the banking sector. The other half went into the tradable sector, in particular production of machines, tools, electronic equipment, and cars (SITC7).

The average productivity growth of SITC7 in the second half of the transition period (2001–2005) reached 11%, exceeding the manufacturing sector average by 3%, and was the dominant driver of the steadily improving export performance.

The significant Czech export performance was recently analyzed by Fabrizio et al. (2007). They show that the Czech Republic, similarly to the other transition economies in the region, increased its share of world exports (by 2 p.p. over the analyzed decade), while recording significant real exchange rate appreciation (approx. 3% p.a.). They attribute this to an improvement in the unit value of exports. The ratio of the unit value of Czech exports to the unit value of world exports recorded an average improvement of 5% p.a. In combination with the change in the composition of exports towards high and medium-tech products (2% p.a.), this suggests that the quality of Czech output increased significantly over the analyzed decade.

The real exchange rate appreciation of the Czech koruna was 3.2% p.a. during 1995–2005. By using the standard tool, the Balassa-Samuelson model, for explaining the exchange rate dynamics, the empirical studies conducted on CEE countries report rather a small size of this effect. In the case of the Czech Republic, the size is especially low, close to 0.5% p.a. (see the survey of 14 studies by Égert et al., 2006).

In order to address the aforementioned Czech economic transition we need to model four basic principles. These are: consumption smoothing (external borrowing), transfer of technology and productivity (foreign direct investment), self-selection into the export market (productivity based), and investment in quality (influencing observed price indices and thus the real exchange rate).

We build our model on postulates developed by Ghironi and Melitz (2005) containing self-selection into the export market and consumption smoothing (portfolio investment). We extend the framework with a vertical investment margin (investment in quality) and cross-border asset ownership (foreign direct investment) where the firm's productivity level pertains to ownership. Our calibrated model is solved for the perfect-foresight transition dynamics of the Czech economy, which converges to its more advanced counterpart (the Euro area). Thus, it aims at explaining the long-run convergence trend. This contrasts with the standard DSGE models, which aim at modeling deviations from exogenously given long-run trends.

The rest of the paper is organized as follows. Section 2 presents the two-country model. Section 3 is on calibration and explains the dynamics of some of the endogenous variables, and Section 4 presents the conclusions. The Appendix contains a detailed derivation of the model, and its reformulation using a recursive form and discusses the numerical techniques used to solve the model.

2 The Two-Country Model

There are two countries that are modeled in discrete time that runs from zero to infinity. Each country is populated by a representative competitive household that has recursive preferences over consumption streams. Production takes place in heterogeneous firms.

2.1 Firms

In both countries, there is a large number of firms, which may be owned by either the domestic or the foreign household (i.e., cross-border ownership of firms is possible). In each period there is an unbounded mass of potential, ex-ante identical, entrants. Each entrant has to pay the fixed entry cost c. The cost is paid in terms of the aggregate consumption bundle. The actual number of entrants is determined by households: households consider the expected present value of the profit stream of a new entrant relative to the expected entry costs; see Section 2.2 for the exposition.

Firms differ ex-post entry by an idiosyncratic variation of total factor productivity: when a firm enters, it draws a shock z from a distribution $G(z)$. At the end of each period, there is an exogenous probability that a firm is hit by an exit shock. This probability δ is assumed to be independent of aggregate as well as individual states. Hit firms shut down.

The production of a firm is characterized by two features: physical quantity x and product quality h. If the firm j wishes to produce its product with the quality level h_j, it has to pay the fixed quality investment at the level h_j. Similarly to the entry cost, the quality investments take the form of the aggregate consumption bundle. The quality choice is a once-and-for-all decision undertaken at the entry time (but after the idiosyncratic productivity is revealed).

In addition to the quality input h, the production requires a variable input – labor l. The production of the final bundle q is described using the neoclassical production function f and the firm's total factor productivity $A_t z_j$, $q = z_j A_t f(h_j, l)$. The quality of the final bundle is h_j, and therefore the physical quantity is given simply as $x = q/h$. Such a distinction between the final bundle (quality included) and the physical quantity is standard in the literature; e.g., Young (1998). We explicitly distinguish the quality-quantity bundle from the physical quantity, since the explanation for the observed real exchange rate appreciation is based on a dichotomy between quality-adjusted and -

unadjusted prices[1].

The productivity of a firm $A_t z_j$ has two components: (a) an idiosyncratic component z_j, which is i.i.d. across firms and which follows distribution $G(z)$ introduced above, and (b) a common component A_t. The total factor productivity (TFP) A_t pertains to ownership[2]: firms owned by the domestic household enjoy at time t productivity A_t, while firms owned by the foreign household enjoy productivity A_t^*. Productivity does not depend on the location of production or on the time of entry of firms (the time of entry is henceforth called the *vintage*).

A firm may export only if it invests special fixed costs, which may represent the expenditures associated with acquiring necessary expertise such as in the legal, business, or accounting issues of foreign markets. If a firm at the entry time decides to invest the fixed export costs, then it becomes eligible to export in all subsequent periods, otherwise it is never eligible to export. Therefore, we distinguish exporters from non-exporters. The fixed export costs are again paid in terms of the aggregate consumption bundle and are denoted as c^x. This assumption implies, as in Melitz (2003), that in equilibrium there is a cut-off productivity value $\overline{z}$ such that firms with lower idiosyncratic productivity $z_j < \overline{z}$ will not invest to become eligible to export, while firms with a sufficiently high productivity level $z_j \geq \overline{z}$ will do so.

We derive **optimal production and investment plans** using backward induction. We present the derivation for a firm located in the domestic country and owned by the domestic household, which is easily generalized for other firm types. A detailed derivation for a specific parametrization (the Cobb-Douglas production function f, and the CES market structure) is given in Appendix A.1.

Let $\mathbb{P}^{de}_{j\tau t}$ denote the t-period *real operating profit* of a *d*omestic *e*xporter of

[1] The theoretically-consistent price indexes (and hence also the real exchange rate) are based on physical-quality bundles, i.e. they measure quality. These theoretically-consistent indexes might differ from the CPI-based real-world indexes by *the quality effect* (where the real-world price index does not completely capture the quality change) and by the *love-for-variety* effect (where consumers are willing to pay higher prices for consumption baskets with more varieties; see Melitz, 2003, for the CES case). Both the average product quality and the number of varieties increase during the convergence in reality, and our model replicates this feature.
[2] This is an important assumption. If the productivity pertained to location, the model would predict perverse FDI flows (i.e., from the transition country to the advanced country).

vintage τ enjoying the idiosyncratic productivity z_j, and be given as follows:

$$\mathbb{P}^{de}_{j\tau t} = \left[\kappa_t \frac{p^d_{j\tau t}}{P_t} + (1-\kappa_t)\frac{\eta_t}{1+\varsigma}\frac{p^x_{j\tau t}}{P^*_t}\right] A_t z_j f(h_{j\tau}, l_{j\tau t}) - \mathbb{W}_t l_{j\tau t},$$

where $0 \leq \kappa_t \leq 1$ is the output share sold in the domestic market [3], $p^d_{j\tau t}$ is the nominal price of its product on the domestic market, $p^x_{j\tau t}$ is the export price (therefore, we allow for pricing-to-market), P_t is the domestic price level, P^*_t is the foreign price level, η_t is the *real exchange rate*, which is linked to the nominal exchange rate s_t as $\eta_t = s_t P^*_t / P_t$, $\varsigma \geq 0$ represents the unit iceberg transportation cost, $\mathbb{W}_t$ is the *real wage*, and $l_{j\tau t}$ is the labor demand of the firm. Similarly, the *real operating profit* of a domestic non-exporter is given as follows:

$$\mathbb{P}^{dn}_{j\tau t} = \frac{p^d_{j\tau t}}{P_t} A_t z_j f(h_{j\tau}, l_{j\tau t}) - \mathbb{W}_t l_{j\tau t}.$$

The operating profits depend on wages, productivity, the chosen quality level, exporting status, and on idiosyncratic productivity. Let us assume the problem of maximizing the value of a firm. Given its exporting status, a firm chooses the level of quality to maximize its value. Therefore, if we denote the marginal rate of substitution between dates τ and t as μ^t_τ, the value of an exporter is given as:

$$\mathbb{V}^{de}_\tau(z_j) = \max_{h_{j\tau}\geq 0}\left[\sum_{t=\tau}^{\infty} \mu^t_\tau (1-\delta)^{t-\tau} \mathbb{P}^{de}_{j\tau t} - (c + h_j)\right] - c^x,$$

while the value of a non-exporter is given as:

$$\mathbb{V}^{dn}_\tau(z_j) = \max_{h_{j\tau}\geq 0}\left[\sum_{t=\tau}^{\infty} \mu^t_\tau (1-\delta)^{t-\tau} \mathbb{P}^{dn}_{j\tau t} - (c + h_j)\right].$$

The value functions $\mathbb{V}^{dn}_\tau(z_j)$, $\mathbb{V}^{de}_\tau(z_j)$ implicitly define the cut-off value $\overline{z}$, which is the lowest idiosyncratic shock which makes the export-eligibility investment profitable. Thus, it is defined as

$$\overline{z}^d_\tau = \arg\min_{z_j}(\mathbb{V}^{de}_\tau(z_j) \geq \mathbb{V}^{dn}_\tau(z_j)).$$

Note that the cut-off value $\overline{z}^d_\tau$ depends on the vintage. The explanation is straightforward: as the macroeconomic conditions (such as productivity) change,

[3] We show in the Appendix (Lemma 1) that in the equilibrium, all exporters export at a particular date t the same share of their production to the foreign market, regardless of their vintage τ or productivity j. Therefore, we shall simply write κ_t. The vintage and productivity only determine whether a particular firm is an exporter or not.

the incentive of firms to become exporters changes as well.

The value of a firm is given by

$$\mathbb{V}^d_\tau(z_j) = \max_{\xi\in\{n,e\}} \mathbb{V}^{d\xi}_\tau(z_j) = \begin{cases} \mathbb{V}^{de}_\tau(z_j) & \text{if } z_j \geq \overline{z}^d_\tau \\ \mathbb{V}^{dn}_\tau(z_j) & \text{if } z_j < \overline{z}^d_\tau \end{cases},$$

and the expected value of a new entrant $\mathcal{V}^d_\tau$ is:

$$\mathcal{V}^d_\tau = \int_{z_L}^{z_u} V^d_\tau(z)\, G(\mathrm{d}z). \tag{1}$$

This completes the backward induction.

The optimal production plan derived above induces a measure over firms. Denote by $\widetilde{\mathbb{P}}^d_{\tau t}$ the t-time expected[4] real operating profit of a domestic firm, which enters at time τ, $\widetilde{\mathbb{P}}^d_{\tau t} = \int_{z_L}^{\overline{z}^d_\tau} \mathbb{P}^{dn}_{j\tau}\, G(\mathrm{d}z) + \int_{\overline{z}^d_\tau}^{z_U} \mathbb{P}^{de}_{j\tau}\, G(\mathrm{d}z)$, and $\widetilde{c}^d_\tau$ represent expected real investment cost under such measure. Then, another way of expressing $\mathcal{V}^d_\tau$ is:

$$\mathcal{V}^d_\tau = \sum_{\sigma\geq 0} \mu_\tau^{\tau+\sigma}(1-\delta)^\sigma \widetilde{\mathbb{P}}^d_{\tau,\tau+\sigma} - \widetilde{c}^d_\tau,$$

where the expected real investment cost consists of three terms:

$$\widetilde{c}^d_\tau = c + c^x(1 - G(\overline{z}^d_\tau)) + \widetilde{h}^d. \tag{2}$$

The first term is the fixed entry cost c paid by all entrants prior to entry. The second term $c^x(1 - G(\overline{z}^d_\tau))$ is the expected export-eligibility cost (recall that only firms with $z_j \geq \overline{z}^d_\tau$ pay the cost). And the final term $\widetilde{h}^d$ is the expected quality investment, given by: $\widetilde{h}^d = \int\limits_{z_L}^{\overline{z}^d_\tau} h^{opt,dn}_{j\tau}\, G(\mathrm{d}z) + \int\limits_{\overline{z}^d_\tau}^{z_U} h^{opt,de}_{j\tau}\, G(\mathrm{d}z)$.

To summarize the sequencing, the timing proceeds first with the domestic and foreign households' decision about the number of new entrants in both countries. This decision is based on consideration of the expected value of firms relative to the expected investment costs and is described in more detail in Section 2.2 below (see equations 6 and 7). Then, each new entrant draws a productivity level from the distribution G and the owner decides the amount of investment in quality and whether to invest in export eligibility. Then labor demand and production (of both entrants and incumbents) take place. At the end of the period, some firms experience an exit shock and shut down.

[4] This expectation is taken with respect to the measure given by the optimal production plan.

2.2 Households

The domestic household has recursive preferences over streams of consumption of the aggregate good. Leisure does not enter the utility and so labor is supplied inelastically. The aggregate labor supply in the domestic country is $\mathcal{L}$. Households can trade bonds denominated in the foreign currency.

The domestic household maximizes

$$\max U = \sum_{t=0}^{\infty} \beta^t u(C_t),$$

subject to

$$B_t = (1+r^*_{t-1})B_{t-1} + \frac{\mathbb{W}_t\mathcal{L} - C_t}{\eta_t} + \frac{\Xi^d_t - \left[\widetilde{c}^d_t n^d_t + \frac{\Psi_d}{2}\left(n^d_t\right)^2\right]}{\eta_t} + \Xi^f_t - \left[\widetilde{c}^f_t n^f_t + \frac{\Psi_f}{2}\left(n^f_t\right)^2\right] - \frac{\Psi_B}{2}B^2_t + \mathcal{T}_t, \tag{3}$$

where B_t is the real bond holding of the domestic household[5], r^*_{t-1} is the real interest rate on the internationally traded bond. $\Xi^d_t = \sum_{\sigma\leq t}(1-\delta)^{t-\sigma} n^d_\sigma \widetilde{\mathbb{P}}^d_{\sigma,t}$ is the flow of profits from domestic firms owned by the domestic household (and Ξ^f_t is the analogous profit flow from firms located in the foreign country and owned by the domestic household[6]), $\widetilde{c}^d_t n^d_t$ are the expected[7] investment costs of new domestically located entrants (see Equation 2 for the definition of $\widetilde{c}^d_t$), and $\widetilde{c}^f_t n^f_t$ are the expected investment costs of new entrants located in the foreign country and owned by the domestic household.

The parameters Ψ_B, Ψ_d, and Ψ_f represent portfolio adjustment costs, and $\mathcal{T}_t$ is the lump-sum rebate of these costs to the household. The bond adjustment costs Ψ_B are used as in Schmitt-Grohe and Uribe (2003) to stabilize the model[8]. The parameter Ψ_d is the adjustment cost of investing in the resident

[5] Bonds are denominated in foreign currency by our convention. However, since the model is deterministic, this assumption is completely innocent.

[6] The variables pertaining to firms located in the foreign country and owned by the domestic household are distinguished by the superscript f.

[7] This expectation is taken with respect to the measure induced by the optimal production plan. Because of the law of large numbers and of perfect foresight, the *ex-ante* expected values of the key variables for household decisions (such as investment costs or profit flows) coincide with the *ex-post* realizations.

[8] In a strict sense, the model is stable even without bond adjustment costs, i.e., with $\Psi_B = 0$. The model is deterministic and therefore it would not exhibit unit-root behavior even under such a condition. Nevertheless, if $\Psi_B = 0$, then the model would exhibit steady state dependence on the initial asset holding and we do not like such a model property. We use the bond adjustment costs $\Psi_B > 0$ to give up the dependence of the steady state on the initial asset holding.

country (i.e., in the domestic country for the domestic household and in the foreign country for the foreign household), while the parameter Ψ_f is the adjustment cost of investing in the non-resident country[9].

The first-order conditions for the domestic household are as follows:

$$1 + \Psi_B B_t = \frac{\eta_{t+1}}{\eta_t}(1 + r_t^*)\mu_t^{t+1}, \tag{4}$$

$$\lim_{t\to\infty} \beta^t B_t u'(C_t) = 0, \tag{5}$$

$$n_t^d = \Psi_d^{-1} \underbrace{\left[\sum_{\upsilon\geq 0} (1-\delta)^{\upsilon} \mu_t^{t+\upsilon} \widetilde{\mathbb{P}}_{t,t+\upsilon}^d - \widetilde{c}_t^d\right]}_{=\mathcal{V}_\tau^d}, \tag{6}$$

$$n_t^f = \Psi_f^{-1} \underbrace{\left[\sum_{\upsilon\geq 0} (1-\delta)^{\upsilon} \frac{\eta_{t+\upsilon}}{\eta_t} \mu_t^{t+\upsilon} \widetilde{\mathbb{P}}_{t,t+\upsilon}^f - \widetilde{c}_t^f\right]}_{=\mathcal{V}_\tau^f}. \tag{7}$$

The marginal rate of substitution is defined as: $\mu_{t_1}^{t_2} \equiv \beta^{t_2-t_1}\frac{u'(C_{t_2})}{u'(C_{t_1})}$. Although there is an idiosyncratic variance at the firm level, the model is deterministic at the aggregate level, thus the dynasty problem is deterministic too. Therefore, the marginal rate of substitution does not involve the expectation operator. Equation (4) is the Euler consumption equation, Equation (5) is a combination of the transversality condition and the non-Ponzi game condition, and Equations (6) and (7) determine the number of entrants.

The part of the model related to the foreign household is defined analogously.

2.3 General Equilibrium

The general equilibrium is defined as a time profile of prices and quantities such that all households optimize and all markets clear. Since there is no price stickiness, nominal prices are indeterminate. Therefore, only relative prices matter. The general equilibrium requires that the market-clearing conditions hold. These conditions include the GDP identity in both countries, labor market clearing in both countries, a net zero supply of international bonds, and balance-of-payments equilibrium.

The definition of the general equilibrium is standard. A more complicated task is to simulate the dynamic path, because the model is effectively a vintage type

[9] Their purpose is to mitigate the corner solution of household investments in new varieties. Without these costs, households would invest only in one country, and FDI modeling would not be possible.

model. However, the model can be rewritten in the recursive (first-order) form, which makes it convenient for application of a variety of efficient numerical methods. It turns out that the domain-truncation approach seems to be the most efficient approach. The full set of equations of the model in the recursive form and a discussion of the methods are available in the Appendix.

3 Calibration and Projections

This section discusses the model's calibration, the in-sample simulation (1995–2005), and the long-run convergence projection for the Czech economy. The choice of the start date is motivated by the fact that by 1995 full external (trade and financial) and price liberalization had been completed in the Czech Republic (see Roland, 2004, for a comparison of transition EBRD indexes of liberalization and reforms). The data for the Czech Republic and the EU15 economies were compiled from various sources: the Czech Statistical Office, the Czech National Bank, the Ministry of Finance, and Eurostat.

3.1 Calibration

When we calibrate the model, we use the Cobb-Douglas production function $f(k,l) = k^{\alpha}l^{1-\alpha}$ for production of the quality-quantity bundle. The goods are aggregated into final production using the CES function with the parameter of intratemporal substitution θ. We chose the standard value of the parameter $\theta = 4.70$. Indeed, this value delivers a mark-up over average costs close to the observed mark-up in the Czech manufacturing industry (20–25% on average over 1995–2005; see Podpiera and Raková, 2008). The momentary utility function is parameterized using the common constant-relative-risk-aversion form $u(C) = (1-\varepsilon)^{-1}C^{1-\varepsilon}$, with the parameter of intertemporal substitution ε, which is calibrated at a standard value of 2. The distribution G is calibrated to be uniform on the interval [0,1]. We choose $\mathcal{L}^*/\mathcal{L} = 6$, which implies that the transition economy is much smaller than its advanced counterpart, so that the convergence effect on the advanced economy is negligible.

Domestic productivity evolves according to the logistic curve $A_t = A_{\mathrm{SS}}\frac{1+\mathbf{m}\exp(-(t-1990)/\tau)}{1+\mathbf{n}\exp(-(t-1990)/\tau))}$, which has been fitted to our in-sample data. The logistic curve assumes average growth in total factor productivity during 1995–2005 of 1% p.a. This is roughly in line with the other empirically found values [10]. The parameters $\mathbf{m}$ and $\mathbf{n}$ anchor the initial gap in productivities and the parameter τ determines the

[10] The Czech Ministry of Finance (2006), for instance, found growth in TFP of between 1% and 3% during the period 1995–2005.

speed of productivity convergence. Domestic productivity converges to the value A_{SS}, which may differ from foreign productivity A^*. The reason for allowing $A_{\text{SS}} \neq A^*$ is that models with monopolistic competition (including ours) exhibit the scale effect. Hence, economies differing in terms of their size only attain different levels of per capita output in general. We set $A^* = 1$, which is an arbitrary normalization, and choose the value of $A_{\text{SS}} = 1.1$ so that both economies attain the same level of per capita output in the steady state.

Besides domestic productivity A_t, there are two other parameters that are inconstant during the convergence. These are the fixed exporting cost c^x and the portfolio adjustment costs Ψ_f. The former parameter decreases from an initial value of 0.35 to a terminal value of 0.25 and reflects the gradually increasing openness of the Czech economy to international trade. The initial value of Ψ_f is 10 times greater than its steady state value. Its decline is driven by a twofold intuition. Firstly, during the first years of the economic transition, foreign investors faced significant uncertainty about the business and legal environment in transition countries, including fear of political reversals, which gradually vanished. Secondly, the Czech government launched an incentive scheme to attract FDI (such as temporary tax exemptions, financing infrastructure, etc.) during 2000–2005. After the achievement of full integration of the Czech Republic into the EU (the EU entry occurred in May 2004), both transitory parameters reach the terminal (steady state) value.

And finally, a typical feature of a transition economy is higher exit and entry rates. Therefore, we chose the value of $\delta = 0.45$. The discount factor β takes the conventional value for yearly frequency of 0.95. A summary of the parameters can be found in Table 1.

3.2 In-sample Simulations

Figure 1 displays the yearly frequency of the main macroeconomic aggregates for the period 1995–2005 and their respective trends derived using the Hodrick-Prescott filter with the smoothing parameter $\lambda = 100$ (the recommended value for yearly data). The figure also contains the trajectories implied by the model.

First, we interpret the convergence of *output per capita* to the average of the EU15. Starting with Czech GDP per capita at 60% of the EU15 average in the mid-1990s (measured in PPP), and remaining at that level for the rest of the 1990s, in the early 2000s the Czech economy started to converge more apparently, reaching roughly 65% in 2005. The model outcome, along with the data, is displayed in the Figure 1.1. The model implies similar trend dynamics as observed in the data.

Second, *the real exchange rate* has been appreciating and was approximately

30% stronger in 2005 than in 1995. Figure 1.2 compares the actual real exchange rate and the model's trajectory. The series are rebased such that the value of the average for 1997 and 1998 of the original data equals the model's outcome. Although this is an arbitrary normalization, the reason behind it is that in order to facilitate comparison of price indexes, we need to choose a benchmark equilibrium year. Since all the available estimates of the equilibrium or parity of the real exchange rate fall into these two years (a summary of the evidence is provided by Babetskii and Égert, 2005), we choose it as the benchmark equilibrium year.

Recall that the theoretically-consistent price indexes and the corresponding real exchange rate η may differ from their real-world counterparts by two effects: (i) the quality effect, and (ii) the love-for-variety effect [11]. The two features are responsible for the explanation of the real exchange rate appreciation. First, love for variety (which is a feature of *inter alia* CES aggregation) means that the expansion of the number of domestic production varieties can be considered a quality improvement of the domestic goods basket. This is the effect which is responsible for persistent deviations of the real exchange rate from PPP in Ghironi and Melitz (2005). However, Brůha and Podpiera (2008, 2009) show by simulations that it is unlikely that this effect alone can achieve the size of real exchange rate appreciation observed in the Visegrad-4 countries [12]. This is why we introduced the second feature: *quality investment*. Our calibration implies that the quality input is quite intensive in production ($\alpha = 0.32$) and causes an increase in the portion of quality in quality-quantity bundle. The accumulation of quality brings about the empirically observed exchange rate appreciation.

In a strict sense, the quality of the goods basket increases in both countries. However, this effect is much stronger in the transition country and it is amplified by trade and financial openness, therefore the perceived quality of domestic goods increases relatively more. The quality improvement of the domestic composite basket is the very explanation why the transition country becomes able to sell more and, at the same time, for relatively higher price as its total factor productivity increases.

It is worth noting that the pace of real exchange rate appreciation in the model

[11] Therefore, to get the correct counterpart of the *measured real exchange rate in reality*, we use the following index $\eta_t^e = \left(\frac{\nu_t^*}{\nu_t}\right)^{\frac{1}{\theta-1}} \frac{\mathcal{H}_t^*}{\mathcal{H}_t}\eta_t$, where ν_t is the number of varieties available in the domestic country (both domestically produced and imported), $\mathcal{H}_t$ is the average quality content in goods available in the domestic country, and ν_t^* and $\mathcal{H}_t^*$ are the foreign-country counterparts.

[12] Brůha and Podpiera (2009) find that the love-for-variety effect can account for only about one third of the observed real exchange rate appreciation in the Czech Republic. A similar pattern holds also for the Slovak Republic, Hungary and Poland.

is obtained without any explicit assumption of an exogenous productivity differential in the tradable and non-tradable sectors (although the model displays an endogenous productivity differential between traded and non-traded goods). In fact, the reason for the appreciation comes from the improvement of the domestic composite good through variety expansion and explicit investment in quality.

Third, the initial trade deficit of *trade balance* slowly decreased, and the trade balance reached positive values by the end of the period (see Figure 1.3). The standard consumption-smoothing mechanism is responsible for that pattern. The initial smoothing of consumption, represented by an excess in imports of goods and services (goods for final consumption in the early stages, later moderated by an increasing share of investment goods imports) over exports, was replaced with a stronger export performance and an excess of exports over imports.

Fourth, Figure 1.4 shows the foreign direct investment in the Czech Republic. Due to declining adjustment costs Ψ_f, the FDI in the model follows a similar pattern as observed in the data. As a consequence of the increasing net inflow, foreign-owned companies increased their share quite rapidly. Based on a financial survey conducted by the Czech Statistical Office among non-financial companies, in 1998, foreign-owned companies represented only one tenth of the total number of firms, while in 2004 they exceeded one quarter by a large margin (28%).

3.3 The Long-Run Convergence Projection

We carried out a long-run projection of the Czech economy's convergence using the calibrated model. We present the scenario for the two key variables, i.e., output convergence and the real exchange rate path. The scenario, shown in Figures 2.1 and 2.2, assumes that Czech GDP per capita will reach the EU15 average in 2030. The path of the 'equilibrium' output suggests fast growth in the coming decade (in excess of the EU15 long-term growth). Around 2015 it is expected to moderate towards the EU15 growth. As for the real exchange rate, the trend appreciation projected by the model is slowly moderated and stabilizes around 2010 at a level which is roughly 45% stronger than the real exchange rate in 1997.

4 Conclusion

In this paper, we aim at providing an essential input for Czech policymakers – the long-run trend in key variables. Unlike a developed economy, which exhibits standard and settled characteristics for a sufficiently long period of time and for which long-run values (usually called *equilibrium*) can be obtained by averaging past observations, transition economies fall short in this respect. In order to find and assess these variables for a transition economy, one needs a specific model that delivers simultaneously determined long-term trajectories. We present a two-country model in which a transition economy is converging to its large and developed counterpart. The presented model adds a vertical investment margin to the models available in the literature. This seems to be the crucial ingredient for successful simultaneous replication of the GDP per capita convergence and real exchange rate appreciation.

The model, calibrated for the Czech economy and the EU15, shows that the symptoms of convergence can be explained by decreasing export and investment costs and by growing productivity and investment in quality in the transition country. The development of the economy is described by a quartet of endogenously determined trajectories for GDP, foreign direct investment, the trade balance, and the real exchange rate, which relate to their trends in the observed data. The long-run projections suggest that when the GDP per capita convergence is completed, the real exchange rate stabilizes at 45% of the base level in 1997.

A Detailed Derivation of the Model

This appendix contains a detailed description of the model outlined in Section 2. The first part of the appendix contains a detailed derivation of firms' production and investment plans, while the second part shows how to rewrite the model into the recursive form.

A.1 Model Equation under Particular Functional Forms

In this part of the paper, we derive the main model equation for particular functional forms of the production function, utility function and investment cost functions. In particular, as a benchmark calibration, we use the Cobb-Douglas production function $f(h,l) = h^{\alpha} l^{1-\alpha}$ for production of quality-quantity bundle. The momentary utility function is parameterized using the common constant-relative-risk-aversion form $u(C) = (1-\varepsilon)^{-1} C^{1-\varepsilon}$, with the parameter of intertemporal substitution ε. The distribution G of idiosyncratic shocks is uniform on the interval $[0,1]$.

The short-run cost function associated with the production function $A_t z_j f(h,l)$ is given as follows:

$$\mathbb{C}(q, \mathbb{W}_t, A_t, z_j, h_{j\tau}) = \mathbb{W}_t \left[\frac{q}{A_t z_j h_{j\tau}^{\alpha}} \right]^{\frac{1}{1-\alpha}} .$$

First, we derive the maximizing behavior of non-exporters[13]. The period t supply decision of a vintage τ non-exporter, who enjoys the productivity z_j and who has invested in the product quality $h_{j\tau}$, is a solution to the following program[14]:

$$\max_{q^d_{j\tau t}} \left\{ \left[q^d_{j\tau t} \right]^{\frac{\theta-1}{\theta}} Q_t^{\frac{1}{\theta}} - \mathbb{C}(q^d_{j\tau t}, \mathbb{W}_t, A_t, z_j, h_{j\tau}) \right\} .$$

[13] We derive expressions only for domestic firms owned by the domestic household. The expression for other types of firms are easily derived analogously.

[14] In the derivation, we use the properties of the CES market structure: the real turnover is $\frac{p^d_{j\tau t}}{P_t} q^d_{j\tau t} = \left(q^d_{j\tau t} \right)^{\frac{-1}{\theta}} Q_t^{\frac{1}{\theta}} q^d_{j\tau t} = \left(q^d_{j\tau t} \right)^{\frac{\theta-1}{\theta}} Q_t^{\frac{1}{\theta}}$ by the residual demand function: $\frac{p^d_{j\tau t}}{P_t} = \left(\frac{q^d_{j\tau t}}{Q_t} \right)^{\frac{-1}{\theta}}$.

A simple algebra yields the optimal supply:

$$q^d_{j\tau t} = \left(\left[\frac{\theta-1}{\theta}(1-\alpha)\,\mathbb{W}_t^{-1}\left[A_t z_j h^{\alpha}_{j\tau}\right]^{\frac{1}{1-\alpha}}\right]^{\theta} Q_t\right)^{\frac{(1-\alpha)}{\alpha(\theta-1)+1}},$$

and the optimal labor demand:

$$l_{j\tau t} = \left[\frac{q^d_{j\tau t}}{A_t z_j h^{\alpha}_{j\tau}}\right]^{\frac{1}{1-\alpha}} = \left(\left[\frac{\theta-1}{\theta}(1-\alpha)\,\mathbb{W}_t^{-1}\right]^{\theta}\left[A_t z_j h^{\alpha}_{j\tau}\right]^{\theta-1} Q_t\right)^{\frac{1}{\alpha(\theta-1)+1}}. \tag{A.1}$$

Now, using the CES market structure, it is easy to derive the real turnover:

$$RT^{dn}_{j\tau t} = z_j^{\frac{\theta-1}{\alpha(\theta-1)+1}} h_{j\tau}^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}}\left[\frac{\theta-1}{\theta}(1-\alpha)\,\mathbb{W}_t^{-1}A_t^{\frac{1}{1-\alpha}}\right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} Q_t^{\frac{1}{\alpha(\theta-1)+1}}, \tag{A.2}$$

and the real operating profit [15]:

$$\mathbb{P}^{dn}_{j\tau t} = \frac{p^d_{j\tau t}}{P_t} q^d_{j\tau t} - \mathbb{C}(q^d_{jt}, \mathbb{W}_t, A_t, z_j, h_{j\tau}) = \mathcal{W}_1 z_j^{\frac{\theta-1}{\alpha(\theta-1)+1}} h_{j\tau}^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}} \mathbb{W}_t^{\frac{-(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} A_t^{\frac{(\theta-1)}{\alpha(\theta-1)+1}} Q_t^{\frac{1}{\alpha(\theta-1)+1}}.$$

Second, the optimal production decisions of exporters is derived. The problem can be characterized as follows:

$$\max_{\kappa_{j\tau t}, q_{j\tau t}} \left\{(\kappa_{j\tau t} q_{j\tau t})^{\frac{\theta-1}{\theta}} Q_t^{\frac{1}{\theta}} + \left(\frac{\eta_t}{1+\varsigma}\right) Q_t^{*\frac{1}{\theta}}\left((1-\kappa_{j\tau t})q_{j\tau t}\right)^{\frac{\theta-1}{\theta}} - \mathbb{C}(q_{j\tau t}, \mathbb{W}_t, A_t, z_j, h_{j\tau})\right\}.$$

The solution yields that $\kappa_{j\tau t} q_{j\tau t} = \left[\frac{\theta-1}{\theta}\left(\frac{\partial \mathbb{C}}{\partial q_{j\tau t}}\right)^{-1}\right]^{\theta} Q_t$, and $(1-\kappa_{j\tau t})q_{j\tau t} = \left[\frac{\theta-1}{\theta}\frac{\eta_t}{1+\varsigma}\left(\frac{\partial \mathbb{C}}{\partial q_{j\tau t}}\right)^{-1}\right]^{\theta} Q_t^*$. Some simple, but tedious, algebraic manipulations yield:

$$\kappa_{j\tau t} q_{j\tau t} \equiv \left[\frac{\theta-1}{\theta}(1-\alpha)\,\mathbb{W}_t^{-1}\left(A_t z_j h^{\alpha}_{j\tau}\right)^{\frac{1}{1-\alpha}}\right]^{\theta} \frac{Q_t}{q_{jt}^{\frac{\alpha\theta}{1-\alpha}}},$$

and

$$(1-\kappa_{j\tau t}) q_{j\tau t} \equiv \left[\frac{\theta-1}{\theta}(1-\alpha)\frac{\eta_t}{1+\varsigma}\mathbb{W}_t^{-1}\left(A_t z_j h^{\alpha}_{j\tau}\right)^{\frac{1}{1-\alpha}}\right]^{\theta} \frac{Q_t^*}{q_{j\tau t}^{\frac{\alpha\theta}{1-\alpha}}}.$$

This implies that $\kappa_{j\tau t} = \frac{Q_t}{Q_t + Q_t^*\left(\frac{\eta_t}{1+\varsigma}\right)^{\theta}}$. Observe that $\kappa_{j\tau t}$ does not depend on individual characteristics of firms: z_j and $h_{j\tau}$; it depends only on relative

[15] We define $\mathcal{W}_1 \equiv \left[\frac{\theta-1}{\theta}(1-\alpha)\right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} - \left[\frac{\theta-1}{\theta}(1-\alpha)\right]^{\frac{\theta}{\alpha(\theta-1)+1}} = \frac{\alpha(\theta-1)+1}{(\theta-1)(1-\alpha)}\left[\frac{\theta-1}{\theta}(1-\alpha)\right]^{\frac{\theta}{\alpha(\theta-1)+1}}$.

tightness of both markets and on the real exchange rate corrected for transport costs ς. Therefore, we just proved the following Lemma:

Lemma 1 *It is never optimal for exporters to export all production and not to export in a given period. Moreover, the optimal exporting share κ_t depends only on the current macroeconomic conditions, and – given the exporting status of a firm – it does not depend on its vintage or on its productivity.*

Thus, we will simply write κ_t instead of $\kappa_{j\tau t}$. Define $\xi_t \equiv Q_t + Q_t^* \left(\frac{\eta_t}{1+\varsigma}\right)^\theta = \frac{Q_t}{\kappa_t}$. The total production of eligible firms can be written as follows:

$$q_{j\tau t} = \left(z_j^\theta h_{j\tau}^{\alpha\theta}\right)^{\frac{1}{\alpha(\theta-1)+1}} \left\{\left[\frac{\theta-1}{\theta}(1-\alpha)\mathbb{W}_t^{-1} A_t^{\frac{1}{1-\alpha}}\right]^\theta \xi_t\right\}^{\frac{(1-\alpha)}{\alpha(\theta-1)+1}},$$

and the optimal labor demand:

$$l_{j\tau t} = \left[\frac{q_{j\tau t}}{A_t z_j h_{j\tau}^\alpha}\right]^{\frac{1}{1-\alpha}} = \left(\left[\frac{\theta-1}{\theta}(1-\alpha)\mathbb{W}_t^{-1}\right]^\theta \left[A_t z_j h_{j\tau}^\alpha\right]^{\theta-1} \xi_t\right)^{\frac{1}{\alpha(\theta-1)+1}}. \tag{A.3}$$

Thus, the real operating profit in a period t is given as:

$$\mathbb{P}_{j\tau t}^{de} = \mathcal{W}_1 z_j^{\frac{\theta-1}{\alpha(\theta-1)+1}} h_{j\tau}^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}} A_t^{\frac{(\theta-1)}{\alpha(\theta-1)+1}} \mathbb{W}_t^{\frac{-(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} \xi_t^{\frac{1}{\alpha(\theta-1)+1}}.$$

Now, we are able to derive the expected present value of profit stream. We start with an exporter $\mathbb{P}_{j\tau}^{de} = \sum_{t=\tau}^{\infty} \mu_\tau^t (1-\delta)^{t-\tau} \mathbb{P}_{j\tau t}^{de}$, whose expected present value satisfies:

$$\mathbb{P}_{j\tau}^{de} = z_j^{\frac{\theta-1}{\alpha(\theta-1)+1}} h_{j\tau}^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}} \underbrace{\mathcal{W}_1 \sum_{t=\tau}^{\infty} (1-\delta)^{t-\tau} \mu_\tau^t A_t^{\frac{(\theta-1)}{\alpha(\theta-1)+1}} \mathbb{W}_t^{\frac{-(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} \xi_t^{\frac{1}{\alpha(\theta-1)+1}}}_{\varpi_\tau^{de}}, \tag{A.4}$$

while the expected present value of a non-exporter $\mathbb{P}_{j\tau}^n = \sum_{t=\tau}^{\infty} \mu_\tau^t (1-\delta)^{t-\tau} \mathbb{P}_{j\tau t}^{de}$ satisfies:

$$\mathbb{P}_{j\tau}^n = z_j^{\frac{\theta-1}{\alpha(\theta-1)+1}} h_{j\tau}^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}} \underbrace{\mathcal{W}_1 \sum_{t=\tau}^{\infty} (1-\delta)^{t-\tau} \mu_\tau^t A_t^{\frac{(\theta-1)}{\alpha(\theta-1)+1}} \mathbb{W}_t^{\frac{-(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} Q_t^{\frac{1}{\alpha(\theta-1)+1}}}_{\varpi_\tau^{dn}}. \tag{A.5}$$

The value of an exporter, who enjoys a productivity level z_j, is determined by quality investment and is given by:

$$\mathbb{V}^{de}_{\tau}(z_j) = \max_{h_{j\tau}\geq 0}\left[z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} h_{j\tau}^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \varpi^{de}_{\tau} - (c + h_{j\tau})\right] - c^x;$$

and similarly for a non-exporter:

$$\mathbb{V}^{dn}_{\tau}(z_j) = \max_{h_{j\tau}\geq 0}\left[z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} h_{j\tau}^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \varpi^{dn}_{\tau} - (c + h_{j\tau})\right].$$

Firms' manager maximizing the value of the firm chooses the following quality level:

$$h^{opt,e}_{j\tau} = z_j^{\theta-1}\left[\frac{\alpha(\theta-1)\varpi^{e}_{\tau}}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)+1}, \tag{A.6}$$

and the value of an exporting firm is[16]:

$$\mathbb{V}^{de}_{\tau}(z_j) = z_j^{(\theta-1)}\left[\varpi^{e}_{\tau}\right]^{\alpha(\theta-1)+1}\mathcal{G} - (c + c^x), \tag{A.7}$$

similarly, the value of a non-exporting firm is

$$\mathbb{V}^{dn}_{\tau}(z_j) = \max_{h\geq 0}\mathbf{V}^{dn}_{\tau}(h|z_j) = z_j^{\theta-1}\left[\varpi^{n}_{\tau}\right]^{\alpha(\theta-1)+1}\mathcal{G} - c, \tag{A.8}$$

and the optimal investment to quality is:

$$h^{opt,n}_{j\tau} = z_j^{\theta-1}\left[\frac{\alpha(\theta-1)\varpi^{n}_{\tau}}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)+1}. \tag{A.9}$$

The value functions $V^{dn}_{\tau}(z_j)$, and $V^{de}_{\tau}(z_j)$ implicitly define the cut-off value $\overline{z}$, which is the least idiosyncratic shock, which makes the export-eligibility investment profitable, i.e. $\overline{z}^{d}_{\tau} = \arg\min_{z_j}(V^{de}_{\tau}(z_j) \geq V^{dn}_{\tau}(z_j))$, which for the particular parametrization is given as follows:

$$\overline{z}_{\tau} = \left(\frac{c^x}{\mathcal{G}[[\varpi^{de}_{\tau}]^{\alpha(\theta-1)+1} - [\varpi^{dn}_{\tau}]^{\alpha(\theta-1)+1}]}\right)^{\frac{1}{\theta-1}}. \tag{A.10}$$

Note that the definition of the cut-off value is correct, only if the expected present value of profit is increasing in z_j. The lemma below demonstrates that this is indeed the case:

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[16] Define $\mathcal{G} \equiv \left[\left(\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}\right)^{\alpha(\theta-1)} - \left(\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}\right)^{\alpha(\theta-1)+1}\right]$, which can be simplified to $\mathcal{G} = \frac{1}{\alpha(\theta-1)+1}\left(\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}\right)^{\alpha(\theta-1)}$.

Lemma 2 *: The net present value of the stream of exporter's real operating profits $\mathbb{P}^e_{j\tau}$ is increasing in z_j, and similarly for non-exporters. Moreover, for any z_j and τ: $\mathbb{P}^e_{j\tau} > \mathbb{P}^n_{j\tau}$.*

PROOF. The first part of the claim is a direct application of the envelope theorem. Indeed, the envelope theorem ensures that $\frac{d\mathbb{P}^n_{j\tau t}}{dz_j} = \frac{\partial \mathbb{P}^n_{j\tau t}}{\partial z_j}$. The maximization of operating profit in a given period (when the quality level and export eligibility status is already decided) implies that $\frac{\partial \mathbb{P}^n_{j\tau t}}{\partial z_j} = \frac{\theta-1}{\theta z_j} \left[A_t z_j f(h_{j\tau}, l_{jt})\right]^{\frac{\theta-1}{\theta}} Q_t^{\frac{1}{\theta}}$, which is clearly positive for any finite z_j, A_t, and Q_t. Therefore $\frac{d\mathbb{P}^d_{j\tau}}{dz_j} = \sum_{t=\tau}^{\infty} \mu^t_\tau (1-\delta)^{t-\tau} \frac{d\mathbb{P}^e_{j\tau t}}{dz_j} = \sum_{t=\tau}^{\infty} \mu^t_\tau (1-\delta)^{t-\tau} \frac{\partial \mathbb{P}^e_{j\tau t}}{\partial z_j} > 0$. The exactly analogous reasoning applies for exporters. This proves the first part of Lemma. To prove the second part of Lemma, observe that the exporter can secure at least as high profit as the non-exporter by choosing $\kappa \equiv 1$, and by choosing the same level of the quality investment $h_{j\tau}$. Therefore $\mathbb{P}^e_{j\tau} \geq \mathbb{P}^n_{j\tau}$. The strict inequality follows from the fact that $0 < \kappa_t < 1$ by Lemma 1.

A.2 Model in the Recursive Form

In this part of the paper, we transform the model into the recursive form, which is suitable for efficient application of numerical methods. We do it for parametrization used in A.1. The recursive form consists of dynamic (first-order difference) equations and static (algebraic) equations. These are listed below. To solve the transition dynamics, we then apply the LBJ approach (due to Lafargue, 1990, Boucekkine, 1995, and Juillard et al., 1998) to the recursive form.

Since the presented model involves several kinds of goods and firms, we will use indexes to distinguish among them. To make reading of the paper easier, we introduce the following convention. Firms differ by location, ownerships, and vintage. Location of firms is distinguished by superscripts *d* and *f*, where the former stands for the *domestic* and the latter for the *foreign* country. Firms owned by household from the foreign country are denoted by the superscript *, while the ownership of domestic household is given no special superscript.

Also here, we show variables related to the domestic household and domestic location. The other variables can be represented analogically.

Intertemporal Marginal Rate of Substitution

$$\mu_t^{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{\varepsilon}.$$

The value of a new entrant $\mathcal{V}_\tau^d$ can be easily expressed as follows:

$$\Omega_\tau^d = \left[\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)} \left(\varpi_t^{n^{\alpha(\theta-1)+1}} \int_{z_L}^{\overline{z}_t^d} z^{\theta-1}\, G(\mathrm{d}z) + \varpi_t^{e^{\alpha(\theta-1)+1}} \int_{\overline{z}_t^d}^{z_U} z^{\theta-1}\, G(\mathrm{d}z)\right),$$

where the cut-off values are given by (A.10) and the profit flows satisfies:

$$\begin{aligned}
\varpi_t^{nd} &= \mathcal{W}_1 \left([A_t]^{(\theta-1)}\, \mathbb{W}_t^{-(\theta-1)(1-\alpha)} Q_t\right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{t+1}\varpi_{t+1}^{nd},\\
\varpi_t^{ed} &= \mathcal{W}_1 \left([A_t]^{(\theta-1)}\, \mathbb{W}_t^{-(\theta-1)(1-\alpha)} \xi_t\right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{t+1}\varpi_{t+1}^{ed}.
\end{aligned}$$

To get the representation of realized profit flows Ξ_t, the value of exports, the labor demand, and of the CPI-based real exchange rate, it is necessary to define the 'weighted' numbers of exporters and non-exporters, where the weights are based on firms' size. The weighted number of exporters $\widehat{n}_t^e$ obeys the following recursive relation:

$$\widehat{n}_{t+1}^e = (1-\delta)\widehat{n}_t^e + n_{t+1}^e \left[\frac{\alpha(\theta-1)\varpi_{t+1}^e}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)} \int_{\overline{z}_{t+1}}^{z_U} z^{\theta-1}\, G(\mathrm{d}z), \tag{A.11}$$

while a similar recursive equation holds for non-exporting firms:

$$\widehat{n}_{t+1}^n = (1-\delta)\widehat{n}_t^n + n_{t+1}^n \left[\frac{\alpha(\theta-1)\varpi_{t+1}^n}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)} \int_{z_L}^{\overline{z}_{t+1}} z^{\theta-1}\, G(\mathrm{d}z). \tag{A.12}$$

To get the recursive representation for the *actual realized profits* Ξ_t, we have to split it into two parts (according to the export-status): $\Xi_t = \Xi_t^e + \Xi_t^n$. Then:

$$\begin{aligned}
\Xi_t^e &= \mathcal{W}_1 \widehat{n}_t^e \left(A_t^{\theta-1}\mathbb{W}_t^{-(\theta-1)(1-\alpha)}\xi_t\right)^{\frac{1}{\alpha(\theta-1)+1}}, \qquad \text{(A.13)}\\
\Xi_t^n &= \mathcal{W}_1 \widehat{n}_t^n \left(A_t^{\theta-1}\mathbb{W}_t^{-(\theta-1)(1-\alpha)}Q_t\right)^{\frac{1}{\alpha(\theta-1)+1}}.
\end{aligned}$$

The *balance-of-payments equilibrium* condition requires that:

$$B_{t+1} = (1+r_t^*)B_t + \eta_t^{-1}X_t + \left(\Xi_t^f - \widehat{\chi}(n_t^f)\right) - \eta_t^{-1}\left(\Xi_t^{d*} - \widehat{\chi}(n_t^{d*})\right), \tag{A.14}$$

where X_t is the value of *net* real exports of the domestic country expressed in the domestic currency.

The value of net exports is necessary for the balance-of-payments equilibrium condition, and is defined as the difference between the value of gross domestic exports minus the value of gross domestic imports $X_t = X_t^d + X_t^{d*} - \eta_t\left(X_t^f + X_t^{f*}\right)$. The value of gross domestic exports by the firms owned by the domestic household (expressed in the domestic currency) X_t^d satisfies the following equation:

$$X_t^d = \widehat{n}_t^e (1-\kappa_t)^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}} \left(\frac{\eta_t}{1+\varsigma}\right)^{\frac{\theta}{\alpha(\theta-1)+1}} \left[\frac{\theta-1}{\theta}(1-\alpha)\mathbb{W}_t^{-1} A_t^{\frac{1}{1-\alpha}}\right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} Q_t^{*\frac{1}{\alpha(\theta-1)+1}}.$$

The household's *budget constraint* (3), household's *Euler equations* (4)-(7), and their foreign counterparts. These equations are already in the form of first-order difference equations.

The *expected investment costs* obey:

$$\widetilde{c}_t^d = c + (1 - G(\overline{z}_t^d))c^x + \dots$$
$$+ \left[\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)+1} \left(\varpi_t^{dn\,\alpha(\theta-1)+1} \int_{z_L}^{\overline{z}_t^d} z^{\theta-1}\, G(\mathrm{d}z) + \varpi_t^{de\,\alpha(\theta-1)+1} \int_{\overline{z}_t^d}^{z_U} z^{\theta-1}\, G(\mathrm{d}z)\right).$$

The rest of equations are mainly market clearing conditions and definitions. The market clearing conditions include the *clearing of the goods markets*:

$$C_t + n_t^d \widetilde{c}_t^d + n_t^{d*} \widetilde{c}_t^{d*} = Q_t,$$

and its foreign counterpart, *international bond market clearing*:

$$B_t + B_t^* = 0,$$

and *labor market clearing* conditions. The labor market clearing conditions can be restated into the recursive form as follows: define $\eth_t$ as

$$\eth_t^n = \left(\left[\frac{\theta-1}{\theta}(1-\alpha)\right]^\theta A_t^{\theta-1}\mathbb{W}_t^{-\theta} Q_t\right)^{\frac{1}{\alpha(\theta-1)+1}},$$
$$\eth_t^e = \left(\left[\frac{\theta-1}{\theta}(1-\alpha)\right]^\theta A_t^{\theta-1}\mathbb{W}_t^{-\theta} \xi_t\right)^{\frac{1}{\alpha(\theta-1)+1}}.$$

Then the domestic labor demand is given as $\mathcal{L}_t = \sum_{\xi\in\{e,n\}} \eth_t^\xi \widehat{n}_t^\xi$. The labor demands should be equal to inelastic labor supply.

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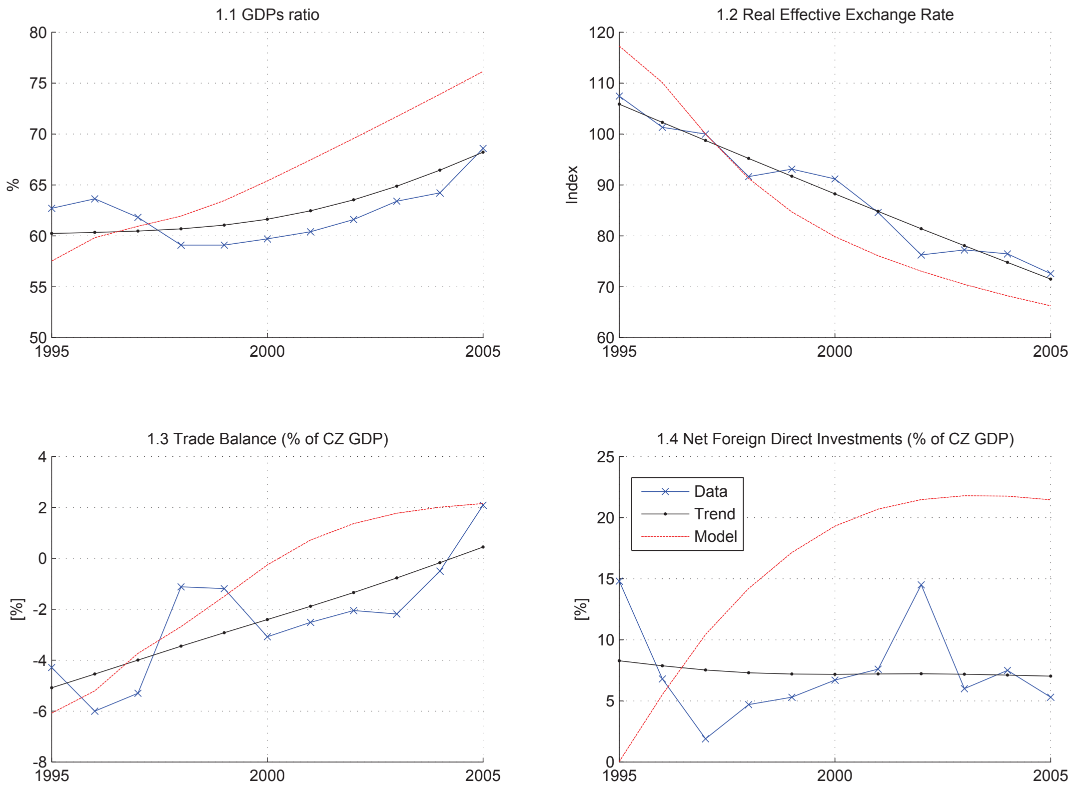
Table 1
Summary of model parameters

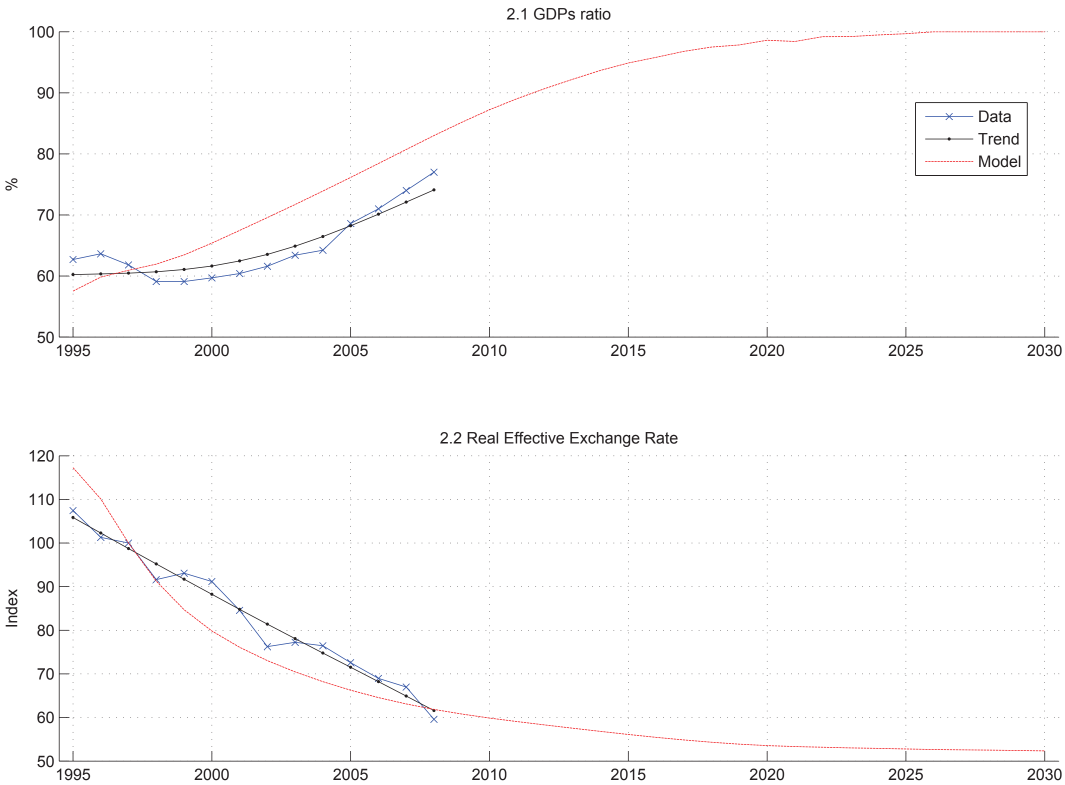
Parameter	Description	Value
	Production and utility functions	
θ	elasticity of substitution	4.70
β	discount factor	0.95
α	quality intensity	0.32
δ	exit rate	0.45
ς	iceberg costs	0.04
ε	elasticity of intertemporal substitution	2.00
	Productivity	
$\mathbf{m}$	auxiliary parameter for A_t	9.0
$\mathbf{n}$	auxiliary parameter for A_t	12.0
τ	auxiliary parameter for A_t	5.0
A_{ss}	terminal value of domestic productivity	1.1
A^*	foreign productivity	1
	Investment costs	
c	fixed entry cost	0.45
c^x	fixed exporting cost	0.50
$1/\Psi_d$	the inverse of the adjustment cost parameter (domestic investments)	0.22
$1/\Psi_f$	the inverse of the adjustment cost parameter (cross-country investments)	0.01
Ψ_B	adjustment cost parameter (bond holding)	0.01

Fig. 1. Main macroeconomic aggregates – data and trends



Source: Czech National Bank, Czech Statistical Office, Czech Ministry of Finance, Our computation.

Fig. 2. Main macroeconomic aggregates – projections



Source: Czech National Bank, Czech Ministry of Finance, Our computation.