Inverse Rate-Dependent Prandtl–Ishlinskii Model for Feedforward Compensation of Hysteresis in a Piezomicropositioning Actuator

Mohammad Al Janaideh and Pavel Krejčí

***Abstract*—Piezomicropositioning actuators, which are widely used in micropositioning applications, exhibit strong rate-dependent hysteresis nonlinearities that affect the accuracy of these micropositioning systems when used in open-loop control systems, and may also even lead to system instability of closed-loop control systems. Feedback control techniques could compensate for the rate-dependent hysteresis in piezomicropositioning actuators. However, accurate sensors over a wide range of excitation frequencies and the feedback control techniques inserted in the closed-loop control systems may limit the use of the piezomicropositioning and nanopositioning systems in different micropositioning and nanopositioning applications. We show that open-loop control techniques, also called feedforward techniques, can compensate for rate-dependent hysteresis nonlinearities over different excitation frequencies. An inverse rate-dependent Prandtl–Ishlinskii model is utilized for feedforward compensation of the rate-dependent hysteresis nonlinearities in a piezomicropositioning stage. The exact inversion of the rate-dependent model holds under the condition that the distances between the thresholds do not decrease in time. The inverse of the rate-dependent model is applied as a feedforward compensator to compensate for the rate-dependent hysteresis nonlinearities of a piezomicropositioning actuator at a range of different excitation frequencies between 0.05–100 Hz. The results show that the inverse compensator suppresses the rate-dependent hysteresis nonlinearities, and the maximum positioning error in the output displacement at different excitation frequencies.**

***Index Terms*—Hysteresis, inverse control, piezomicropositioning actuator, Prandtl–Ishlinskii, rate dependent.**

I. Introduction

PIEZOCERAMIC actuators are becoming increasingly popular for use in micropositioning and nanopositioning applications because of a number of advantages which include nanometer resolution, high stiffness, and fast response. These smart actuators have been used in different applications such as micromachining [1], [2], observing, and manipulating objects at the nanoscale [3], [4], design and control of smart micropositioning and nanopositioning systems [5]–[7], and vibration control [8], [9]. However, piezomicropositioning actuators exhibit hysteresis nonlinearities between the applied input voltage and output displacement [10]. These nonlinearities have been associated with oscillations in the open-loop system's responses, and poor-tracking performance and potential instabilities in the closed-loop system [11]. Different piezomicropositioning actuators show rate-dependent hysteresis nonlinearities when the excitation frequency of the applied input voltage increases, see for example [12]–[16].

A number of rate-independent hysteresis models have been proposed to characterize hysteresis nonlinearities in piezomicropositioning actuators. These models include: the Preisach model, the Prandtl–Ishlinskii model, and Bouc–Wen model. Such models have been widely applied for modeling and compensation of hysteresis effects. Feedback control systems have been proposed with inverse rate-independent hysteresis models to compensate for hysteresis nonlinearities at different excitation frequencies. Ge and Jouaneh [17] used an inverse Preisach model, which was obtained using a numerical algorithm, as a feedforward compensator with a PID feedback control system to reduce the hysteresis nonlinearities in a piezoceramic actuator. Hu *et al.* [18] applied an inverse Preisach model with a dynamic density function in a closed-loop control system to compensate for hysteresis nonlinearities. In a similar manner, Song *et al.* [19] applied the inverse Preisach model to compensate the hysteresis nonlinearities in a piezoceramic actuator with PD-lag and PD-lead controllers in a closed-loop control system. Reduction in hysteresis nonlinearities was demonstrated experimentally for both major and minor hysteresis loops under low excitation frequencies. Esbrook *et al.* [20] applied a servocompensator with an inverse Prandtl–Ishlinskii model in a closed-loop control system to compensate for hysteresis in a micropositioning piezo actuator at different excitation frequencies. Shan and Leang [21] applied a discrete-time repetitive controller combined with an inverse hysteresis compensator based on the Prandtl–Ishlinskii model to minimize the hysteresis effects at different excitation frequencies. Differential equation based hysteresis models such as Bouc–Wen and Dahl models have been also applied with a repetitive proportional integral–derivative feedback control algorithm to compensate for hysteresis nonlinearities in piezoceramic actuators [22]. Ang *et al.* [23] applied the inversion of a modified Prandtl–Ishlinskii model as a feedforward compensator to compensate for hysteresis nonlinearities under different excitation frequencies. In [24], the charge control linearizes the piezoceramic system, and the feedforward controller is applied to invert the linear dynamics of the system.

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M. Al Janaideh is with the Department of Mechatronics Engineering, University of Jordan, Amman 11942, Jordan (e-mail: aljanaideh@gmail.com).

P. Krejčí is with the Mathematical Institute, Academy of Sciences of the Czech Republic, 11720 Prague, Czech Republic (e-mail: krejci@math.cas.cz).

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In this study, we propose novel and crucial compensation algorithm to compensate for rate-dependent hysteresis in piezomicropositioning actuators at different excitation frequencies. Rate-dependent hysteresis nonlinearities are characterized using the rate-dependent Prandtl–Ishlinskii model. The analytical inverse of the rate-dependent Prandtl–Ishlinskii model is formulated and applied as a feedforward compensator to compensate for the rate-dependent hysteresis nonlinearities in a piezomicropositioning actuator. The main advantage of the rate-dependent Prandtl–Ishlinskii model over the other hysteresis models used in the literature is that its inverse can be attained analytically, and it can be implemented as a feedforward compensator to control the piezomicropositioning actuator over different excitation frequencies without necessitating the use of feedback control techniques.

In [25], we present the analytical inverse of the Prandtl–Ishlinskii model constructed with time-dependent thresholds. The explicit inversion formula for the Prandtl–Ishlinskii model presented in [26] remains applicable also for the case of time-dependent thresholds, provided the distances between them do not decrease in time. In this paper, we use this inverse as a feedforward controller in order to compensate for the rate-dependent hysteresis nonlinearities in a piezomicropositioning actuator. We demonstrate experimentally the effectiveness of this technique by carrying out control experiments on a piezomicropositioning actuator.

The contents of the paper are as follows. A background is presented in Section II. Rate-dependent Prandtl–Ishlinskii model and its inverse are presented in Section III, where we also show the conditions necessary to obtain exact inversion of the model. In Section IV, parameter identification of the rate-dependent Prandtl–Ishlinskii model and its inverse are presented. In Sections V, compensation of the rate-dependent hysteresis nonlinearities is carried out at different excitation frequencies. Finally, Section VI concludes the paper.

II. Background

In this section, the time-independent Prandtl–Ishlinskii model is presented. The Prandtl–Ishlinskii model integrates play operators Γ_r with different thresholds r, with positive weights in order to characterize hysteresis nonlinearities in actuators and materials, see [26]. When an input $u(t) \in C[0,T]$ is applied, where $C[0,T]$ is the space of continuous functions on the time interval $[0,T]$, the output of the Prandtl–Ishlinskii model is given, according to [26], by the formula

$$\Pi[u](t) = a_0 u(t) + \sum_{i=1}^{n} a_i \Gamma_{r_i}[u, x_i](t) \tag{1}$$

for $t \in [0,T]$, $i = 1, \ldots, n$, where n represents the number of the play operators, a_0, a_i are positive weights, $x_i \in [-r_i, r_i]$ are given initial conditions, and r_i are positive thresholds. Recall that the play operator Γ_r with constant threshold $r > 0$ is the mapping which with a given function $u(t)$ and initial condition $x \in [-r, r]$ associates the solution $\xi(t) = \Gamma_r[u,x](t)$ of the variational inequality

$$\left.\begin{array}{l} |u(t) - \xi(t)| \le r \\ u(0) - \xi(0) = x \\ \dot{\xi}(t)(u(t) - \xi(t) - y) \ge 0 \quad \forall y \in [-r, r] \end{array}\right\}. \tag{2}$$

If u is not differentiable, the variational inequality has to be stated in the Stieltjes integral form to avoid the differentiation. If the function $u(t)$ is monotone (nondecreasing or nonincreasing) in an interval $[t_0, t_1]$, then the output of the play operator $\xi(t) = \Gamma_r[u,x](t)$ admits a particularly simple representation

$$\xi(t) = P(u(t), r, \xi(t_0)) \text{ for } t \in [t_0, t_1] \tag{3}$$

where the function $P(U,R,Z)$ of real variables $U, R,$ and Z is defined as the shifted dead-zone function

$$P(U,R,Z) = \max(U - R, \min(U + R, Z)). \tag{4}$$

The Prandtl–Ishlinskii model is a rate-independent hysteresis model, attributed to the time-independent play operator that the model employs. This model has a unique advantage since it admits an exact inverse, which has been established in [27]. In [26], the output of the inverse time-independent Prandtl–Ishlinskii model is written in the form

$$\Pi^{-1}[u](t) = b_0 u(t) + \sum_{i=1}^{n} b_i \Gamma_{s_i}[u, y_i](t) \tag{5}$$

with suitably chosen parameters b_i, and s_i, and initial conditions $y_i \in [-s_i, s_i]$ analogous to (14)–(16) below. This inverse has been widely applied as a feedforward controller to compensate for hysteresis nonlinearities in smart-material actuators at low excitation frequencies of the applied input. However, these actuators exhibit hysteresis nonlinearities that are strongly rate dependent. Consequently, the use of the inverse time-independent Prandtl–Ishlinskii model as a feedforward compensator may cause considerable errors in the output displacement. It is therefore necessary to design a model and compensator capable of incorporating time-dependent hysteresis effects. This can be accomplished by including a viscosity term in the constitutive relation [26]. In [28], a feedforward compensator is constructed employing hysteresis and creep operators and implemented using the Preisach model. This model is applied as a feedforward controller to mitigate for complex time-dependent hysteresis nonlinearities and creep dynamics. Alternatively, in this paper, the time dependence of the hysteresis effects is directly included in the Prandtl–Isklinskii model by using thresholds which are functions of time.

III. Rate-Dependent Prandtl–Ishlinskii Model and Its Inverse

In this section, the rate-dependent Prandtl–Ishlinskii model and its inverse are presented to characterize and to compensate for the rate-dependent hysteresis nonlinearities. This model is formulated in [25] using the concept of play operator $\Phi_{r(t)}[u,x]$ with time-dependent threshold $r(t) > 0$. It is defined by the variational inequality (2), with r replaced by $r(t)$ and y by $y(t)$. The model and its inverse are applied to characterize and to compensate for the rate-dependent hysteresis nonlinearities in a piezomicropositioning stage when a periodic input voltage is applied.

A. Rate-Dependent Prandtl–Ishlinskii Model

In this paper, we deal with the space $\mathrm{AC}(0,T)$ of real absolutely continuous functions defined on the interval $[0,T]$. For an input $u(t) \in \mathrm{AC}(0,T)$, the output of the rate-dependent Prandtl–Ishlinskii model, where its construction is based on the rate of the applied input $\dot{u}(t)$, is given by the formula

$$\Psi[u](t) = a_0 u(t) + \sum_{i=1}^{n} a_i \Phi_{r_i(\dot{u}(t))}[u, x_i](t) \tag{6}$$

where n represents the number of the time-dependent play operators considered in the model. We denote the output of the time-dependent play operator as

$$z_i(t) = \Phi_{r_i(\dot{u}(t))}[u, x_i](t) \tag{7}$$

where x_i are given initial conditions for $i = 1, 2, \ldots, n$ such that for $i = 1, \ldots, n-1$, we have

$$\begin{aligned} |x_1| &\le r_1(\dot{u}(0)) \\ |x_{i+1} - x_i| &\le r_{i+1}(\dot{u}(0)) - r_i(\dot{u}(0)). \end{aligned} \tag{8}$$

The dynamic thresholds $r_i(t)$ are defined for $t \in [0,T]$ as

$$0 \le r_1(\dot{u}(t)) \le r_2(\dot{u}(t)) \le \cdots \le r_n(\dot{u}(t)). \tag{9}$$

In the time-dependent play operator, an increase in input $u(t)$ causes the output of the play operator $z(t)$ to increase along the curve $u(t) - r_i(\dot{u}(t))$, while a decrease in input $u(t)$ causes the output to decrease along the curve $u(t) + r_i(\dot{u}(t))$, resulting in a symmetric hysteresis loop. As shown in [14], the rate-dependent Prandtl–Ishlinskii model can characterize rate-dependent hysteresis nonlinearities in piezomicropositioning actuators over a range of different excitation frequencies. In this paper, we show that the inverse of the rate-dependent Prandtl–Ishlinskii model is achievable and can be applied as a feedforward compensator to compensate for rate-dependent hysteresis nonlinearities in real-time systems.

B. Inverse Rate-Dependent Prandtl–Ishlinskii Model

The concept of the open-loop control system used in this paper is to obtain an identity mapping between the input $u(t)$ and the output $v(t)$ such that $u(t) = v(t)$. When the output of the exact inverse of the rate-dependent Prandtl–Ishlinskii model $\Psi^{-1}[u](t)$ is applied as a feedforward controller to compensate for the rate-dependent hysteresis nonlinearities presented by the rate-dependent Prandtl–Ishlinskii model $\Psi[u](t)$, the output of the compensation is expressed as

$$v(t) = \Psi \circ \Psi^{-1}[u](t). \tag{10}$$

The exact inversion formula holds under the condition that the distances between the dynamic thresholds $r_i(\dot{u}(t))$ do not decrease in time [25]. Analytically for $\forall i = 1, \ldots, n-1$

$$\frac{d}{dt}(r_{i+1}(\dot{u}(t)) - r_i(\dot{u}(t))) \ge 0. \tag{11}$$

The proof of the inversion formula is based in a substantial way on the so-called Brokate formula for the superposition of play operators with different thresholds. It was established for constant thresholds in [29], and the extension to moving thresholds has been done in [25]. The inverse of the rate-dependent Prandtl–Ishlinskii is also a rate-dependent Prandtl–Ishlinskii model. The output of the inverse is expressed as

$$\Psi^{-1}[u](t) = b_0 u(t) + \sum_{i=1}^{n} b_i \Phi_{s_i(\dot{u}(t))}[u, y_i](t). \tag{12}$$

We denote the output of the rate-dependent play operator of the inverse model by

$$d_i(t) = \Phi_{s_i(\dot{u}(t))}[u, y_i](t). \tag{13}$$

The thresholds of the inverse model are

$$\begin{aligned} &s_1(\dot{u}(t)) = a_0 r_1(\dot{u}(t)) \\ &s_{i+1}(\dot{u}(t)) - s_i(\dot{u}(t)) = \left(\sum_{j=0}^{i} a_j\right)(r_{i+1}(\dot{u}(t)) - r_i(\dot{u}(t))). \end{aligned} \tag{14}$$

The weights of the inverse model $b_0, b_1, \ldots, b_n$ are

$$b_0 = \frac{1}{a_0}, b_i = \frac{1}{\sum_{j=0}^{i} a_j} - \frac{1}{\sum_{j=0}^{i-1} a_j}. \tag{15}$$

The initial conditions of the inverse model $y_1, y_2, \ldots, y_n$ are

$$y_1 = a_1 x_1, (y_{i+1} - y_i) = \left(\sum_{j=0}^{i} a_j\right)(x_{i+1} - x_i). \tag{16}$$

The rate-dependent Prandtl–Ishlinskii model of a single-rate-dependent play operator $n = 1$ is unconditionally invertible [25], and, analytically, its output can be expressed as

$$\Psi[u](t) = a_0 u(t) + a_1 \Phi_{r_1(\dot{u}(t))}[u, x_1](t). \tag{17}$$

The output of the inverse is

$$\Psi^{-1}[u](t) = b_0 u(t) + b_1 \Phi_{s_1(\dot{u}(t))}[u, y_1](t) \tag{18}$$

where $b_0 = \frac{1}{a_0}$, $b_1 = -\frac{a_1}{a_0(a_0+a_1)}$, $s_1(\dot{u}(t)) = a_0 r_1(\dot{u}(t))$, and $y_1 = a_0 x_1$.

C. The Dynamic Threshold

As shown in Section III, the time-dependent Prandtl–Ishlinskii model Ψ is constructed using a combination of play operators with dynamic thresholds $r_i(\dot{u}(t))$. We propose the following dynamic thresholds:

$$r_i(\dot{u}(t)) = \alpha_i + g(\dot{u}(t)). \tag{19}$$

This can be shown to be mathematically equivalent to modeling hysteresis and creep by means of an analogical model with elastic, plastic, and viscous elements, as demonstrated in [28] and [30]. We also have

$$\alpha_i - \alpha_{i-1} \ge \sigma \tag{20}$$

where σ is a positive constant. The constants α_i in (19) represent the rate-independent hysteresis effects, while the function

$g(\dot{u}(t))$ is proposed to characterize the rate-dependent hysteresis effects. With this choice, we have

$$r_{i+1}(\dot{u}(t)) - r_i(\dot{u}(t)) = \alpha_i - \alpha_{i-1} \tag{21}$$

and

$$\frac{d}{dt}(r_{i+1}(\dot{u}(t)) - r_i(\dot{u}(t))) = 0. \tag{22}$$

From these equations, it can be concluded that the exact inversion formula for the time-dependent Prandtl–Ishlinskii model holds. The dynamic threshold of the rate-dependent play operator can be presented for $i = 1, 2, \ldots, n$

$$\alpha_i = \zeta i \tag{23}$$

where ζ is a positive constant. The function $g(\dot{u}(t))$ can be chosen as

$$g(\dot{u}(t)) = \beta|\dot{u}(t)| \tag{24}$$

where β is a positive constant and $r_{i+1}(\dot{u}(t)) - r_i(\dot{u}(t)) = \zeta$. This can be shown to be mathematically equivalent to modeling hysteresis and creep by means of an analogical model with elastic, plastic, and viscous elements as in [28]. It should be mentioned that the proposed formulation for the dynamic threshold reduces the rate-dependent Prandtl–Ishlinskii model $\Psi[u](t)$ into the rate-independent Prandtl–Ishlinskii model $\Pi[u](t)$ for $g(\dot{u}(t)) = 0$. However, the choice of function $g(\dot{u}(t))$ would depend upon the nature of rate-dependent hysteresis of a particular material and actuator. At low excitation frequencies, the rate-dependent Prandtl–Ishlinskii model (6) and its inverse (12), constructed based on the dynamic threshold (23), are reduced to the rate-independent Prandtl–Ishlinskii model and its inverse. Analytically, the dynamic thresholds are reduced to

$$r(\dot{u}(t)) \approx \zeta i. \tag{25}$$

Then, it can be concluded that

$$\Pi[u](t) \approx \Psi[u](t). \tag{26}$$

Consequently, the model can be applied to characterize and to compensate rate-independent hysteresis nonlinearities. However, the model cannot be applied to characterize creep effects at low excitation frequencies.

D. *Modified Rate-Dependent Prandtl–Ishlinskii Model*

The inverse rate-dependent Prandtl–Ishlinskii model can be applied as a feedforward compensator to compensate for convex rate-dependent hysteresis loops. However, different piezomicropositioning actuators exhibit complex rate-dependent hysteresis nonlinearities that increase as the excitation frequencies of the applied input increases; see,for example [12], [28]. In this section, we show that that the rate-dependent Prandtl–Ishlinskii model can be extended to asymmetrical hysteresis loops in piezomicropositioning actuators by inserting functional nonlinearities applied by Kuhnen [31], Visone and Sjöström [32], Al Janaideh *et al*. [33], and Al Janaideh and Krejčí [34].

Kuhnen [31] proposed a modified Prandtl–Ishlinskii model, in the section we introduce the modified rate-dependent Prandtl–Ishlinskii model. The output of this modified model is expressed as

$$\Psi_{\lambda}[u](t) = \Lambda \circ \Psi[u](t) \tag{27}$$

where Λ is a memoryless, continuous, and strictly monotone function. This model can characterize nonconvex, and asymmetrical rate-dependent hysteresis loops. The output of the inverse modified model can be expressed as

$$\Psi_{\lambda}^{-1}[u](t) = \Psi^{-1} \circ \Lambda^{-1}[u](t). \tag{28}$$

The functions $\Lambda(w)$ and $\Lambda(-w)$ of variable $w \in [0, \infty)$ can be approximated by compositions of dead-zone functions of the form

$$\Lambda(w) = \sum_{i=0}^{m} g_i S_{\rho_i}(w) \tag{29}$$

where S_{ρ_i} are dead-zone functions

$$S_{\rho_i}(w) = \max(w - \rho_i, 0) \tag{30}$$

g_i are constants, and ρ_i are thresholds such that $0 = \rho_0 \leq \rho_1 \leq \cdots \leq \rho_m$. The exact inverse of Γ can be expressed as

$$\Gamma^{-1}(w) = \sum_{i=0}^{m} \hat{\delta}_i S_{\hat{\rho}_i}(w). \tag{31}$$

The parameters of the exact inverse model for $i = 0, \ldots, m$, are [31]

$$\hat{\rho}_i = \sum_{j=0}^{i} \delta_j(\rho_i - \rho_j) \tag{32}$$

$$\hat{\delta}_0 = \frac{1}{\delta_0}, \quad \hat{\delta}_i = \frac{-\delta_i}{(\delta_0 + \sum_{j=1}^{i} \delta_j)(\delta_0 + \sum_{j=1}^{i-1} \delta_j)}. \tag{33}$$

The modified rate-dependent Prandtl–Ishlinskii model (27) can be applied to characterize asymmetrical and nonconvex rate-dependent hysteresis loops in piezomicropositioning actuators. Consequently, the output of the inverse modified rate-dependent Prandtl–Ishlinskii model can be used as a feedforward compensator to compensate for asymmetrical rate-dependent hysteresis loops of the piezomicropositioning actuators.

E. *Numerical Implementation*

The numerical implementation of the proposed rate-dependent Prandtl–Ishlinskii model, and its inverse is presented for an input $u(t)$ with sampling time h. The rate of input $\dot{u}(t)$ can be estimated from the discrete input $k = 1, 2, 3, \ldots, K = T/h$ as

$$u_s(k) = \frac{u(k) - u(k-1)}{h} \tag{34}$$

where the sampling time h is defined as

$$h = t(k) - t(k-1). \tag{35}$$

Then, the discrete dynamic threshold is presented as

$$r_i(u_s(k)) = \alpha_i + g(u_s(k)). \tag{36}$$

The discrete output of the rate-dependent play operator is, by virtue of (3), expressed recurrently as

$$z_i(k) = \Phi_{r_i(u_s(k))}[u](k) = P(u(k), r_i(u_s(k)), z_i(k-1)) \tag{37}$$

where $P(U, R, Z)$ is the shifted dead-zone function defined in (4), and the initial conditions $z_i(0)$ are chosen in such a way that $|u(0) - z_1(0)| \leq \alpha_1$, $|z_{i+1}(0) - z_i(0)| \leq \alpha_{i+1} - \alpha_i$ for $i = 1, \ldots, n-1$. The discrete output of rate-dependent Prandtl–Ishlinskii model can thus be derived as

$$\Psi[u](k) = a_0 u(k) + \sum_{i=1}^{n} a_i \Phi_{r_i(u_s(k))}[u](k). \tag{38}$$

The discrete output of the inverse model can thus be expressed as

$$\Psi^{-1}[u](k) = b_0 u(k) + \sum_{i=1}^{n} b_i \Phi_{s_i(u_s(k))}[u](k). \tag{39}$$

F. Example

An input signal of the form $u(t) = 22\sin(2\pi f t) + 25\sin(6\pi f t)$ is considered to evaluate minor as well as major hysteresis loops, while fundamental frequencies of $f = 10$ and 60 Hz are considered. The parameters of the rate-dependent Prandtl–Ishlinskii model are $\zeta = 3$, $\beta = 28/40000$, $a_0 = 0.5$, $n = 4$, $a_1 = 0.3667$, $a_2 = 0.2690$, $a_3 = 0.1973$, and $a_4 = 0.1447$. In this example, the results obtained from the model show an increase in hysteresis effects as the fundamental frequency of the applied input increases. Fig. 1(a) and (b) show the output of the play operator $\Phi_{r_i(t)}[u](t)$ at excitation frequencies of 10 and 60 Hz. The parameters of the inverse rate-dependent Prandtl–Ishlinskii model are given by (14)–(16). Fig. 1(c) and (d) show the outputs of the rate-dependent Prandtl–Ishlinskii model and its inverse at excitation frequencies of 10 and 60 Hz. The outputs of the compensation at 10 and 60 Hz are shown in Fig. 1(e) and (f).

In this example, we also apply the output of the inverse rate-independent Prandtl–Ishlinskii model $\Pi^{-1}[u](t)$ constructed with $r_i = \zeta i$ as a feedforward compensator to compensate for the rate-dependent hysteresis nonlinearities presented by the rate-dependent Prandtl–Ishlinskii model $\Psi[u](t)$ at an excitation frequency of $f = 60$ Hz. In this case, the output of the compensation is expressed as

$$w(t) = \Psi \circ \Pi^{-1}[u](t). \tag{40}$$

The error of the compensation is expressed analytically as

$$e(t) = u(t) - \Psi \circ \Pi^{-1}[u](t). \tag{41}$$

Fig. 2(c) and (d) show strong hysteresis nonlinearities in the output of the compensation $\Psi \circ \Pi^{-1}[u](t)$ and high compensation error $u(t) - \Psi \circ \Pi^{-1}[u](t)$. It can be concluded that the use of the inverse rate-independent Prandtl–Ishlinskii model to compensate for rate-dependent hysteresis nonlinearities yields high compensation errors in the output. It is obvious that the output of the compensation $\Psi \circ \Pi^{-1}[u](t)$ exhibits hysteresis nonlinearities. Also, we do not classify the compensation error as a result of uncertainties or disturbance.

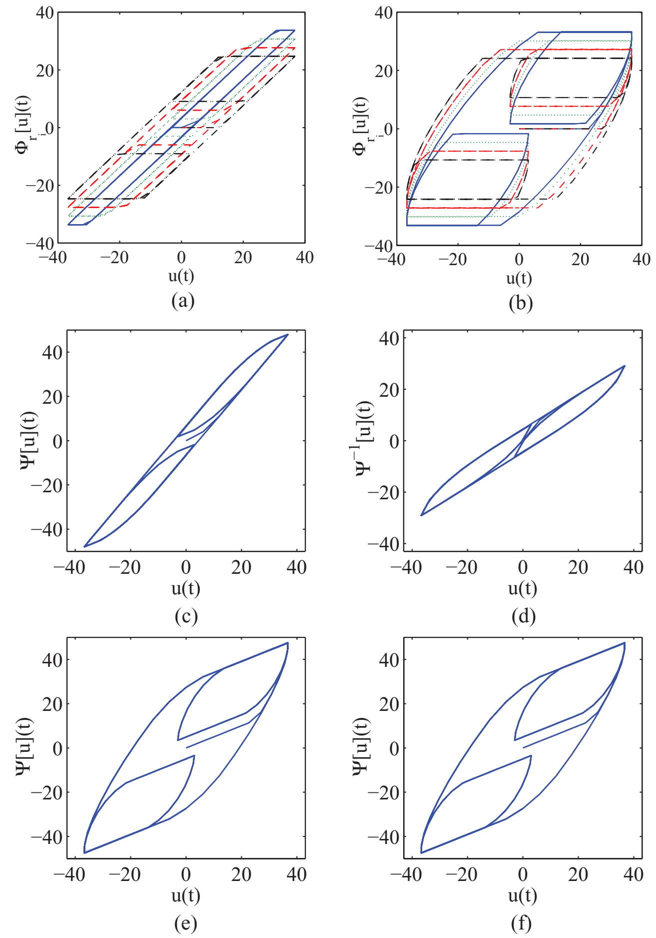

Fig. 1. Output of the rate-dependent play operator at (a) 10 Hz, and (b) 60 Hz. The output of the rate-dependent Prandtl–Ishlinskii model at (c) 10 Hz and (d) 60 Hz. The output of the inverse rate-dependent Prandtl–Ishlinskii model at (e) 10 Hz and (f) 60 Hz.

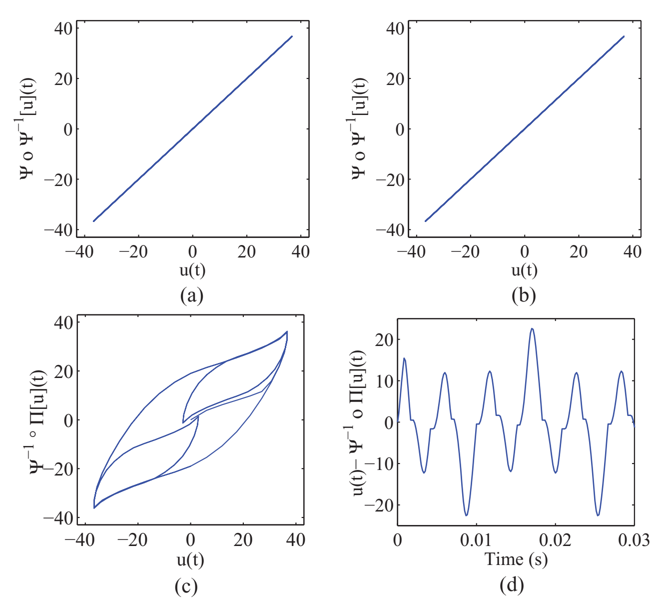

Fig. 2. Output of the inverse compensation a (a) 10 Hz and (b) 60 Hz. (c) Output of the $\Psi \circ \Pi^{-1}[u](t)$. (d) Time history of $u(t) - \Psi \circ \Pi^{-1}[u](t)$.

IV. Experimental Results and Hysteresis Modeling

Experiments were performed for a piezomicropositioning actuator to characterize the rate-dependent hysteresis properties under harmonic excitations in the 1–100 Hz frequency range.

A. Experimental Setup

The experiments were performed on a piezomicropositioning stage (P-753.31C) from Physik Instrumente Company. The natural frequency of the actuator is 3 kHz. The actuator provided a maximum displacement of 100 μm from its static equilibrium position. We used a capacitive sensor C-23C manufactured by the Lion Precision Company for measurement of the stage displacement response. The bandwidth of the sensor is 15 kHz. The excitation module comprises of a voltage amplifier (LVPZT, E-505) with a fixed gain of 10, which provides the excitation voltage to the actuator. The actuator displacement response signal was acquired by a DSpace DS1104 controller board. This setup is used to measure the output of the piezomicropositioning stage at different excitation frequencies between 0.1 and 100 Hz. It should be mentioned that the amplifier adds a delay phase of 0.5° and 1.5° at 50 Hz and 100 Hz, respectively. This phase eliminated from the measured displacement of the capacitive sensor. The applied input voltage and measured output displacement signals were acquired at a sampling frequency of 10 kHz.

B. Experimental Results

The measurements with the piezomicropositioning stage were performed under a harmonic input voltage of $u(t) = 40\sin(2f\pi t)$ V at three excitation frequencies (1, 50, and 100 Hz). The measured data were further analyzed to quantify the hysteresis and displacement attenuation as a function of the applied excitation frequency. The resulting hysteresis loops relating displacement responses to the input voltage are shown at various excitation frequencies in Fig. 3. It is evident that the micropositioning stage exhibits rate-dependent hysteresis effects between the input voltage and the output displacement when the frequency increases form 1 to 100 Hz. The hysteresis percent is 13.03%, 20.28%, and 26.18% for 1, 50, and 100 Hz, respectively.

C. Parameter Identification

Measured rate-dependent hysteresis loops of the piezomicropositioning actuator presented in Fig. 3 are used to identify the parameters of the rate-dependent Prandtl–Ishlinskii model and its inverse. We choose the dynamic threshold of (19) and (24) to model the rate-dependent hysteresis in the piezomicropositioning actuator. The characterization error of the rate-dependent Prandtl–Ishlinskii model is defined as

$$e_c(k) = \Psi[u](k) - y(k) \tag{42}$$

where $y(k)$ represents the measured displacement of the piezomicropositioning actuator when an input voltage at a particular excitation frequency is applied and $\Psi[u](k)$ is the output of the rate-dependent Prandtl–Ishlinskii model under the same

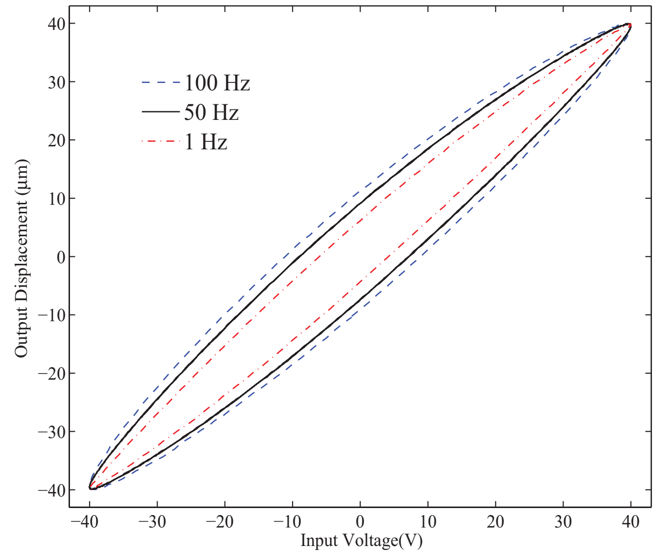

Fig. 3. Measured hysteresis loops of the piezomicropositioning actuator when a sinusoidal input voltage is applied at different excitation frequencies.

input voltage. The index k ($k = 1, \ldots, K$) refers to the number of data points considered in computing the error for one complete hysteresis loop. The parameter vector $X = \{\beta, \zeta, a_0, a_1, a_2, \ldots, a_n\}$ of the rate-dependent Prandtl–Ishlinskii model Ψ, was identified through minimization of characterization error function over different excitation frequencies, given by

$$Q(X) = \Theta(k). \tag{43}$$

The model error function Θ is used to identify the parameters of the rate-dependent Prandtl–Ishlinskii model Ψ. The error function Θ is expressed as

$$\Theta(k) = \sum_{p=1}^{P}\sum_{k=1}^{K} A_p(\Psi[u](k) - y(k))^2. \tag{44}$$

The model error function is constructed through summation of squared errors over a range of input frequencies, denoted by p ($p = 1, 2$, and 3). Two hundred data points ($K = 200$) were available for each measured hysteresis loop. Owing to the higher error at higher excitation frequencies, a weighting constant A_p was introduced to emphasize the error minimization at higher excitation frequencies. The weights A_p for $p = 1, 2$, and 3, are calculated based on the hysteresis percentage presented as

$$A_p = \frac{H_p}{H_1} \tag{45}$$

where H_p represents the hysteresis percent for the pth excitation frequency. The weights A_p are obtained as 1, 1.56, and 2.00 for 1, 50, and 100 Hz, respectively. The error minimization is performed using the MATLAB-constrained optimization toolbox and subject to the following constraints:

$$(\beta, \zeta, a_0, a_1, a_2, a_3) > 0,\ \zeta \gg \beta.$$

The results suggest three rate-dependent play hysteresis operators ($n = 3$) to characterize the rate-dependent hysteresis nonlinearities in the actuator. The parameters of the model are:

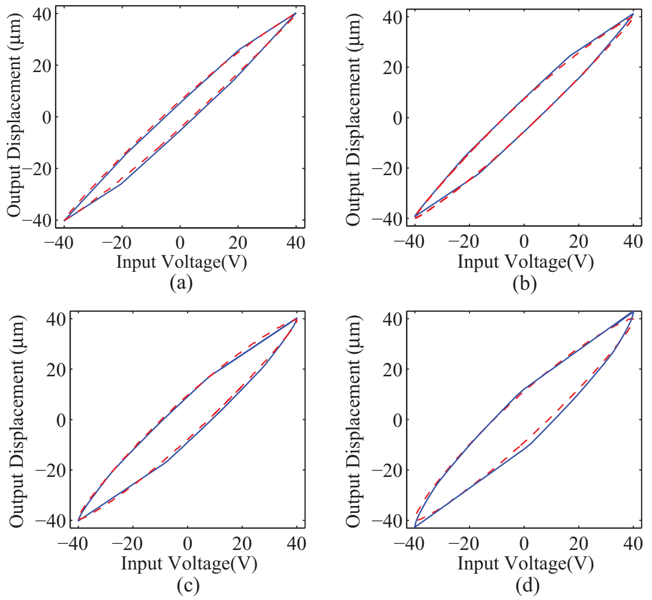

Fig. 4. Comparison between the output of the rate-dependent Prandtl–Ishlinskii model (solid line) and the measured hysteresis nonlinearities (dashed line) in the piezomicropositioning actuator at (a) 1 Hz, (b) 20 Hz, (c) 60 Hz, and (d) 100 Hz.

$n = 3$, $\zeta = 9.715$, $\beta = 8.572 \times 10^{-4}$, $a_0 = 0.7124$, $a_1 = 0.2906$, $a_2 = 0.0461$, and $a_3 = 0.1512$. The rate-dependent Prandtl–Ishlinskii model is used to characterize the rate-dependent hysteresis nonlinearities of the piezomicropositioning actuator.

D. Hysteresis Modeling

The validity of the model was examined by comparing the model displacement responses to the measured data. The results suggest that the model can effectively predict the hysteresis properties of the piezomicropositioning actuator at a range of different excitation frequencies between 1 and 100 Hz. Fig. 4 shows the capability of the rate-dependent Prandtl–Ishlinskii model to characterize rate-dependent hysteresis nonlinearities at different excitation frequencies between 1 and 100 Hz.

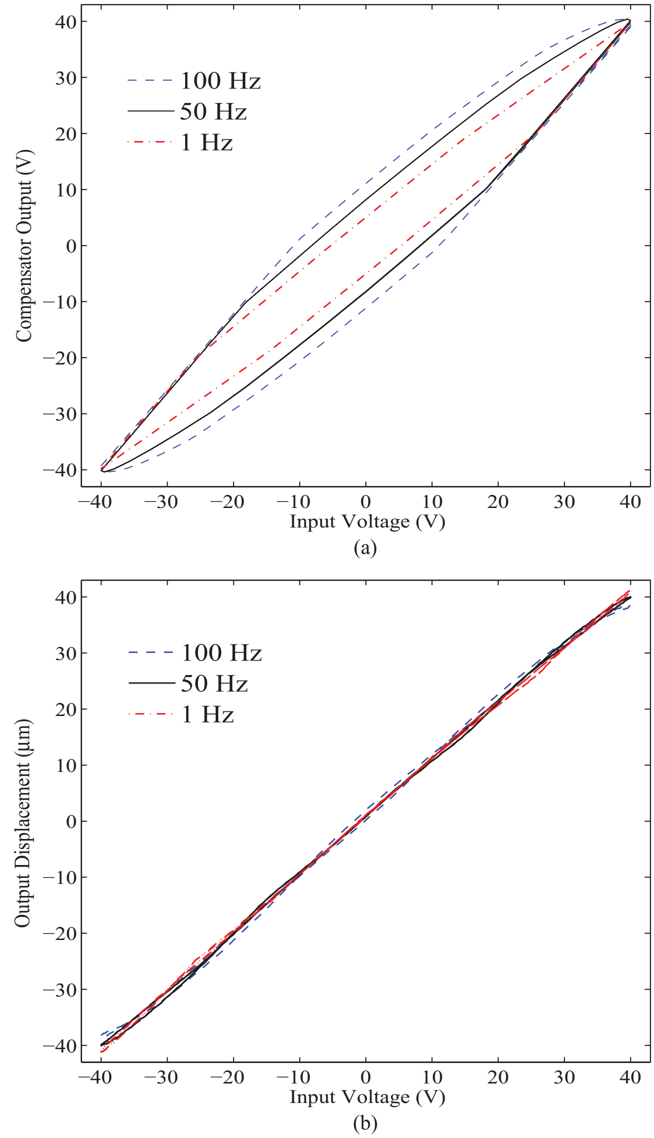

Fig. 5. (a) Output of the inverse rate-dependent Prandtl–Ishlinskii model at excitation frequencies of 1, 50, and 100 Hz, and (b) measured output displacement of the piezomicropositioning actuator when the inverse rate-dependent Prandtl–Ishlinskii model is applied as a feedforward compensator at 1, 50, and 100 Hz.

V. Feedforward Compensation of Rate-Dependent Hysteresis Nonlinearities

The inverse rate-dependent Prandtl–Ishlinskii model is applied as a feedforward compensator to compensate for hysteresis nonlinearities under different excitation frequencies.

A. Hysteresis Compensation

The parameters of the inverse model obtained by (14), (15), and (16) are $b_0 = 1.3892$, $b_1 = -0.3961$, $b_2 = -0.038$, $b_3 = -0.1187$, $s_1 = 6.2024 \times 10^{-4}|\dot{u}(t)| + 7.0293$, $s_2 = 6.2024 \times 10^{-4}|\dot{u}(t)| + 16.8823$, and $s_3 = 6.2024 \times 10^{-4}|\dot{u}(t)| + 27.1395$. The input–output characteristics of the inverse rate-dependent Prandtl–Ishlinskii model at excitation frequencies of 1, 50, and 100 Hz are shown in Fig. 5(a). The figure shows that hysteresis nonlinearities in the output of the inverse increase as the excitation frequency of the input voltage increases. The measured output–input characteristics of the piezomicropositioning actuator with the inverse compensator are illustrated in Fig. 5(b). The results show that the inverse rate-dependent model can effectively compensate the hysteresis effects at different excitation frequencies.

The positioning error is computed as the deviation between the input voltage, which represents the desired displacement, and output voltage of the capacitive sensor, which represents the output displacement, of the piezomicropositioning actuator. Fig. 6 gives a comparison of the maximum positioning errors with and without the inverse compensator at different excitation frequencies between 0.05 and 100 Hz. Without the inverse compensator, the maximum positioning errors are between 5.9 and 10.6 μm. The measured responses with the inverse

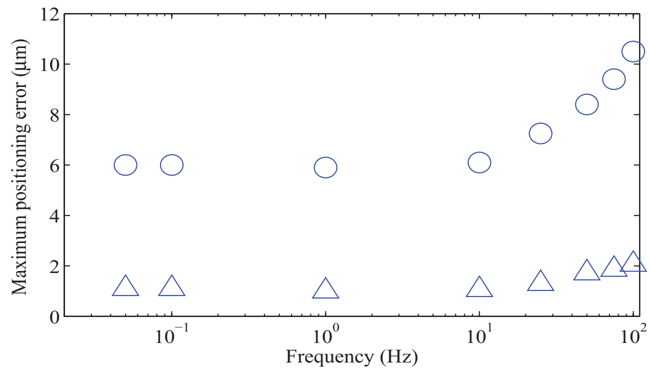

Fig. 6. Comparison between the maximum positioning error without, circle, and with, triangular, the inverse rate-dependent Prandtl–Ishlinskii model for different excitation frequencies between 0.05 and 100 Hz.

compensator show maximum positioning errors between 1.0 and 2.0 μm across the entire 0.05–100-Hz band. It is obvious that the inverse rate-dependent Prandtl–Ishlinskii model suppresses the error due to hysteresis regardless of the excitation frequency of the input voltage, while also preserving consistency in the tracking accuracy. Since the inverse compensate for hysteresis nonlinearities at low excitation, it can also be concluded that the inverse rate-dependent Prandtl–Ishlinskii model can be applied to compensate for rate-independent hysteresis nonlinearities.

B. Major and Minor Hysteresis Loops

Two different periodic input voltages, triangular and harmonic, are considered as input voltages for the piezomicropositioning actuator. A periodic input voltage with different amplitudes is applied to show the capability of the inverse model to compensate for rate-dependent major and minor hysteresis loops. Fig. 7 shows the time history of the output of the piezomicropositioning with and without the inverse compensator. The results demonstrate that the inverse compensator effectively compensates for both major and minor rate-dependent hysteresis nonlinearities. Fig. 7 also shows the time history of the positioning error with and without the inverse compensator. The results show the effectiveness of the inverse compensator for major and minor rate-dependent hysteresis loops. As shown in Fig. 8, the inverse compensator compensates for the hysteresis nonlinearities when a triangular input voltage is applied. The figure also shows the time history of the positioning error with and without the inverse compensator. The results show the effectiveness of the compensator when a parodic triangular input voltage is applied as an input signal.

C. Discussion

The above analysis shows that the inverse rate-dependent Prandtl–Ishlinskii model is capable of suppressing the error due to hysteresis, regardless of the excitation frequency of the input voltage, while maintaining consistency in the tracking accuracy. The results manifest the effectiveness of the inverse rate-dependent model in compensating for hysteresis under both low and high excitation frequencies.

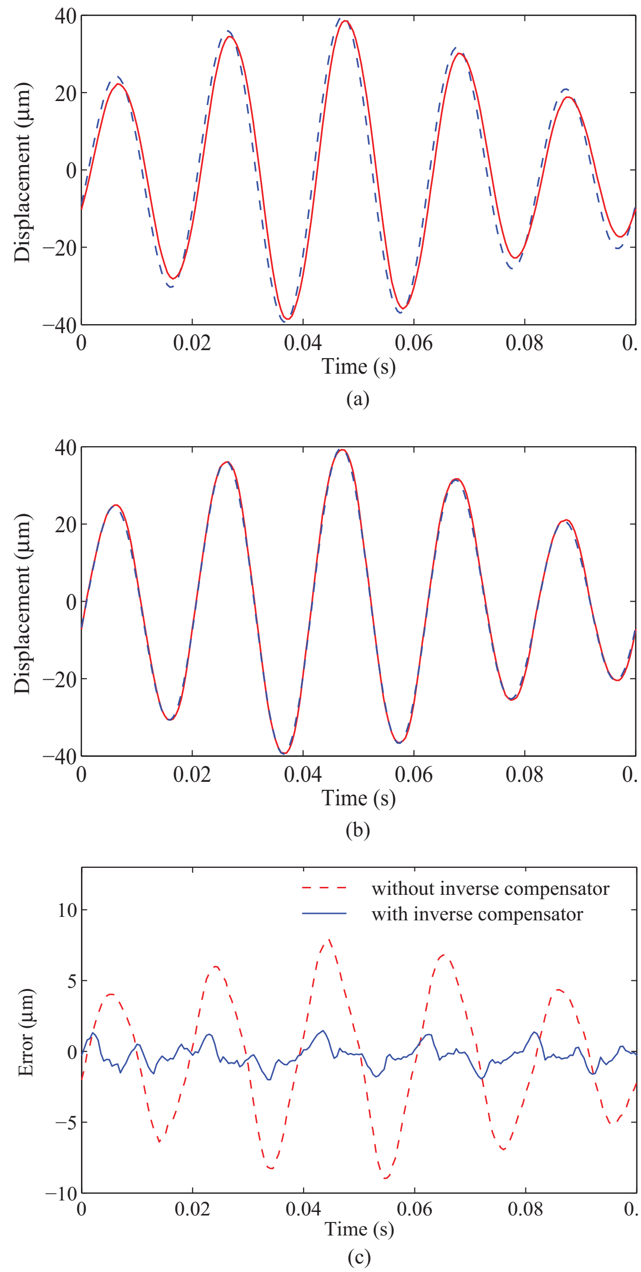

Fig. 7. Time history of the applied input voltage (dashed line) and the output displacement (solid line) of the piezomicropositioning actuator: (a) without the inverse compensator, (b) with the inverse compensator, and (c) time history of the positioning error with and without the inverse compensator.

The rate-dependent Prandtl–Ishlinskii model and its inverse are easy to use since the model and its inverse can be implemented with few rate-dependent play operators, and few parameters are needed to be defined in order to construct the model and the inverse. In this paper, we use three rate-dependent play operators, $n = 3$, and six parameters, $\zeta, \beta, a_0, a_1, a_2, a_3$, to control a piezomicropositioning actuator excited between 0.05 and 100 Hz. However, the inverse compensator does show some deviation in the output, which can be attributed to prediction errors attained between the output of the rate-dependent model and the measured displacement of the piezomicropositioning actuator.

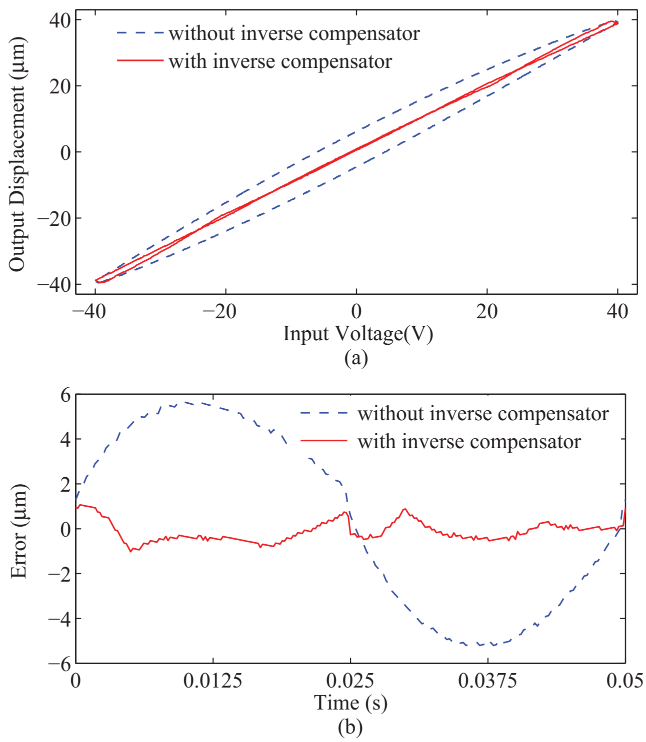

Fig. 8. (a) Output displacement of the piezomicropositioning actuator when a triangular input voltage is applied as an input signal, and (b) time history of the positioning error with and without the inverse compensator.

VI. Conclusion

This paper has presented the analytical inverse of the rate-dependent Prandtl–Ishlinskii model and its ability to compensate for rate-dependent hysteresis in a peizomicropositioning actuator over a range of different excitation frequencies between 0.05–100 Hz. Since the inverse is obtained analytically, the proposed inverse rate-dependent compensator is attractive for compensation of rate-dependent hysteresis nonlinearities. The exact inversion of the model holds under the condition that the distances between the dynamic thresholds do not decrease in time. The results also show that the inverse rate-dependent model can compensate for hysteresis nonlinearities at low excitation frequencies of 0.1 and 0.05 Hz. The inverse rate-dependent Prandtl–Ishlinskii model is exact, analytical, and unique. This makes the inverse model attractive for control piezomicropositioning actuators at different operating conditions in real-time system. It can be concluded that the proposed compensation algorithm is easy to use and can be applied to compensate for rate-dependent hysteresis nonlinearities in micropositioning and nanopositioning applications where the use of feedback sensors and feedback control techniques are not easy and difficult.

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Mohammad Al Janaideh received the M.S. and Ph.D. degrees in mechanical engineering and mechatronics from Concordia University, Montreal, QC, Canada, in 2004 and 2009, respectively.

He is currently with the Department of Mechatronics Engineering, University of Jordan, Amman, Jordan. His research interests include control, smart actuators, modeling rate-dependent and rate-independent hysteresis nonlinearities, and compensation of hysteresis.

Pavel Krejčí was born in 1954. He received the Ph.D. degree in mathematics in 1984 from the Institute of Mathematics, Czechoslovak Academy of Sciences, Prague, Czech Republic.

He was a Researcher at the Weierstrass Institute, Berlin, Germany from 1997 to 2000 and from 2004 to 2009. From 2002 to 2009, he was the Editor-in-Chief of *Applications of Mathematics*. Since 2009, he has been the Director of the Institute of Mathematics, Academy of Sciences of the Czech Republic, Prague. He is the author or coauthor of two monographs and about 80 research papers on evolution differential equations, hysteresis, and mathematical models of elastoplasticity, smart materials, and phase transitions.