NUMERICAL SIMULATION OF 3D STRATIFIED FLOW IN ABL

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Introduction

The work deals with the numerical solution of 3D turbulent stratified flows in atmospheric boundary layer over the cosine function shaped hill. Mathematical model for the turbulent stratified flows in atmospheric boundary layer is the Boussinesq model - Reynolds averaged Navier-Stokes equations (RANS) for incompressible turbulent flows with addition of the transport equation for density and that is coupled with the RANS system by the source term at the right hand side of the momentum equation. The artificial compressibility method and the finite volume method have been used in all computed cases and Lax-Wendroff scheme (MacCormack form) has been used together with the Cebecci-Smith algebraic turbulence model.

Governing equations

The incompressible Navier-Stokes equations with addition of transport equation for density. There has been taken into account transport equation for density without diffusive terms on the RHS. In other words the diffusion coefficient of the transport equation for density is equal 0. This model has been published in [7] and arises from the Boussinesq model.

$$\frac{\partial u_j}{\partial x_j} = 0 \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} + \frac{1}{\rho_0}\frac{\partial p}{\partial x_i} = \frac{\partial}{\partial x_j}\left(\nu \frac{\partial u_i}{\partial x_j}\right) + \frac{\rho}{\rho_0} g_i \tag{2}$$

$$\frac{\partial \rho}{\partial t} + u_j \frac{\partial \rho}{\partial x_j} = 0 \tag{3}$$

where g_i is the component gravitational acceleration vector. The model has to be completed by hydrostatic pressure equilibrium function (4) for simulation of atmospheric boundary layer flows:

$$\frac{\partial p}{\partial z} = -\rho \cdot g_z, \tag{4}$$

where g_z denotes the z component of the gravitational acceleration vector (assuming the z axis is perpendicular to the ground).

Let me introduce a vector form of the previous system at this place, which will be very useful in the following chapters:

$$\mathbb{M} \cdot \mathbf{W}_t + \mathbf{F}_x + \mathbf{G}_y + \mathbf{H}_z = \mathbf{R}_x + \mathbf{S}_y + \mathbf{T}_z + \mathbf{K}, \tag{5}$$

where $\mathbb{M}$ is diagonal matrix with following entries:

$$\mathbb{M} = diag \parallel 0,\ 1,\ 1,\ 1,\ 1 \parallel, \tag{6}$$

$\mathbf{W}$ is a vector of conservative variables:

$$\mathbf{W} = \| \, p, \; u, \; v, \; w, \; \rho \, \|^T , \tag{7}$$

$\mathbf{F}$, $\mathbf{G}$, $\mathbf{H}$ are vectors of convective fluxes:

$$\mathbf{F} = \left\| \begin{array}{c} u \\ u^2 + p \\ u \cdot v \\ u \cdot w \\ u \cdot \rho \end{array} \right\| , \quad \mathbf{G} = \left\| \begin{array}{c} v \\ v \cdot u \\ v^2 + p \\ v \cdot w \\ v \cdot \rho \end{array} \right\| , \quad \mathbf{H} = \left\| \begin{array}{c} w \\ w \cdot u \\ w \cdot v \\ w^2 + p \\ w \cdot \rho \end{array} \right\| , \tag{8}$$

$\mathbf{R}$, $\mathbf{S}$, $\mathbf{T}$ are vectors of diffusive fluxes:

$$\mathbf{R} = \nu \left\| \begin{array}{c} 0 \\ u_x \\ v_x \\ w_x \\ 0 \end{array} \right\| , \quad \mathbf{S} = \nu \left\| \begin{array}{c} 0 \\ u_y \\ v_y \\ w_y \\ 0 \end{array} \right\| , \quad \mathbf{T} = \nu \left\| \begin{array}{c} 0 \\ u_z \\ v_z \\ w_z \\ 0 \end{array} \right\| \tag{9}$$

and $\mathbf{K}$ is the source term defined as follows in this case:

$$\mathbf{K} = \frac{\rho}{\rho_0} \| \, 0, \; 0, \; 0, \; -g, \; 0 \, \|^T . \tag{10}$$

One can split the pressure into sum of its initial value and its perturbation as follows:

$$p(z) = p''(z) + p_0(z), \tag{11}$$

and the same could be done with density:

$$\rho(z) = \rho''(z) + \rho_0(z). \tag{12}$$

Lets make a substitution in (4) by (11) and (12):

$$\frac{\partial p''}{\partial z} + \frac{\partial p_0}{\partial z} = -\left(\rho'' + \rho_0\right) g_z, \tag{13}$$

and because the initial values of pressure p_0 and initial values of density ρ_0 has to be in equilibrium i.e.:

$$\frac{\partial p_0}{\partial x} = 0, \tag{14}$$

$$\frac{\partial p_0}{\partial y} = 0, \tag{15}$$

$$\frac{\partial p_0}{\partial z} = -\rho_0 g_z, \tag{16}$$

so if one substitute the pressure p in equation (2) by (11), one obtains the system in which the unknowns are the pressure perturbations:

$$\frac{\partial u_j}{\partial x_j} = 0, \tag{17}$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\nu \frac{\partial u_i}{\partial x_j}\right) - \frac{1}{\rho_0}\frac{\partial p''}{\partial x_i} + \frac{\rho''}{\rho_0} g_i, \tag{18}$$

$$\frac{\partial \rho}{\partial t} + u_j \frac{\partial \rho}{\partial x_j} = 0. \tag{19}$$

This system has been used in a numerical model for all the computations of stratified flows that have been presented in this paper.

Cebecci-Smith Algebraic Turbulence Model

Presented Cebbeci-Smith algebraic turbulence model has been taken from [2]. Cebecci-Smith algebraic turbulence model could be used to compute the turbulent viscosity ν_t. Domain Ω is divided into two subdomains. In the inner subdomain (near walls) the inner turbulent viscosity ν_{t_i} is computed. In the outer subdomain the outer turbulent viscosity ν_{t_o} is computed. Most common procedure is to compute both turbulent viscosities and then to use the minimal one:

$$\nu_t = \min\left(\nu_{t_i},\, \nu_{t_o}\right). \tag{20}$$

For turbulent viscosity computing is necessary to use local systems of coordinates $(X,\, Y)$. Where X is parallel with the nearest wall and Y is perpendicular to the nearest wall (distance from the wall). In the inner subdomain the turbulent viscosity is defined as follows:

$$\nu_{t_i} = L_m^2 \left|\frac{\partial U}{\partial Y}\right|, \tag{21}$$

where $(U,\, V)$ are components of velocity vector in direction of $(X,\, Y)$ and L_m is given by equation:

$$L_m = \kappa Y F_D, \tag{22}$$

where:

$$F_D = 1 - \exp\left(-\frac{1}{A^+} u_r Y Re\right), \tag{23}$$

u_r is so called friction velocity:

$$u_r = \left(\nu \left|\frac{\partial U}{\partial Y}\right|\right)_\omega^{\frac{1}{2}}. \tag{24}$$

In outer subdomain the turbulent viscosity is defined by Clauser‘s equation:

$$\nu_{t_o} = \rho \alpha \delta^* U_e F_k, \tag{25}$$

where:

$$F_k = \left[1 + 5.5\, \left(\frac{Y}{\delta}\right)^6\right]^{-1},\; U_e = U(\delta) \tag{26}$$

and δ is the boundary layer thickness and:

$$\delta^* = \int_0^\delta \left(1 - \frac{U}{U_e}\right) dY. \tag{27}$$

Following values of the constants have been used:

$$\kappa = 0.4,\ \alpha = 0.0168,\ A^+ = 26. \tag{28}$$

Numerical Solution

The artificial compressibility method together with the finite volume method have been used to discretize the RANS system and the Lax-Wendroff predictor-corrector scheme (MacCormack form) has been used to compute the solution. The scheme has been used in a following form:

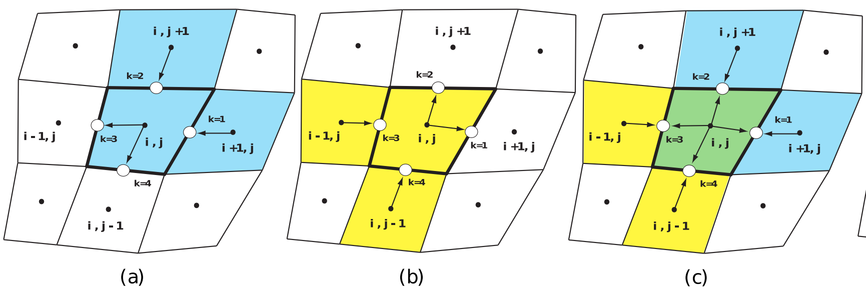
$$\mathbf{W}_i^{n+\frac{1}{2}} = \mathbf{W}_i^n - \Delta t\, \mathbf{L}(\mathbf{W}_{ij}^n), \tag{29}$$

$$\mathbf{W}_i^{n+1} = \frac{1}{2}(\mathbf{W}_i^{n+\frac{1}{2}} + \mathbf{W}_i^n) - \frac{\Delta t}{2}\, \mathbf{L}(\mathbf{W}_{ij}^{n+\frac{1}{2}}), \tag{30}$$

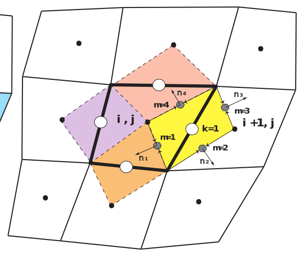
where

$$\mathbf{L}(\mathbf{W}_{ij}) = \frac{1}{V_i} \sum_{j \in A_i} (\tilde{\mathbf{F}}_{ij} - \tilde{\mathbf{R}}_{ij},\ \tilde{\mathbf{G}}_{ij} - \tilde{\mathbf{S}}_{ij},\ \tilde{\mathbf{H}}_{ij} - \tilde{\mathbf{T}}_{ij}) \vec{n}_{ij} \Delta S_{ij} + \tilde{\mathbf{K}}_i, \tag{31}$$

and $\tilde{\mathbf{F}}_{ij}$, $\tilde{\mathbf{G}}_{ij}$, $\tilde{\mathbf{H}}_{ij}$ are numerical convective fluxes computed in a backward direction in the predictor step and in a forward direction in a corrector step, as reader can see in the figure (1) $\tilde{\mathbf{R}}_{ij}$, $\tilde{\mathbf{S}}_{ij}$, $\tilde{\mathbf{T}}_{ij}$ are numerical diffusive fluxes computed centrally in each step, as reader can see in the figure (2).



Obrázek 1: Stencil for inviscid fluxes computation, (a) predictor step, (b) corrector step, (c) predictor + corrector



Obrázek 2: Stencil for diffusive fluxes computation

Neutrally and stably stratified incompressible turbulent results of the flow over 3D cosine shaped hills

The 3D flow past cosine shaped hills is considered in following cases. The height of the hill is 10% of its basis length. The basis length of the hill is $1000\,[m]$ so the height h of the hill is $h = 100\,[m]$. In the last case the flow over two cosine shaped hills is considered. The height of the first hill is 10% of its basis lenght and the second one is 15% of its basis lenght high.

The Reynolds number has been set to $Re = 6.67 \cdot 10^7$ which corresponds approximately to the inlet velocity $u_\infty = 1.0\ [m \cdot s^{-1}]$ if one considers that the kinematic density of the air is about $\nu = 1.5 \cdot 10^{-5}$ $[m^2 \cdot s^{-1}]$. The uniform velocity distribution in the inlet has been considered there.

The fine computational mesh (200 cells in x direction, 100 cells in y direction, 80 cells in z direction), with the near wall resolution $\Delta z_{min} < 1/\sqrt{Re}$, has been considered in the neutrally stratified case.
The twice coarser computational grid as in the neutrally stratified case (100 cells in x direction, 50 cells in y direction, 40 cells in z direction), with the near wall resolution $\Delta z_{min} \approx 10/\sqrt{Re}$, has been considered in stably stratified cases.
The gravitational acceleration vector has been set to $\mathbf{g} = (0,\, 0,\, -10)$ approximately as the gravitational acceleration of the Earth for stably stratified cases.

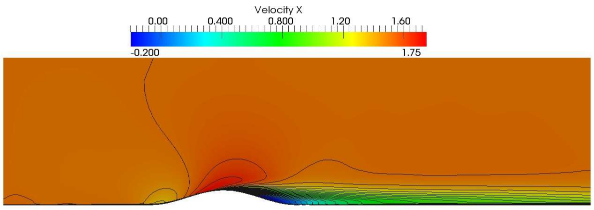
The inlet boundary condition in the inlet has been set according to the equation:

$$\rho(z) = \frac{\rho_H - \rho_{0_w}}{H} z + \rho_{0_w}, \tag{32}$$

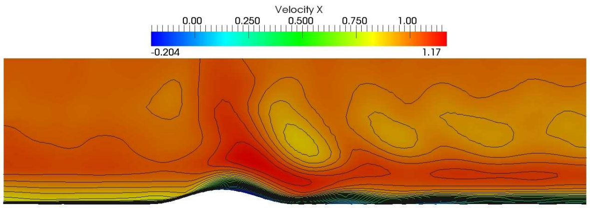
where the density near the ground $\rho_{0_w} = 1.2 [kg/m^3]$ and the density at the top of the domain $\rho_H = 0.6$ $[kg/m^3]$.
The wall boundary condition for density ρ_w has been set as in Case 1 using the relation:

$$\rho_w(x,\, y) = \frac{\rho_H - \rho_{0_w}}{H} z_0(x,\, y) + \rho_{0_w}. \tag{33}$$

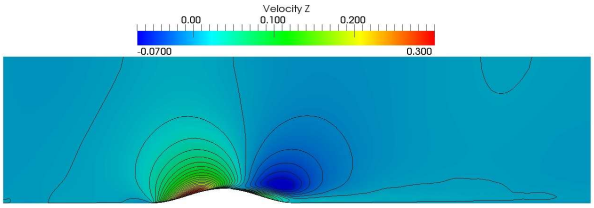
where H is the height of the domain, $z_0(x,\, y)$ is the absolute z coordinate of the ground (let's say altitude) and ρ_{0_w} is the density in $z = 0$.



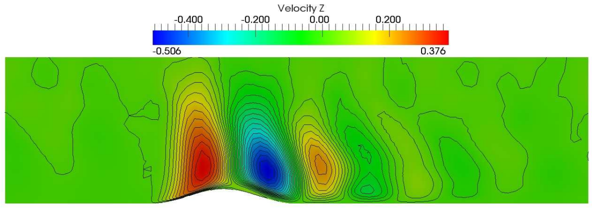
Obrázek 3: Neutrally stratified flow - Contours of velocity X $[m \cdot s^{-1}]$ - XZ plane in the middle of the domain



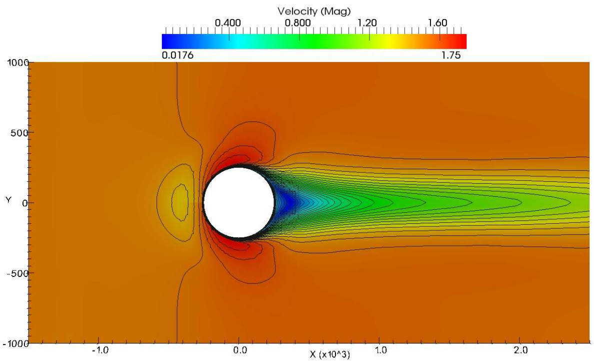
Obrázek 4: Stratified flow - Contours of velocity X $[m \cdot s^{-1}]$ - XZ slice in the middle of the domain



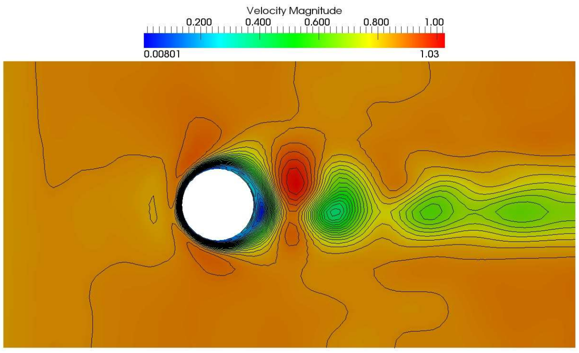
Obrázek 5: Neutrally stratified flow - XZ slice in the middle of the hill - Contours of velocity Z $[m \cdot s^{-1}]$



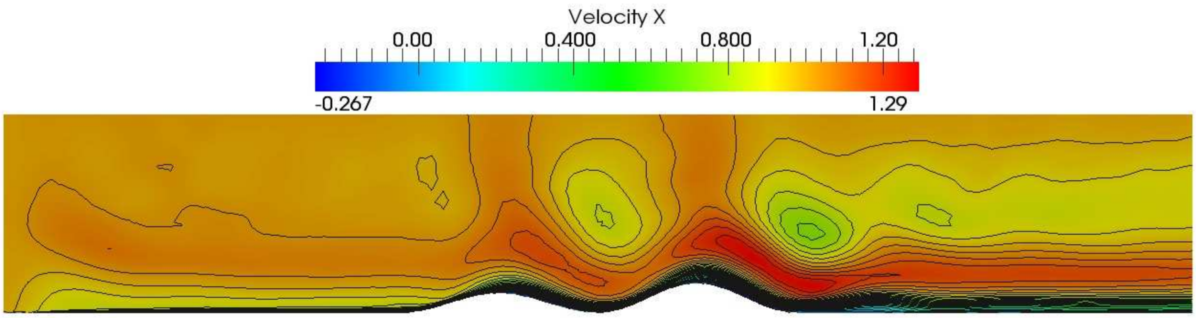
Obrázek 6: Stratified flow - XZ slice in the middle of the hill - Contours of velocity Z $[m \cdot s^{-1}]$



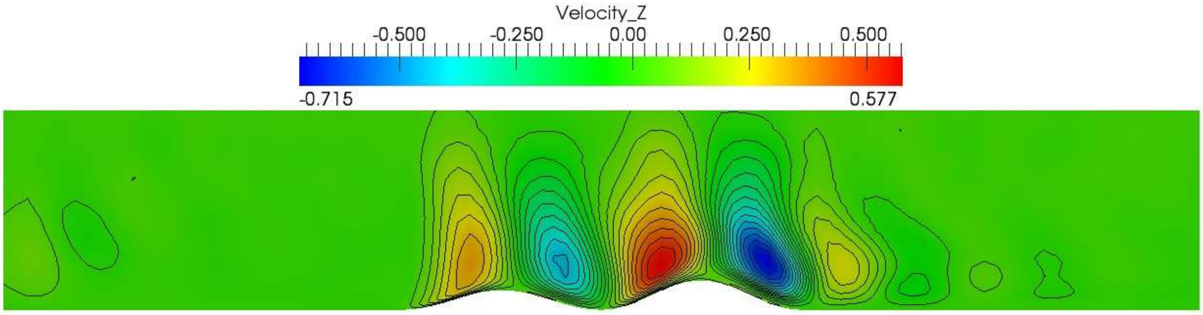
Obrázek 7: Neutrally stratified flow - XY slice in the middle of the hill height - Contours of velocity magnitude $[m \cdot s^{-1}]$



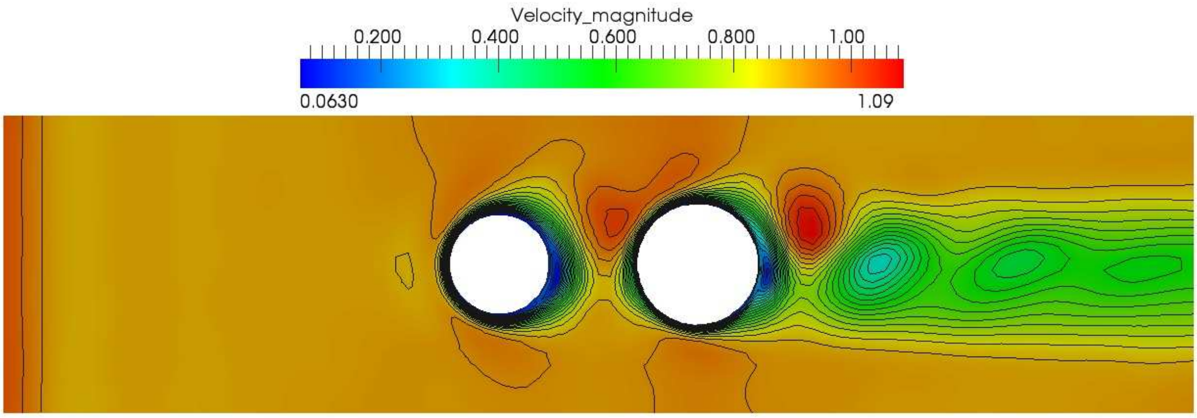
Obrázek 8: Stratified flow - XY slice in the middle of the hill height - Contours of velocity magnitude $[m \cdot s^{-1}]$



Obrázek 9: 3D cosine 10%, cosine 15% hills - XZ slice in the middle of the domain - Contours of velocity X $[m \cdot s^{-1}]$ ($\rho \in [0.6; 1.2]$, $Re = 6.67 \cdot 10^7 \approx U_\infty = 1.0\, m \cdot s^{-1}$)



Obrázek 10: 3D cosine 10%, cosine 15% hills - XZ slice in the middle of the domain - Contours of velocity Z $[m \cdot s^{-1}]$ ($\rho \in [0.6; 1.2]$, $Re = 6.67 \cdot 10^7 \approx U_\infty = 1.0\ m \cdot s^{-1}$)



Obrázek 11: 3D cosine 10%, cosine 15% hills - XY slice in the middle of the hill height - Contours of velocity magnitude $[m \cdot s^{-1}]$
($\rho \in [0.6; 1.2]$, $Re = 6.67 \cdot 10^7 \approx U_\infty = 1.0\ m \cdot s^{-1}$)

Conclusion

The so called ”lee waves” should appear in stratified flows [3]. One can see the creation of lee waves in neutrally stratified cases, especially in the figure where the distribution of z component of the velocity vector is shown (6). One can see the periodic changes of the vertical direction of the flow there, that are damped because of the presence of viscous forces in the fluid.

The next phenomenon visible in the presented figures showing XY plane (8) is that the flow pattern is not symmetrical in stratified flows. One can see in the figure (7) that the flow is symmetrical in the case with neutral stratification. It means that the asymmetry is the most probably caused by the stratification of the flow in our cases.

Acknowledgements

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