Physica B 430 (2013) 95–98

Contents lists available at ScienceDirect

Physica B

journal homepage: www.elsevier.com/locate/physb

A hybrid model for the play hysteresis operator

Mohammad Al Janaideh [a,*], Roberto Naldi [b], Lorenzo Marconi [b], Pavel Krejčí [c]

[a] Department of Mechatronics Engineering, The University of Jordan, 11942 Amman, Jordan
[b] CASY Center for Research on Complex Automated Systems, Department of Electronics, Computer Science and Systems (DEIS), University of Bologna, Via Risorgimento 2, 40136 Bologna, Italy
[c] Mathematical Institute, Academy of Sciences of the Czech Republic, Žitná 25, CZ-11567 Praha 1, Czech Republic

ARTICLE INFO

Article history:
Received 24 April 2013
Received in revised form
22 June 2013
Accepted 3 July 2013
Available online 10 July 2013

Keywords:
Hysteresis
Hybrid
Play

ABSTRACT

A hybrid model is proposed for characterization of the hysteresis nonlinearity of the well-known play operator. The proposed model holds the hysteresis nonlinearity and the memory effects of the play operator. Simulation results are also presented to show the capability of the hybrid model to present the hysteresis nonlinearity with memory effects.

1. Introduction

Smart actuators and ferromagnetic materials invariably exhibit hysteresis, which is a path-dependent memory effect where the output relies not only on the current state but also on the past output history [1–8]. The presence of the hysteresis in smart actuators, such as piezoceramic, magnetostrictive and shape memory alloy (SMA) actuators has been widely associated with various performance limitations. These include the oscillations in the responses of the open-as well as closed-loop systems, and poor tracking performance and potential instabilities in the closed-loop system [5–12]. Considerable continuing efforts are thus being made to seek methods for effective compensation of hysteresis effects in order to enhance the tracking performance of smart actuators. The characterization and modeling of the hysteresis properties of smart actuators, however, is vital for designing efficient compensation algorithms.

A number of models have been reported to describe hysteresis properties in different smart actuators and ferromagnetic materials. The reported hysteresis models may be classified into physics-based models and phenomenological models [5]. The physics-based models are generally derived on the basis of a physical measure, such as energy, displacement, or stress–strain relationship. Phenomenological models include the Preisach model and the Prandtl–Ishlinskii model have been widely applied for modeling and compensation of hysteresis effects, see for example Refs. [2,3,13–15]. In this paper, a hybrid model is proposed for characterization of the hysteresis nonlinearity of the well-known play operator.

2. The play operator

In this section, we give an alternative characterization in terms of hybrid systems of the play operator, which is the main building block of all rate independent hysteresis models with return point memory (cf. [2]), in particular the Preisach and the Prandtl–Ishlinskii models.

2.1. Classical approach

According to Ref. [4], the output of the play operator $\Gamma_r[v,\eta](t)$ with threshold $r>0$ and initial condition $\eta\in[-r,r]$ is defined first for continuous inputs $v(t)$ that are monotone (non-decreasing or non-increasing) in each interval $t\in[t_{i-1},t_i]$ of a partition $0=t_0< \ldots <t_m=T$ by the formula

$$\Gamma_r[v,\eta](t)=\max\{v(t)-r,\min\{v(t)+r,\Gamma_r[v,\eta](t_{i-1})\}\} \quad (1)$$

for $t\in[t_{i-1},t_i]$, with initial condition $\Gamma_r[v,\eta](0)=v(0)-\eta$. Fig. 1 shows a digram for the play operator.

The definition is then extended to the whole space $C[0,T]$ of continuous functions by density, see Ref. [4]. The argument of the operator is written in square brackets to indicate the functional dependence, since it maps a function to another function. The play operator can be visualized by the motion of a piston within a cylinder of length $2r$. The position of the center of the piston

* Corresponding author.
E-mail addresses: aljanaideh@gmail.com (M. Al Janaideh), roberto.naldi@unibo.it (R. Naldi), lorenzo.marconi@unibo.it (L. Marconi), krejci@math.cas.cz (P. Krejčí).

0921-4526/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.physb.2013.07.002

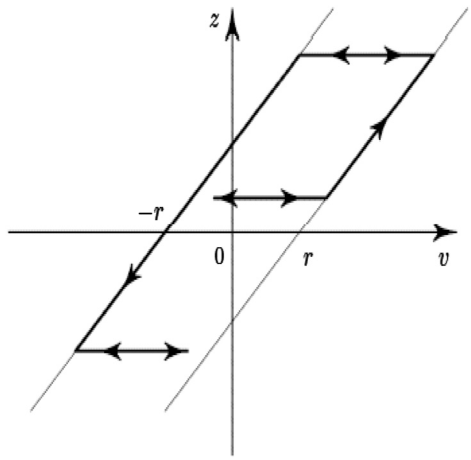

Fig. 1. A diagram of the play operator.

is represented by the coordinate $v(t)$, while the cylinder position is given by $\Gamma_r[v,\eta](t)$.

An equivalent variational approach, see Ref. [14], defines the play as the solution operator, which with a given absolutely continuous function $v:[0,T]\to\mathbb{R}$ and initial condition $\eta\in[-r,r]$ associates the solution $z(t)=\Gamma_r[v,\eta](t)$ of the variational inequality

$$\left.\begin{array}{l} |v-z|\le r, \\ z(0)=v(0)-\eta, \\ \dot z(t)(v(t)-z(t)-\rho)\ge 0 \quad \forall\rho\in[-r,r], \end{array}\right\} \tag{2}$$

to be satisfied almost everywhere in $(0,T)$.

2.2. Hybrid dynamical systems

Hybrid systems are a special class of dynamical systems that exhibit characteristics of both continuous time and discrete time systems. In this work, in particular, we consider the notion of hybrid systems given in Refs. [16,18]. Driven by Ref. [18], a hybrid system $\mathcal{H}$ is given by

$$\mathcal{H}\quad\begin{cases} \dot x = f(x,u) & (x,u)\in C \\ x^+ = g(x,u) & (x,u)\in D \\ y = h(x,u) \end{cases} \tag{3}$$

with state $x\in\mathbb{R}^n$, input $u\in\mathbb{R}^m$ and output $y\in\mathbb{R}^p$. The sets $C\subset\mathbb{R}^n\times\mathbb{R}^m$ and $D\subset\mathbb{R}^n\times\mathbb{R}^m$ define the flow and jump sets, respectively; the maps $f:\mathbb{R}^n\times\mathbb{R}^m\to\mathbb{R}^n$ and $g:\mathbb{R}^n\times\mathbb{R}^m\to\mathbb{R}^n$ define the flow map and jump map, respectively. Finally, the function $h:\mathbb{R}^n\times\mathbb{R}^m\to\mathbb{R}^p$ defines the output y.

Solutions to hybrid systems $\mathcal{H}$ are given by pairs of *hybrid arcs* and *hybrid inputs* defined over extended time domains called *hybrid time domains*. A set $S\subset\mathbb{R}_{\ge 0}\times\mathbb{N}$ is a hybrid time domain if for all $(T,J)\in S$, the set $S\cap([0,T]\times\{0,1,\dots J\})$ can be written as

$$\bigcup_{j=0}^{J-1}([t_j,t_{j+1}],j)$$

for some finite sequence of times $0=t_0\le t_1\le t_2\dots\le t_J$. A hybrid arc $x:\mathrm{dom}\,x\to\mathbb{R}^n$ is such that $\mathrm{dom}\,x$ is a hybrid time domain and $t\mapsto x(t,j)$ is absolutely continuous on the interval $\{t(t,j)\in\mathrm{dom}x\}$. A hybrid arc is parameterized by (t,j), where t is the ordinary-time component and j is the discrete-time component that keeps track of the number of jumps. A hybrid input $u:\mathrm{dom}\,u\to\mathbb{R}^m$ is such that $\mathrm{dom}\,u$ is a hybrid time domain and, for each $j\in\mathbb{N}$, the function $t\mapsto v(t,j)$ is Lebesgue measurable and locally essentially bounded on the interval $\{t:(t,j)\in\mathrm{dom}\,u\}$. Then, given a hybrid input $v:\mathrm{dom}\,u\to\mathbb{R}^m$ and an initial condition ξ, a hybrid arc $\phi:\mathrm{dom}\,\phi\to\mathbb{R}^n$ defines a *solution pair* (ϕ,u) *to the hybrid system* $\mathcal{H}$ in (3) if $\phi(0,0)\in\overline{C}\cup D$, where $\overline{C}$ denotes the closure of the set C, and the following conditions hold:

(S1) For each $j\in\mathbb{N}$ such that
$I_j:=\{t:(t,j)\in\mathrm{dom}\,(\phi,u)\}$ has nonempty interior $\mathrm{int}(I_j)$,
$(\phi(t,j),u(t,j))\in C$ for all $t\in\mathrm{int}(I_j)$,
and, for almost all $t\in I_j$,
$(d/dt)\phi(t,j)=f(\phi(t,j),u(t,j))$;

(S2) For each $(t,j)\in\mathrm{dom}\,(\phi,u)$ such that $(t,j+1)\in\mathrm{dom}\,(\phi,u)$, $(\phi(t,j),u(t,j))\in D$, $\phi(t,j+1)=g(\phi(t,j),u(t,j))$.

2.3. A hybrid model of the play operator

Let us consider a hybrid system $\mathcal{H}_P$ with state $x=[q\ m]^T$ in which $q\in\{-1,0,1\}$ is a discrete state and $m\in\mathbb{R}$ is the *memory*, input $u\in\mathbb{R}^2$ given by $u=[v\ \dot v]^T$ and output $w\in\mathbb{R}$ described by the following equations:

$$\mathcal{H}_P\quad\begin{cases} \dot x = 0 & (x,u)\in C \\ x^+ = \begin{bmatrix} q-\mathrm{sgn}(\dot v) \\ |q|v+rq \end{bmatrix} & (x,u)\in D \\ w=|q|v+rq+(1-|q|)m \end{cases} \tag{4}$$

in which $r>0$ is the threshold,

$$\begin{aligned} &C=\{(x,u)\in\{-1,0,1\}\times\mathbb{R}\times\mathbb{R}^2\} \\ &D=\{(x,u)\in\{-1,0,1\}\times\mathbb{R}\times\mathbb{R}^2: \\ &\quad(v\ge m+r \text{ and } q=0 \text{ and } \dot v\ge 0)\text{or} \\ &\quad(v\le m-r \text{ and } q=0 \text{ and } \dot v\le 0)\text{or} \\ &\quad(q\ne 0 \text{ and } q\dot v>0)\} \end{aligned} \tag{5}$$

are the flow and jump domains and the function $\mathrm{sgn}:\mathbb{R}\to\{-1,0,1\}$ is the standard *sign* function defined as

$$\mathrm{sgn}(s)=\begin{cases} 1 & s>0 \\ 0 & s=0 \\ -1 & s<0. \end{cases}$$

Observe that, in the above hybrid model, the input u includes both the standard continuous input v and its first order derivative $\dot v$ which should then be available to properly define the system $\mathcal{H}_P$.

The output of the hybrid system $\mathcal{H}_P$ can be shown to match the output of the operator provided that the internal state of the hybrid model satisfies some conditions. This is precisely stated by means of the following result.

Theorem 1. *Let $\Gamma_r[v,\eta]$ be the play operator defined by (1), and let w be the output of the hybrid system $\mathcal{H}_P$ defined in (4) with initial states $q(0)$, $m(0)$ such that $v(0)-w(0)=\eta$. Assume that $\dot v(t)$ is a left continuous piecewise constant function, that is,*

$$\dot v(t)=u_i \text{ for } t\in(t_{i-1},t_i],\ i=1,\dots,h, \tag{6}$$

continuously extended to t=0, where $0=t_0<\dots<t_h=T$ is a fixed partition of $[0,T]$, *and $u_1,\dots,u_h$ are fixed constants. Then*

$$w(t)=z(t)\ :=\Gamma_r[v,\eta](t)\quad\forall t\in[0,T].$$

Proof. By the choice of initial conditions, we have $w(0)=z(0)$. We continue by induction over $i=1,\dots,h$. Assuming that $w(t_{i-1})=z(t_{i-1})$ for some i, we check that $w(t_i)=z(t_i)$. This is obvious if $\dot v_i=0$. Then w is in the flow regime and all state variables q, m and the input v are constant, hence $w(t)$ is constant in the whole interval $[t_{i-1},t_i]$ and remains equal to $z(t)$ by virtue of (6).

Assume now that $u_i > 0$. We have by (6) for $t \in [t_{i-1}, t_i]$ that

$$z(t) = \max\{z(t_{i-1}), v(t) - r\}. \quad (7)$$

We now classify all possible cases by the value of $q(t_{i-1})$.

(i) $q(t_{i-1}) = -1$. Then $w(t_{i-1}) = z(t_{i-1}) = v(t_{i-1}) - r$, w is in the flow regime, and we conclude that $w(t) = z(t) = v(t) - r$ for all $t \in [t_{i-1}, t_i]$.

(ii) $q(t_{i-1}) = 1$. Then w is in the jump regime, and we have $v(t_{i-1}+) = v(t_{i-1})$, $q(t_{i-1}+) = 0$, $m(t_{i-1}+) = v(t_{i-1}) + r$, and $w(t_{i-1}+) = w(t_{i-1}) = v(t_{i-1}) + r$. For $t > t_{i-1}$, w is in the flow regime as long as $v(t) - m(t_{i-1}+) < r$ (note that v is increasing in $[t_{i-1}, t_i]$). Hence, $w(t) = z(t) = v(t_{i-1}) + r$ as long as $v(t) - r < v(t_{i-1})$. If $v(t^*) - r = v(t_{i-1})$ for some $t^* \in [t_{i-1}, t_i)$, then w switches to the jump regime at $t = t^*$ with $v(t^*+) = v(t^*)$, $q(t^*+) = -1$, $m(t^*+) = 0$, and we argue as in (i).

(iii) $q(t_{i-1}) = 0$. Then $w(t_{i-1}) = z(t_{i-1}) = m(t_{i-1})$. Similarly as in (ii), w is in the flow regime as long as $v(t) - m(t_{i-1}) < r$. The same argument as in (ii) yields the assertion.

Repeating the same procedure for $u_i < 0$, we complete the proof. □

Remark. For $r = 0$, the play operator reduces to the identity mapping $\Gamma_0[v, \eta] = v$. This is true also for the hybrid operator (4). □

Remark. As a consequence of Theorem 1, we see that the hybrid operator is a rate-independent hysteresis operator, that is, the values of the output do not depend on the input speed. Furthermore, Fig. 1 illustrates the meaning of the state variables q and m: $q = 1$ ($q = -1$) characterizes the irreversible motion along the descending (ascending) branch $z = v + r$ ($z = v - r$), $q = 0$ corresponds to the reversible horizontal motion inside the dead region $v - r < z < v + r$. The value of m keeps the information about the last motion of z (return point!). □

Remark. For the proposed hybrid dynamical model of hysteresis, the requirement on the knowledge of both the input v and its first order derivative $\dot{v}$ can be satisfied by means of a dynamic extension of the input v itself. In particular by defining a new input $u' \in \mathbb{R}$ and new continuous state $v' \in \mathbb{R}$, it is possible to add the continuous dynamics $\dot{v}' = u'$ to the hybrid system and use the state v' as the input for the hysteresis operator. □

3. Numerical example

In this example we compare the output of the play operator z (1) and output of the proposed hybrid system w (4) considering the harmonic input $v(t) = \sin(t) + 0.5\sin(2.4t)$, where $t \in [0, 20]$, and the threshold $r = 0.5$. The initial conditions are $z(0) = w(0) = -0.4$, $m(0) = -0.4$, and $q(0) = 0$. The selected input signals $v(t)$ can show minor reversible branches in the input-output characteristics of the play operator. Fig. 2 shows the time history of q of the hybrid hysteresis operator. As shown in Figs. 3 and 4, the hybrid operator and the play operator have undistinguishable outputs. In Fig. 5, in order to show the rate-independency of the hybrid operator, we use the input $v(t) = \sin(10t) + 0.5\sin(24t)$, where $t \in [0, 2]$. Fig. 6 shows the output of the hybrid operator with $r = 0$. The simulation results in this example are carried out using

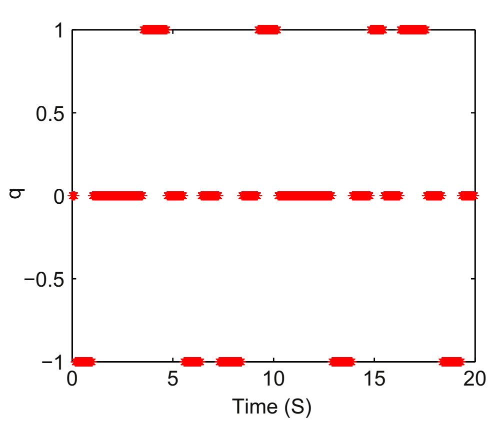

Fig. 2. Shows the time history of q of the hybrid hysteresis operator.

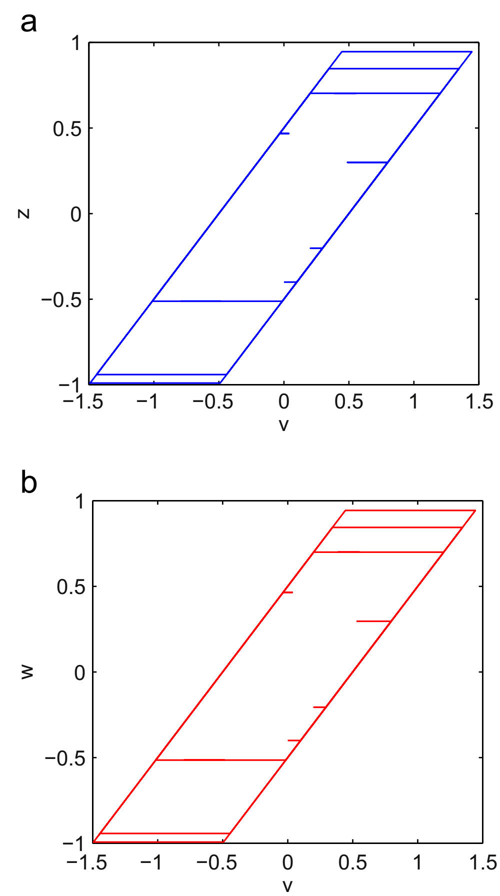

Fig. 3. (a) Shows the output of the play operator z (1) with $r = 0.5$ and $v(t) = \sin(t) + 0.5\sin(2.4t)$, (b) shows the output of the hybrid hysteresis operator w defined in (4) with $r = 0.5$ and $v(t) = \sin(t) + 0.5\sin(2.4t)$.

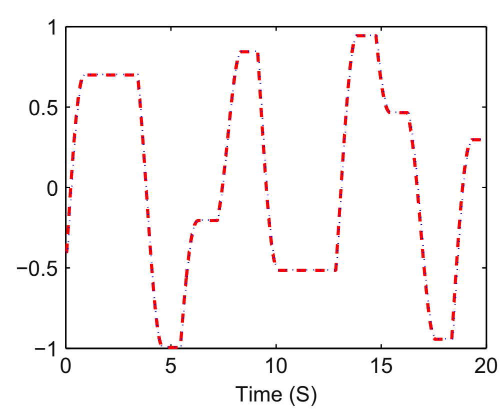

Fig. 4. Shows the time history of outputs $z(t)$ (dotted line) and $w(t)$ (dashed line).

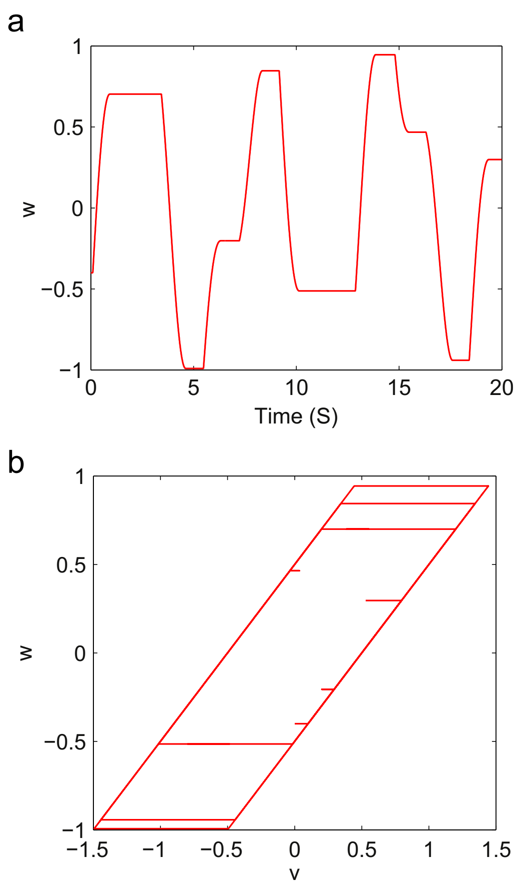

Fig. 5. (a) Shows the time history of the output w with $r=0.5$ (1) and $v(t)=\sin(10t)+0.5\sin(24t)$, (b) shows the input–output relationship for the hybrid hysteresis operator presented in (a).

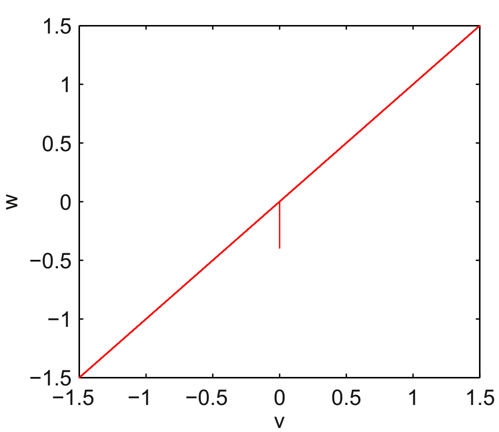

Fig. 6. Shows the output of the hybrid hysteresis operator with $v(t)=\sin(t)+0.5\sin(2.4t)$ and $r=0$.

the Hybrid Simulator V-6 available in [17] for hybrid systems defined as in [16].

4. Conclusion

This work has shown how hybrid dynamical systems can be employed to characterize a class of hysteresis nonlinearities with memory effects. The proposed hybrid model in particular is proved to be able to represent the well-known play hysteresis operator.

Acknowledgment

Supported by GACR Grant P201/10/2315 and RVO: 67985840. The work of R. Naldi and L. Marconi is framed within the collaborative project SHERPA (ICT 600958) supported by the European Community under the 7th Framework Programme.

References

[1] C. Visone, J. Phys. Conf. Ser. 138 (2008).
[2] M. Brokate, J. Sprekels, Hysteresis and Phase Transitions, Springer, New York, NY, 1994.
[3] I.D. Mayergoyz, Mathematical Models of Hysteresis, Elsevier, New York, NY, 2003.
[4] M. Krasnoselskii, A. Pokrovskii, Systems with Hysteresis, Springer-Verlarg, Nauka, Moscow 1983, 1989.
[5] R.C. Smith, Smart Material Systems, Springer-Verlag, Philadelphia, PA, 2005.
[6] C. Visone, M. Sjöström, Phys. B 343 (2004) (2004) 148.
[7] L. Riccardi, D. Naso, B. Turchiano, H. Janocha, D. Palagachev, in: The Proceedings of the American Control Conference, 2012, pp. 1670–1675.
[8] M. Al Janaideh, S. Rakheja, C-Y. Su, Mechatronics 19 (2009) 656.
[9] M. Al Janaideh, S. Rakheja, C-Y. Su, IEEE/ASME Trans. Mechatronics 16 (2011) 734.
[10] X. Tan, J. Baras, Automatica 40 (2004) 1469.
[11] D. Davino, A. Giustiniani, C. Visone, Phys. B 407 (2012) 1433.
[12] G. Tao, P.V. Kokotović, Adaptive Control of Systems with Actuator and Sensor Nonlinearities, Wiley, New York, NY, 1996.
[13] P. Krejčí, M. Al Janaideh, F. Deasy, Phys. B 407 (2012) 1354.
[14] A. Visintin, Differential Models of Hysteresis, Springer-Verlag, New York, NY, 1994.
[15] M. Al Janaideh, P. Krejčí, Phys. B 406 (2011) 1528.
[16] R. Goebel, R.G. Sanfelice, A.R. Teel, IEEE Control Syst. Mag. 29 (2009) 28.
[17] R.G. Sanfelice, Hybrid Simulator, 〈http://www.u.arizona.edu/~sricardo/index.php?n=Main.Software〉, 2011.
[18] R. Goebel, R.G. Sanfelice, A.R. Teel, Hybrid Dynamical Systems Modeling, Stability, and Robustness, Princeton University Press, 2012.