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Essays on Interest Rates and Credit Risk

Martin Vojtek

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Preface

This dissertation addresses inefficiencies and problems in the financial markets of post-transition countries, which denies the use of standard estimation techniques. It focuses on interest rate markets and empirically analyzes the situation in the countries that joined the EU in May 2004. These countries underwent significant changes over the last two decades and markets in these countries are often not stable and not developed. In my dissertation I conduct research in areas where empirical results are very scarce. A deeper understanding of the specifics in the markets of post-transition countries can be very helpful for example in designing policy measures touching these markets.

Chapter 1 (which was published in the CERGE-EI Working Papers Series) deals with one the specifics of post-transition countries, namely non-existent or very small markets with certain types of financial products, in this case the derivatives of interest rates. These products (or the implied volatility derived using the prices acquired from the market) can be used for the correct calibration of the models of interest rates. However, due to infrequent or non-regular trading, the prices do not contain sufficient information (or the prices are not quoted at all) and the implied volatility approach to calibration cannot be used. The paradigm used in Chapter 1 is the Brace-Gatarek-Musiela model (Brace, Gatarek and Musiela (1997)) of interest rates that models the evolution of LIBOR (London InterBank Offered Rates)-type market interest rates together with the Orthogonal GARCH model proposed by Alexander (2002), and further generalized by van der Weide (2002). The BGM model is among the most widely used no-arbitrage type of model and its correct calibration is crucial in the calculation of the correct prices of financial instruments based on interest rates. An exact methodology is important not only for business and traders, but also for regulators to avoid market failures. The paper builds on calibrated models for the Visegrad 4 countries (the Slovak

Republic, the Czech Republic, Hungary and Poland) and an analysis of interest rate markets with shorter-end maturities is performed. It answers the question to what extent are these models reliable for pricing derivatives in transition markets.

Chapter 2 is a part of the research conducted at the Czech National Bank to measure how market participants perceive the prospects of enlarging the euro area for the four Visegrad countries that joined the EU in May 2004. The paper has been submitted for publication in the Czech National Bank Working Paper Series. The traditional methods to estimate the probability of EMU enlargement for post-transition countries cannot be used or there are serious limitations. Therefore a method based on the state space model from Lund (1999) is developed and used in this chapter. Lund (1999) builds on the equilibrium interest rate model of the Vasicek (1977) type, where first the so-called "true local spreads", i.e. the spreads of the local (domestic) interest rates to the euro interest rates that would occur in the case that no anticipated entry of a domestic country into the EMU is possible are estimated. Based on the knowledge of the true local spread and the actual spread it is possible using the Kalman filter to infer how likely is the entry of the domestic country into the EMU zone. Using this method the date of EMU enlargement for Slovakia was correctly predicted at the time of the analysis.

One can say that there are two approaches to measure the market perceptions of EMU entrance. They may be extracted from the market information (such as prices or interest rates) or they may rely upon the beliefs of experts (or market participants). In Chapter 2 also both sources of results are compared, i.e. a market-data-based approach with the results of the Reuters opinion survey.

The last paper (written together with Professor Evžen Kočenda, currently under review for the Journal of Financial Services Research) presented in Chapter 3 contributes to the literature that is still very rare, meaning empirical studies analyzing the problem of differentiating between "good" and "bad" debtors prior to granting credit, with emphasis placed on credit scoring related to retail loans in post-transition countries that became EU members. To develop and to use a precise credit-scoring system is crucial in the banking sector. A bank with an accurate and powerful credit scoring model not only decreases its costs connected with bad loans, but also strengthens a bank's risk management in general.

In this paper we develop an optimal specification of the credit scoring model to analyze data on loans at the Czech retail banking market. We employ two approaches: parametric (logistic regression) and non-parametric (Classification and Regression Trees, or CART), as described in Vojtek and Kočenda (2006). Along with analyzing our results we also aimed to assess the determinants of default behavior. We construct three different models using logistic regression and one model using CART and compare these models in terms of efficiency and power in discriminating between bad and good clients. We were able to detect the most important characteristics of default behavior. Both methods are robust: they found similar variables as determinants. We therefore show that parametric as well as non-parametric methods can produce successful models. Further, we show that socio-demographic variables are important in the process of granting credit and therefore such variables should not be excluded from credit scoring model specification.

Chapter 1

Calibration of Interest Rate Models - Post-Transition Markets Case

Abstract

A methodology to calibrate a multifactor interest rate model for post-transition countries is proposed. The usual methodology of calibration with implied volatility cannot be used as there are no markets for regularly traded derivatives. The existence of such markets is essential for this calibration. The paradigm used is the Brace-Gatarek-Musiela model of interest rates (Brace et al. (1997)), which models the evolution of LIBOR (London InterBank Offered Rate) market interest rates, together with the Orthogonal GARCH model proposed by Alexander (2002), and further generalized by van der Weide (2002). The estimated model is used for the analysis of interest rate markets with shorter-end maturities in the four Visegrad countries (the Slovak Republic, the Czech Republic, Poland and Hungary).

Keywords: interest rate, interest rate models, calibration, transition countries
JEL classification codes: C13, C32, C82, E43, G14

1.1 Introduction

In this paper the Brace et al. (1997) model is used for the interest rates markets in emerging countries together with the G(O)-GARCH model proposed in Alexander (2002), and further generalized by van der Weide (2002). Also issues connected with estimating the parameters of the mentioned interest rate model are analyzed. The analysis is done on daily data from four Visegrad countries (Poland, Slovakia, Hungary and the Czech Republic), i.e., post-transition countries where the institutional reforms of the economy are the most advanced. We use data originating prior the accession of these countries to the EU as our main concern is the analysis of markets before entry into the EU. The entry is connected with a higher reliability or credibility of markets in the eyes of investors and as such causes a structural shift.

Theory about modelling the term structure of interest rates (IR) has evolved over the last 30 years, and since then a number of different approaches have been developed. This theory represents one of the most dynamic parts of the study of finance, where a lot of research is still going on with interesting practical applications, and therefore the theory is widely used by both academics and practitioners.[1]

A lot of research has been done in the field of calibrating various models of IR to the market data of developed countries, however, there is a gap in the field of calibrating models of IR to transition countries' markets. There are a few reasons why this work has not been done yet such as that models of IR usually contain strong assumptions about the efficiency of IR markets and there is a problem with access to data (due to frequent changes in the recording of statistics).

The main reason for the failure of the calibration of more complex models of IR to post-transition markets is that these models are usually calibrated to exactly match the prices of some frequently traded derivatives, for example swaptions or caps and floors.[2] But there is no market for these derivatives in emerging markets, or at least these derivatives are traded very rarely and thus their prices are not reliable and they cannot be taken as benchmark prices. Therefore alternative techniques for the calibration of the models of IR or for the estimation of their parameters are needed. An exact methodology is important, not only for business and traders, but also for regulators to avoid market failures. For the market

[1] For a survey of research in this area, see Rebonato (2003).

[2] In other words, the models are calibrated to implied volatility.

regulator it is essential that correct pricing rules can be set up. This paper should answer the question to what extent are these models reliable for pricing derivatives in post-transition and emerging markets. To achieve this, a method for calibrating multi-factor models of term structure for transition markets is proposed in this paper.

In what follows I will depict the motivation for the research in this area, the aim of the research, the main literature dealing with the area of research, the proposed methodology and the estimation results and their analysis.

1.2 Literature review

The main streams of research in term structure modelling are (general equilibrium) models of short rate, no-arbitrage models of term structure and stochastic volatility modelling.

The general equilibrium models are the oldest and are based on the modelling of the interest rate over the smallest possible time interval, the so-called *short rate*. The whole term structure one can obtain from the predicted future paths of the short rate. The first to use a general equilibrium approach was Merton (1973) to derive a model of discount bond prices. His model was simply a Brownian motion with constant drift. The next to use a model of IR was Vasicek (1977), and his model is one of the most used models of IR using this approach. Vasicek made the following assumptions: "(A.1) The instantaneous (spot) interest rate follows a diffusion process; (A.2) the price of a discount bond depends only on the spot rate over its term; and (A.3) the market is efficient". Under these assumptions, he showed by means of an arbitrage argument that the expected rate of return on any bond in excess of the spot rate is proportional to its standard deviation. This property is then used to derive a partial differential equation for bond prices. The solution to that equation is given in the form of a stochastic integral representation.

This general equilibrium model has a big disadvantage in that it allows for negative interest rates due to a constant coefficient for volatility. This setting was changed by Cox, Ingersoll and Ross (1985), who use an inter-temporal general equilibrium asset pricing model to study the term structure of interest rates. In this model, anticipations, risk aversion, investment alternatives, and preferences about the timing of consumption all play a role in determining bond prices. The volatility of the short rate depends on its value. Many of the factors traditionally mentioned as influencing the term structure are thus included in a way which is fully consistent with maximizing behavior and rational expectations. The model leads to specific formulas for bond prices, which are well suited for empirical testing.

Calibration methodologies for these models are known and widely used. One of the approaches is the estimation of the parameters of models using the Generalized Method of Moments, pioneered by Chan, Karolyi, Longstaff and Sanders (1992). They found for U.S. treasury data that the models most successful in capturing the dynamics of the short rate are those that allow the volatility of IR changes to be highly sensitive to the level of these riskless rates. It is clear that these results have important implications for the use of the different term structure models in valuing interest rate dependent derivatives. The problem is that the GMM method can give imprecise results. That was the motivation of Nowman (1997), who proposed a method of estimation based on the Gaussian estimation method of continuous time dynamic models (which means a method based on using the maximum likelihood technique). He found that for U.K. data, the findings of Chan et al. (1992) are not valid and that the volatility of the short rate is not sensitive to the level of the rate in this case; for the U.S. data these findings are similar to Chan et al. (1992). Nowman (1997) uses another method of estimation, as his model allows the use of an exact maximum likelihood estimator, which can help reduce some of the temporal aggregation bias.

The next approach to modelling the IR is called no-arbitrage pricing. It evolved from the previous approach. One of the differences is that this approach describes the whole term structure, not only one of its points (as the short rate does).[3] It has two purposes in relation to the term structure of interest rates. The first is to price all zero coupon bonds of varying maturities from a finite number of economic fundamentals, called state variables. The second is to price all interest rate-sensitive contingent claims, taking as given the prices of zero coupon bonds. Heath, Jarrow and Morton (1992) presented a unifying theory for valuing contingent claims under a stochastic term structure of interest rates. This methodology, based on the equivalent martingale measure technique, takes as given an initial forward rate curve and a family of potential stochastic processes for its subsequent movements.

[3]These models are thus more complex and have more accurate pricing implications (Rebonato (1998)).

A no-arbitrage condition restricts this family of processes, yielding valuation formulae for interest rate sensitive contingent claims, which do not explicitly depend on the market prices of risk.

In most developed markets, caps and floors are the most traded derivatives. A cap is a strip of caplets each of which is a call option on a forward rate. Market practice is to price the option assuming that the underlying forward rate process is lognormally distributed with zero drift. Consequently, the option price is given by the Black-Scholes formula (Black and Scholes (1973)). In an arbitrage-free framework, however, forward rates over consecutive time intervals are related to one another and cannot all be lognormal under one measure. Brace et al. (1997) show that the mentioned market practice can be made consistent with an arbitrage-free term structure model and they construct the so-called BGM model of InterBank Offered Rates (IBOR).[4]

Calibration of these models to the market data is much more problematic than in the case of short rate models. These models are driven by more independent factors and each forward rate has its own volatility parameters for these factors, which are interdependent. The correlation matrix of forward rates is also important in these models. All these parameters (volatilities and the correlation matrix) have to be estimated from the market data consistently in order to preserve all the relationships. The BGM model offers a closed solution for the price of caps, where the parameters are volatilities of some forward rates. By inverting this formula, one can obtain the implied volatilities from the prices of the cap. This calibration methodology is described by Rebonato (1999).[5]

The calibration of these advanced models of IR to transition markets is very problematic. For example, the pricing approach of Rebonato (1999) cannot be used for transition countries as it is based on the prices of caps and the volatility implied by these caps, and as mentioned in the previous section, these products are either

[4]Various IBOR rates are usually measured as "the best rates among the best banks" and based on an everyday survey. During the recent turbulence on the market, however, major questions have been raised about the reliability of these rates, even for advanced and liquid markets. One of the problems is that the data being supplied may not be reliable. Given that some banks, especially those in trouble, are paying higher rates for the short-term loans they need to finance operations, they should be reporting these higher rates, but they may not be willing to signal to their investors that they are in trouble. This may be even more true for the markets that are much less developed and much less liquid, such as the Polish or Hungarian IR markets.

[5]It can be extended to numerical simulations for determining the prices of path-dependent derivatives that are sensitive to interest rates.

not traded or their prices often have no explanatory power. The methodology of the calibration of short-rate models can be used without exception to calibrate the models for transition country data because they are based only on levels of interest rates.

The last approach to the modelling of the term structure is based on the so-called stochastic volatility assumption. It means that the volatility of the stochastic process itself follows a stochastic process. This method allows one to estimate a short rate process without loss of efficiency and consistency and uses the quasi-maximum likelihood method. The first to apply this approach was Ait-Sahalia (1996). Ball and Torous (1999) estimate a stochastic volatility model of short-term risk-free interest rate dynamics. Estimated interest rate dynamics are broadly similar across a number of countries and reliable evidence of stochastic volatility is found throughout. In contrast to stock returns, interest rate volatility exhibits faster mean-reverting behavior, and innovations in interest rate volatility are negligibly correlated with innovations in interest rates. The less persistent behavior of interest rate volatility reflects the fact that interest rate dynamics are impacted by transient economic shocks such as central bank announcements and other macroeconomic news.

As regards modelling the term structure of interest rates in the Visegrad 4 countries, one can mention Slavík (2001), which builds on previous research by Kotlan (1999) in the case of the Czech Republic. The Polish interest rate market is analyzed for example in Konstantinou (2005) or Serwa (2004) and some modelling results for Slovakia can be found for example in Stehlíková (2005). A similar effort in the field of the calibration of multifactor IR models in the transition countries with a comparison to advanced European countries was performed by Urbánová-Csajková (2007).

1.3 Models and methodology

The first step is to get data on interest rates from various transition countries and to calculate the term structure over certain time periods. The parameters of the BGM model[6] are possible to obtain either using the prices of traded derivatives[7] or (the approach proposed in this paper) using the information on conditional

[6] They are volatilities and correlations of interest rates with various maturities.
[7] That means to calibrate using implied or historical volatilities.

volatilities extracted using some model of conditional volatility (such as various types of GARCH models). In this work it is proposed to use the (G)O-GARCH model and describe how it can be used to achieve the calibration of the parameters of the BGM model.[8]

1.3.1 Definitions and relationships

The most basic contract based on the interest rate is an agreement to borrow a particular amount now in exchange for a promise to repay a bigger amount later. In general, the value of such an agreement depends on the credibility of the debtor and factors other than the time value of money. However, in this paper it is not in our interest to find answers to these other questions, and the introduction of a credit risk spread would significantly complicate our analysis. IBOR rates used in this work are considered by market participants as default free rates. Thus it will be assumed that there is no possibility of default, as is the standard assumption in similar models, see Baxter and Rennie (1996) for example. Let us now define the basic concepts:

Let $T^* > 0$ be a fixed time horizon for all activities in the market. By *discount bond with maturity* $T \leq T^*$ let us mean the contract that pays out the owner of a unit of cash in a fixed time T in the future. The price of the discount bond will be denoted as $P(t,T)$. Clearly, $P(T,T) = 1$. For every maturity T it will be assumed that the price of the bond $P(.,T)$ follows a stochastic, strictly positive process.

The curve $P(t,.)$ describes the price of the whole spectrum of bonds with various maturities. Let us define the process $R(t,T)$, called *yield to maturity*. Formally,

$$R(t,T) = -\frac{1}{T-t} \ln P(t,T) \quad \forall t \in \langle 0,T).$$

By *term structure* let us mean the functional relationship of yield $R(t,T)$ as a function of the maturity T. *A forward contract* is an agreement negotiated in time t about paying out cash at some later time T_1 and receiving the payment back in time $T_2 > T_1$. This claim can be replicated in time t by buying a T_2 bond and selling k units of T_1 bonds. The initial costs are $P(t,T_2) - kP(t,T_1)$ in time t; the debtor pays k in time T_1 and he receives the 1$ payment in T_2. To give this

[8]GARCH models are based on the works of Engle (1982) and Bollerslev (1986) and are very popular in finance. They are more than suitable to capture the changes in volatilities in models with high frequency data.

contract a zero value, k has to be equal to

$$k = \frac{P(t, T_2)}{P(t, T_1)}.$$

Let us call the adequate payoff *a forward rate covering the period* $\langle T_1, T_2 \rangle$ and will denote it as $f(t, T_1, T_2)$. So,

$$\frac{P(t, T_2)}{P(t, T_1)} = e^{-f(t,T_1,T_2)(T_2-T_1)} \quad \forall t \leq T_1 \leq T_2,$$

or

$$f(t, T_1, T_2) = -\frac{\ln P(t, T_2) - \ln P(t, T_1)}{T_2 - T_1}.$$

If $T_2 \rightarrow T_1$, we get *an instantaneous forward rate*

$$f(t, T) = -\frac{\partial}{\partial T} \ln P(t, T),$$

or equivalently

$$P(t, T) = \exp\left(-\int_t^T f(t, u) du\right) \quad \forall t \in \langle 0, T \rangle. \tag{1.1}$$

Let r_t be *an instantaneous interest rate* over the interval $\langle t, t + dt \rangle$.

1.3.2 Models of interest rates

The Heath, Jarrow, Morton (1992) model

The earlier models of term structure were based on the explicit modelling of short rate evolution. This approach arose from the need to price simple derivatives of the term structure, for example options or swaps, which depend on one underlying bond. The approach by Heath et al. (1992), which models term structure evolution, is, on the other hand, based on the explicit specification of the dynamics of instant forward rates $f(t, T)$. This method is the generalization of older models, as shown in Baxter and Rennie (1996).

Let W be d-dimensional Brownian motion defined on the filtered (in the sense of the Wiener filtration) probability space $(\Omega, \mathcal{F}, \mathbb{P})$. With the dot symbol $(\cdot)$ let us denote the standard product of vectors.

The HJM model characterizes the term structure by the following theorem:

Theorem 1 *For an arbitrary maturity $T \leq T^*$, under the assumption of the non-existence of arbitrage, the dynamics of the price of bond $P(t,T)$ under the risk-neutral measure $\mathbb{P}^*$ is*

$$dP(t,T) = P(t,T)\,(r_t dt - b(t,T)) \cdot dW_t^* \tag{1.2}$$

and the forward rate $f(t,T)$ satisfies

$$df(t,T) = \sigma(t,T)b(t,T)dt + \sigma(t,T) \cdot dW_t^*, \tag{1.3}$$

where $b(t,T) = -\int_t^T \sigma(t,u)du$.

The Brace, Gatarek, Musiela (1997) model

The common feature of earlier models of interest rates (up to the HJM model) is the fact that (explicitly or implicitly) they include a specification of the stochastic behavior of non-observable financial quantities, as for example instantaneous forward rates. The calibration of these models to the set of market data thus needs some transformation of these data through the "black-box" of the model to the dynamics of non-observable quantities.

This picture has radically changed with the introduction of the BGM (Brace et al. (1997)) model, which describes directly observable market quantities as discrete LIBOR forward rates.

Let us fix a positive real number δ. Following the definition, the forward δ-LIBOR rate $L(t,T)$ is a discrete forward rate over the interval $\langle T, T+\delta\rangle$ and is given by the relationship

$$1 + \delta L(t,T) = \frac{P(t,T)}{P(t,T+\delta)} \quad \forall t \in \langle 0, T\rangle. \tag{1.4}$$

The derivation of the dynamics of $L(t,T)$ under the risk-neutral measure is beyond the scope of this paper and can be found in Brace et al. (1997).

1.3.3 The calibration of the BGM model using the (G)O-GARCH model

The BGM model can be discretized in the following way

$$y_i^t = \frac{L(t+1,T_i) - L(t,T_i)}{L(t,T_i)} = \mu_i(t)\Delta t + \sum_{k=1}^{r} a_{ik}(t)\Delta W_t^k, \tag{1.5}$$

where ΔW_t^k is an increase at time t of the kth Brownian motion, $a_{ik}(t)$ are instantaneous volatilities of the ith LIBOR rate belonging to the kth factor (or Brownian motion) and $\mu_i(t)$ is the drift of the ith LIBOR rate.

Let us suppose that we have T observations y_i^t on the returns of k interest rate series with various maturities (i.e., one week, two weeks, one month, etc.). (G)O-GARCH models are based on so-called principal component analysis, each component being a simple linear combination of the original returns series. The weights in these linear combinations are determined by the eigenvectors of the correlation matrix of the returns matrix. The principal components are ordered according to the size of the eigenvalues (which are in fact variances of the principal components) so that the first principal component, the one corresponding to the largest eigenvalue (i.e., the one with the largest variance), explains most of the variation. If the system is highly correlated (as is assumed for interest rates with various maturities), only the first few eigenvalues will be significantly different from zero. This means that one can simplify the task by taking just a few principal components into account to represent the original variables with a fairly high degree of accuracy.

The following text is based on Alexander (2002). Let us have the original returns in a $T \times k$ matrix $\mathbf{Y}$. One can normalize these k series into series with zero mean and unit variance, to get matrix $\mathbf{X}$. Now, let matrix $\mathbf{W}$ be the matrix of eigenvectors of $\mathbf{X}'\mathbf{X}/T$, and $\mathbf{\Lambda}$ be the associated diagonal matrix of eigenvalues, ordered according to the decreasing magnitude of the eigenvalue. The principal components of $\mathbf{Y}$ are given by the matrix $\mathbf{P}$:

$$\mathbf{P} = \mathbf{XW}. \tag{1.6}$$

It can be shown that the matrix $\mathbf{P}$ is orthogonal. Because of the orthogonality of matrix $\mathbf{W}$, (1.6) can be rewritten as $\mathbf{X} = \mathbf{P}\mathbf{W}'$, which means

$$x_i = w_{i1}p_1 + \cdots + w_{ik}p_k$$

or

$$y_i = \mu_i + \omega_{i1}p_1 + \cdots + \omega_{ir}p_r + \epsilon_r, \tag{1.7}$$

where $\omega_{ij} = w_{ij}\sigma_i$, μ_i, σ_i are the mean and standard deviation of y_i and the error term ϵ_i means the approximation from using only r out of the k principal factors. When variances of (1.7) are taken into account, one gets

$$\mathbf{V} = \mathbf{A}\mathbf{D}\mathbf{A}' + \mathbf{V}_\epsilon,$$

where $\mathbf{D} = \text{diag}(V(p_1), \ldots, V(p_r))$ is a diagonal (because of orthogonality) covariance matrix of chosen r principal factors, $\mathbf{A} = (\omega_{ij})$ and $\mathbf{V}_\epsilon$ is the covariance matrix of the errors. Ignoring the error term gives us the approximation that forms the basis for the model of covariance matrix $\mathbf{V}$

$$\mathbf{V} \approx \mathbf{A}\mathbf{D}\mathbf{A}'.$$

Because matrix $\mathbf{A}$ is known, it is enough to model matrix $\mathbf{D}$, which can be achieved by running r simple GARCH models on the first r principal components from $\mathbf{P}$. This is the basis of the O-GARCH model.[9]

Now, let us closely look at specifications (1.5) and (1.7). Because the series $p_1, \ldots, p_r$ are generated from the series with zero mean and unit variance, one can consider them as increases of r Brownian motions, so that the estimates of coefficients ω_{ij} are actually estimates of the conditional volatility belonging to the jth Brownian motion.[10]

[9] The main limitation of this approach is that the principal components are only unconditionally uncorrelated so the assumption that off-diagonal elements of $\mathbf{D}$ are zero may be unnecessarily strong. This assumption has been relaxed by van der Weide (2002), who develops a generalization of the model called Generalized O-GARCH. In this model the univariate GARCH specifications are applied to transformed variables $\mathbf{P}^* = \mathbf{P}\mathbf{U}$, where $\mathbf{U}$ is an orthonormal matrix that can be estimated using conditional information from the observed data.

[10] The last step in the calibration process is to perform a numerical simulation of future term structure evolution. One possible simulation algorithm is described by Brace, Musiela and Schloegl (1998). These numerical simulations then can be used for the construction of the processes of derivatives as well as replicating portfolios needed for their comparison.

1.4 Data description

In this research, data from the four Visegrad countries are used: the Slovak Republic, the Czech Republic, Hungary and Poland. Various time spans have to be used as the quotations vary across the countries. The datasets for the Slovak Republic and the Czech Republic come from the web pages of the central banks, the datasets for Hungary and Poland come from Reuters' databases. All interest rate series used are the analogue of LIBOR (London InterBank Offered Rate). In this work only data for the shorter end of the term structure are used (the tenor with maturities from 1 week to 1 year). There are a few reasons for such a restriction.

The markets in transition countries are often imperfect and undeveloped and the interest rate market is not an exception. Although the countries mentioned in the previous paragraph have the most developed markets among the transition countries, they are still not at the level of developed countries. The interest rate market is a very good example as banking institutions lend and borrow mostly with the shortest maturities and official quotations of interest rates exist only for maturities of up to 1 year. The pricing of instruments with longer maturities is based on swaps and rates calculated from swaps. These rates are quoted by Reuters (except for the Slovak Republic), but as mentioned above, with longer maturities the market is even more imperfect.[11]

So the reasons for restricting the data-sets to maturities of up to one year are market imperfections and liquidity (no trading with longer maturities - the volume of trades is often zero for longer maturities) and data availability. These estimations concentrate on the interbank offered rates, not on the rates that are implied in the prices of government bonds. There are again a few reasons for this. Firstly, the BGM model describes the evolution of interbank offered rates. Secondly, these rates, although in general not risk-free (there is always a risk of the bankruptcy of a bank, which is incorporated in the rates), are used by banks and other financial institutions as the lending and borrowing rates, and therefore they are used for the pricing of derivatives.

The following data are used. For the Czech Republic, the Prague InterBank Offered Rate (PRIBOR) is used, which is an analogue of LIBOR rates; the time

[11] Mostly there exist only government bonds with higher maturities; municipal or corporate bonds exist only rarely.

span is from 2 January 1998[12] to 28 November 2003, which comprises 1492 daily observations. In this case the eight PRIBOR interest rate time series with maturities from 1 week to 1 year are used. The basic characteristics of the data are in Table 1.2. In Table 1.3 one can see the eigenvectors of the unconditional variance matrix, i.e., the weights of respective interest rates in principal factors. There are also the eigenvalues of the unconditional variance matrix (equal to the unconditional variances of principal factors) and the fraction of the total variance explained by the concrete principal factor.

For the Slovak Republic, the Bratislava InterBank Offered Rate (BRIBOR) is used, which is an analogue of LIBOR rates; the time span is from 5 June 2000 to 28 November 2003 (earlier data are not usable as the rates were quoted only up to a six month maturity), which consists of 871 daily observations. The data used were eight BRIBOR interest rates time series with maturities from 1 week to 1 year. The basic characteristics of the data are in Table 1.5. Table 1.6 shows the eigenvectors of the unconditional covariance matrix together with eigenvalues.

For Hungary, the Budapest InterBank Offered Rate (BUBOR) is used, which is an analogue of LIBOR rates; the time span is from 2 May 2002 to 28 November 2003, which includes 406 daily observations. In the case of Hungary seven interest rate time series with maturities from 1 week to 1 year are used (the series with a two month maturity was not available). The time span is much shorter than in the previous cases. It is due to the fact that until May 2002 only rates for one, three and six month maturities were quoted in the market. The basic characteristics are in Table 1.8. Table 1.9 shows the eigenvectors and eigenvalues.

For Poland, the Warsaw InterBank Offered Rate (WIBOR) is used, which is an analogue of LIBOR rates; the time span is from 2 January 2001 to 28 November 2003 (again, no quoting of longer rates occurred beforehand), which comprises 739 daily observations. Six interest rate time series with maturities from 1 week to 1 year were used (again, the series for two week and two month maturities were not quoted). The their basic characteristics are in Table 1.11. Table 1.12 shows the eigenvectors and eigenvalues.

[12] Although it would be possible to use a longer time span, it would not be very useful. The interbank market was underdeveloped and unstable before 1998 as there were few institutions that needed large amounts of credit and they were able to unbalance the whole market. For more information see Hájková, Hanousek and Něměček (2002).

1.5 Estimation Techniques

For all currencies the original interest rate time series were transformed in order to fit the BGM model specification. The time series used in estimations are constructed as yields of these original interest rate processes. For all currencies three principal factors were chosen, as they explain more than 95% of the variance in all cases (except Poland, where it is 93%). Although the third factor was sometimes relatively unimportant (e.g., Hungary, where it explained only 2.1 % of the variance), it was chosen for modelling to have uniform results and due to the possibility of a change in the shape of the term structure.[13]

The time series of principal factors were calculated using the procedure described in the Methodology section. When there was a suspicion of autocorrelation in the principal factors,[14] the correction for it was used in the mean equation (using the lags of the dependent variable) for the O-GARCH model. All model specifications were tested using a battery of specification tests. The specification tests used in the selection of the number of lags in the mean equations and the number of parameters in the O-GARCH specifications were the Ljung-Box test, the Likelihoood Ratio (LR) test and the Lagrange Multiplier (LM) test on squared standardized residuals proposed by Engle (1982). Standardized residuals were also tested by the Sign Bias test, the Negative Bias test and the Positive Bias test, proposed in Engle and Ng (1993).[15]

As autocorrelation is present for the rates with higher maturities in the Czech Republic, the correction for autocorrelation in the modelling of principal factors is reasonable. The mean equation for the principal components is specified with constant and one (for the first and third components) or two (for the second component) lags. The conditional variances were specified as the GARCH(1,2) processes for the second and third component and as the GARCH(1,1) process for

[13] From the tables of principal component weights one can see, as is usual in principal component analysis, that the first component directs the horizontal movements of the term structure, the second directs the changes in slope and the third directs the changes in curvature.

[14] For example, when the Ljung-Box statistics were sufficiently high for some of the interest rate time series.

[15] These tests are designed to examine whether the volatility models are not misspecified in the sense that they are able to deal with potential asymmetry in the reaction to positive and negative shocks (Sign Bias test) or the potential asymmetry in the reaction to the magnitude of a shock (Negative Bias test and Positive Bias test).

the first component, so that

$$p_t = c + b_1 p_{t-1} \, (+b_2 p_{t-2}) + \mu_t, \;\; \mu_t \sim N(0, H_t), \tag{1.8}$$

where the diagonal elements of H_t are described by

$$h_{i,t} = \alpha_0 + \alpha_1 \mu_{i,t-1}^2 + \beta_1 h_{i,t-1} \, (+\beta_2 h_{i,t-2}) \, , \;\; i = 1, 2, 3, \tag{1.9}$$

and the off-diagonal elements of H_t are 0.[16] The results of the O-GARCH procedures on these principal factors are in Table 1.4, as well as the values of the log-likelihood function and the Schwarz Bayes Information Criterion. There are also the values of Ljung-Box statistics for standardized residuals (Q_{10}) and squared standardized residuals (Q_{10}^2) for 10 lags. They have an χ^2 distribution with 10 degrees of freedom. The last row is the value of the statistics for the LM test with five lags; the distribution of these statistics is χ^2 with five degrees of freedom.[17] All specifications of the model that were used in the estimation minimized the Schwarz criterion among all tested specifications. Also, the LR tests rejected the null hypothesis that any coefficient has zero value. The Ljung-Box test based on the Q_{10} statistics does not reject the null hypothesis, thus there is no autocorrelation present in the standardized residuals, i.e., more lags are not necessary in the mean equation. The Ljung-Box test on squared standardized residuals is the test for the presence of second order dependence in the residuals. From Table 1.4 one can conclude that the models are well specified, so there is no additional heteroscedasticity in the residuals. This conclusion is further supported by the LM test of the null hypothesis that no ARCH process is presented in the residuals. This hypothesis cannot be rejected at any reasonable level of significance. All three specifications were tested using the above-mentioned bias tests with the result that one cannot reject the hypothesis that the volatility processes are not misspecified.

When taking the case of the Slovak Republic, it is possible to see that there is a strong autocorrelation present in the data (from the Q statistics in Table 1.5). The above-mentioned specification tests were performed and three lags in the mean equation and GARCH(1,1) for variance process were chosen as the op-

[16]The terms in brackets were added for the second factor.

[17]The critical values at the 95% significance level are 18.31 and 11.07 for 10 and five degrees of freedom, respectively, for this distribution.

timal specification for the second and the third factor. The first factor is much more problematic. In the time evolution of interest rates in the Slovak case one can see a sharp drop around the end of November 2002, caused by a decrease in the central bank's discount rate, which amounted to 1.5%. This drop is represented by extremely large values in the first factor, which caused either some numerical instability in the GARCH parameters estimation or even the impossibility of numerical estimation. Due to numerical instability it is not possible to change this outlier as it would change dramatically the estimated values of parameters. Under these conditions, the one with two lags in the mean equation and GARCH(2,1) parametrization was chosen as the best specification for the first factor. The results of the estimation are in Table 1.7. The Negative and Positive Bias tests show that the processes are not misspecified. The Sign Bias test returns significant statistics for the third factor, where the residuals are slightly asymmetric. The second order effects in the residuals are however captured, and all other tests are positive for this specification.

For Hungary, there is no autocorrelation presented as the Q statistics are statistically not significant. Thus in this case it is possible to use the mean equation without lags of dependent variables. After the usual specification testing procedure, one can conclude that the most appropriate model is GARCH(1,1) in all cases. However, the LR tests and the Schwarz criterion indicated other feasible specifications, but in order to have a more parsimonious model I decided for the simple GARCH(1,1) process without lags in the mean equation. The other possibility was the specifications with two or three lags in the mean equation and GARCH(2,0) or GARCH(1,2) processes. The former specification can be used because not only the Sign Bias tests indicated that it is the correct specification but also the Ljung-Box and the LM tests have fairly low (insignificant) statistics. The estimation results are in Table 1.10.

The rates for Poland show a significant presence of autocorrelation as the Q statistics are statistically significant for all rates except the one with three-month maturity. However, after the specification testing procedure, one can decide to employ one lag of the dependent variable on the RHS of the mean equation only for the first and the third factor. For the second factor it was not necessary, as was shown by the LR test and the Ljung-Box on the residuals. So, the changes in the shape of the yield curve in this case does not depend on past observations. The standard deviation of the changes in interest rates are higher than in the previous cases. Also, the weights of the principal components are higher. These facts signal that the volatilities for interest rates in Poland are of higher levels than those for the previous currencies: one could observe this mainly at the shortest examined rate. The Sign Bias tests again do not show any misspecification of the model, as the histogram of residuals is symmetric around 0.

1.6 Estimation results

1.6.1 Czech Republic

As expected, the constant coefficients in the mean equations are statistically unimportant. However, the sums of the β coefficients for the second and the third factor are large, showing that there are fairly persistent volatilities of components and that these components in the Czech case are almost nonreactive to the inflow of new information. The first factor is the opposite case. The constant term in the GARCH specification is large together with a high value of the α_1 coefficient. It follows that the volatility of the first factor is high and is very dependent on the past income of news. A higher shock is followed by a period of higher volatility. This suggests that after a horizontal movement of the whole yield curve (which is the consequence of a shock in the first factor) one can expect more intensive trading in the next trading day. This result is consistent with the observed facts that the interest rates in the Czech Republic show a very low level of volatility and they remain relatively fixed for a number of days and the periods of relatively higher volatilities follow mainly after drops in the yield curve levels. This is also supported by the levels of weights for the first principal component, whose values vary around 0.35, so only one third of the shock is translated into the movement in the horizontal direction. The high minimal value of the variance of shock in the first factor (i.e., α_0) indicate that it is much more probable to have an unexpected shock in the horizontal movement of the yield curve than in the change of its slope or shape.

Using the estimated results, it is possible to generate the time evolution of instantaneous conditional covariance and correlation matrices. Figure 1.3 shows the estimated instantaneous conditional volatility of the 1 month PRIBOR rate.

Although the overall volatility level is low, the periods of “increased” volatility in Figure 1.3 correspond to the periods with higher market activity (mostly after significant drops in the interest rate, which also confirms the conclusions from the previous paragraphs). Similarly, the correlation surface calculated is consistent with the market development seen around the end of November 2003.

From these facts is it clear that the calibration procedure is able to reveal the true market development and as such can be used in the pricing of IR-sensitive derivatives. We are able to ascertain the development of conditional correlations and volatilities among the rates, which are factors influencing the prices of such derivatives.

1.6.2 Slovak Republic

The analysis of these results is different from the previous case. The constant coefficients are again statistically unimportant, as one may expect. What is more interesting is the fact that the lag coefficients are (except the second lag for the first factor) negative. This means that greater changes (in absolute values) of the factors tend to be followed by smaller changes of the opposite sign. So, for example, a higher increase in the level of interest rates should be followed by a smaller decrease in the level (or vice versa), which is a correction often observable in financial markets. For the first factor there is some autocorrelation left in the residuals, as one can observe from the value of $Q(10)$ statistics, however, using more lags does not help. We are interested mainly in the volatility analysis. The LM test statistics are fairly low in this case, similar to the $Q^2(10)$ statistics, which is a sign that heteroscedasticity or second order effects were well captured in this case. From the estimation results one can observe that the processes for variances of the shocks ($h_{i,t}$) look like there are strong suspicions of a unit root presence, as the sum of α and β coefficients is very close to zero. The reaction to the inflow of new information concerning the variance of the first component is very high, so there is a high probability that a shock in the first component will influence the volatilities of interest rates significantly. The persistence of the variance is fairly high for the last two components and thus the variance stays high for a longer time after a high shock. All these results suggest that the volatility of the interest rates in Slovakia could be fairly high after high shocks.

The time evolution of the volatility of the one month BRIBOR rate is in Figure 1.6. The volatilities higher than 0.06 are not in this graph in order to increase the resolution of the figure. Here it is possible to see the confirmation of the conclusions from the previous paragraph that the high shock in the first factor will influence very significantly the volatility of the rates. The highest shock is dated 18 November 2002, when the Slovak Central Bank lowered the discount rate (at that time the official rate for the refinancing of banks) from 8% to 6.5%. Thus in the next few days the conditional volatility was affected for all rates, as can be seen in the data. Otherwise, the conditional volatility is low, mostly under 1%. Due to this sharp change in level, it would be more suitable to use some type of switching regime GARCH. However, there are only 250 observations after the break, which are not enough to ensure the stability of the estimated coefficients. This use of a switching regime would also be more reasonable from the pricing point of view. The estimated volatility levels are unnecessarily high and it took some time for the shock to disappear.

With the exception of this break in the level of interest rates, our approach is able to reveal the market course of events. For example, the correlation surface can reveal the fact that in the last examined trading days the longer maturities (from 6 to 12 months) were stable, while the shorter ones were increasing. This is in accordance with Figure 1.5, where there is a small negative correlation between short-term and long-term IR rates. Also the model is able to capture the second order effects of all factors, as the residuals show no evidence for heteroscedasticity. However, the estimation problems and numerical instability are drawbacks in the case of the Slovak Republic. This is mainly caused by the great change in the level of interest rates.

1.6.3 Hungary

The results of the estimation are quite interesting. Both Q statistics are almost equal to zero, similarly to the LM test statistics. This is a sign that there is no autocorrelation hidden in the residuals as well as no second order effects. Coefficients for the innovation term (α_1) are very high for all three estimated principal components. This means that the impact of shocks to instantaneous variance in the next period is large. This fact can indicate that after significant shocks, the intensity of trading increases. On the other hand, the persistence of the variance is even lower than in the case of the Slovak crown. The α_0 coefficient for the first factor, meaning the constant in the volatility process, is relatively high so that the variance or volatility connected with the first coefficient should be higher than with the previous currency. With the next figures, it is possible track the consequences of these facts. The shocks are much more frequent than in the previous cases, and they also do not have a long duration. Moreover the rates have higher volatilities after the shocks (probably due to more intensive trading). Also the estimated correlations at the end of November 2003 are in line with the evolution of the market (Figures 1.8, and 1.9), where the rates are decreasing simultaneously. The correlation between the shortest and the longest rate is around 0.6. Figure 1.9 is again adapted in such a way that the volatilities higher than 0.2 are not shown in this graph in order to increase the resolution of the figure.

The previously stated facts may cause problems when using this calibration for pricing IR derivatives, as the external shocks to volatility are too high and too frequent. When comparing the estimated periods of high volatility with data, it is possible to conclude that the model is again able to capture the periods of high volatility in the data, however, these periods may cause the instability of prices dependent on these factors.

1.6.4 Poland

From the results of the regression in Table 1.13, it is clear that in this case, there is no suspicion of the presence of unit root in the GARCH processes. The first and the third components have a fairly persistent variance, however, the second component shows a higher responsiveness to random shocks. This means that the trend in the changes in the level is stable (with small influences from innovations), while the changes in the slope are more chaotic but last only for a short time. This is also in accordance with the conclusion from the last paragraph that the second factor does not account for autocorrelation and thus does not depend on past movements. The total variance explained by the first factor is relatively low in the case of Poland; it suggests that the volatility of rates are more connected with higher factors. The high weights of the second factor for the shorter maturities correspond with the observed volatile movements of these maturities.

In Figure 1.12 one can see that in November 2003, the one-month interest rate had a higher volatility than the long-term average. This may be the reason for lower correlations of shorter interest rates (week and month) with the longer interest rates, as is observable in Figure 1.11. Also, the predicted behavior of the volatility can be seen. The high persistence of the variance (showed by the large values of β_1 parameters) is visible, as periods with higher volatility alternate with periods with lower volatility. This fact can be seen as confirmation that in this case the calibration methodology is suitable and the conditional parameters coincide well with market development. Out of all four countries the Polish market seems to be the most suitable for the development of interest rate derivatives. There are no sudden breaks in the levels, the estimation procedures work very well in this case and the specifications of factors are very parsimonious.

1.7 Conclusions

In this paper a new methodology is proposed for the calibration of the Brace-Gatarek-Musiela (BGM) model of interest rates. The BGM model is chosen because it is one of the most sophisticated models of interest rates and it has very good pricing implications. A new way to perform the calibration is needed in cases where the standard calibration technique (calibrate to fit the prices of caps or swaptions, i.e., implied volatility) can not be used. The methodology is used for the calibration of this model to the markets of four post-transition countries (the Czech Republic, the Slovak Republic, Hungary and Poland) and an analysis of these markets is carried out based on the calibration.

The methodological contribution of the paper is that instead of calibrating the model to match the prices of some frequently traded derivatives, such as swaptions or caps and floors, it uses an alternative estimation that does not require trading in such derivatives.

There are more reasons to perform such research. The exact pricing of derivatives is very important not only for business but also for policymakers. An example of the importance of exact pricing for business is hedging. Hedging based on derivatives can be successful only if the correct prices of derivatives are available; otherwise, there is a possibility of arbitrage. For the market regulator it is essential that correct pricing rules can be set up to avoid market failure. The amount of derivatives in the books of financial institutions is increasing. A correct daily mark-to-market reevaluation of these portfolios can be done only under the condition of the possibility of the extracting of correct prices from market. The regulator therefore needs an independent pricing mechanism for the derivatives that are not traded regularly and their prices are either not quoted or are not reliable. The analysis in this paper hints that one can rely on more advanced models such as the BGM model in countries with more developed markets such as the Czech Republic and Poland. However, more attention needs to paid to the reported prices in the case of other countries.[18] Another notable outcome of the research is that it may help to start up trades with derivatives in the emerging markets.

The estimation results and estimated evolution of conditional volatilities and correlations (which are in fact the parameters of the BGM model) are generally in correspondence with true market development. However, only the Czech and Polish markets are developed to such a degree that it is possible to use the calibrated interest rate model for pricing IR sensitive derivatives. The other two countries (Slovakia and Hungary) do not have markets developed enough to use such a strong model for pricing IR derivatives.

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[18] Brigo and Mercurio (2006) used a similar approach to calibration with piecewise-constant volatility to the U.S. market with very satisfactory results, similar to the calibration to the implied volatility using prices of caps and swaptions.

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A.1 The models of interest rates

The Heath, Jarrow, Morton (1992) model

The HJM model is based on the assumptions **(HJM.1)** and **(HJM.2)**. The definition of forward rates $f(t,T)$ allows us to write the equation for the instant interest rate $r_t = f(t,t)$. Then, the savings account satisfies the following equation:

$$B_t = \exp\left(\int_0^t f(u,u)du\right) \quad \forall t \in \langle 0, T^* \rangle. \tag{10}$$

The following lemma describes the dynamics of the prices of bonds $P(t,T)$ under the actual (real) probability measure $\mathbb{P}$.

Lemma 1 *The prices of bonds $P(t,T)$ satisfy the relationship*

$$dP(t,T) = P(t,T)\left(a(t,T)dt + b(t,T)\cdot dW_t\right), \tag{11}$$

where a and b are defined as

$$a(t,T) = f(t,t) - \alpha^*(t,T) + \frac{1}{2}|\sigma^*(t,T)|^2, \quad b(t,T) = -\sigma^*(t,T),$$

and for arbitrary $t \in \langle 0, T\rangle$ is

$$\alpha^*(t,T) = \int_t^T \alpha(t,u)du, \quad \sigma^*(t,T) = \int_t^T \sigma(t,u)du. \tag{12}$$

Proof: Let's denote $I_t = \ln P(t,T)$. From (1.1) we get

$$I_t = -\int_t^T f(0,u)du - \int_t^T \int_0^t \alpha(v,u)dvdu - \int_t^T \int_0^t \sigma(v,u)\cdot dW_v du.$$

From the Fubini theorem and the technical conditions of the HJM model it follows that

$$I_t = -\int_t^T f(0,u)du - \int_0^t \int_t^T \alpha(v,u)dvdu - \int_0^t \int_t^T \sigma(v,u)\cdot dW_v du,$$

or equivalently

$$I_t = -\int_0^T f(0,u)du - \int_0^t \int_v^T \alpha(v,u)dvdu - \int_0^t \int_v^T \sigma(v,u)\cdot dW_v du +$$

$$+\int_0^t f(0,u)du + \int_0^t \int_v^t \alpha(v,u)dvdu + \int_0^t \int_v^t \sigma(v,u)\cdot dW_v du.$$

The instantaneous interest rate can be written as

$$r_u = f(u,u) = f(0,u) + \int_0^u \alpha(v,u)dv + \int_0^u \sigma(v,u)\cdot dW_v. \tag{13}$$

From this follows

$$I_t = I_0 + \int_0^t r_u du - \int_0^t \int_u^T \alpha(u,v)dvdu - \int_0^t \int_u^T \sigma(u,v)dv\cdot dW_u.$$

Taking into account (12) one gets

$$I_t = I_0 + \int_0^t r_u du - \int_0^t \alpha^*(u,T)du - \int_0^t \sigma^*(u,T)\cdot dW_u.$$

It is enough now to use the Ito theorem to yield the claim of the lemma. □

Let us now consider T as a particular fixed maturity. If one defines the discounted bond process as $Z(t,T) = B_t^{-1}P(t,T)$, then it satisfies the following equation:

$$dZ(t,T) = Z(t,T)\left(b(t,T)\cdot dW_t + \left(a(t,T) - r_t\right)dt\right).$$

Also let us define process γ_t as such a change of drift of process $Z(t,T)$ that it becomes a martingale. Then with the help of the Girsanov theorem, there exists a measure $\mathbb{P}^*$ equivalent with the real measure $\mathbb{P}$ such that $W_t^* = W_t + \int_0^t \gamma_s ds$ is $\mathbb{P}^*$-Brownian motion. These measures will be denoted as *risk-neutral*. Then the process for the discounted bond can be written as

$$dZ(t,T) = Z(t,T)b(t,T)dW_t^*.$$

The dynamics of the bond prices are under the risk-neutral measure

$$dP(t,T) = P(t,T)\left(b(t,T)dW_t^* + r_t dt\right). \tag{14}$$

Let us assume that there is a claim X, which pays in time $S < T$. Let us define the process E_t as a $\mathbb{P}^*$ martingale:

$$E_t = \mathbb{E}_{\mathbb{P}^*}\left(B_S^{-1} X | \mathcal{F}_t\right).$$

Now, one can use the martingale representation theorem. This theorem states that there exists an $\mathcal{F}$-predictable process ϕ such that

$$E_t = E_0 + \int_0^t \phi_s dZ(s,T).$$

Let us now define a replication portfolio formed with the T-bond and savings account B_t such that the portfolio is replicating claim X in the time S. More concretely, in time t one holds

- ϕ_t units of the T-bond and
- $\psi_t = E_t - \phi_t Z(t,T)$ units of the savings account.

This portfolio has this value in time t:

$$\begin{aligned} V_t &= \phi_t P(t,T) + \psi_t B_t = \phi_t P(t,T) + (E_t - \phi_t Z(t,T))\, B_t = B_t E_t \\ V_t &= B_t \mathbb{E}_{\mathbb{P}^*}\left(B_S^{-1} X | \mathcal{F}_t\right). \end{aligned} \tag{15}$$

So, if X is the payment of a derivative maturing in time S then its value in time t is

$$V_t = \mathbb{E}_{\mathbb{P}^*}\left(\exp\left(-\int_t^T r_s ds\right) X | \mathcal{F}_t\right). \tag{16}$$

However, the bond maturing in time S is the claim $X = 1$. Using (16), it follows that its non-arbitrage value has to be $B_t \mathbb{E}_{\mathbb{P}^*}\left(B_S^{-1} | \mathcal{F}_t\right)$, respectively,

$$P(t,S) = \mathbb{E}_{\mathbb{P}^*}\left(\exp\left(-\int_t^S r_s ds\right) | \mathcal{F}_t\right) \quad t \leq S \leq T. \tag{17}$$

The discounted S-bond then can be written as

$$Z(t,S) = B_t^{-1} P(t,S) = \mathbb{E}_{\mathbb{P}^*}\left(B_S^{-1} | \mathcal{F}_t\right).$$

It means that even the process $Z(t,S)$ is a martingale under the $\mathbb{P}^*$ measure. From this fact follows that the process γ_t has to be equal for all maturities and thus it is independent from T. If one rewrites the definition of the process γ_t and he uses that $\frac{\partial \gamma_t}{\partial T} = 0$ he gets

$$\begin{aligned} \int_t^T \alpha(t,u)du &= \frac{1}{2}b^2(t,T) - b(t,T)\gamma_t \\ \alpha(t,T) &= \sigma(t,T)\left(\gamma_t - b(t,T)\right). \end{aligned}$$

This proves theorem 1. This theorem actually states that under the risk-neutral measure the forward rates cannot have arbitrary drifts but only drifts derived from the volatility process.

The Brace, Gatarek, Musiela (1997) model

In this section the dynamics of $L(t,T)$ rates, defined in 1.4, is derived under the risk-neutral measure. This derivation is based on the original Brace et al. (1997) article. The advantage of BGM is that the $L(t,T)$ rates can be modelled as lognormal.

From (1.4) and (1.1) one gets

$$L(t,T) = \frac{\exp\left(\int_T^{T+\delta} f(t,u)du\right) - 1}{\delta}. \tag{18}$$

In (1.3) one wants to choose the volatility $\sigma(t,T)$ such that it would be possible to obtain the equation for $L(t,T)$ in the following form:

$$dL(t,T) = (\cdots)dt + L(t,T)\gamma(t,T) \cdot dW_t^*$$

for some $\gamma(t,T)$. From (1.3) follows:

$$\begin{aligned} d\int_T^{T+\delta} f(t,u)du &= \int_T^{T+\delta} df(t,u)du = \\ &= \int_T^{T+\delta} \sigma(t,u)b(t,u)du + \int_T^{T+\delta} \sigma(t,u)dW_t^* = \\ &= \int_T^{T+\delta} \frac{1}{2}\frac{\partial b^2(t,u)}{\partial u}du + \left[b(t,T) - b(t,T+\delta)\right] dW_t^* = \\ &= \frac{1}{2}\left[b^2(t,T) - b^2(t,T+\delta)\right] dt + \left[b(t,T+\delta) - b(t,T)\right] dW_t^*. \end{aligned} \tag{19}$$

Then

$$\begin{aligned} dL(t,T) &= d\frac{\exp\left(\int_T^{T+\delta} f(t,u)du\right) - 1}{\delta} \\ &= \frac{1}{\delta}\exp\left(\int_T^{T+\delta} f(t,u)du\right) d\int_T^{T+\delta} f(t,u)du + \\ &\quad + \frac{1}{2\delta}\exp\left(\int_T^{T+\delta} f(t,u)du\right)\left(d\int_T^{T+\delta} f(t,u)du\right)^2 = \\ &\overset{(19)}{=} \frac{1}{\delta}\left[1+\delta L(t,T)\right]\left[\frac{1}{2}\left[b^2(t,T+\delta) - b^2(t,T)\right] dt + \right. \\ &\quad \left. + \left[b(t,T) - b(t,T+\delta)\right] dW_t^* + \frac{1}{2}\left[b(t,T) - b(t,T+\delta)\right]^2 dt\right] \\ &= \frac{1}{\delta}\left[1+\delta L(t,T)\right]\left[b(t,T) - b(t,T+\delta)\right]\left[-b(t,T+\delta)dt + dW_t^*\right]. \end{aligned} \tag{20}$$

If one defines now the process $\lambda(t,T)$ as

$$\lambda(t,T)L(t,T) = \frac{1}{\delta}\left[1+\delta L(t,T)\right]\left[b(t,T) - b(t,T+\delta)\right], \tag{21}$$

he obtains

$$dL(t,T) = -\lambda(t,T)L(t,T)b(t,T+\delta)dt + \lambda(t,T)L(t,T)dW_t^*. \tag{22}$$

Equation (22) can be conveniently rewritten as

$$dL(t,T) = \lambda(t,T)L(t,T)\left[-b(t,T+\delta)dt + dW_t^*\right]. \tag{23}$$

If one combines the previous condition (22) with the Girsanov theorem, he obtains

$$dL(t,T) = \lambda(t,T)L(t,T)dW_t^{T+\delta}, \tag{24}$$

where for all $t \in \langle 0, T+\delta \rangle$,

$$W_t^{T+\delta} = W_t^* - \int_0^{T+\delta} b(u, T+\delta)du. \tag{25}$$

The process $W_t^{T+\delta}$ is a Brownian motion under the measure $\mathbb{P}_{T+\delta} \sim \mathbb{P}^*$, defined with the help of the Radon-Nikodym derivative as

$$\frac{d\mathbb{P}_{T+\delta}}{d\mathbb{P}^*} = \exp\left(\int_0^{T+\delta} b(u, T+\delta) \cdot dW_u^* - \frac{1}{2}\int_0^{T+\delta} |b(u, T+\delta)|^2 du\right). \tag{26}$$

Let us denote the measure $\mathbb{P}_{\mathbb{T}+\delta}$ as the *forward rate connected with the maturity* $T+\delta$. Musiela and Rutkowski (1998) show that if the price of some tradable asset (with no dividends or coupons), expressed in $P(t,T)$ units, is a martingale under the $\mathbb{P}_{\mathbb{T}}$ measure, so as numeraire under this measure is the price of a bond maturing at time T.

In the following part the model of forward LIBOR rates for the case of discrete time tenor will be constructed, based on the following assumptions:

(LR.1): For the arbitrary maturity $T \leq T^* - \delta$ is given a bounded, deterministic function $\lambda(\cdot, T) \in \mathbb{R}^d$, which represents the volatility of the forward rate $L(\cdot, T)$ process.

(LR.2) Let us assume the existence of a strictly decreasing and positive initial term structure $P(0,T), T \in \langle 0, T^* \rangle$, which means also the existence of the initial curve $L(0,T)$ of forward rates.

$$L(0,T) = \delta^{-1}\left(\frac{P(0,T)}{P(0,T+\delta)} - 1\right) \quad \forall T \in \langle 0, T^* - \delta \rangle.$$

Discrete tenor

Let us assume that the time horizon T^* is a multiple of δ; let us say $T^* = M\delta$ for some natural M. In this subpart we will concentrate on the forward LIBOR rates with maturities in discrete time tenor $\{0, T_{(M-1)\delta}, T_{(M-2)\delta}, \cdots,$
$T_\delta, T^*\}$, where $T_{m\delta} = T^* - m\delta$ for $m = 1, 2, \cdots, M-1$. This procedure is based

on backward induction, when one begins with the definition of the LIBOR rate with the longest maturity possible, $L(t, T_\delta)$. Let us assume that we have specified lognormal volatilities $\lambda(t, T_{m\delta})$ for $m = 1, 2, \cdots, M-1$. Let us postulate that the rate $L(t, T_\delta)$ is under the probability measure $\mathbb{P}_{T^*}$ driven by the following stochastic differential equation:

$$dL(t, T_\delta) = L(t, T_\delta)\lambda(t, T_\delta) \cdot dW_t^{T^*}, \tag{27}$$

with initial condition

$$dL(0, T_\delta) = \delta^{-1}\left(\frac{P(0, T_\delta)}{P(0, T^*)} - 1\right). \tag{28}$$

Because the initial term structure is strictly decreasing, it is clear that $L(t, T_\delta)$ is positive and for fixed $t \leq T^* - \delta$ the random variable $L(t, T_\delta)$ has a lognormal distribution under $\mathbb{P}_{T^*}$. This way the dynamics of LIBOR rates with maturity in the last date of our tenor is defined.

In the next step the forward LIBOR rate for date $T^*_{2\delta}$ with the use of (21) will be defined, where $T = T_\delta$, so that mean and volatility are specified as

$$\begin{aligned}
\lambda(t, T_\delta) &= \frac{1 + \delta L(t, T_\delta)}{\delta L(t, T_\delta)}\left[b(t, T_\delta) - b(t, T^*)\right] \\
\mu(t, T_\delta, T^*) &= \frac{\delta L(t, T_\delta)}{1 + \delta L(t, T_\delta)}\lambda(t, T_\delta),
\end{aligned} \tag{29}$$

where as $\mu(t, T, T+\delta)$ is denoted $b(t, T) - b(t, T+\delta)$. Let us define process $W_t^{T_\delta}$, corresponding with date T_δ as

$$W_t^{T_\delta} = W_t^{T^*} - \int_0^t \mu(u, T_\delta, T^*)du \quad \forall t \in \langle 0, T_\delta \rangle.$$

This process is connected with date T_δ (due to (25), it describes the relationship between Brownian motions under measures $\mathbb{P}_{T+\delta}$ and $\mathbb{P}^*$).

Because $\mu(t, T_\delta, T^*)$ is bounded, the existence of this process follows both the Girsanov theorem and the probability measure associated to it is $\mathbb{P}_{T_\delta} \sim \mathbb{P}_{T^*}$ under which process W_{T_δ} is a Brownian motion. It is given by the Radon-Nikodym derivative

$$\frac{d\mathbb{P}_{T\delta}}{d\mathbb{P}_{T^*}} = \exp\left(\int_0^{T_\delta} \mu(u, T+\delta) \cdot dW_u^{T^*} - \frac{1}{2}\int_0^{T_\delta} |\mu(u, T+\delta)|^2 du\right).$$

From (26) one can see that it is the forward rate connected with maturity T_δ. Now it is possible to specify the dynamics of the LIBOR rate for the maturity $T_{2\delta}$ under the measure $\mathbb{P}_{T\delta}$. Analogically as in (27) let's define

$$dL(t, T_{2\delta}) = L(t, T_{2\delta})\lambda(t, T_{2\delta}) \cdot dW_t^{T_\delta}, \tag{30}$$

with the initial condition

$$L(0, T_{2\delta}) = \delta^{-1}\left(\frac{P(0, T_{2\delta})}{P(0, T_\delta)} - 1\right). \tag{31}$$

From (21) we get the value of the needed change of Brownian motion $W_t^{T_\delta}$ in order to get to the values connected with the date $T_{2\delta}$:

$$\mu(t, T_{2\delta}, T_\delta) = \frac{\delta L(t, T_{2\delta})}{1 + \delta L(t, T_{2\delta})}\lambda(t, T_2\delta) = b(t, T_{2\delta}) - b(t, T_\delta).$$

If we have defined the process $\mu(t, T_{2\delta}, T_\delta)$, we can define the pair $(W^{T_{2\delta}}, \mathbb{P}_{T_{2\delta}})$, connected with the maturity $T_{2\delta}$, and so on. With backward induction to the first relevant date $T_{(M-1)\delta}$, we can construct the class of forward LIBOR rates $L(t, T_{m\delta})$, $m = 1, \cdots, M-1$. With this procedure the lognormal distribution of each process $L(t, T_{m\delta})$ is assured under the corresponding forward probability measure $\mathbb{P}_{T_{(m-1)\delta}}$. We have for all $m = 1, \cdots, M-1$

$$dL(t, T_{m\delta}) = L(t, T_{m\delta})\lambda(t, T_{m\delta}) \cdot dW_t^{T_{(m-1)\delta}}, \tag{32}$$

where $dW_t^{T_{(m-1)\delta}}$ is some Brownian motion under $\mathbb{P}_{T_{(m-1)\delta}}$.

This finishes the derivation of the lognormal model of forward LIBOR rates under discrete tenor. Before the end, let us bring in the explicit relationship among Brownian motions connected with the adjacent maturities:

$$W_t^{T_{m-1\delta}} = W_t^{T_{m\delta}} + \int_0^t \frac{\delta L(u, T_{m\delta})}{1 + \delta L(u, T_{m\delta})}\lambda(u, T_{m\delta})du. \tag{33}$$

A.2 Market development in transition countries

In this paper a link between the goodness of fit of the model and the degree of the development of the market is made and I do suggest that the problems that the estimation has encountered in Hungary and Slovakia are due to the lower level of the development of the IR market in those two countries. To support this argument, the World Bank's Bond Market Development Indicators can be used.[19] The FSDI project, as part of its objective to comprehensively assess financial systems, introduces indicators for monitoring bond markets according to four dimensions of the financial system: size, access, efficiency and stability. The four-dimension analytical capacity provided by FSDI serves as a powerful mechanism for identifying strengths and weaknesses in bond markets and can be utilized effectively for the purposes of policy formulation and reforms. I use bond market indicators as the bond market is the closest to the IR market. It is possible to see that the bond markets in Hungary and Slovakia clearly lagged behind those in Poland and the Czech Republic, especially in the Efficiency and Size indicators. This fact can be taken as confirmation of the hypothesis stated in the paper about the lower level of the development of IR markets in Slovakia and Hungary.

Country	Size	Efficiency	Access	Stability	Overall
SVK	4.32	4.51	5.00	5.00	4.71
HUN	5.01	3.12	3.52	5.38	4.26
CZE	5.18	5.30	5.00	5.00	5.12
POL	4.49	5.87	3.76	6.09	5.05

Table 1.1: Bond market development indicators
Source: World Bank

[19] Accessible from HTTP://WWW.WORLDBANK.ORG/ as a part of the FSDI project, with data as of 2005.

A.3 Figures and Tables

Figures

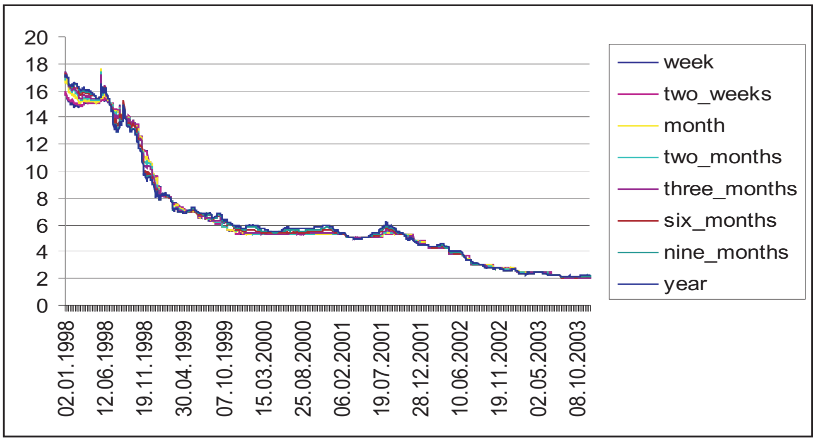

Figure 1.1: Time evolution of BRIBOR rates

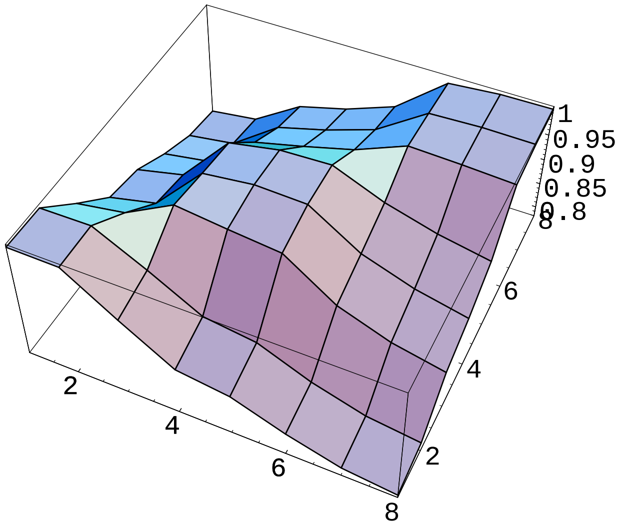

Figure 1.2: Estimated correlation surface of CZK as of 28 November 2003

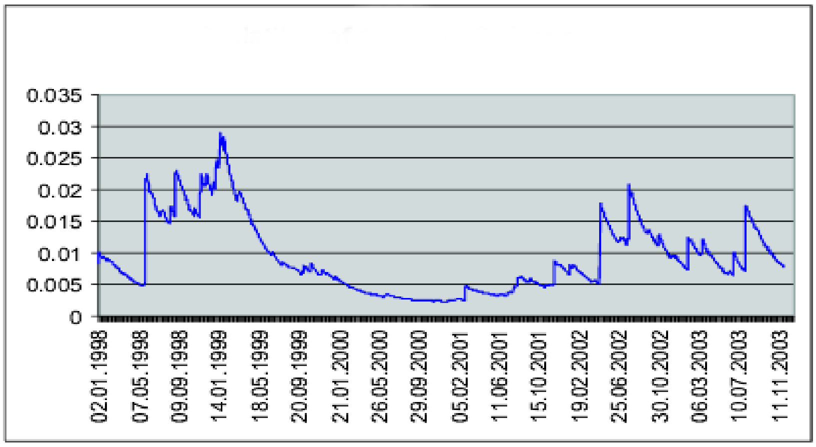

Figure 1.3: Estimated conditional volatility of one month PRIBOR rate

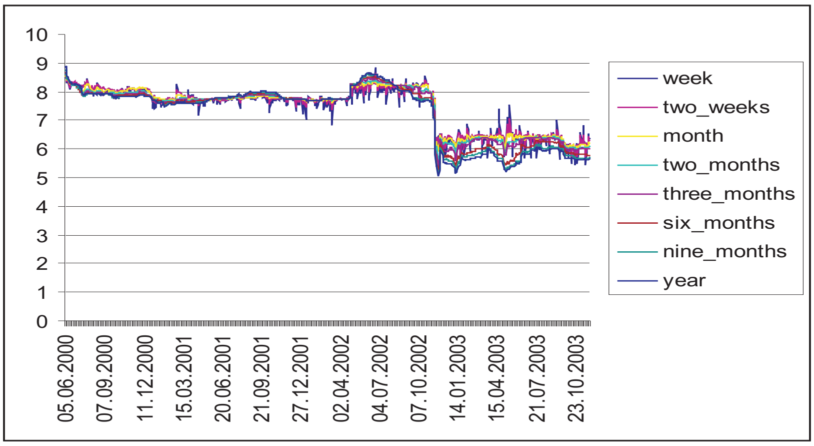

Figure 1.4: Time evolution of BRIBOR rates

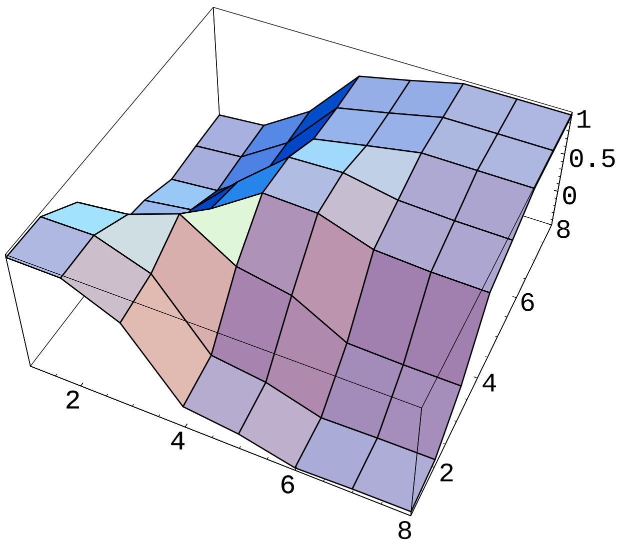

Figure 1.5: Estimated correlation surface of SKK as of 28 November 2003

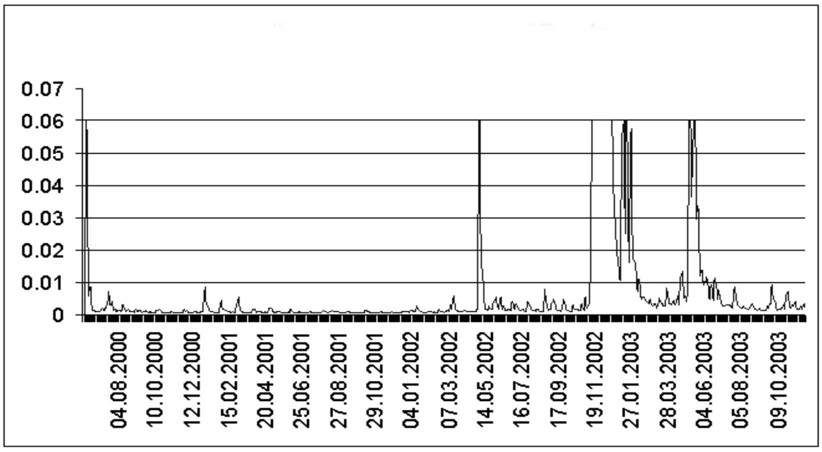

Figure 1.6: Estimated conditional volatility of one month BRIBOR rate

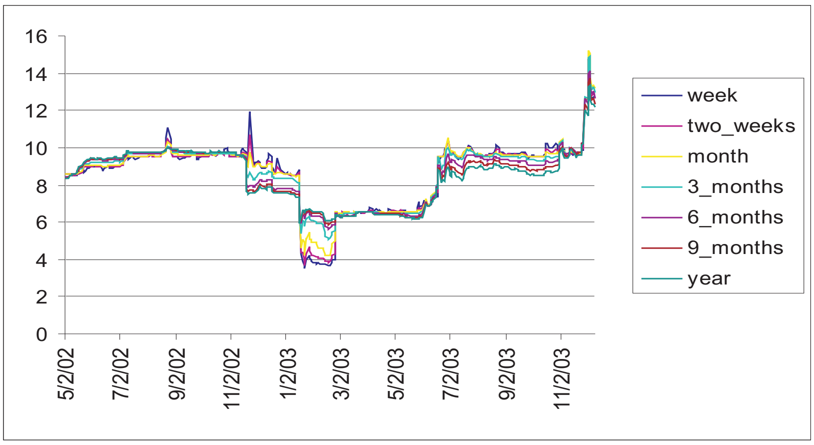

Figure 1.7: Time evolution of BUBOR rates

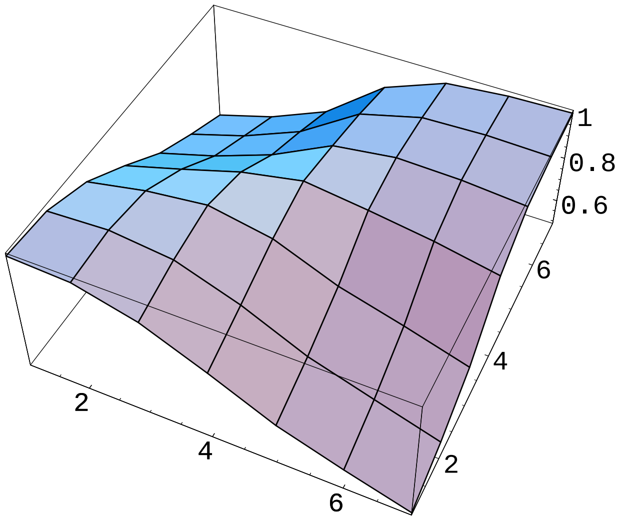

Figure 1.8: Estimated correlation surface of HUF as of 28 November 2003

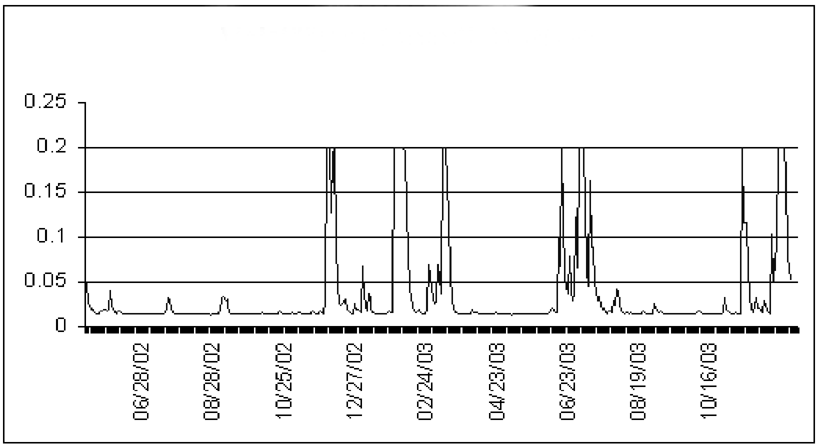

Figure 1.9: Estimated conditional volatility of one month BUBOR rate

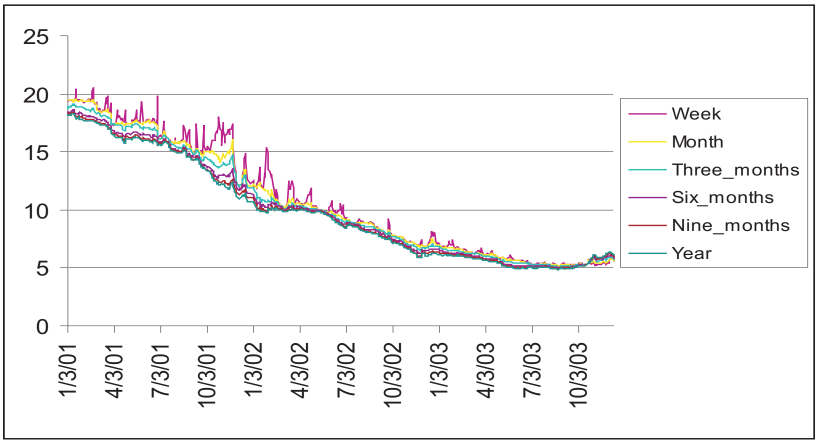

Figure 1.10: Time evolution of WIBOR rates

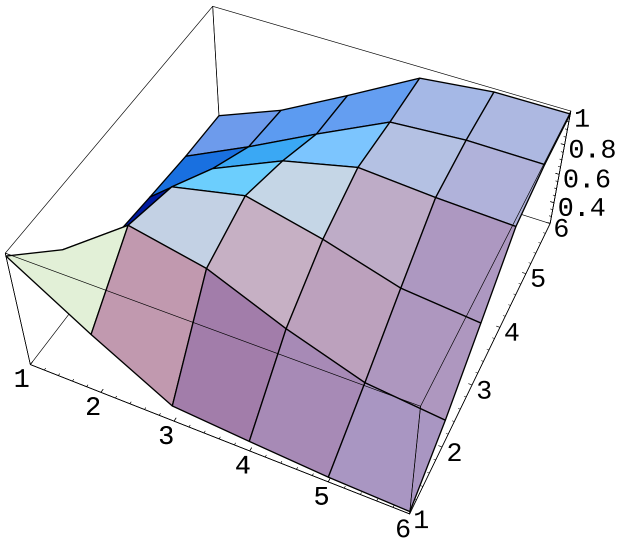

Figure 1.11: Estimated correlation surface of PLZ as of 28 November 2003

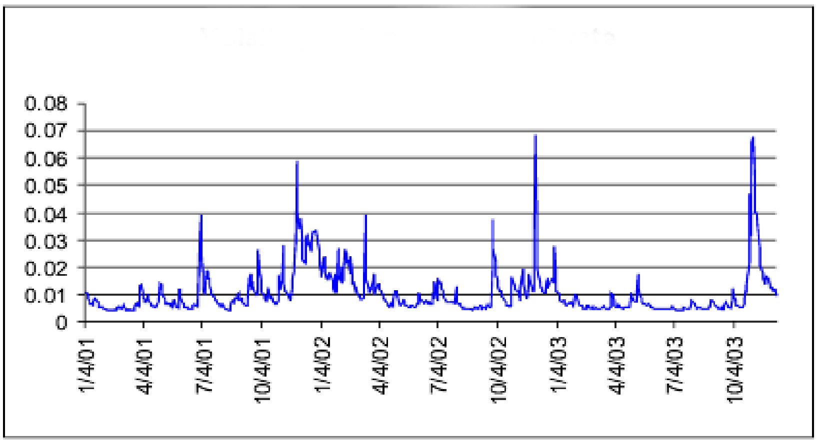

Figure 1.12: Estimated conditional volatility of one month WIBOR rate

Tables

Series	Mean	Std. Dev.	Minimum	Maximum	Sum	Q_{10} stat
Y_WEEK	-0.00132	0.00883	-0.112	0.0798	-1.978	9.59
Y_2W	-0.00132	0.00877	-0.109	0.0915	-1.982	8.99
Y_M	-0.00135	0.00916	-0.107	0.1101	-2.018	22.1
Y_2M	-0.00136	0.00860	-0.107	0.1031	-2.036	11.4
Y_3M	-0.00137	0.00838	-0.103	0.0832	-2.044	15.9
Y_6M	-0.00135	0.00848	-0.084	0.0853	-2.024	28.8
Y_9M	-0.00133	0.00822	-0.079	0.0724	-1.990	72.3
Y_Y	-0.00131	0.00842	-0.079	0.0726	-1.956	83.4

Table 1.2: Characteristics of time series for CZK

Component	1	2	3	4	5	6	7	8
Y_WEEK	0.322	-0.495	0.411	-0.249	0.012	0.133	0.631	0.050
Y_2W	0.332	-0.477	0.316	-0.024	0.037	-0.056	-0.742	-0.074
Y_M	0.365	-0.245	-0.303	0.730	-0.169	-0.342	0.183	0.010
Y_2M	0.376	-0.058	-0.521	-0.021	0.206	0.729	-0.064	0.015
Y_3M	0.376	0.057	-0.423	-0.577	0.162	-0.558	0.019	-0.007
Y_6M	0.361	0.327	0.097	-0.143	-0.791	0.115	-0.080	0.292
Y_9M	0.350	0.408	0.238	0.082	0.045	0.036	0.073	-0.798
Y_Y	0.339	0.432	0.334	0.206	0.522	-0.052	-0.011	0.517
Eigenvalue	6.314	1.142	0.262	0.099	0.068	0.051	0.030	0.030
Total variance expl.	0.789	0.932	0.964	0.977	0.985	0.992	0.996	1.000

Table 1.3: Principal component weights for CZK

Component	First		Second		Third	
	Coefficient	t-stat	Coefficient	t-stat	Coefficient	t-stat
Constant	-0.033	-0.269	0.024	1.297	0.004	0.319
b_1	0.414	2.548	0.214	2.780	0.081	2.025
b_2	-	-	0.077	2.084	-	-
α_0	3.952	2.921	0.006	0.502	0.007	1.335
α_1	0.905	2.486	0.069	0.831	0.187	2.651
β_1	0.011	0.217	0.268	2.298	0.059	1.547
β_2	-	-	0.658	6.792	0.743	13.534
Log Likelihood	-3368.12		-1975.57		-877.21	
Schwarz B.I.C.	3386.58		2001.14		899.12	
Q_{10} stat	10.8		17.9		9.68	
Q^2_{10} stat	0.472		10.8		6.11	
LM test stat	0.309		9.37		2.78	

Table 1.4: Regression results for CZK

Series	Mean	Std. Dev.	Minimum	Maximum	Sum	Q_{10} stat
Y_WEEK	0.000221	0.0298	-0.161	0.1661	0.193	72.9
Y_2W	-0.000064	0.0179	-0.179	0.1101	-0.056	53.8
Y_M	-0.000300	0.0095	-0.187	0.0616	-0.261	34.6
Y_2M	-0.000352	0.0092	-0.193	0.0615	-0.307	63.3
Y_3M	-0.000365	0.0093	-0.179	0.0694	-0.317	65.5
Y_6M	-0.000443	0.0084	-0.141	0.0675	-0.386	189
Y_9M	-0.000490	0.0088	-0.158	0.0978	-0.427	180
Y_Y	-0.000503	0.0090	-0.159	0.0992	-0.438	165

Table 1.5: Characteristics of time series for SKK

Component	1	2	3	4	5	6	7	8
Y_WEEK	0.196	-0.646	0.294	-0.155	0.151	-0.256	0.582	0.083
Y_2W	0.256	-0.586	0.123	-0.039	0.059	0.212	-0.720	-0.085
Y_M	0.373	-0.224	-0.502	0.442	-0.550	0.137	0.203	-0.020
Y_2M	0.395	0.102	-0.479	-0.057	0.388	-0.644	-0.184	0.044
Y_3M	0.399	0.138	-0.256	-0.446	0.328	0.633	0.221	0.012
Y_6M	0.381	0.231	0.269	-0.543	-0.610	-0.216	-0.099	0.072
Y_9M	0.386	0.231	0.363	0.297	0.129	-0.020	0.087	-0.743
Y_Y	0.383	0.228	0.384	0.439	0.154	0.096	-0.044	0.652
Eigenvalue	5.604	1.763	0.314	0.123	0.081	0.048	0.039	0.027
Total variance expl.	0.701	0.921	0.960	0.976	0.986	0.992	0.997	1.000

Table 1.6: Principal component weights for SKK

Component	First		Second		Third	
	Coefficient	t-stat	Coefficient	t-stat	Coefficient	t-stat
Constant	-0.017	-0.215	0.074	1.297	0.011	0.900
b_1	-0.245	-1.277	-0.115	2.780	-0.079	-1.714
b_2	0.077	0.413	-0.248	2.084	-0.115	-2.501
b_3	-	-	-0.157	2.084	-0.011	-0.239
α_0	0.259	1.601	0.004	0.502	0.003	1.308
α_1	0.325	1.431	0.037	0.831	0.066	2.427
α_2	0.264	0.822	-	-	-	-
β_1	0.405	2.382	0.957	6.792	0.919	27.134
Log Likelihood	-1404.41		-1219.72		-426.24	
Schwarz B.I.C.	1428.10		1243.40		449.92	
Q_{10} stat	28.2		14.5		6.58	
Q_{10}^2 stat	2.09		14.2		7.07	
LM test stat	0.656		6.917		6.410	

Table 1.7: Regression results for SKK

Series	Mean	Std. Dev.	Minimum	Maximum	Sum	Q_{10} stat
Y_WEEK	0.00201	0.0487	-0.404	0.620	0.815	7.63
Y_2W	0.00183	0.0431	-0.389	0.506	0.746	10.6
Y_M	0.00163	0.0371	-0.321	0.313	0.663	16.6
Y_3M	0.00129	0.0274	-0.197	0.204	0.524	16.6
Y_6M	0.00104	0.0229	-0.184	0.179	0.425	17.3
Y_9M	0.000997	0.0216	-0.180	0.150	0.404	13.9
Y_Y	0.000966	0.0218	-0.190	0.127	0.392	7.42

Table 1.8: Characteristics of time series for HUF

Component	1	2	3	4	5	6	7
Y_WEEK	0.337	-0.509	0.593	0.114	0.018	0.488	-0.155
Y_2W	0.367	-0.445	0.109	-0.015	0.029	-0.785	0.196
Y_M	0.389	-0.282	-0.570	-0.595	-0.144	0.245	-0.095
Y_3M	0.414	0.035	-0.435	0.596	0.518	0.113	-0.030
Y_6M	0.401	0.277	0.007	0.243	-0.613	0.144	0.554
Y_9M	0.380	0.393	0.114	0.046	-0.281	-0.227	-0.745
Y_Y	0.352	0.481	0.329	-0.464	0.505	0.029	0.255
Eigenvalue	5.567	1.219	0.142	0.040	0.012	0.011	0.008
Total variance expl.	0.795	0.969	0.990	0.996	0.997	0.999	1.000

Table 1.9: Principal component weights for HUF

Component	First		Second		Third	
	Coefficient	t-stat	Coefficient	t-stat	Coefficient	t-stat
Constant	-0.238	-1.990	0.105	1.147	0.023	0.507
α_0	1.040	1.255	0.074	0.956	0.010	0.554
α_1	0.585	7.438	0.537	5.420	0.626	0.956
β_1	0.386	4.738	0.451	4.512	0.351	0.539
Log Likelihood	-825.01		-472.68		-1.061	
Schwarz B.I.C.	837.02		484.69		13.074	
Q_{10} stat	0.06		0.89		0.704	
Q^2_{10} stat	0.05		0.09		0.096	
LM test stat	0.021		0.037		0.041	

Table 1.10: Regression results for HUF

Series	Mean	Std. Dev.	Minimum	Maximum	Sum	Q_{10} stat
Y_WEEK	-0.00127	0.0298	-0.166	0.196	-0.938	27.0
Y_M	-0.00163	0.0113	-0.0811	0.0543	-1.207	40.3
Y_3M	-0.00160	0.00816	-0.0523	0.0540	-1.183	8.9
Y_6M	-0.00154	0.00755	-0.0455	0.0516	-1.140	32.0
Y_9M	-0.00151	0.00741	-0.0431	0.0544	-1.118	55.7
Y_Y	-0.00148	0.00755	-0.0326	0.0573	-1.097	66.1

Table 1.11: Characteristics of time series for PLZ

Component	1	2	3	4	5	6
Y_WEEK	0.259	0.717	-0.590	0.254	-0.060	0.003
Y_M	0.387	0.453	0.418	-0.659	0.182	-0.001
Y_3M	0.445	0.060	0.539	0.507	-0.499	0.025
Y_6M	0.451	-0.201	0.029	0.360	0.782	0.112
Y_9M	0.442	-0.326	-0.264	-0.169	-0.142	-0.760
Y_Y	0.429	-0.357	-0.337	-0.288	-0.285	0.639
Eigenvalue	4.091	1.154	0.331	0.194	0.154	0.074
Total variance expl.	0.682	0.874	0.930	0.962	0.988	1.000

Table 1.12: Principal component weights for PLZ

Component	First		Second		Third	
	Coefficient	t-stat	Coefficient	t-stat	Coefficient	t-stat
Constant	0.063	.929	0.055	1.472	-0.048	-0.269
b	0.168	3.617	0.072	1.224	-0.125	-2.465
α_0	0.203	1.636	0.365	2.598	0.015	0.767
α_1	0.115	2.577	0.399	2.579	0.071	1.146
β_1	0.831	13.233	0.363	2.477	0.878	7.223
Log Likelihood	-1458.68		-1053.24		-591.38	
Schwarz B.I.C.	1475.19		1066.45		607.89	
Q_{10} stat	10.8		3.56		8.74	
Q^2_{10} stat	8.69		6.09		6.42	
LM test stat	0.575		1.439		2.993	

Table 1.13: Regression results for PLZ

Chapter 2

Financial Markets Assessment, Official Strategies and Survey Interpretations of EMU Enlargement: The Case of Central Europe

Abstract

In this paper the views of markets on the enlargement of the EMU are measured, discussed and compared to the Reuters market survey concerning EMU enlargement. The analysis is conducted for the Visegrad 4 countries, i.e. Slovakia, the Czech Republic, Poland and Hungary. We use an approach based on Lund (1999)'s idea to employ a state space specification of an international interest rate model. The results of the analysis seem to be in accordance with the Reuters survey. Also, the date of EMU enlargement for Slovakia was correctly predicted at the time of the analysis.

Keywords: EMU enlargement, EMU calculators, international interest rate model, Vasicek model
JEL Classification: B41, C13, C51, E43, F36

2.1 Introduction

In this paper, an attempt to measure how market participants perceive the prospects of enlarging the euro area is performed for the four Visegrad countries that joined the EU in May 2004. Already before the European monetary union (EMU) was realized in 1999 considerable interest had been attached to methods of extracting market views of the project. EMU probability calculators were also designed to infer the probability, as it was perceived by market participants, that a particular country would become a member of the EMU.

Bates (1999) surveys the methods and categorizes them into those based on currency option contracts, e.g. Butler and Cooper (1997) and Aguilar and Hördahl (1998), and methods utilizing European forward interest rates, e.g. Morgan (1997), Favero, Giavazzi, Iacone and Tabellini (2000), Angeloni and Violi (1997) and Lund (1999). While all of these approaches use cross-section data, other indicators based on time series were devised, too. For example, Aguilar and Hördahl (1998) estimated exchange rate volatility and correlations using a generalized autoregressive conditional heteroscedasticity model and interpreted a decline in bilateral volatility and increased correlation in terms of EMU convergence.

However, there are several limitations that do not allow using most of the above-mentioned methods for the countries that have recently entered the European Union. The option-based approaches are not applicable due to a lack of data. At the time of this analysis, the earliest new EMU entrants could be expected in several years' time, but the maturity of interbank currency option contracts for the analysed countries do not extend over one year. Also, it is too early for the application of time series exchange rate models. Most of the analysed countries operate in a floating exchange rate regime and too much may happen to this kind of exchange rate over several years. For example, Aguilar and Hördahl (1998) shows that GARCH volatility estimates fell to low levels only approximately two years before the EMU was launched.

The EMU calculators that are based on the term structure of interest rates seem to be more feasible. In these methods EMU entry is treated as a random event and the observed interest rates are used to estimate its probability. In particular, the forward interest rate differentials are viewed as a weighted average of zero value stemming from the union being realized and some non-zero value conditional on the non-EMU scenario for a given pair of countries. The relationship among forwards, the probability of EMU membership[1] at time τ and the conditional expected interest rate differential is usually written as

$$f_{t,\tau,T} - f^*_{t,\tau,T} = \left(1 - \pi^{EMU}_{t,\tau}\right) E_t \left(r_{\tau,T} - r^*_{\tau,T} | nonEMU\right). \quad (2.1)$$

National and foreign (euro area) interest rates as of time τ and with maturity T are denoted $r_{\tau,T}$ and $r^*_{\tau,T}$, respectively, while $f_{t,\tau,T}$ and $f^*_{t,\tau,T}$ are national and foreign interest rate forwards, respectively, as of time t, with horizon τ and maturity T.

There are several challenges regarding Equation (2.1). First, the forward rates of the long horizons on the left hand side might not be observable and in these cases they need to be estimated. Nevertheless, under the no arbitrage assumption this estimation should be rather straightforward and reliable.

Second, a more crucial problem is how to determine the expected spread of future interest rates conditional on non-EMU membership at time τ. This is the major point in which the term-structure-based calculators differ.

Thirdly, as Bates (1999) notes, the EMU calculators are most robust when national and foreign (euro area) interest rates differ substantially in the case that the country does not join. In other words, formula (2.1) can be a base for the estimate of $\pi^{EMU}_{t,\tau}$ only if the expected future spread $E_t \left(r_{\tau,T} - r^*_{\tau,T} | nonEMU\right)$ is large enough in absolute value. Otherwise, forecast errors and other potential biases would make the EMU and non-EMU cases hard to distinguish. Therefore, before the EMU was introduced, research on the topic of the EMU calculators concerned mainly Italy and several other countries with a history of substantial interest rate differentials that could have been extrapolated into the future as non-EMU interest rate paths. For instance, at the beginning of 1996, three years before the EMU was launched, the long horizon spreads for Italy were close to four percentage points. The current situation for countries of our sample is rather different. A low inflationary environment prevailed in Europe and potential euro area entrants have independent central banks that adopted inflation targeting regimes with medium- to long-run inflation targets close to the ECB target. Furthermore, there is a very close relationship between these countries and the euro area as most trade is with euro area countries and the majority of investment comes from the EU. Under these circumstances, one would assume that a low inflationary

[1] Before the EMU was launched the star usually denoted Germany; in the current context the star denotes European rates.

monetary policy environment would be sustained and interest rate spreads would remain low regardless of whether these countries join the euro area or not.[2] The Czech Republic, on which we focus, is a good example of a country for which the EMU and non-EMU scenarios could be too close for making reliable assessments about the EMU probabilities using only forward rate levels.

One can say that there are two approaches to measuring market perceptions of EMU entrance. They may be extracted from the market information (such as prices or interest rates) or they may be based on the beliefs of experts (or market participants). The primary motivation of this paper is to use the market data to estimate the perception of EMU entrance. The study relates yield curve data to the expectations of EU accession in order to derive the probabilities of accession for the Visegrad countries. The robustness of this estimation is confirmed with the results of the Reuters opinion survey. If these two approaches would be found inconsistent, it may imply that market participants' assessments are irrational, however, without further analysis it would not be possible to conclude this.

We use the state space model based on Lund (1999) to estimate the probability of joining the EMU for selected countries using market data.[3]

We decided to select the post-transition countries that have recently entered the European Union, as they are obliged by their accession treaties to adopt the Euro (although the time of adoption is not specified in the treaties).

This chapter is organised in the following way. Section 2.2 discusses the estimation of forward rates (based on interest swap rates) that are not directly observable in the market. Section 2.4 presents market surveys and discusses official Euro strategies valid at the time of this analysis. These are natural reference points for methods aiming at market view measurement. The theory behind the dynamic term structure model is shown in Section 2.3. Next, the results of this approach applied to the problem are shown in Section 2.5, and Section 2.6 concludes.

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[2] Kočenda, Kutan and Yigit (2007) provide evidence of the decline and convergence of inflation and interest rates in new EU members.

[3] Cincibuch and Horníková (2008) uses the short-term dynamics of forward spreads to estimate the probability of joining the EMU using the same data as in this paper.

2.2 Estimation of Forward Spreads

For some maturities and horizons forward contracts are traded, but most of them have to be estimated. The estimation is possible using government bond yields or interest rate swap rates. This issue is mostly technical and compared to other potential difficulties it is relatively easy to tackle. But it may gain importance when the absolute difference between forward rates is low relative to the potential errors introduced by the estimation methods.

While Favero et al. (2000) estimate instantaneous forward rates from government bonds using the specification of Svensson (1994), Lund (1999) derives instantaneous forwards from the zero-coupon curve estimated using the bootstrap method with linear interpolation from interest rate swaps. Others like Morgan (1997) or Angeloni and Violi (1997) used directly forward rates with finite maturity (five- and one-year maturities, respectively) also derived from interest rate swap rates. As is shown by Favero et al. (2000), since the forward rate with horizon τ and maturity T, i.e. $f_{t,\tau,T}$, is the average of instantaneous forward rates over the period between τ and $\tau + T$, the estimated probability in this case is rather the average "instantaneous" probabilities over the period weighted by the interest rate differentials.

To estimate forward rates we use benchmark interest swap rates, which are preferable to government bond yields. They are standardized and they have a favorable structure allowing the derivation of precise zero coupon curves. It is important because we have to deal with relatively narrow forward spreads and therefore we tried to avoid any interpolation or ad hoc specification. Therefore, we did not follow Favero et al. (2000) or Lund (1999) in estimating instantaneous forwards from the Nelson-Siegel specification but rather adapted the approach of Angeloni and Violi (1997).

We estimated one-year forwards directly from the benchmark interest rate swaps that are quoted in annual maturities. The daily data are available from Bloomberg. First, to extract the term structure of the interest rate (the zero coupon curve) we used the bootstrapping procedure, which hinges only on the assumptions of a liquid and well-arbitraged market. Then we calculated the implied synthetic one-year forward rates for different horizons. In general, the data on the benchmark IRS curves are of very good quality, but some large outliers may occur. We checked the data very carefully and cleaned these obvious data errors.

The technical details of the bootstrapping procedure are provided here.

In order to find out the forward interest rate implied by the swap curve, it is convenient first to derive the zero coupon term structure of interest rates. As is shown below it is possible without any approximation only for some types of swaps, but fortunately in practice the suitable swaps are often used. Let $B_{t,M}$ be the price of the discount bond with maturity M applicable at time t. Further, let $I_t^M(m,v)$ denote the rate of the interest rate fixed for the floating swap with maturity M as of trade date t, which is based on the floating rate with maturity m. Let the fixed leg of the swap be settled v times a year and let all interest rates be expressed in terms of annual compounding. We abstract from any credit risk in constructing the forward interest rates implied by the swap curve, for the sake of simplicity, as we do care about the spreads of domestic and Euro interest rates.[4]

The present value of the cash flow for the fixed leg of the swap $I_t^M(m,v)$ on the unity notional amount is given by

$$PV_{fixed} = \left(\left[1 + I_t^M(m,v)\right]^{\frac{1}{v}} - 1 \right) \sum_{k=1}^{vM} B_{t,\frac{k}{v}}. \tag{2.2}$$

Let further $f_{t,\tau,T}$ be the forward rate as of trade date t in horizon τ and maturity T based on the term structure of risk-free bonds. Using this term structure of forward rates swap sellers[5] may hedge their exposure to interest rate risk. The present value of the cash flow of the floating leg of the interest rate swap is then

$$PV_{floating} = \sum_{j=0}^{\frac{M}{m}} \left[(1 + f_{t,jm,m})^m - 1\right] B_{t,(j+1)m}. \tag{2.3}$$

The non-existence of arbitrage opportunities further dictates the relationship be-

[4]We are using swap rates that are not entirely default-free rates but are considered by market participants as such. Although the importance of credit risk has been highlighted during the recent global financial turbulence we do abstract from credit risk as the claims in the domestic interest rate market can always be repaid by the government by printing more money. We left for further research the possibility to correct (or amend) the swap rates in Euro for the spread resulting from the default risk of the domestic country. We also need to stress that the model analyzed in the next chapter itself does consider also credit risk spreads.

[5]Swap sellers receive a fixed rate and pay floating rate payments.

tween discount rates and forward rates obviously expressed as

$$\frac{B_{t,jm}}{B_{t,(j+1)m}} = (1 + f_{t,jm,m})^m , \tag{2.4}$$

so when this is in (2.3) substituted for $(1 + f_{t,jm,m})^m$ one may write

$$PV_{floating} = \sum_{j=0}^{\frac{M}{m}} B_{t,jm} - B_{t,(j+1)m}.$$

and since most of the terms in this series cancels it is possible to conclude that

$$PV_{floating} = 1 - B_{t,M}. \tag{2.5}$$

Since both the fixed leg and hedged floating leg represent streams of certain payments, the no arbitrage condition on the swap rate is that the present values of both payment streams are equal, therefore

$$\left(\left[1 + I_t^M (m, v)\right]^{\frac{1}{v}} - 1 \right) \sum_{k=1}^{vM} B_{t,\frac{k}{v}} = 1 - B_{t,M}. \tag{2.6}$$

This formula relates the prices of discount bonds and interest rate swap rates. Further, if a sufficient number of interest rate swaps are traded then it is possible to use this formula to calculate recursively the prices of discount bonds. After some straightforward algebraic manipulations it follows from (2.6) that

$$B_{t,\frac{1}{v}} = \frac{1}{\left[1 + I_t^{\frac{1}{v}} (m, v)\right]^{\frac{1}{v}}} \tag{2.7a}$$

$$B_{t,M} = \frac{1 + \sum_{k=1}^{vM-1} B_{t,\frac{k}{v}}}{\left[1 + I_t^M (m, v)\right]^{\frac{1}{v}}} - \sum_{k=1}^{vM-1} B_{t,\frac{k}{v}}, \text{ for } M > \frac{1}{v}. \tag{2.7b}$$

(2.7) shows that if the prices of discount bonds with maturities $\frac{1}{v}, \frac{2}{v}, \frac{3}{v}, ..., \frac{vM-1}{v}$ are known then the knowledge of the swap rate $I_t^M (m, v)$ enables the determination of the discount factor $B_{t,M}$.

If these discount bond prices are known, then, similarly to (2.4), one may obtain the implied forward rates of maturity $\frac{1}{v}$ as

$$f_{t,\frac{k-1}{v},\frac{1}{v}} = \left(\frac{B_{t,\frac{k-1}{v}}}{B_{t,\frac{k}{v}}} \right)^{\frac{1}{v}} - 1. \tag{2.8}$$

The possibility of this procedure hinges on the condition that there are enough points on the swap curve in relation to the settlement frequency of the fixed part of the swap contracts. In particular, there must be vT equally spaced swap rates to allow the determination of the vT discount factors. On the contrary, there is no such condition on the maturity of the underlying floating rate. Fortunately and perhaps not surprisingly, swap rates are often quoted for maturities in whole years and with annual settlements, i.e. $v = 1$, which facilitates empirical analysis.

Figure 2.1 graphs the IRS yield curves for several countries and Figure 2.2 shows the dynamics of the Czech forward rates in relation to the Euro rates. Similar graphs for other countries are in Figures 2.3 to 2.5.

2.3 EMU Perception Using the State Space Model

In this section we describe the approach based on Lund (1999) to estimate the probability of joining the EMU for selected countries. We attempt to estimate this probability using the state-space model with the help of the Kalman filter. The idea to use the Kalman filter in the context of the estimation of stochastic models of interest rates is not new; see for example papers by Duan and Simonato (1999) or Jagedeesh and Pennacchi (1996). However, Lund (1999) innovated this idea and developed a stochastic model of interest rates with an EMU effect parameterized also using the probability of the EMU, and he used non-linear Kalman filtration to estimate this probability.

With some differences we followed the approach of Lund (1999). Our task was simplified by the fact that the monetary union already exists and thus we could use the spreads of Euro interest rates.[6] Also, contrary to Lund (1999), who allows for nonlinear specification, we estimated the parameters of the model using linear Kalman filtration. The reason was the instability of the coefficients when we tried the non-linear specification. This might have been caused by more volatile and

[6] Lund (1999) used spreads of German interest rates as the analysis was performed before the formation of the EMU.

unstable markets in the post-transition countries.

The EMU model of Lund (1999) is based on the international term-structure model, which is augmented by the EMU effect (meaning that domestic bonds will be redenominated to Euros). The stochastic models used are so-called equilibrium models such as Vasicek (1977) or CIR Cox et al. (1985), where the prices of bonds are derived from the stochastic processes for the short rate and for the market prices of risk.

An important step is the estimation of the so-called "true local spreads", i.e. the spreads of the local (domestic) interest rates to the Euro interest rates, that would occur in the case that no anticipated entry of a domestic country into the EMU is possible. Based on the knowledge of the true local spread and the actual spread it is possible to infer how likely is the entry of the domestic country into the EMU zone. In the following subsection the theoretical basis for the mentioned estimation is described.

2.3.1 A Formal Model

The setup of the model is as follows. There are two currencies in the model, the domestic currency and the Euro. All stochastic quantities are driven by an m-dimensional standard Brownian motion W_t. The short rate in the domestic country (denoted r_t) as well as the short rate in the EMU (denoted as R_t) are both functions of a $d \times 1$ vector of state variables X_t, which is driven by the following SDE:

$$dX_t = \mu(X_t)dt + G(X_t)dW_t, \tag{2.9}$$

where $\mu(X_t)$ is a $d \times 1$ vector and $G(X_t)$ is a $d \times m$ matrix. The exchange rate between the domestic currency and the Euro is denoted by $Z(t)$. Let us define also a local spread relative to the Euro y_t as

$$r_t = R_t + y_t. \tag{2.10}$$

The most critical assumption in the setup is the assumption that R_t and y_t are driven by independent stochastic processes.[7]

[7] In other words the Brownian motion W_t can be partitioned into two subvectors, W_{1t} and W_{2t} such that R_t is driven exclusively by W_{1t} and the dynamics of other factors (y_t and Z_t) is driven only by W_{2t}.

When the domestic country joins the EMU at time τ, future claims are redenominated to Euros at the exchange rate prevailing at time τ. We will treat τ as a random variable and specify its distribution later. We assume that its distribution is independent of the stochastic processes governing R_t, y_t and Z_t.

Let us consider a zero-coupon bond, denominated in the domestic currency, maturing at time T with price $P(t,T)$. The expression for $P(t,T)$ that takes into account the possibility of joining the EMU will be derived. First, let us condition on the date of EMU membership τ. At time τ one zero-coupon is converted into $1/Z_i(\tau)$ Euro bonds with value $P_E(t,T)/Z_i(\tau)$ in Euros, or with value

$$Z(\tau)\left(P_E(\tau,T)/Z_i(\tau)\right) = P_E(\tau,T)$$

in the domestic currency. The time t bond price for a given EMU membership date, denoted as $P(t,T,\tau)$, is given by the expectation of its payoff:

$$P(t,T,\tau) = E_t^Q\left[e^{-\int_t^\tau (R_u+y_u)du} P_E(\tau,T)\right],$$

where Q is the risk-neutral probability measure for the domestic currency.[8] Similarly,

$$P_E(\tau,T) = E_t^{Q_E}\left[e^{-\int_t^\tau R_u du}\right],$$

where Q_E is a Euro risk-neutral measure (different from Q). However, due to the independence of R_t and y_t, it can be shown that in the last formula the Q_E measure can be interchanged with the Q measure. Then

$$\begin{aligned} P(t,T,\tau) &= E_t^Q\left[e^{-\int_t^\tau (R_u+y_u)du} \cdot E_t^Q\left(e^{-\int_\tau^T R_u du}\right)\right] = && (2.11)\\ &= E_t^Q\left[e^{-\int_t^T R_u du}\right] \cdot E_t^Q\left[e^{-\int_t^\tau y_u du}\right] = && (2.12)\\ &= P_E(t,T)\cdot D(t,\tau). && (2.13) \end{aligned}$$

This means that the bond price can be expressed as a Euro bond multiplied by a domestic discount factor $D(t,\tau)$, where discounting is done only until the domestic country joins the EMU.

Now let us define the distribution for τ. Because of tractability and parsimony the survivor function with one parameter θ was chosen, i.e. the probability that

[8] More about stochastic calculus can be found in Oksendal (2002).

the domestic country will not join the EMU by time s is specified as

$$Pr_t(\tau > s) = \exp\left(-\int_t^s \pi_u du\right),\ \pi_u > 0 \tag{2.14}$$

with the density function

$$p_t(s) = \pi_u \exp\left(-\int_t^s \pi_u du\right).$$

Now we are in a position to calculate the bond price $P(t,T)$ as the expectation of (2.13) over all possible τ. We get

$$\begin{aligned} P(t,T) &= \int_t^T P(t,T,\tau)p_t(\tau)d\tau + P(t,T,\tau)Pr_t(\tau > T) = && (2.15)\\ &= P_E(t,T)\cdot F(t,T), && (2.16)\end{aligned}$$

where the multiplicative factor $F(t,T)$ is defined as

$$F(t,T) = \int_t^T D(t,T)\pi_u e^{-\int_t^\tau \pi_u du} d\tau + D(t,T)e^{-\int_t^T \pi_u du}. \tag{2.17}$$

The stochastic model chosen to specify the short rate spread to Euro y_t has to permit negative values for y_t and therefore we cannot use some models developed to keep non-negativity. Therefore Gaussian models (i.e. models of type (2.9), where $G(X_t)$ does not depend on X_t, such as the Vasicek model) can be used. Because the Gaussian class of models belongs to the exponential-affine class (see Duffie and Kan (1996)), the local discount factor can be calculated as

$$D(t,T) = E_t^Q\left[\exp\left(\int_t^T y(X_s)ds\right)\right] = \exp\left[A(T-t) + B(T-t)'X_t\right], \tag{2.18}$$

where the scalar $A(T-t)$ and the $m \times 1$ vector $B(T-t)$ depend on the choice of the specification of the process governing y_t. Note that both are functions of time to maturity.

We also need to specify the EMU membership date distribution function. For the Central and Eastern European countries we have to take into account that no EMU membership could happen before May 1, 2004, when they became members of the European Union. Therefore (similarly as Lund (1999)) we define

$$\pi_t = \begin{cases} 0 & \text{for t} < \tau^* \\ \theta & \text{for t} \geq \tau^*, \end{cases} \tag{2.19}$$

where τ^* corresponds to May 1, 2004 to capture the fact that no EMU membership could happen earlier. This specification is also important because it allows us to estimate the true local spread to the Euro from the interest rates maturing before $\tau*$ (i.e. the spread that would occur when there is no EMU membership possible). This is due to the fact that formula (2.17) simplifies to

$$F(t,T) = D(t,T)$$

for $T \leq \tau^*$.

As it is more convenient to work with the yield curve than with the prices of bonds, here we derive the zero-coupon curve with the EMU effect. From the formula (2.16) using the well known formula for the yield with continuous compounding we have the equation for the zero-coupon yield of the domestic country:

$$Y(t, T-t) = -\frac{\ln P_E(t,T)}{T-t} - \frac{\ln F(t,T)}{T-t},$$

or

$$Y(t,T^*) = Y_E(t,T*) + S(t,T^*), \tag{2.20}$$

where $Y(t,T*)$ is the yield of the zero-coupon bond with maturity $t + T*$, i.e. where T^* is time to maturity. $Y_E(t,T)$ is the yield of the Euro zero-coupon bond and $S(t,T^*)$ is domestic country-specific spread relative to the Euro. This equality enables us to model the country-specific spread using the Kalman filter.

The main idea of the estimation is to use yields maturing before May 1, 2004 and estimate the parameters of the stochastic process governing y_t, i.e. to estimate the “true” spread of domestic yields to Euro yields. Using the estimated process and comparing the observed spreads in the second part of the data with the estimated the “true” spreads we can compute the implied EMU probability. We will now proceed with the setup of the Kalman filtration problem.

Let the spread of domestic and Euro yields y_t be governed by an m-dimensional vector of state variables X_t. Let the unknown parameters of the stochastic process governing X_t be stacked in the vector ψ. The observed data consists of zero-coupon yield spreads of domestic and Euro yield curves for J different maturities, observed at n different dates, denoted as $\widetilde{S}(t_k, T_j^*)$, where $k = 1, \ldots, n$ and $j = 1, \ldots, J$. Because in practice the zero-coupon yields are not observed, some deviations caused by rounding, bid-ask spreads, etc. can be expected and therefore (if putting all maturities observed at time t_k in a vector $\widetilde{S}_k$), we can write a measurement equation for the Kalman filter (for the details about Kalman filtration, see for example Harvey (1989)).

$$\widetilde{S}_k = S_k(X_{t_k}, \psi) + \epsilon_k, \ \epsilon \sim N(0, \sigma_\epsilon^2 I),$$

where the jth row of $S_k(X_{t_k}, \psi)$ is given by $S(t_k, T_j^*)$ in equation (2.20). In the first part of the data it can be easily seen that the measurement equation reduces due to formula (2.18) to

$$\widetilde{S}_k = d_k(\theta) + Z_k(\theta) X_k + \epsilon_k,$$

where the jth row of d_k and Z_k are given by $-A(T*_j)/T_j^*$ and $-B(T*_j)/T_j^*$, respectively, from equation (2.18).

The update equation follows from equation (2.9), see Arnold (1974) or Oksendal (2002). In general, the update equation in this case is a VAR autoregression of the first order

$$X_k = c_k(\theta) + \Phi_k(\theta) X_{k-1} + u_k, \ u_k \sim N(0, V_k(\theta)),$$

where vector c_k and matrices Φ_k and V_k depend on the chosen process in (2.9).

In order to have comparable results with Lund (1999) and because of parsimony, we decided to keep the same model for the short rate spread as Lund (1999). This model is actually the Vasicek model with another process for the stochastic market price of risk (dW_{1t} and dW_{1t} are uncorrelated Brownian motions):

$$\begin{aligned} dy_t &= (\kappa_1(\mu_1 - y_t) + \kappa_1 \lambda_t)\, dt + \sigma_1 dW_{1t} \\ d\lambda_t &= \kappa_2(\mu_2 - \lambda_t) dt + \sigma_2 dW_{2t}. \end{aligned}$$

In this case, the formulas for c_k and matrices Φ_k are

$$\begin{aligned} c_k(\theta) &= \left(I - e^{-K(t_k - t_{k-1})}\right)\Theta \\ \Phi_k(\theta) &= e^{-K(t_k - t_{k-1})}, \end{aligned}$$

where $K = (\kappa_1, \kappa_2)'$ and $\Theta = (\mu_1, \mu_2)'$. The discount factor in this case is calculated as (from equation (2.18))

$$D(t, t + T^*) = \exp\left(A(T^*) + B(t^*)y_t + C(T^*)\lambda_t\right).$$

Functional forms for $A(T^*)$, $B(T^*)$ and $C(T^*)$ can be found in Appendix B of Lund (1999).

After estimating the parameters of the process for the true local spreads, we can compare the prices of zero-coupon bonds that stem from the data (i.e. with the effect of the EMU) with the prices of bonds calculated using the estimated spread (i.e. without the effect of the EMU). From the differences in the prices we can estimate using non-linear regression techniques the unknown parameter of probability that a domestic country will join the EMU.

2.4 Market Surveys and the Official Strategies of Euro Adoption in 2006

Since the late 1990s, Reuters has conducted market surveys on several issues concerning the European Union. Before the start of the EMU, market polls were run on a monthly basis for the old EU Member States. Nowadays these surveys are conducted bianually for the new EU Member States. Reuters surveys around 30 strategists and political analysts across Europe for their views on the timing of the EMU and the ERM II entry of these countries and also the ERM II parity rates against the Euro. Table 2.1 presents a summary of the results on the question of the expected timing of EMU accession for the Czech Republic, Hungary, Poland and Slovakia valid at the time of this analysis. Although the survey is a very helpful insight into the market view of EMU participation dates, its results have to be taken into account with some caution, mainly due to some outliers and also because of internal inconsistency among some answers.

Looking at the results we can see some differences between the answers for the individual countries. The range between the maximum and minimum year of entry is quite broad (around six years). In particular, Slovakia was at that time supposed to join already in 2009, followed by the Czech Republic in 2010 and somewhat later by Hungary (2011) and Poland (2012). One may expect the Reuters polls to match approximately the official strategies adopted by these countries.[9] Nevertheless, at the time of the analysis the national Euro adoption targets were postponed mainly regarding the fiscal situation in these countries and at the same time the targets became somewhat unclear for all the countries, except for Slovakia. The Slovak Ministry of Finance reported that Slovakia's preparations for entering the Euro zone were proceeding according to the government's plan to adopt the single currency in 2009.

The central bank of Poland commented in 2006 that Poland should not set a target date for adopting the Euro until it was sure of meeting the criteria for joining the single currency. At the same time, the Polish government proclaimed that they expect Poland to enter in 2011. Also in the case of Hungary, the current state of finance did not allow the country to set a credible Euro target date and the country aimed to set a date in the second half of 2008 somewhere in the 2011-2013 range. Also the Czech government abandoned a plan to adopt the Euro in 2010, citing an outlook for widening budget deficits and rising inflation. The government had officially dropped any target date due to the need for fiscal reforms and huge government deficits.

2.5 Data and Estimation Results

2.5.1 Data

In this subsection the data used in the estimation process are described. The maximum maturity of the yield curve used is 10 years, therefore the maximum horizon of the estimation is April 2016 (as the yield curves from April 2006 are the latest data available at the time of the estimation). As mentioned above, we estimate the parameters for the Visegrad 4 countries, as only for them is the assumption of no entry before May 1, 2004 legitimate. For the countries already in the European Union (such as Great Britain or Denmark) and countries that

[9]We refer to the national Euro-strategies in this section that were valid in mid-2006 in order to be comparable with the Reuters poll valid at that time as well as with the results of the estimation in the following sections.

will not join the EU (Switzerland or Norway) we would have to use the year 1999 (EMU creation) as the earliest possible date. This would complicate the analysis as we would have to use both the German Mark and Euro yield curve (the German Mark was an approximation of the Euro before the EMU was officially created). The data used are the zero-coupon yield curves of the Euro, Czech crown, Slovak crown, Polish zloty and Hungarian forint derived from the swap rates prevailing at the market with maturities from one to ten years. We decided to estimate the forward rates from interest rate swaps instead of government bonds due to their standardised and favorable structure, which allows an estimation of precise zero-coupon yield curves. Data from Bloomberg was used.

2.5.2 Estimation Results

In this subsection we present the estimated probability parameters for the countries newly accessing the European Union. In Table 2.2, there are estimated the values of parameter θ for all the mentioned countries together with the standard errors (in parentheses). The interpretation of the θ parameter is that it defines the probability distribution of τ, i.e. of the date of EMU accession. This distribution is related to May 1, 2004 (due to definitions (2.14) and (2.19)). That means that no entry is possible before May 1, 2004.

In Table 2.3 we calculate the probabilities implied by the values of the parameters from the previous table. In general, these values do not fully reflect the dates from the Reuters pool referred to in Table 2.1, but are somehow lower. This is mainly due to the fact that parameter θ of the distribution function is estimated as constant over the period May 2004 - April 2006, although there are some indications that it was increasing over this period for all countries (as follows from the informal analysis using just a part of data).[10] The yields for all four countries were quite volatile in the past, which may have caused the instability in the estimation of the true spread to the Euro. The fact is that these estimates were quite sensitive to the change in the functional form and as we use only a two-factor model, quite a lot of volatility stays in the residuals and this may have caused the underestimation of the θ parameter. Another problem in the data is that the

[10] However, the advantage of this approach is that short-term market fluctuations are not observable and thus only long-term effects are apparent. All four countries expected general elections around the time when the analysis was performed and the uncertainty about the election results was mirrored in the short-term end of the yield curve.

markets of the Visegrad 4 countries are fairly small with few big players. This means that the prices contain a lot of information other than convergence to Euro markets.

The Reuters survey suggested that Slovakia was likely to be the first country to join the Euro area. According to the survey it should be a member in 2011 at the latest. The dynamic method employed in this paper indicated 80% for 2011 and 95% in 2015. According to Cincibuch and Horníková (2008), their method indicated that the market seemed to be pricing in Slovakian Euro area membership for 2011 and for latter horizons. This was in line with the actual development, when Slovakia became an EMU member starting from January 2009. Although the estimated results were showing a bit later date (the estimated probability for entry in 2009 was 62%), one has to take into account that the estimated coefficients are actually averages of their values over the estimation range. There were indication in the data that the actual value of the θ coefficient was increasing towards mid-2006.

For the Czech Republic, all respondents from the May 2006 Reuters survey expected that the country would become an EMU member before 2015, with the mean of the expectations at 2010. The analysis in our paper indicated a probability of about 75% for 2010 and 92% for 2015, which is comparable with the survey. Cincibuch and Horníková (2008) observed that the market was expecting the Czech Republic to join the EMU in 2014.

On average the poll expected Hungary to enter the EMU in 2011 and in 2016 at the latest. For Poland, the respective dates were 2012 and 2015. The dynamic model indicated probabilities at 50-60% for the mean years and 70-80% for the maximum years of the survey. Cincibuch and Horníková (2008) found a much weaker tendency of the Hungarian and Polish rates to revert towards European rates than in the case of Slovakia and the Czech Republic and according to their estimation, the Euro adoption for these countries is supposed to be after 2015. Both empirical methodologies tend to be more pessimistic than the market survey for these two countries.

2.5.3 Official Strategies and Survey in 2009

For comparison we include also the results of the latest (at the time of writing) Reuters opinion poll, dated January 2009, on the views of 30 emerging market strategists on when the countries analyzed in this paper[11] are likely to adopt the Euro. The results are in Table 2.4.

The preferences for early Euro adoption in the three countries (other than Slovakia) changed when the governments were confronted with fiscal and monetary problems. The political costs of Euro adoption (and of the much needed reforms before) started to be very high, as the Maastricht criteria requires significant tightening of fiscal and monetary policies. It also follows that the fiscal performance is a fundamental factor in the process of Euro adoption, as continuing large fiscal deficits can create significant inflationary pressures, see e.g. Berger, Kopits and Szekly (2004). However, several observers have raised concerns about the poor fiscal performance of some new EU members and ongoing reform of the public finance system in the whole EU27 is an agenda that is timely and should not be underestimated, see e.g. Kočenda, Kutan and Yigit (2008). They also suggest the need to design further policies to improve fiscal performance.

The Czech Republic was in a very good position to fulfill the Maastricht criteria, however the Euro is not at the top of political agenda, especially because of the current crisis. The government had to officially relinquish the target date due to the needs of fiscal reforms, caused by huge government deficits. The recent estimates from the Czech Ministry of Finance project the full-year fiscal gap at 4.5 percent of GDP for 2009, well over the relevant Maastricht criteria. The new Euro-adoption strategy has not specified any date for Euro adoption, preferring to wait for future fiscal developments and the future impact of fiscal reforms. The positive perception of the general public of the strong appreciation of the Czech currency and the benefits of a sovereign monetary policy are other reasons why the Euro adoption is expected at 2013 or later.

At the beginning of 2009 Poland started official talks with the European Central Bank on joining a fixed exchange-rate program that is a stepping stone to Euro adoption. However, the recent economic slowdown will make it difficult for the country to meet the currency's strict adoption criteria. The official strategy calls for entering the program by the middle of 2009 and adopting the Euro by 2012.

The recent economic slump in Hungary indicates that the fiscal consolidation will take much time. Currently, there is no fixed target date and the government

[11]One obvious exception is Slovakia, which adopted the Euro on January 1, 2009.

is preparing a new Euro adoption strategy that should be finished in mid-2009.

2.6 Conclusions

This paper deals with the financial market perception of the timing of the entry of new EU Member States to the Euro area. It presents a way of measuring market views based on the state space model and compares the obtained results with the opinion poll conducted by Reuters as well as with plans presented by national authorities.

A simplified version of the Lund (1999) dynamic term structure model was estimated. The method was applied to financial market data in Central European countries, including the Czech Republic, Hungary, Poland and Slovakia. The results of the method used in this paper are in line with with market surveys. We also compared the results of this paper with the results of the model based on the short-term dynamics of forward spreads developed in Cincibuch and Horníková (2008).

According to this analysis, Slovakia was supposed to be the first country to join the EMU. Poland and Hungary are estimated to trail the other two countries. Slovakia and the Czech Republic had fairly high chances to join the EMU before or in 2011 according to the information contained in the swap market rates. Also in the data it was possible to observe a high convergence to Euro interest rates even for mid-maturities (around 5 years). This means that pricing in the long end of the yield curve was already performed with respect to Euro market rates. The speed of convergence could be explained by institutional reforms (in the case of Slovakia) and by a good macroeconomic situation (for both countries).

On the other hand, the market did not expect EMU participation for Hungary and Poland before 2013 or at least the degree of convergence to Euro market rates was too low even for higher maturities (up to 10 years). Also note quite high estimation errors for both these countries, indicating a high degree of uncertainty about the date of EMU accession.

Entry into the EMU is influenced mainly by the fiscal policies of the government and it would be interesting to analyze substantive factors such as fiscal performance, structural reforms or monetary policy influencing the market perception of EMU entry. However, these issues are beyond the scope of the current paper and are left for future research.

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A.1 Figures and Tables

Figures

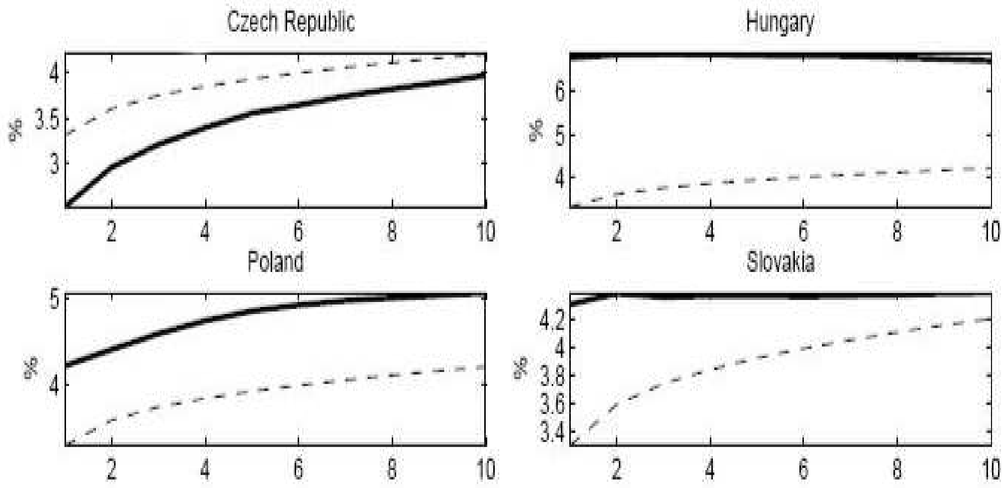

Figure 2.1: Swap curves as of 28 April 2006

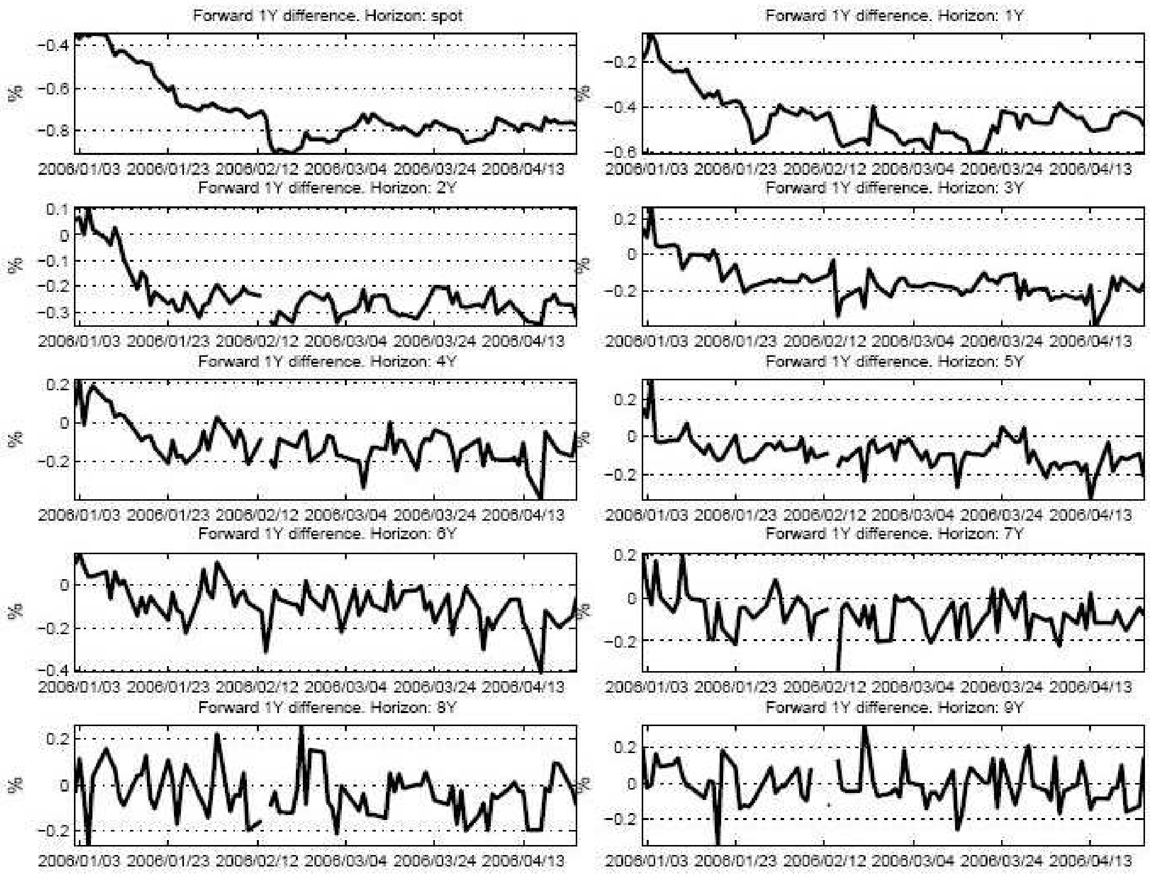

Figure 2.2: EMU and Czech Republic forward rates

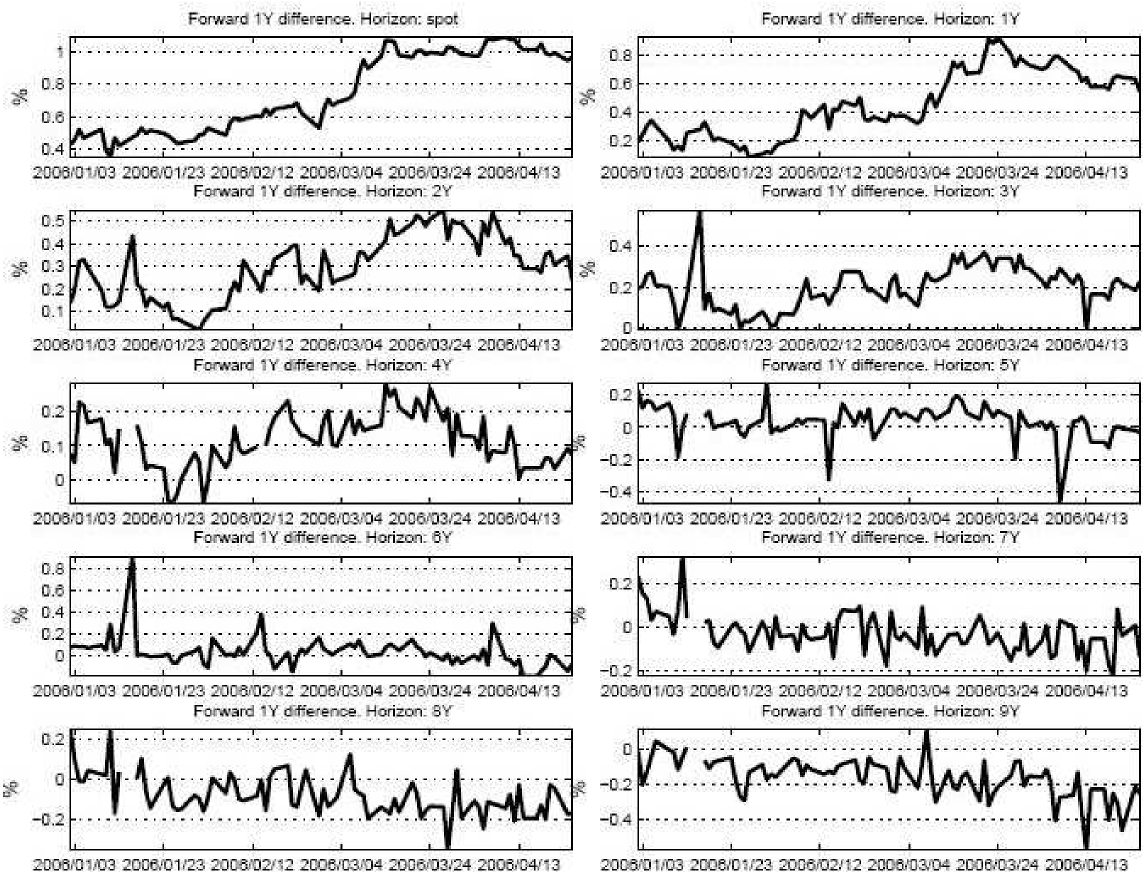

Figure 2.3: EMU and Slovak Republic forward rates

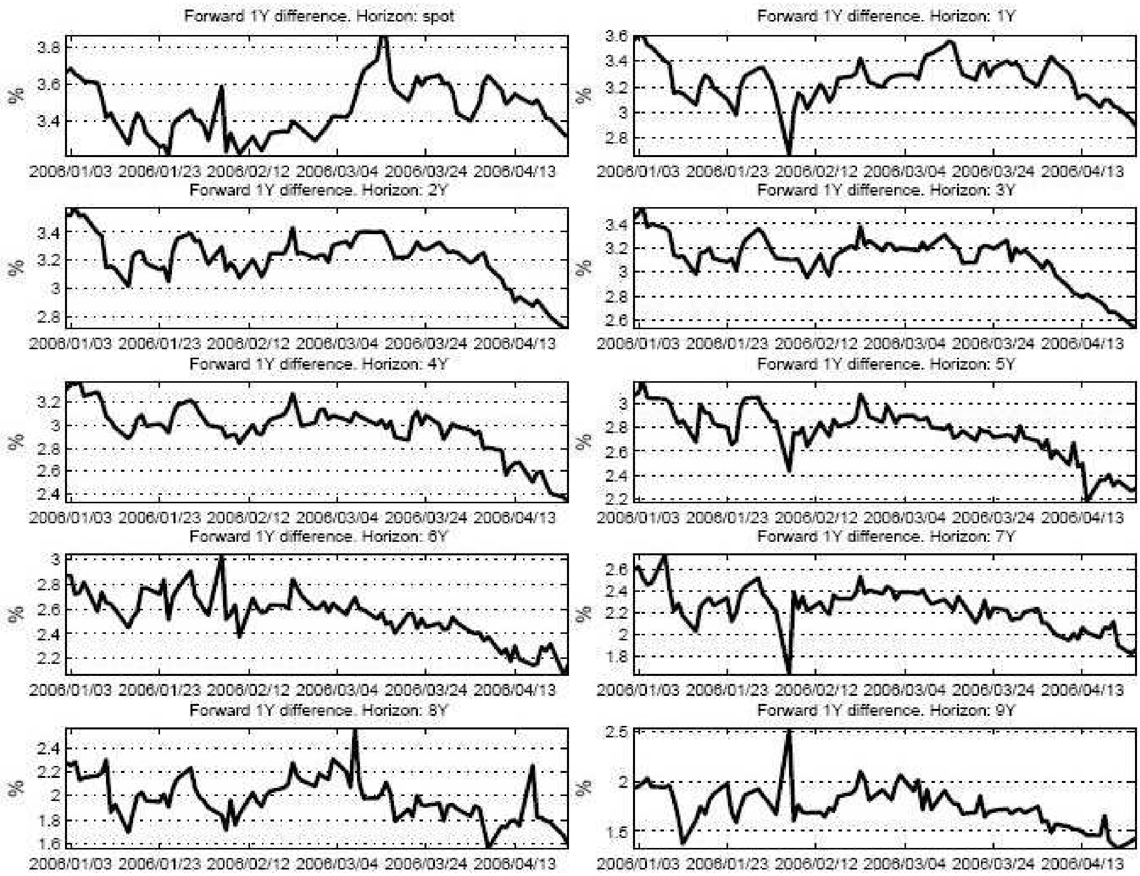

Figure 2.4: EMU and Hungary forward rates

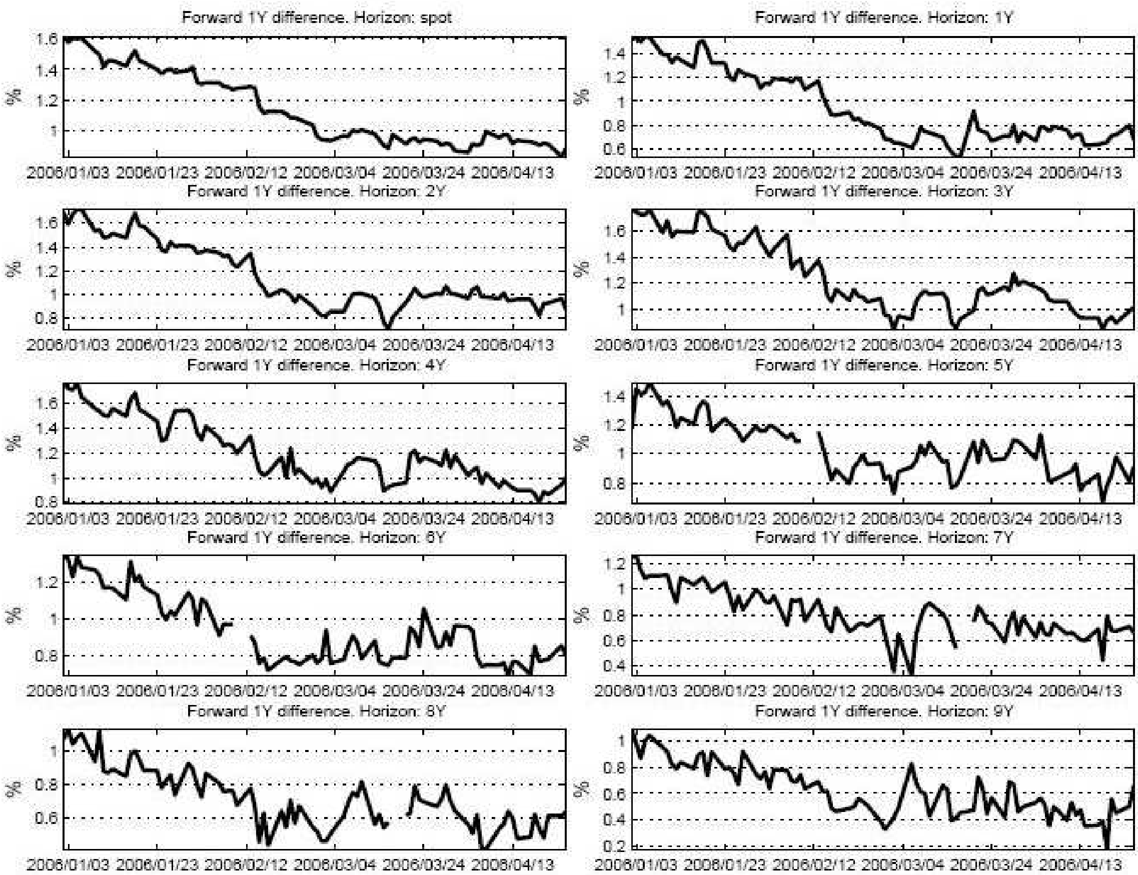

Figure 2.5: EMU and Poland forward rates

Tables

	Median	Mean	Mode	Maximum	Minimum
Czech Rep.	2010	2010	2010	2015	2009
Slovakia	2009	2009	2009	2011	2008
Hungary	2010	2011	2010	2016	2010
Poland	2012	2012	2012	2015	2010

Table 2.1: Expected EMU entry dates according to the Reuters Poll of 38 professional respondents in May 2006.

Note: Reuters Poll May 2006 question was: "In what year do you expect the following countries to enter the EMU, i.e. formally adopt the Euro?".

Country	θ
Czech Republic	0.27439
(p-value)	(0.0635)
Slovak Republic	0.32456
(p-value)	(0.0734)
Hungary	0.15134
(p-value)	(0.1182)
Poland	0.19753
(p-value)	(0.0953)

Table 2.2: Estimated parameters of the distribution of τ.

Note: standard errors are in parenthesis.

Country	Horizon				
	2009	2010	2011	2013	2015
Czech Republic	0.5609	0.6663	0.7463	0. 8535	0.9153
Slovak Republic	0.6223	0.7269	0.8026	0.8968	0.9461
Hungary	0.3649	0.4541	0.5307	0.6533	0,7438
Poland	0.4471	0.5462	0.6275	0.7491	0.8309

Table 2.3: Implied probabilities of each country joining the EMU for a given horizon

	Median	Mean	Mode	Maximum	Minimum
Czech Rep.	2013	2013	2013	2015	2011
Hungary	2013	2014	2013	2017	2010
Poland	2013	2013	2013	2015	2011

Table 2.4: Expected EMU entry dates according to the Reuters Poll of 30 professional respondents in January 2009.

Note: Reuters Poll January 2009 question was: "In what year do you expect the following countries to enter the EMU, i.e. formally adopt the Euro?".

Chapter 3

Default Predictors and Credit Scoring Models for Retail Banking in the Czech Republic

Abstract

This paper develops a specification of the credit scoring model with high discriminatory power to analyze data on loans at the retail banking market. Parametric and non-parametric approaches are employed to produce three models using logistic regression (parametric) and one model using Classification and Regression Trees (CART, non-parametric). The models are compared in terms of efficiency and power to discriminate between low and high risk clients by employing data from a new European Union economy. We are able to detect the most important characteristics of default behavior: the amount of resources the client has, the level of education, marital status, the purpose of the loan, and the number of years the client has had an account with the bank. Both methods are robust: they found similar variables as determinants. We therefore show that parametric as well as non-parametric methods can produce successful models. We are able to obtain similar results even when excluding a key financial variable (amount of own resources). The policy conclusion is that socio-demographic variables are important in the process of granting credit and therefore such variables should not be excluded from credit scoring model specification.

Keywords: credit scoring, discrimination analysis, banking sector, pattern recognition, retail loans, CART
JEL Classification: B41, C14, D81, G21, P43

3.1 Introduction

Despite the wide variety of banking services, lending to corporate clients and the public still constitutes the core of the income of commercial banks and other lending institutions. Due to asymmetric information, lending carries a risk in terms of defaulted loans. Hasan and Zazzara (2006) stress that under the new Basel II rules that are grounded in recognizing an individual credit risk through internal rating systems banks' managers must correctly measure risk and price it accordingly. Credit scoring greatly reduces the risk provided a capable model is applied and reliable data are available as firmly shown by Dinh and Kleimeier (2007). Both of these requirements might be hard to come by during periods of massive economic change (e.g. the economic transformation of the countries in Central Europe and their integration into the European Union (EU)) or turbulent instabilities (e.g. the financial crisis that erupted in 2008 and spread worldwide). During these periods financial data may be unreliable predictors. Indeed, Caselli, Gatti and Querci (2008) show that there is relation between the loss given default rate on bank loans and macroeconomic conditions.

In this paper we build two parametric and one non-parametric credit scoring models and test them on a large dataset of retail loans containing financial as well as behavioral and socio-demographic variables from a new EU economy.[1] Based on various tests and out-of-sample testing we show that our models deliver efficient results in terms of potential default identification and that socio-demographic data are useful predictors of the future characteristics relevant to the loan granting process. This is certainly good news as the findings of Jacobson, Lind and Roszbach (2005) show that retail portfolios are usually riskier than corporate credit.[2]

3.1.1 Literature

From a technical perspective, the lending process is a relatively straightforward series of actions involving two principal parties. These actions go from the initial

[1] We did not incorporate macroeconomic variables into our analysis, as our main area of interest was to focus on socio-demographic variables. Also, our data sample reflects only a period of steady macroenomic growth in the Czech Republic and to estimate the impact of macroeconomic developments on individual defaults would require at least whole economic cycle.

[2] The models developed in this paper may not be transferable to banking markets in the other new EU member countries due to the specificity of the data used. Each bank has its own processes and ways to deal with clients and defaulted credits and therefore models used in the respective bank may be highly specific.

loan application to the successful repayment of the loan or its default. Although retail lending is among the most profitable investments in lenders' asset portfolios (at least in developed countries), increases in the amount of loans also bring increases in the number of defaulted loans. Thus, the primary problem of any lender is to differentiate between "low risk" and "high risk" debtors prior to granting credit. Due to the asymmetric information between the lender and borrower such differentiation is not a trivial task. However, it is possible by using parametric or non-parametric credit-scoring methods.

The practice of credit scoring began in the 1960's, when the credit card business matured and automatic decision-making processes became necessary. Later, the use of credit scoring techniques was extended to other classes of customers, in particular to small and medium enterprises. In this respect, Myers and Forgy (1963) compared discrimination analysis with regression in credit scoring applications and Beaver (1967) introduced a bankruptcy prediction model. The two works above both focused on two aspects: predictions of failure as well as on the classification of credit quality. This is an important distinction in empirical analysis as it is often not clear which aspect to focus on. Altman (1980) described the basic bank lending process as an integrated system and analyzed a procedure for how the criteria for the assessment of commercial loans is set.[3]

Most of the credit-scoring literature deals with non-retail loans, i.e. loans to firms, as the data are more readily available. Corporate credit scoring-also known as rating assignment-is different from scoring for retail loans for several reasons. Primarily, the amounts lent are much smaller in the case of retail lending, and therefore from the point of view of risk management retail loans are dealt with using a portfolio approach, while corporate loans are managed on an individual basis. Most importantly, there are different types of variables used in the process of constructing a model as well as the decision process for each type of loan. For example, for corporate loans, various ratios of financial indicators are typically used in corporate failure models since they are usually very powerful in determining the quality of a client.[4] As regards collateral, for example Blazy and Weill (2006) state that it might be that riskier loans are more likely to be collateralized,

[3]For a more thorough exposition of the credit scoring literature, see Renault and de Servigny (2004).

[4]Altman and Narayanan (1997) provide a broad review of corporate failure models and their classification.

otherwise these projects would not be financed. In retail lending, the bank has to collect various socio-demographic characteristics, as well as various behavioral indicators (e.g. indicators of a client's behavior on his current account) to make a decision about the client's portfolio.

As an example of early methodologies concerning retail loans, Long (1976) studied a selection of the empirically best credit scoring techniques and proposed criteria for the optimal updating cycle of a credit scoring system. Apilado, Warner and Dauten (1974) empirically studied two hypotheses: that there is a limited set of variables discriminating between low and high risk loans with a high degree of accuracy and that profitability can be increased without increasing risk for most lenders. Gropp, Scholz and White (1997) examined how personal bankruptcy and personal bankruptcy exemptions affect the supply of and demand for credit. They found that bankruptcy exemptions redistribute credit towards borrowers with a high level of assets. As an example of recent work in the area of retail credit scoring, Avery, Calem and Canner (2004) examine the potential costs of failing to incorporate into consumer credit evaluations situational data, such as information about the economic or personal circumstances of individuals. They also discuss practical difficulties associated with the development of credit scoring models that incorporate situational data. For further examples of the uses of credit scoring in retail banking see Jacobson and Roszbach (2003); Allen, DeLong and Saunders (2004); Wagner (2004); Jacobson et al. (2005); Bofondi and Lotti (2006); Dinh and Kleimeier (2007); Saurina and Trucharte (2007). Finally, Hand and Henley (1997) provide an excellent survey of the statistical techniques used in the process of building a credit scoring model.

3.1.2 Objective

In this paper we focus on an analysis of the determinants of defaults of retail loans in an environment where "classical" models might not perform well. One such region is that of the new EU members, where there has been a sharp increase in the amount of this type of loan recently, and the increase is expected to continue. Hilbers, Johnsen, Otker-Robe and Pazarbasioglu (2005) review trends in bank lending to the private sector, with a particular focus on Central and Eastern European countries, and find that the rapid growth of private sector credit may create a key challenge for most of these countries in the future. Take for example two countries on the forefront of the EU integration process: in the last few years, banks in the Czech Republic and Slovakia have allocated a significant part of their lending to retail clientele. Even before the integration of both countries into the EU, the financial liabilities of households between years 1999-2004 (which is covered by our data) increased more than twice in both countries (relative to GDP). Later on, in 2006, Czech and Slovak banks recorded 30.5% and 32% increases in retail loans, respectively. In 2007 these increases amounted to 35.2% and 27.8%, respectively. In the Czech Republic and Slovakia the financial liabilities of households formed 15.6% and 15.7%, respectively, of the GDP in 2006. In 2007 these liabilities increased to 18.8% and 16.4%, respectively.[5] The average ratio of financial liabilities to GDP in the older 15 members of the European Union is about three times higher than in the Czech Republic and Slovakia;[6] it is expected that the amount of loans to retail clientele will continue to increase, as there is a lot of space for expansion in the financial liabilities of households in both countries (even though the household sectors in at least some of the older EU countries clearly took on too much debt).

In light of these recent developments, we address the primary problem of lenders: how to determine between low and high risk debtors prior to granting credit. That means we aim to build an application type of model that would primarily be suitable for the pre-scoring of clients.[7] One of our goals is to look at the importance of socio-demographic variables as determinants of default. The reason is that this type of variable provides useful information in times of change. This is particularly true in new EU members that recently underwent an unprecedented economic transformation and have integrated into the EU. Socio-demographic variables evolve in a stable manner over time and a well-designed credit scoring model based on socio-demographic and behavioral variables might perform as well as a model based on historic or current financial characteristics.

[5] These numbers, which originate from the financial stability reports of the central banks of both countries, cover only the banking sector and not other types of lending institutions.

[6] As of 2006; EU Economic Data Pocketbook

[7] The models constructed in this paper are not appropriate for example for the ongoing and regular calculation of regulatory capital as they rely mostly on the application characteristics of clients valid at the time of loan application. Application characteristics are usually not updated during the life of the loan and they grow more imprecise as time elapses and therefore are not suitable for the assessment of the current riskiness of a portfolio of bank loans. Also, as our main concern is the probability of default models, we do not take into account the loss given default parameter of defaulted loans.

In this paper we contribute to the literature in several ways. First, we construct two types of credit scoring model, one based on logistic regression and the other on Classification and Regression Trees (CART). Both methods are often used for developed countries and we are interested in whether they are able to construct a powerful credit scoring model for new EU markets that due to their economic history differ from the old EU members. Second, we test our models on an empirical dataset from one of the banks operating in the retail loan business in a new EU market (the Czech Republic). Based on out-of-sample testing we compare the efficiency of the two methods and identify the key determinants of default behavior, with socio-demographic variables being important.[8] We show that with the logistic regression model we were able to build a specification that does not contain the single most important financial variable (available resources) but still performs only marginally worse than the specification with this variable.

The rest of the paper is organized as follows. In Section 3.2 we describe the data used in the estimation process. Section 3.3 describes the empirical methodology and results and Section 3.4 concludes.

3.2 Data

In this section we briefly introduce our dataset. We intentionally deviate from standard practice and introduce our data prior to describing the models. This helps us describe our models in a more lucid way. Some details about the data are also introduced in the model section, where they fit more naturally.

The dataset used for the estimation in this paper comes from a new EU member (the Czech Republic) and was provided by a bank that specializes in providing small- and medium-sized loans to retail clientele in the area of real property purchase and reconstruction.[9] The same data have been used for the bank's own as-

[8] To the best of our knowledge, the empirical studies analyzing this type of problem, with emphasis placed on credit scoring related to retail loans, are non-existent in post-transition countries that became EU members. Part of the lack is due to the fact that commercial banks in post-transition EU countries, especially the biggest ones, are not willing to share their credit-related data. This is understandable since having datasets connected with the default behavior of retail clients can be a competitive advantage over other banks because these datasets enable the bank to construct better credit models. A bank with an accurate and powerful credit scoring model not only decreases its costs connected with bad loans, but also strengthens a bank's risk management in general.

[9] The bank does not wish to be explicitly identified and we honor this request as specified in the contract to provide us with the data.

sessment and scoring modeling. The dataset contains various socio-demographic characteristics and other information collected by the bank on 3403 individual clients who were granted loans during 1999-2004. The observation period ends in 2006. Out of these, 1695 clients defaulted on loans and 1708 performed well, i.e. the sample is artificially balanced to have approximately 50% of defaults. The loans are evenly distributed during the analyzed period. There is no concentration of defaults in any period. Each individual client had no more than one loan, so there was no need to aggregate several loans for one individual, as is often the case for companies. The definition of default follows the Bank for International Settlement standard: the client is in default if he or she is more than 90 days overdue with any payment connected with the loan. The definition of a good/bad variable is derived based on the performance of the client, i.e. the client is considered "bad" in the case of his/her default. What follows in the next paragraphs is the economic motivation for including the various variables.

For all clients we have a number of variables that we present in Table 1 along with the variable definitions and whether they are categorized or continuous. The first part of the characteristics are socio-demographic variables and they characterize the client at the moment of loan application. Among others, there are several categorized variables related to the client's employment situation. The bank does not record information about the client's income and expenditures; instead the bank calculates and records the relevant credit ratios. The first ratio is the percentage of income that is spent on expenditures (Credit Ratio 1). The second ratio is the ratio of a client's available income to the official minimum wage valid at the time of the loan application (Credit Ratio 2). The client's region is designated by the postal code of the region of the client's address.

The other part of the variables characterizes the relationship between the client and the bank. The Loan Protection variable records the credit risk mitigation used, i.e. whether collateral, a guarantor or another type of mitigation was used. . It is important to take into account collateral or guarantee of loan as a riskier but well-collateralized loan may be more profitable for a bank than a somewhat less risky loan without collateral. The Points variable is the only behavioral characteristic available.[10] It is a variable constructed by the bank and describes the client's

[10]Behavioral characteristics are very powerful indicators of the type of client. However, the client needs to have a history with the bank in order to use these indicators. Hence, we do not possess other behavioral variables such as delinquency. A new client has to be scored almost

behavior on his or her own current account. It quantifies the frequency at which the client deposits money into the account as well as whether the deposits follow a regular pattern. Hence, the Points variable depends on the amount of a client's savings as well as on how regular saving deposits are made. The Own Resources variable is the amount of resources the client declares to have at the time of loan application available to use for the purpose defined in the Purpose of Loan variable. For example, it is the amount of money a client can allocate as a down payment for the purchase of an apartment. The Years of Having Account variable is the number of years between when the loan was granted and when the client opened an account with the bank.[11] We have also tested the sample on the possible multicollinearity of the Years of Having Account variable and the Date of Account Opening, but found no significant results.

Our data sample did not contain information on rejected applicants, i.e. clients who applied for credit but were rejected, as the bank did not collect this data. The true creditworthiness status of the rejected applicants is unknown and therefore a selection bias may usually be introduced into the results.[12] However, Banasik, Crook and Thomas (2003) compared the classification accuracy of a model based only on accepted applicants, relative to one based on a sample of all applicants. They found only a minimal difference. Also, Hand and Henley (1993) concluded that a reliable rejection inference is impossible and improvements in scoring models achieved by reject inference are based on luck, the use of additional information (for example using expert skill) or ad hoc adjustment of the rules in a direction likely to lead to a reduced bias.

3.3 Estimation techniques

In this section we introduce two distinct techniques for credit scoring. These are a parametric approach with a logistic regression and a non-parametric Classification and Regression Trees (CART) model. The methods are described in Sections 3.3.1

solely on the basis of her/his socio-demographic characteristics (as there is still no practice in the Czech Republic for individuals to have public credit ratings that banks can use to inform themselves). This is also the reason why we do not take into account the bank's interest rate setting policy.

[11] The Date of Loan variable is an endogenous variable and it is not possible to discriminate on the basis of this variable. Therefore this variable is not used in the subsequent analysis.

[12] Hand and Henley (1993) analyze a "reject inference" process, i.e. a process of attempting to infer the true creditworthiness status of rejected applicants.

and 3.3.2, respectively. More thorough exposition can be found in Vojtek and Kočenda (2006).

As it is not practical to use more than 20 variables in logistic regression or in the process of creating trees, single factor analysis was performed as the first step of the estimation. With single factor analysis we tried to eliminate variables which have no discriminating power. We calculated the so-called "odds ratio" and "information value" for each variable. Both characteristics show the degree of the ability of the variable to discriminate between defaulted and non-defaulted loans. Variables with the lowest information values were then omitted.

The odds ratio can be used to determine the discrimination ability of the variable for the given category. It is defined as

$$Odds_i = \left(\frac{Defaulted_i}{Defaulted}\right)\left(\frac{Good}{Good_i}\right), \tag{3.1}$$

where *Defaulted* and *Good* are the total numbers of defaulted and non-defaulted observations and $Defaulted_i$ and $Good_i$ are the numbers of defaulted and non-defaulted clients in the ith category of a variable. An odds ratio equal to 1 implies that the variable is not able to discriminate between bad and good clients in the given category; other values signal the discrimination ability of a variable.

The overall information value of a variable is the sum of the information values for each category of variable, which are defined as

$$IV_i = \ln(Odds_i)\left(\frac{Defaulted_i}{Defaulted} - \frac{Good_i}{Good}\right). \tag{3.2}$$

This information value symbolizes the predictive power of the variable: the higher the value, the higher the predictive power of the variable with the given categorization. In banking practice a value above 0.2 is taken as a sign of the strong predictability of a given variable.

For our analysis we decided to categorize the continuous variables. Although it is possible to build a model using both continuous and discrete variables, the standard practice in credit scoring is to use categorized continuous variables. We used the following practice.[13] First, the range of values for each continuous variable was split into ten categories according to the following two principles:

[13] There are also other ways to categorize continuous variables, see for example Wermuth and Cox (1998).

1. All categories should have the same number of observations, with one exception.
2. The exception is that observations with the same value for the specific variable have to be in the same category.

The odds ratios and information values were calculated for each category and categories with similar values were merged. This step was also performed for the categorized variables.

The odds ratios and information values for the categories of variables from the sample can be found in the appendix. The total information values for the variables can be found in Table 2. It can be seen that the most significant variables are those that characterize the relationship between the client and the bank, a finding that is in accord with the comprehensive overview in Anderson (2007). The variables that characterize the loan protection and credit quality of the debtor (i.e. both credit ratios) are almost insignificant. This fact is surprising especially in the case of loan protection as one would expect that collateral in the form of real estate would be an effective predictor of good performance. However, this detail can be explained by the fact that the amount of each loan in the data sample is not excessively large and therefore even a defaulted loan does not necessarily result in a loss of property.

It is also interesting that most of the socio-demographic variables are not significant. Only Education is a very strong default predictor since clients with a higher level of education show much less default than other clients. Marital Status, Region, Sex and Employment Position have low information values.[14] Another interesting factor is the difference in the information value of both credit ratios. It seems that the default behavior of clients does not depend on the absolute amount of "savings" (i.e. the difference between income and expenditures) but on relative income (i.e. the ratio of expenditures to income). In other words, even high income clients who also have high expenditures can be risky clients.

We will proceed now with the two discrimination techniques to analyze the determinants of default behavior. In the course of this analysis we will also compare

[14] The low information value of the Sex variable is in contrast to finding in Dinh and Kleimeier (2007), where Sex/Gender was found to have good predicting power, as micro finance literature suggest that women repay more reliably. The low information value of the Sex variable also hints at non-discriminatory practices, which are otherwise documented for example by Alesina, Lotti and Mistrulli (2009) in Italy.

logistic regression with CART (Classification and Regression Trees).

3.3.1 Logistic regression

The theoretical background for using logistic, or logit, regression for classification in credit scoring has been outlined in the literature, and the literature also shows that logistic regression is usually very successful in determining low and high risk loans in tasks similar to ours. For details see for example Gardner and Mills (1989), Lawrence and Arshadi (1995), Hand and Henley (1997) or Charitou, Neophytou and Charalambous (2004).

In our analysis we decided to employ all variables with an information value higher than 0.1. The reason for such low threshold is to begin with employing more variables available for the logistic regression and also to have more socio-demographic variables, despite the fact that in our case these tend to exhibit lower information values. Although there are missing values in several of the variables, this problem was eliminated by categorization, i.e. by creating a category for the missing values. We employed forward-backward stepwise model selection using Akaike Information Criterion (AIC) to select the best model. Logistic regression usually starts with the simplest model, i.e. with a regression on a constant only. After each step, the chosen model is tested and a decision is made on whether any variable can be left out based on the change in the value of the information criterion. Then all the models that differ from the current one by adding a single variable are tested. This procedure should choose the best model among all the models based on the supplied regressors (variables). The coefficients are estimated using the maximum likelihood method. Statistical analysis was performed using S-PLUS 6.2 software.

In order to evaluate the performance of our models we follow a strategy to partition our dataset into two samples: one for development (development sample) and one for validation purposes (validation sample). This way an out-of-sample validation can be performed. The dataset was randomly split such that the development sample contains two-thirds of the observations (2280 observations) and the validation sample contains one-third of the observations (1143 observations). The validation sample will be later used to test the discriminatory power of the model on a sample that was not used in the development stage of the model (out-of-sample testing). The quality of the models was tested using the Receiver

Operating Curve (ROC) and the GINI coefficient. Webb (2002) defines the ROC as the plot of the true positive rate on the vertical axis against the false positive rate on the horizontal axis. All the ROC curves pass through the (0,0) and (1,1) points and as the separation increases the curve moves into the top left corner. The ideal model should perform 100% detection and have a 0% false positive rate. The ROC in the case of the ideal model is characterized by a kinked curve passing through the coordinates (0,0)-(0,1)-(1,1). Different models produce different ROCs, characterizing the performance of the model. The performance is defined as the area under the curve and is usually denoted as the c coefficient. It follows that the ideal model has an area under the curve $c = 1$. For the GINI coefficient g, which is the area under the Lorenz curve, the relationship $g = 2c - 1$ is valid.

However the choice of the model in practice does not always depend only on the ROC curve and the GINI coefficient. It may be important to look at the Type I error (accepting a bad loan as a good loan) and Type II error (rejecting a good loan as a bad loan). It is a generally-accepted fact the misclassification costs of a Type I error are much higher than those of a Type II error. For a Type I error the lender may lose the whole amount of loan and its interest while for a Type II error it is only the expected profit from the loan. Therefore it may be important to look at the full curve not only at the parameter c. In banking practice therefore the choice of model may be based on minimizing misclassification costs.

The logistic regression is based on the following idea. Given a vector of application characteristics x, the probability of default p is related to vector x by the relationship

$$\log\left(\frac{p}{1-p}\right) = w_0 + \sum w_i \log x_i, \tag{3.3}$$

where coefficients w_i represent the importance of specific loan application characteristic coefficients x_i in the logistic regression. Coefficients w_i are obtained by using maximum likelihood estimation. Logistic regression can handle categorized data by employing a dummy variable for each category in the data.

Using this method we first estimate Model 1, which is the output of the stepwise procedure; i.e. the model was selected as the ideal model using the above mentioned forward and backward stepwise technique. The estimates are presented in Table 3.3, which also contains the list of variables used. The score for each client can be calculated by summing the respective coefficient values, where the coefficient has a value of 0 for "reference category". This model has several drawbacks.

First, there are variables that have insignificant coefficients. Second, due to the high number of categories and variables, the model has also high number of degrees of freedom, a property that can lead to serious over-learning.

In Model 2 we eliminate variables with insignificant coefficients. In particular the following variables were dropped: Sector of Employment, Years of Employment, and Purpose of Loan. Results are presented in Table 3.4. The elimination of several variables is justified also by the fact that the decrease in AIC was very slow for the last variables that entered the model. In Model 2 the value of the AIC increased only by about 2% and also the properties of the coefficients are similar to those in Model 1. Thus, Model 2 is able to discriminate among clients with fewer variables.

Finally, we estimate Model 3. The need for the third and last logistic model is driven by the fact that the variable Own Resources is a very strong default predictor. Therefore it might be useful to investigate the properties of other variables, i.e. to try to construct the model without this variable and to compare what the ability of the model is without this strong predictor. Further, the amount of resources a client has is usually very hard to detect, especially if a client would have to declare other funds he or she has outside the bank. Therefore it might be interesting to see whether it is possible to discriminate successfully without the knowledge of what funds the customer has. Model 3 is constructed using the same list of variables as Model 1 but the variable Own Resources is omitted.[15] The coefficients of this model are presented in Table 3.5 and reveal that Model 3 is able to successfully discriminate among clients without a knowledge of the resources the client owns.[16]

Next, we compare the quality of the three models. In Figures 3.11-3.3 we present the comparison using the ROC and c coefficients introduced earlier in the section. There are three figures for each model, the ROC curve (yielding the c coefficient), the empirical cumulative distribution function (CDF) for estimated bad and good clients and a histogram for both distributions. We can see that Models 1 and 2 are very similar in the values of the c coefficients: Model 1 has $c = 0.877$ and Model 2 has $c = 0.864$, which is a difference of a mere 1.49%.

[15] The stepwise procedure also did not choose the Sector of Employment variable.

[16] As a robustness check we also constructed a version of Model 3 using Model 2 with the variable Own Resources omitted. The results were equally strong as those presented in Table 5 for Model 3. Because of limited space we do not report detailed results, although they are available upon request.

That means that both models have very similar characteristics and are able to discriminate with almost the same power. Therefore Model 2 is preferred over Model 1 due to the principle of parsimony. Model 3 has a much higher value of the AIC, but more importantly the value of the c coefficient ($c = 0.832$) is only marginally worse than that of Model 1 or 2. The consequences of this are striking: a bank does not need to know the variable Own Resources to construct a model with very similar power to a model containing the variable. This offers for example the possibility for bank to check for fraud simply by running two different scoring functions: one which accounts for the declared resources the customer owns and one that does not. If there are serious differences in the results it may be worth examining the applicant further.

Another test of the power of a model is out-of-sample testing, i.e. the testing of the discriminatory power of the model on a sample that was not used in the development stage of the model, as we note in Section 3.1.2. In Table 3.6 we see the values of both c and the GINI statistics for all three models. It is possible to see that all models have similar power for both development and validation samples. As expected, Model 3 has lower power because the most important variable is left out. The approximately 11% loss of power does not seem that large in view of its great ability to discriminate in the absence of the single most important variable.

We also tested both constrained models (Models 2 and 3) versus Model 1 using the log-likelihood ratio test (LR test). The LR test is used in place of a standard F-test. The F-test, regularly used in the case of OLS regressions, cannot be employed because the response variable is not normally distributed. The LR test is performed by subtracting the so-called residual deviances of constrained and unconstrained models.[17] The statistics has approximately a Chi-square distribution with n degrees of freedom, where n is the number of constraints. The null hypothesis is that the omitted variables are non-significant, i.e. their coefficients are equal to zero.

The residual deviances for all three models are: $DEV_1 = 2013.015$, $DEV_2 = 2104.823$, and $DEV_3 = 2358.410$. This means that when comparing Model 1 with Model 2 the test statistics is $LR_1 2 = 91.808$ with 23 degrees of freedom, and statistics comparing Model 1 with Model 3 is $LR_1 3 = 345.395$ with 17 degrees of freedom.[18] The values are highly statistically significant, implying that we should

[17] The residual deviances are the analogue of the residual sum of squares in the OLS.

[18] Such a high number of degrees of freedom is implied by the fact that each class of categorized

reject the null hypothesis of the non-significance of omitted variables. This is a sign that the omitted variables have statistical significance; however the power of all of the models is approximately the same. We conclude that all three models can be used for credit scoring. However, because of the high number of categories there is the risk connected with the possible over-learning of Model 1. Therefore, we lean towards Models 2 and 3. The final choice of model should be based on other criteria dictated by special needs such as the results of the out-of-time back-testing of models, requirements for model parsimony and data availability.

We now turn to assessing and interpreting our results. With respect to the variable Own Resources, in both Model 1 and Model 2 it is possible to observe an inverse relationship between the amount of resources a client owns and the probability of default. Since we model the probability of default, a higher score reflects a higher default probability and, as one would expect, clients with more funds show a lower default probability.

Another strong predictor is Education Level, which shows that clients with a higher level of education have much less difficulty paying their debts. Clients with only general secondary education are riskier than those with vocational education at the secondary level who have passed the graduation examination.[19] Frequently general secondary school graduates are not accepted for university education. People without specific vocational education and without a university education have a harder time getting better-paid job. They are also more likely to fail to find permanent employment and to become unemployed, and thus they more often fall into the lowest income category.

The Length of the Relationship between the client and the bank is the most important behavioral characteristic. Evidence from the empirical literature (Hopper and Lewis (1992); Thomas, Ho and Scherer (2001); Anderson (2007)) shows the positive correlation between the length of the client having account with the bank and her or his ability to repay the debt. This is because a bank knows clients with longer histories better than those with shorter histories, and therefore the

variable adds one degree of freedom. Critical values at 1% are 41.638 and 33.409 for 23 and 17 degrees of freedom, respectively.

[19] Vocational education, also called career and technical education, prepares students for specific manual or practical careers. Vocational education can be at the secondary or post-secondary level. In some cases secondary-level vocational education ends with a demanding graduation examination, and having passed such an exam indicates a higher level of achievement than graduating without passing an exam.

bank can better foresee that the former group of clients will not default. It has to be noted that the period from the date an account is opened is potentially an endogenous variable. The results show that clients with accounts opened in the previous few years are not risky at all. However, these clients have had less chance to default than clients with longer histories. The variable makes sense in the assessment of clients who have been with a bank for a longer time. For example, our data show that clients who opened accounts in 1993-1995 are less risky than those who opened accounts in 1996-1997.

Marital status showed to be a relatively strong predictor of default in all the models. We conjecture that clients without a spouse may be considered by banks as riskier than married clients who take responsibility for a partner and perhaps also a family. Further, married clients may be considered as less risky because of the possible dual income available.

The variable Amount of Loan offers interesting findings because of the change in the coefficient's sign for different models. Models 1 and 2, which contain the Own Resources variable, show that small loans appear to be more risky. Contrary to this, when excluding the Own Resources variable as in Model 3, large loans become more risky. The explanation may be that both small loans and large loans when the client owns a low amount of resources are risky. When we account for the client's own resources, we identify a second group of loans (i.e. large loans with the client owning a low amount of resources) and the regression is then able to distinguish small (more risky) loans. However, if we do not have this information, the regression identifies the larger loans as more risky.

The variable termed Points characterizes a client's behavior with respect to the use of his or her current account. It is the behavioral variable constructed by the bank. It quantifies the frequency at which the client deposits money into the account as well as whether the deposits follow a regular pattern. Regularity and higher frequency yield a higher value for Points. This variable showed as significant only in Model 3. There is a relatively high correlation of this variable with the Own Resources variable, which may explain its low predictive power in Models 1 and 2.

The variable Purpose of Loan captures the effect of whether the loan is to be used for simple renovation of a standing housing facility or a new construction. The higher the coefficient is, the greater the probability of default. Hence, a higher coefficient has negative consequences for a client. In our estimation the highest coefficient is recorded for the renovation category and the lowest for the house building category. This means that loans for renovation are in general more risky then those for house construction. The result is in line with observation that the decision to build a house is made mostly by people with more potential to repay their loans as compared to those who renovate older houses.[20]

It is interesting that both credit ratios proved to be non-significant variables. Unfortunately, our dataset does not contain information about the income of applicants, only credit ratios. Because the variables do not have discriminatory power, both can serve only as an initial cut-off criterion to exclude clients whose credibility is very low. Also, variables connected with credit risk mitigation (i.e. the number of co-signers or collateral) were not selected for the final model by the test. This result is unexpected because the existence of collateral is usually a very strong motivation to repay debts. We can only speculate that one of the reasons is that the dataset contains observations of smaller loans (up to 1.5 million CZK), and in the case of default, the bank tries to recover its losses from co-signers rather than by selling collateral.

Our assessment shows that logistic regression can be very successful in creating a powerful model for credit scoring and it is able to capture various features specific to emerging market economies. It is also able to detect the variables with the most discriminating power and combine them so that the bank can detect default behavior in multiple ways that are also partially exclusive.

3.3.2 CART analysis

In this section we provide another analysis of the default behavior of retail clients, using Classification and Regression Trees (CART). The CART tree is a non-parametric approach and consists of several layers of nodes: the first layer consists of a root node and the last layer consists of leaf nodes. Because it is a binary tree, each node (except the leaf nodes) is connected to two nodes in the next layer. The root node contains the entire training set; the other nodes contain subsets of the training set. At each node, the subset is divided into two disjoint groups, based on one specific characteristic x_i from the measurement vector. The split

[20] The recent trend in the Czech Republic is an outflow of city dwellers with higher incomes to new houses built in the suburbs. The decision to renovate older houses is mostly made by people living in the countryside, who tend to have lower incomes.

into two groups is defined by the following inequality: if x_i is an ordinal variable, then the split occurs when $x_i > t$; for some constant threshold t. It follows that an individual j is classified into the right node if the previous statement is true; if not, the individual j is classified into the left node. A similar rule applies when x_i is a categorized variable.

The characteristic x_i is chosen from all possible characteristics and the constant t is chosen such that the resulting sub-samples are as homogeneous in the dependent variable y as possible. In other words, x_i and t are chosen to minimize the diversity of the resulting sub-samples (diversity in this context will be defined presently). The classification process is a recursive procedure that starts at the root node and at each further node (with the exception of leaf nodes) one single characteristic and a splitting rule (or constant t) are selected. First, the best split is found for each characteristic. Then, among these characteristics the one with the best split is chosen. This procedure is replicated until the resulting samples are not homogenous enough. As the trees often become quite large, one needs to simplify them. The procedures that prune the existing trees aim to equalize the classification error in the pruned tree to that in the original tree.

The theory behind CART analysis and some of its applications as a discrimination tool, or pattern recognition technique, can be found in Breiman, Friedman, Olshen and Stone (1984) or Webb (2002). The literature describes many uses of trees in the area of credit scoring.[21] Further, the method has been shown to be very competitive with parametric tools such as logistic regression.[22] Finally, the advantage of CART in credit scoring is that it is very intuitive, easy to explain to management, and able to deal with missing observations.

We use the same short list of variables as in the previous subsection, however there is no need to create categories for the numeric variables. Figure 3.4 illustrates the optimal tree obtained after the pruning procedure that was constructed by using the short list of variables mentioned earlier. The classification rules for each node in Figure 3.4 are described below. All clients that satisfy the classification

[21] As an example, Chandy and Duett (1990) compared CART with logit and LDA and found that these methods are comparable in results to a sample of commercial papers from Moody's and S&P.

[22] See Feldman and Gross (2005), Yeh, Yang and Lee (2007) or Lee, Chiu, Chou and Lu (2006). We acknowledge the fact that CART methodology might be less stable with respect to changes in data than logistic regression (see for example Hastie, Tibshirani and Friedman (2001). However, in our case we obtained very similar results from both types of technique.

rule are assigned to the left child-subtree. For example, in the first node, all observations where Own Resources < 0.385 are assigned to the left child-subtree and all observations where Own Resources > 0.385 are assigned to the right child-subtree. For the finite nodes the classification is "default" or "non-default", based on the actual ratio of defaulted observations in these nodes:

1. Own Resources < 0.385
2. Elementary Education or Secondary Vocational Education
3. Own Resources < 0.345
   (a) Both finite nodes classified as default. There are 714 observations in the left node with 90.9% successful classification. There are 244 observations in the right node with 72.95 % successful classification.
4. Own Resources < 0.025
   (a) Left finite node classified as default. 96 observations with 97.92% successful classification.
5. Years of Having Account smaller or equal to 1 or N/A
   (a) Right finite node classified as non-default. 123 observations with 75.61% successful classification.
6. Purpose of Loan: Purchase of Land or Renovation
   (a) Left finite node classified as default. 336 observations with 73.81% successful classification.
   (b) Right finite node classified as non-default. 144 observations with 55.56% successful classification.
7. Years of Having Account smaller or equal to 1 or N/A
8. Elementary Education or Secondary Vocational Education
9. Own Resources < 0.755
   (a) Right finite node classified as default. 12 observations with 91.67% successful classification.

10. Own Resources < 0.525

    (a) Both nodes classified as non-default. 274 observations in the left node with 54.01% successful classification. 184 observations in the right node with 70.11% successful classification.

11. Purpose of Loan: Purchase of Land, Purchase of House or Renovation

    (a) Both finite nodes classified as non-default. 298 observations in the left node with 70.13% successful classification. 220 observations in the right node with 85.91% successful classification.

12. Amount of Loan < 111.500

    (a) Both finite nodes classified as non-default. 302 observations in the left node with 76.82% successful classification. 456 observations in the right node with 89.04% successful classification.

In Figure 3.5 one can see the ROC plots (yielding the c coefficient) together with the histograms and cumulative distribution functions. The c statistic is $c = 0.830$ for the development sample and $c = 0.815$ for the validation sample. This is a sign that CART can also be very successful in discriminating between default and non-default behavior and thus that it can be used successfully for credit scoring decisions. Another very useful feature of CART is the possibility to use it for sensitivity analysis with respect to different variables. In this respect Own Resources, Education, Years of Having Account, Purpose of Loan and Amount of Loan were identified as the most important variables. These variables play a role at the top nodes and they are identical to those identified by parametric regression. Thus, CART confirmed the selection by logistic regression in the previous subsection.

According to the tree, strong default behavior is connected with the client owning a small amount of resources and having a low level of education. Non-default behavior is linked with the client owning a high amount of resources and having a long-standing relationship with the bank. Both of these predictions are in accord with the selection by logistic regression in the previous subsection.

3.4 Conclusions

In this paper we developed an optimal (in the sense of achieving the highest discriminatory power) specification of the credit scoring model. We employed two approaches: parametric (logistic regression) and non-parametric (Classification and Regression Trees, or CART). Along with analyzing our results we also aimed to assess the determinants of default behavior. Our dataset is rich in socio-demographic and behavioral variables. These variables provide more stable information about client characteristics in times of economic change or financial instability than standard financial variables.

We construct three different models using logistic regression and one model using CART and compare these models in terms of efficiency and power in discriminating between bad and good clients, including out-of-sample testing. We were able to detect the most important financial and behavioral characteristics of default behavior: the amount of resources a client owns, the level of education, marital status, the purpose of the loan, and the years of having an account with the bank. One of our strategic contributions is that in terms of a logistic regression model we identified a specification that does not contain the single most important financial variable (the amount of resources a client owns) but still performs only marginally worse than the specification with this variable. Further, both methods validated similar variables as determinants, which means that both methods are robust and can be used for the delicate task of constructing a credit scoring model interchangeably or complementarily. This is another main contribution of our paper since in practice parametric methods (mostly logistic regression) are used for model construction almost exclusively. This study shows that non-parametric methods can also be successful and are able to create good models. In this respect our analysis is relevant from various perspectives.

This paper contributes to the growing literature on pattern recognition techniques and their use in various fields of economy and finance. We deal with the application scoring model, i.e. we focus only on client characteristics at the time of loan application. This paper shows that socio-demographic variables do have a role in the process of the granting of credit and therefore they should not be excluded from credit scoring model specification. An interesting task would be to assess the efficiency of models based solely on behavioral characteristics (the behavior of the client on his or her current account, the behavior of the client on loans already granted, etc.). Application characteristics are usually not updated during the life of the loan and they grow more imprecise as time elapses. For risk management purposes, such as early warning systems, or managing the current portfolio of loans in general, behavioral models are therefore better. This is left for further research.

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A.1 Figures and Tables

Figures

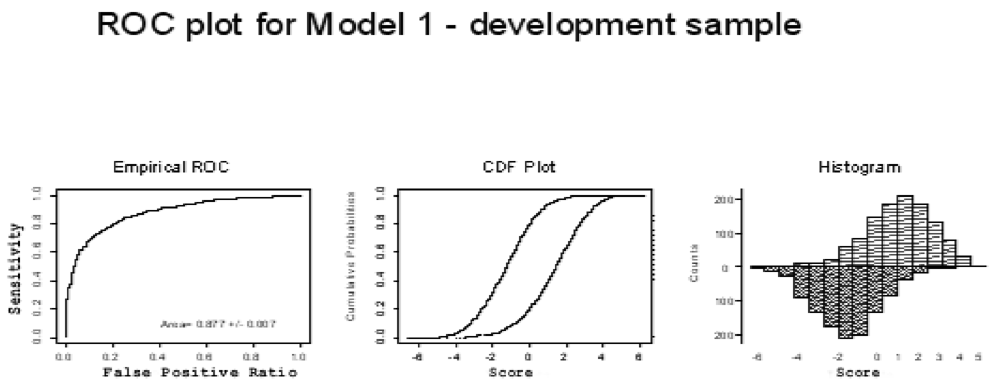

Figure 3.1: ROC curve for Model 1

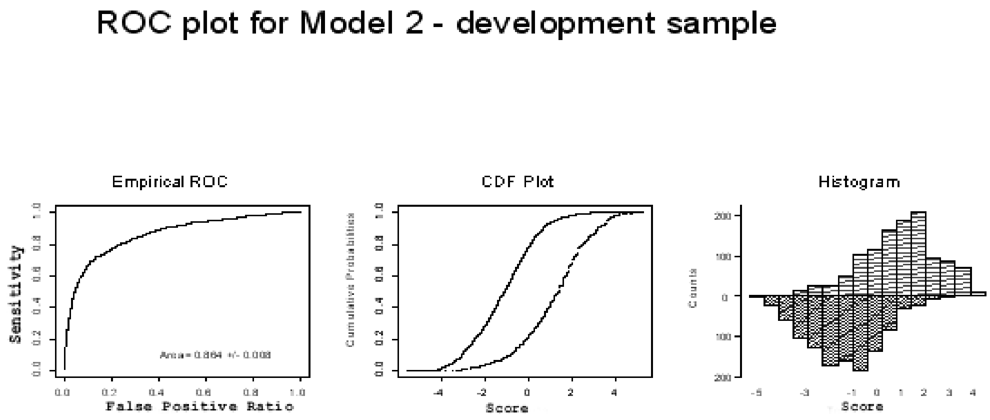

Figure 3.2: ROC curve for Model 2

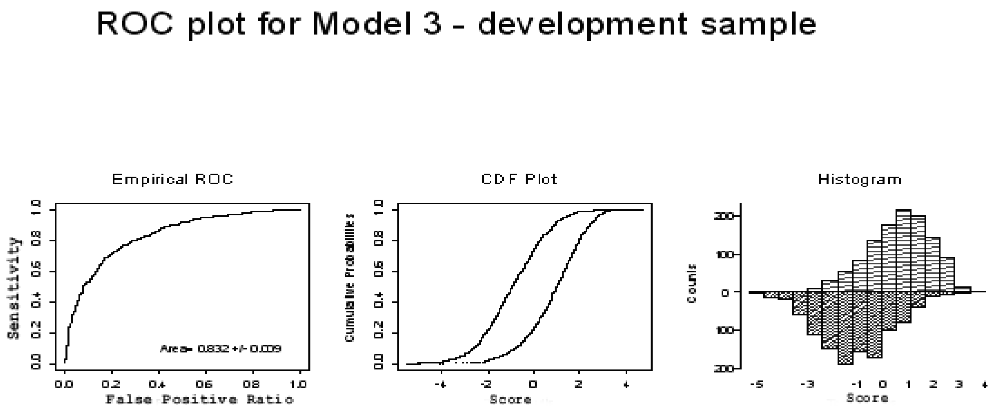

Figure 3.3: ROC curve for Model 3

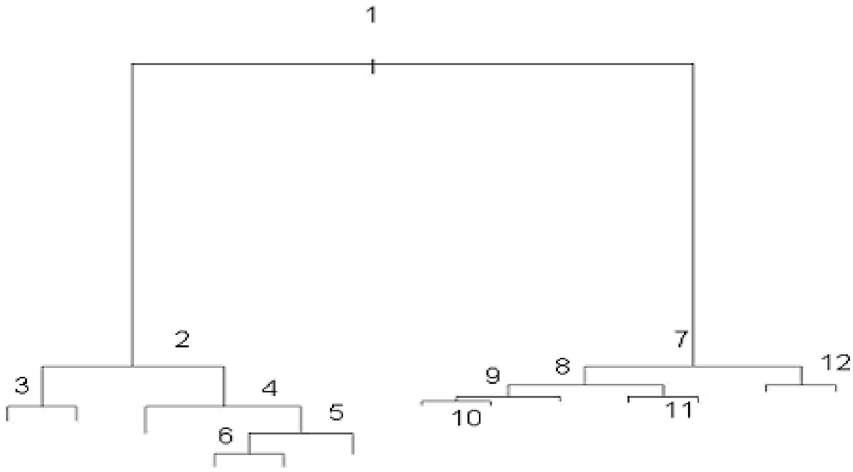

Figure 3.4: The tree model after pruning.

Note: The explanation of classification rules in each node is in the text.

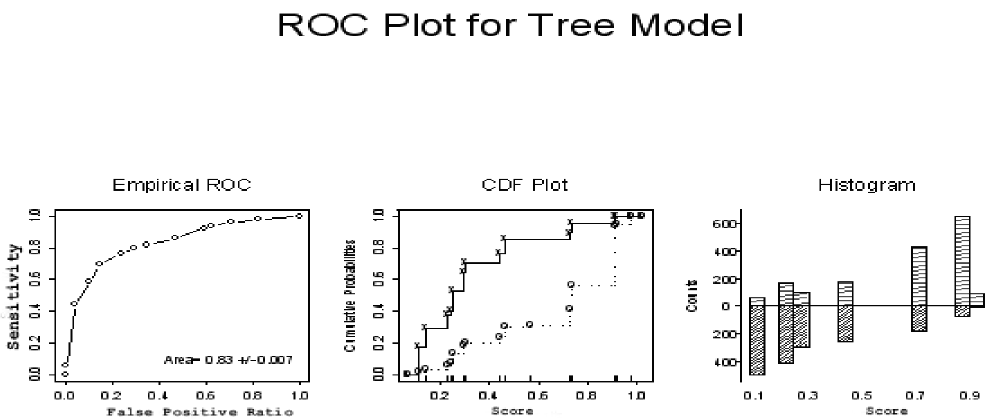

Figure 3.5: ROC plot for the tree model.

Tables

Dependent variable		
Default		Defaulted or not defaulted client
Socio-demographic variables		
Sex	c	Sex of the client, categorized variable
Marital status	c	Status of the client, single/married, categorized variable
Date of Birth		Date of birth of client
Sector of employment	c	The sector in which the client is employed, categorized variable
Type of employment	c	Type of client's employment, categorized variable
Education	c	The highest attained education of client, categorized variable
Number of employments		The total number of employments in the last 3 years
Employment position	c	The position of client in employment, categorized variable
Years of employment		The number of years in the current employment
Credit ratio 1		Ratio of Expenditures/Income of client
Credit ratio 2		Ratio of (Income-Expenditure)/Living Wage of client
Region		Post Code of region of client's address
Bank-client relationship variables		
Type of product		Type of product/loan
Number of co-signers		The Number of co-signers for the current loan
Purpose of loan	c	The declared purpose of loan, categorized variable
Loan Assurance	c	The type of credit risk mitigation, categorized variable
Points		The characteristics of client's behavior on the current account
Own resources		Declared own resources, in percentage of total amount needed
Amount of loan		The total amount of loan granted
Date of account opening		The year when client opened an account in the bank
Date of loan		The year in which the loan was granted
Years of having account		The length of client/bank relationship at the time of loan application

Table 3.1: Variable definitions.

Note : "c" denotes categorized variables.

Variable	Information Value
Own Resources	1.462601
Date of account opening	0.631346
Years of having account	0.601787
Points	0.502122
Education	0.359725
Purpose of loan	0.279959
Years of employment	0.136041
Sector of employment	0.188681
Credit ratio 1	0.175810
Number of co-signers	0.131135
Amount of loan	0.123972
Marital status	0.112809
Region	0.093896
Employment position	0.063872
Type of employment	0.055486
Credit ratio 2	0.052161
Date of Birth	0.047698
Sex	0.039528
Loan Assurance	0.036422
Type of product	0.022380
Number of employments	0.021004

Table 3.2: Information values for variables.

	Value	Coefficient	Std. Error	t value
Intercept		3.78371	0.64390	5.87621
Own resources	0.00+ thru 0.05	reference value		
	0.05+ thru 0.33	-1.54237	0.32630	-4.72682
	0.33+ thru 0.36	-2.29475	0.33569	-6.83584
	0.36+ thru 0.39	-2.87026	0.35403	-8.10729
	0.39+ thru 0.50	-4.02564	0.35085	-11.47404
	0.50+ thru 1.52	-4.64785	0.36855	-12.61131
Education	Elementary	reference value		
	Vocational Education	0.13811	0.26275	0.52564
	Vocational Education with Leaving Exam	-1.27385	0.30249	-4.21123
	Secondary Education	-0.55807	0.27739	-2.01186
	Higher Secondary Education	-1.17440	0.73141	-1.60567
	University Education	-1.44495	0.35028	-4.12518
Years of having account	N/A	reference value		
	0	0.67445	0.30510	2.21062
	0.00+ thru 1	0.32457	0.30735	1.05602
	1.00+ thru 3	-1.09010	0.27888	-3.90892
	3.00+ thru 5	-1.63525	0.26518	-6.16647
	5.00+ thru 10	-1.68684	0.31572	-5.34283
Date of account opening	1993-1995	reference value		
	1996-1997	0.21179	0.25756	0.82228
	1998-1999	-0.17575	0.29988	-0.58609
	2000	-0.45583	0.37718	-1.20851
	2001	-1.23762	0.40064	-3.08911
	2002-2004	-1.84824	0.43655	-4.23372
Purpose of loan	Building of House	reference value		
	Purchase of Apartment	0.57782	0.36337	1.59015
	Purchase of Land	0.68067	0.66512	1.02338
	Purchase of House	0.51811	0.38151	1.35805
	Renovation	0.99526	0.34190	2.91095
	Rest	0.07332	0.37016	0.19807
	N/A	0.27270	0.41299	0.66031
Marital Status	Married	reference value		
	Single	0.45971	0.11689	3.93290
Years of employment	0+ thru 4	reference value		
	4+ thru 5	0.31437	0.20178	1.55793
	5+ thru 6	-0.07598	0.23656	-0.32121
	6+ thru 9	-0.06273	0.16260	-0.38577
	9+ thru 14	-0.18129	0.17992	-1.00761
	14+ thru 60	-0.90223	0.22746	-3.96659
Sector of employment	Building Industry	reference value		
	Mining	0.75255	0.57887	1.30003
	Education	-0.68439	0.41070	-1.66641
	Energy- and Water-supply	-0.40454	0.49881	-0.81101
	Financial Services	-1.08128	0.57359	-1.88510
	Gastronomy and Lodging	0.23238	0.35022	0.66353
	Health Service	-0.14517	0.36312	-0.39980
	Trade	0.08452	0.23730	0.35619
	Agriculture und Forestry	0.07997	0.41040	0.19485
	Communications	-0.28384	0.28931	-0.98108
	N/A	-0.69468	0.36965	-1.87931
	Other Business	0.34166	0.24870	1.37379
	Public Services	-0.32983	0.23067	-1.42986
Points	0.0+ thru 1.0	reference value		
	1.0+ thru 28.0	-0.51537	0.20319	-2.53635
	28.0+ thru 363.0	-0.18748	0.14919	-1.25669
	363.0+ thru 1401.0	0.01587	0.19400	0.08179
Amount of loan	2489+ thru 50000	reference value		
	50000+ thru 69000	0.19988	0.27334	0.73125
	69000+ thru 100000	0.08803	0.19806	0.44446
	100000+ thru 200000	-0.40900	0.20303	-2.01446
	200000+ thru 250000	-0.22937	0.24109	-0.95137
	250000+ thru 1500000	-0.08822	0.21776	-0.40512

Table 3.3: Coefficients for Model 1.

Note: AIC= 2119.02

	Value	Coefficient	Std. Error	t value
Intercept		4.56228	0.51011	8.94381
Own resources	0.00+ thru 0.05	reference value		
	0.05+ thru 0.33	-1.51356	0.31954	-4.73668
	0.33+ thru 0.36	-2.30000	0.32865	-6.99829
	0.36+ thru 0.39	-2.93355	0.34589	-8.48109
	0.39+ thru 0.50	-4.19918	0.34411	-12.20293
	0.50+ thru 1.52	-4.85161	0.36079	-13.44702
Education	Elementary	reference value		
	Vocational Education	0.04582	0.24896	0.18404
	Vocational Education with Leaving Exam	-1.34695	0.28521	-4.72262
	Secondary education	-0.80089	0.25739	-3.11154
	Higher Secondary Education	-1.58778	0.70190	-2.26213
	University Education	-1.76433	0.32876	-5.36660
Years of having account	N/A	reference value		
	0	0.84966	0.29498	2.88041
	0.00+ thru 1	0.42240	0.29531	1.43036
	1.00+ thru 3	-0.91298	0.26804	-3.40609
	3.00+ thru 5	-1.55988	0.25746	-6.05862
	5.00+ thru 10	-1.63651	0.30610	-5.34632
Date of account opening	1993-1995	reference value		
	1996-1997	0.10116	0.24997	0.40468
	1998-1999	-0.31016	0.29192	-1.06248
	2000	-0.62740	0.36594	-1.71450
	2001	-1.43871	0.38669	-3.72053
	2002-2004	-2.00568	0.42097	-4.76445
Marital Status	Married	reference value		
	Single	0.43446	0.11185	3.88427
Amount of loan	2489+ thru 50000	reference value		
	50000+ thru 69000	0.30255	0.26348	1.14829
	69000+ thru 100000	0.23203	0.19109	1.21423
	100000+ thru 200000	-0.38896	0.19412	-2.00365
	200000+ thru 250000	-0.27958	0.22967	-1.21730
	250000+ thru 1500000	-0.09691	0.20469	-0.47345
Points	0.0+ thru 1.0	reference value		
	1.0+ thru 28.0	-0.51402	0.19763	-2.60091
	28.0+ thru 363.0	-0.25143	0.14331	-1.75441
	363.0+ thru 1401.0	-0.02252	0.18889	-0.11922

Table 3.4: Coefficients for Model 2.

Note: AIC= 2164.82

	Value	Coefficient	Std. Error	t value
Intercept		-0.59168	0.47774	-1.23850
Date of account opening	1993-1995	reference value		
	1996-1997	0.55709	0.23483	2.37227
	1998-1999	0.66359	0.26747	2.48099
	2000	0.71870	0.33520	2.14411
	2001	0.55238	0.34562	1.59821
	2002-2004	1.14773	0.35307	3.25069
Education	Elementary	reference value		
	Vocational Education	0.07169	0.23390	0.30648
	Vocational Education with Leaving Exam	-1.40647	0.26712	-5.26538
	Secondary education	-0.85965	0.24180	-3.55521
	Higher Secondary Education	-1.47476	0.69827	-2.11202
	University Education	-1.64829	0.30919	-5.33104
Purpose of loan	Building of House	reference value		
	Purchase of Apartment	0.84813	0.34856	2.43326
	Purchase of Land	0.81182	0.56542	1.43578
	Purchase of House	0.81916	0.36438	2.24807
	Renovation	1.54520	0.32986	4.68444
	Rest	0.35889	0.35419	1.01327
	N/A	0.40644	0.39853	1.01987
Points	0.0+ thru 1.0	reference value		
	1.0+ thru 28.0	-0.71267	0.17700	-4.02641
	28.0+ thru 363.0	-0.82731	0.13231	-6.25299
	363.0+ thru 1401.0	-0.87127	0.16936	-5.14450
Marital Status	Married	reference value		
	Single	0.50590	0.10608	4.76919
Years of having account	N/A	reference value		
	0	-0.29791	0.26704	-1.11563
	0.00+ thru 1	-0.29920	0.27439	-1.09040
	1.00+ thru 3	-1.08482	0.24593	-4.41101
	3.00+ thru 5	-1.34039	0.24019	-5.58059
	5.00+ thru 10	-0.76584	0.26993	-2.83722
Years of employment	0+ thru 4	reference value		
	4+ thru 5	0.25759	0.18265	1.41030
	5+ thru 6	0.02235	0.21297	0.10496
	6+ thru 9	-0.12660	0.14386	-0.88003
	9+ thru 14	-0.26489	0.16047	-1.65074
	14+ thru 60	-0.89137	0.19813	-4.49898
Amount of loan	2489+ thru 50000	reference value		
	50000+ thru 69000	0.03081	0.24944	0.12351
	69000+ thru 100000	-0.01095	0.17532	-0.06245
	100000+ thru 200000	-0.08396	0.17787	-0.47203
	200000+ thru 250000	0.48678	0.20739	2.34718
	250000+ thru 1500000	0.54034	0.18367	2.94193

Table 3.5: Coefficients for Model 3.

Note: AIC= 2430.41

		Development	Validation
Model 1	c	0.877	0.869
	GINI	0.754	0.738
Model 2	c	0.864	0.855
	GINI	0.728	0.71
Model 3	c	0.832	0.814
	GINI	0.664	0.628

Table 3.6: Stability of the models