Introduction to Radio Interferometry

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Radio astronomy - Basic definitions

- Brightness denoted by I(**s**).
- Brightness is defined as the power received per unit frequency dn at a particular frequency n, per unit solid angle dW from direction **s**, per unit collecting area dA.
- Units in terms of (spectral flux density)/(solid angle): watt/(m^2 Hz Ster)

Radio astronomy- Basic definitions

- Flux density S_ν

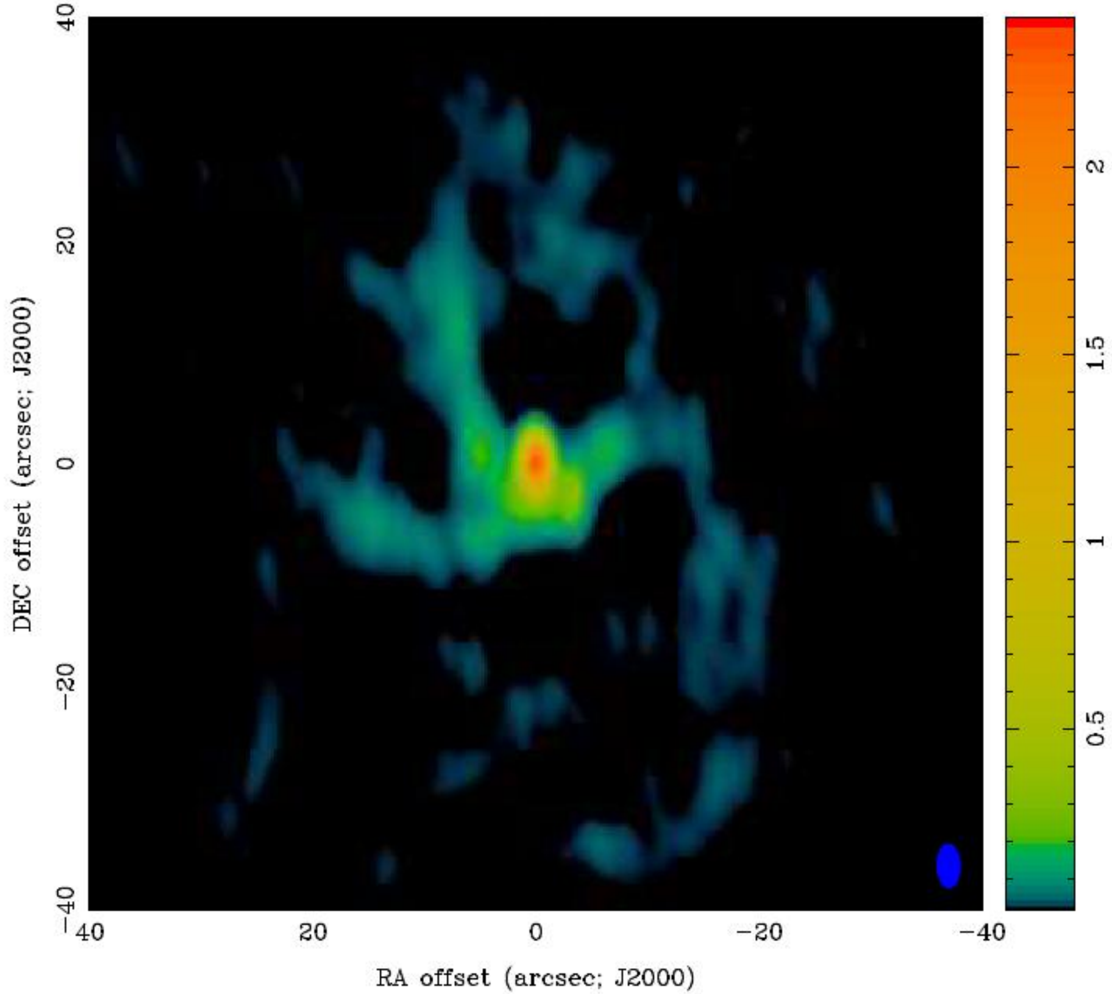
$$S_\nu = \int_{\text{source}} I_\nu d\Omega$$

- 1 Jy = 10^{-26} $Wm^{-2}Hz^{-1}$

- For an extended source, surface brightness measured in Jy/beam,

- Beam is the area of the point source response in the map

3 mm Galactic centre radio map



$3.96'' \times 2.09''$ (P.A.$=1.4^\circ$)

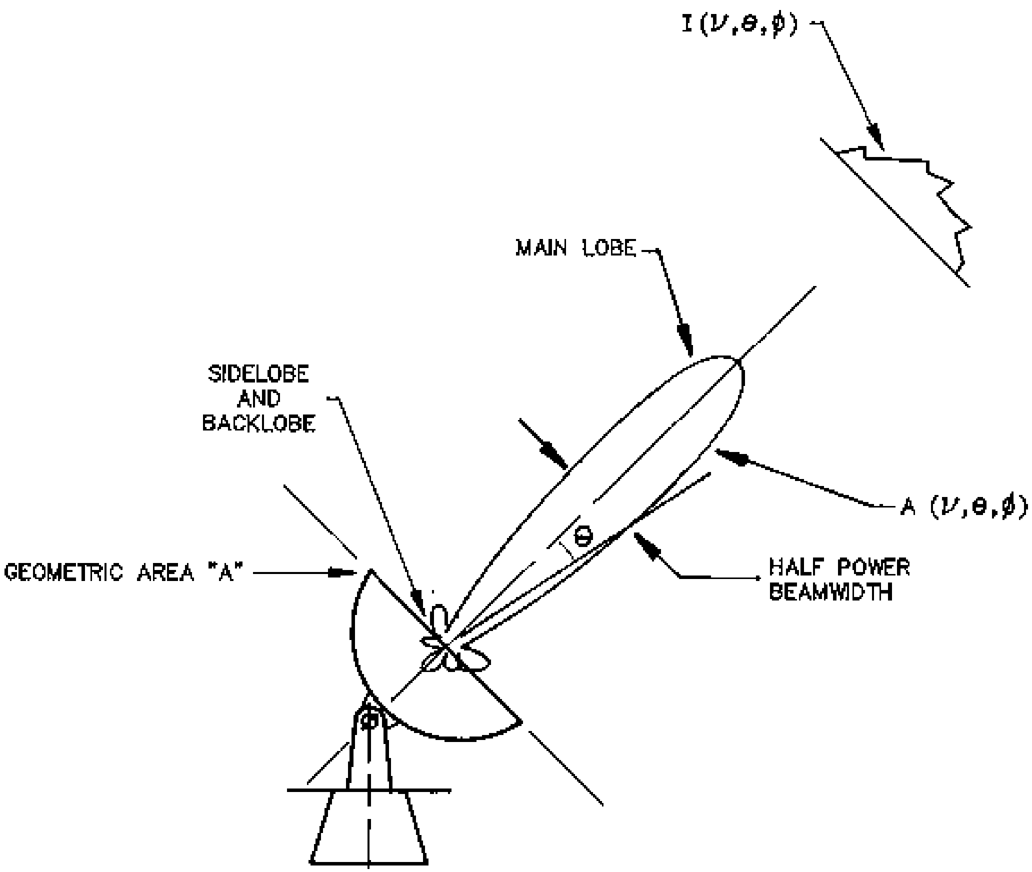
Radio antenna

$$P(\theta,\phi,\nu) = A(\theta,\phi,\nu) I(\theta,\phi,\nu) \Delta\nu \Delta\Omega$$

P Total power received

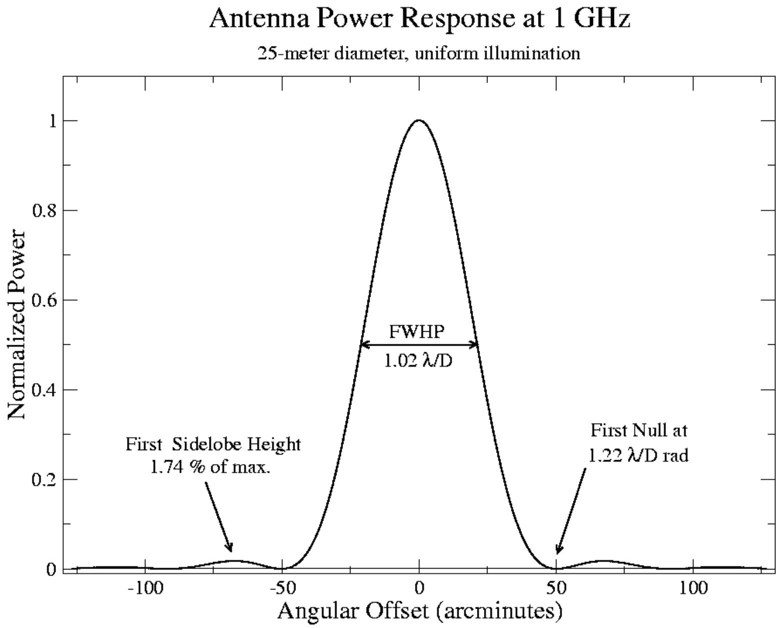
A Antenna power pattern

I Sky brightness distribution



Angular resolution

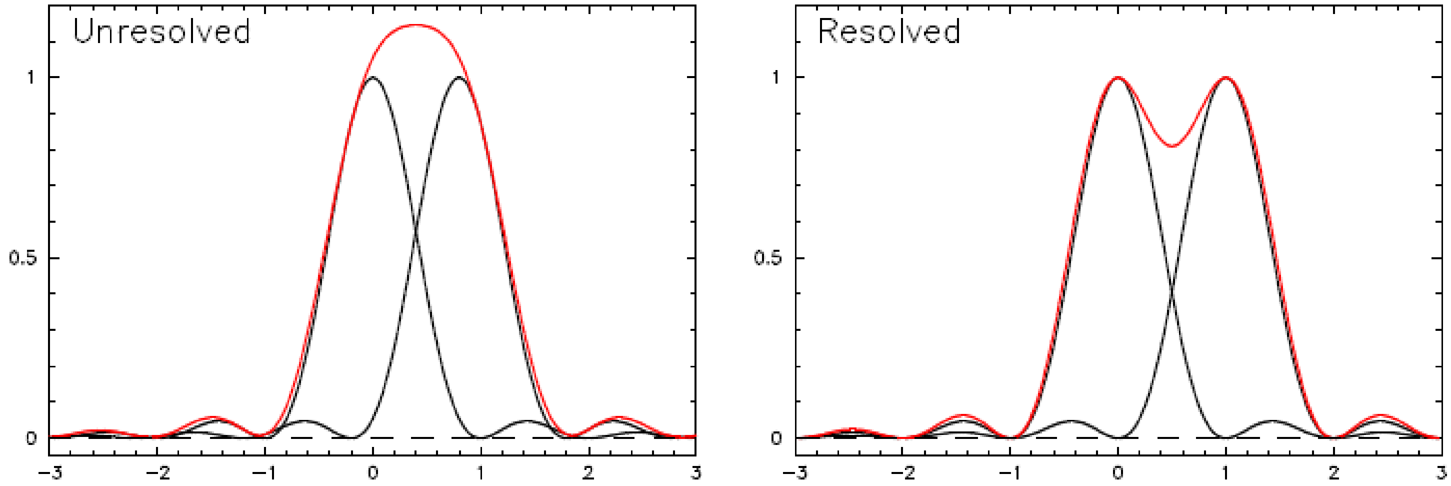
$$\theta \approx \lambda / D$$



http://www.aoc.nrao.edu/events/synthesis/2010/lectures/McKinnon_Antennas.pdf

Increasing angular resolution

Smallest angular separation at which two point sources are recognized as separate



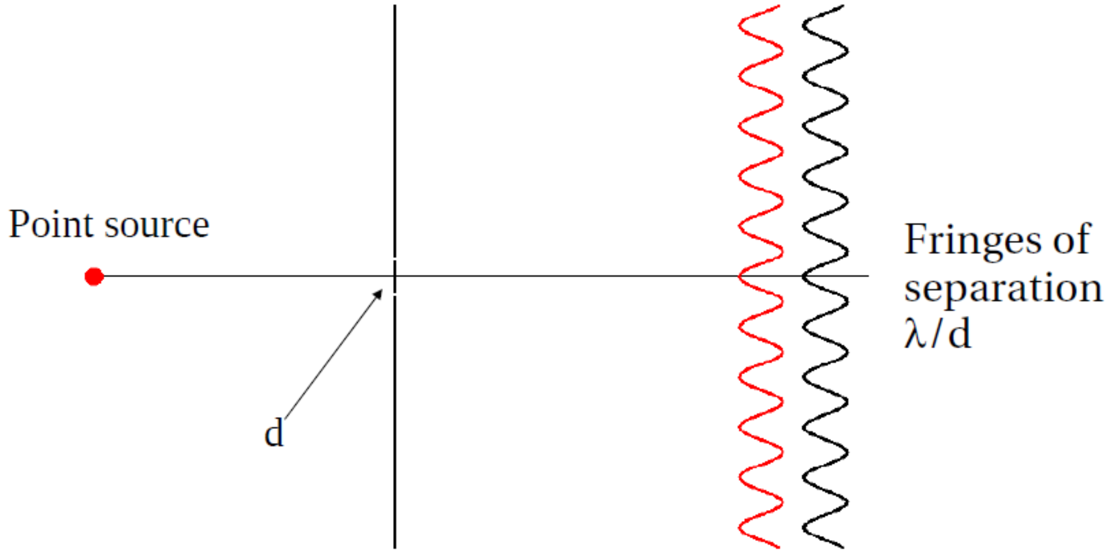
100 m Effelsberg telescope has an angular resolution of 8 arcminutes at 21 cm

$$\theta \approx \lambda / D$$

Replace ***D*** by ***B*** = 30 km, ϑ becomes 1",
where ***B*** is the separation between two telescopes

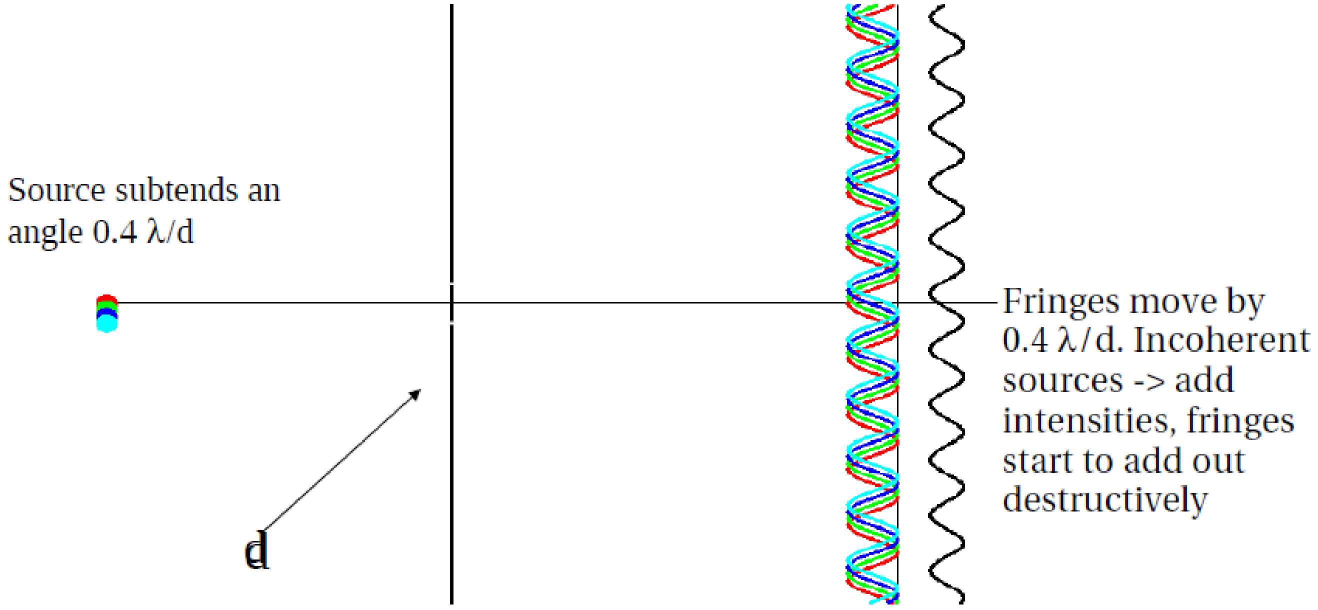
http://www.es.ntnu.edu.tw/tuCASA/novice-ksp/workshop20101120/ALMANovice1_WS.pdf

Young's double slit experiment



http://www.jb.man.ac.uk/~njj/lofardata.pdf

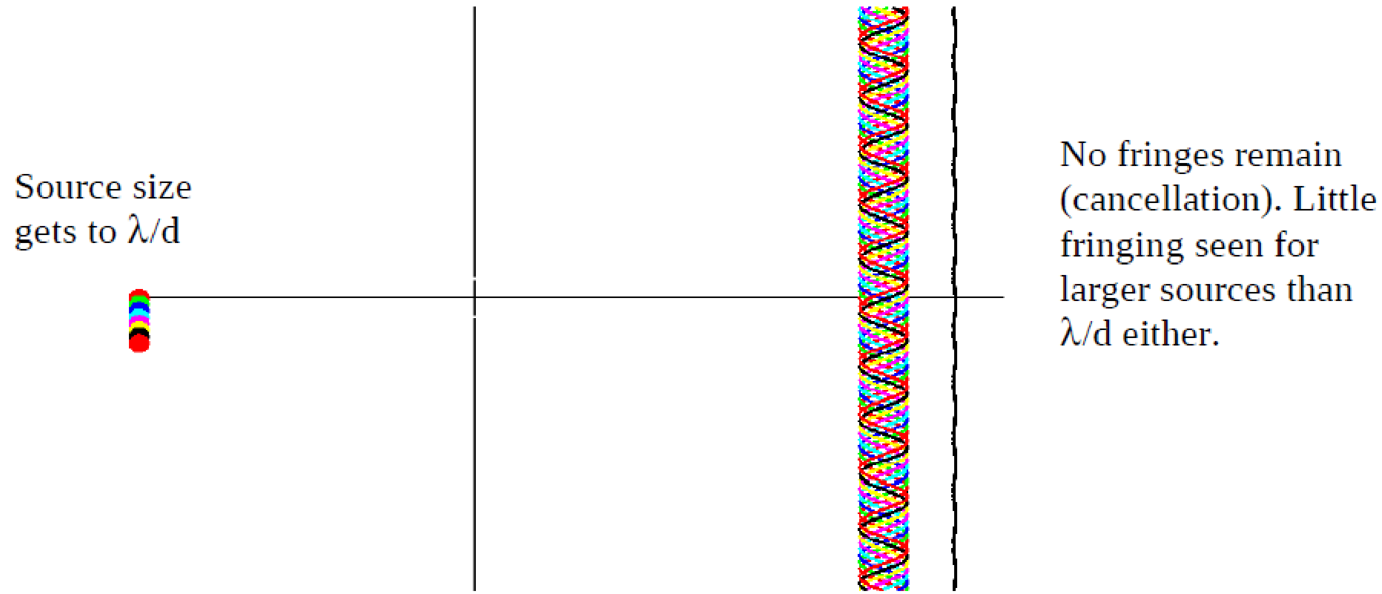
Young's double slit experiment



Visibility of interference fringes decreases with increasing source size

http://www.jb.man.ac.uk/~njj/lofardata.pdf

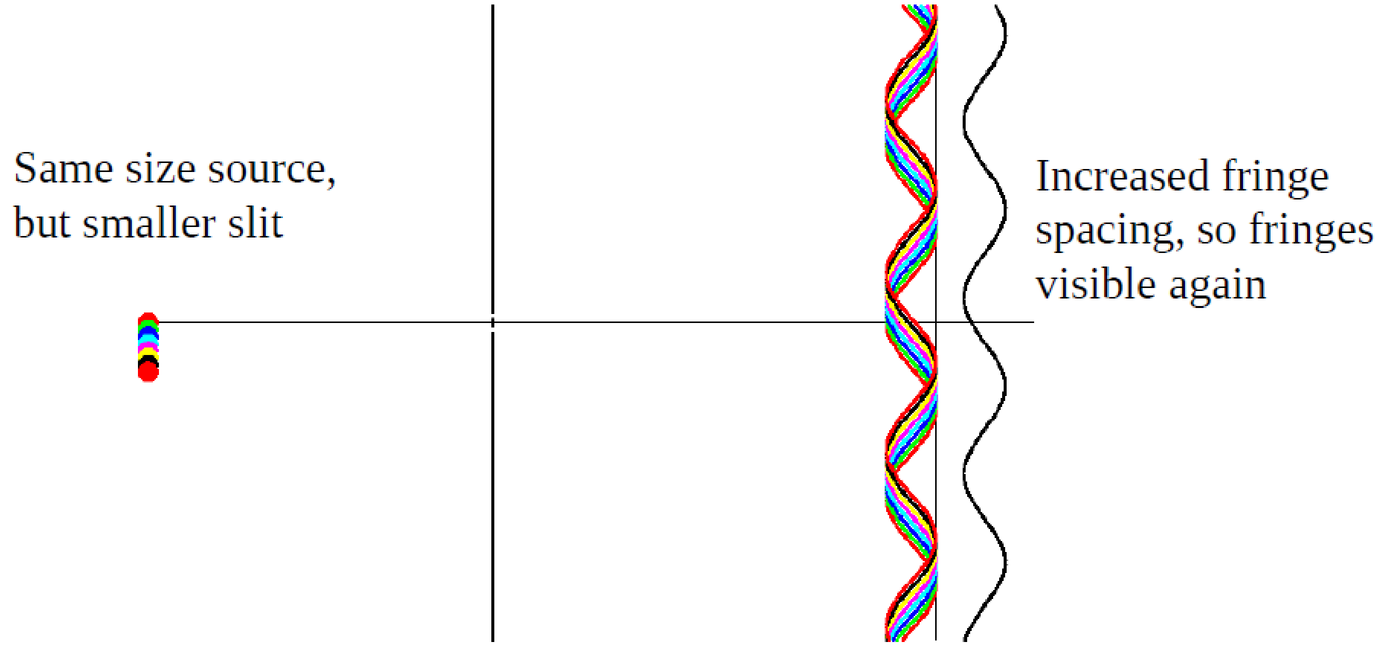
Young's double slit experiment



Visibility of interference fringes goes to zero when source size goes to ***λ/d***

http://www.jb.man.ac.uk/~njj/lofardata.pdf

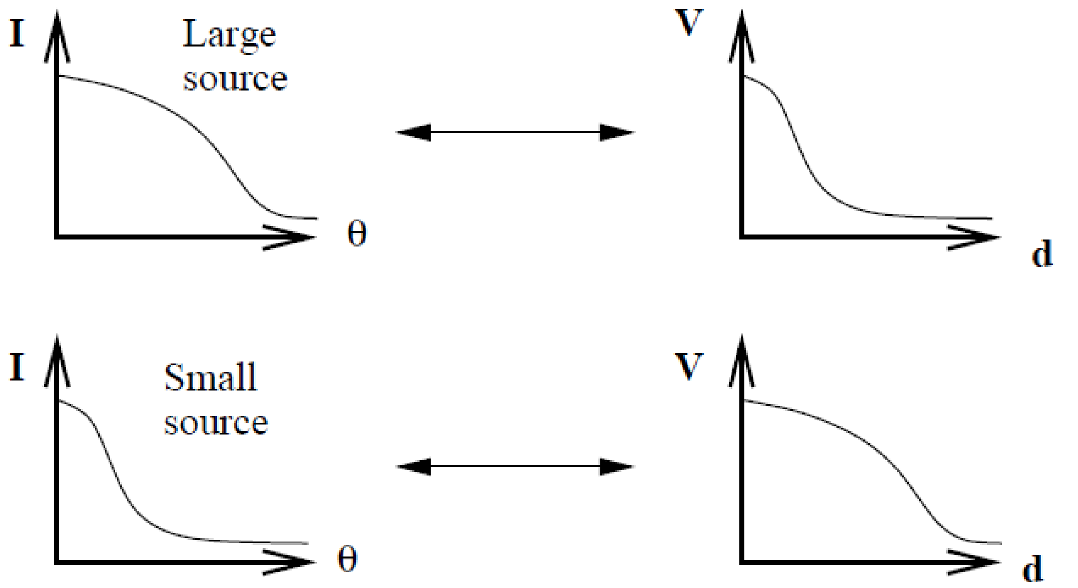
Young's double slit experiment



For given size, visibility increases when separation decreases

http://www.jb.man.ac.uk/~njj/lofardata.pdf

Young's double slit experiment



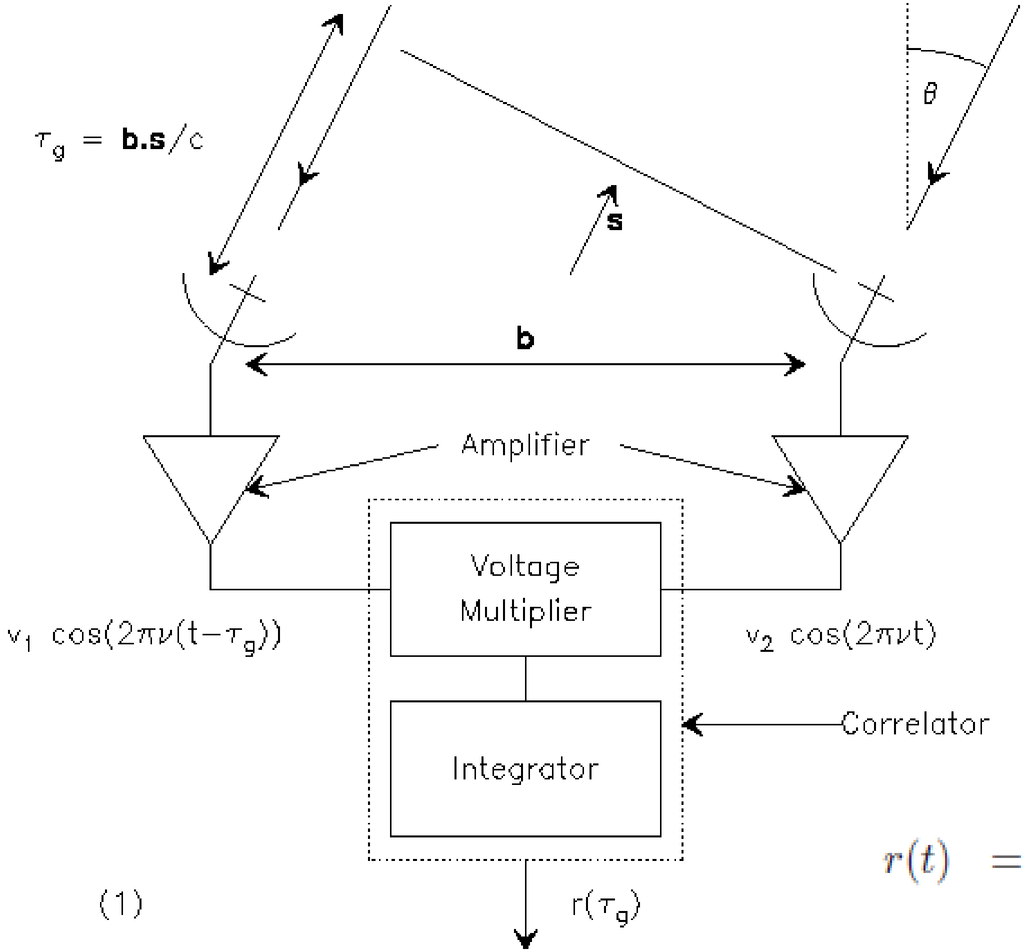
Intensity distribution of source as a function of angle on the sky

Visibility of interference fringes as a function of slit separation ("baseline length")

- Fringe visibility of interferometer gives fourier transform of sky brightness distribution
- Long baselines sensitive to small-scale structure, and short baselines to large-scale structure

http://www.jb.man.ac.uk/~njj/lofardata.pdf

Simple two-element interferometer



Geometric delay

$$\tau_G = \mathbf{b} \cdot \mathbf{s}/c$$

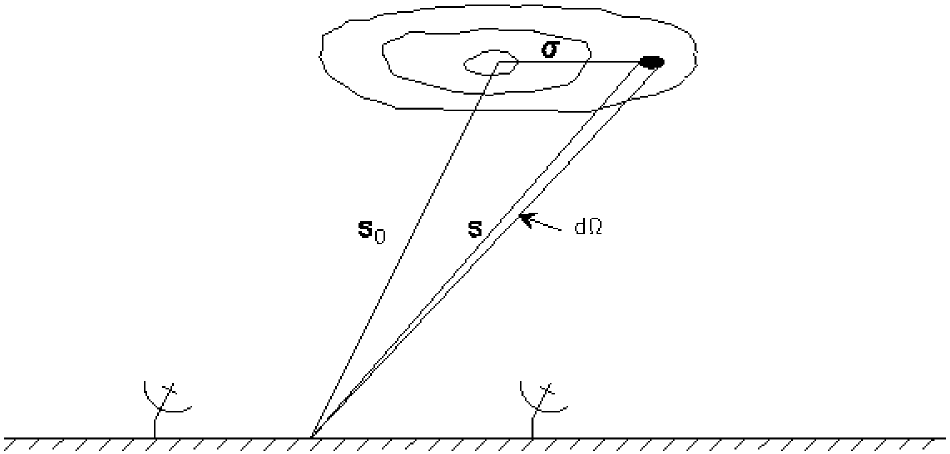
$$R_1(t) = v_1 \cos[2\pi\nu(t - \tau_G)],$$

$$R_2(t) = v_2 \cos(2\pi\nu t)$$

$$\begin{aligned} r(t) &= R_1(t) \times R_2(t) \\ &= [v_1 \cos(2\pi\nu(t - \tau_G))] \times [v_2 \cos(2\pi\nu t)] \\ &= v_1 v_2 \cos(2\pi\nu\tau_G) \end{aligned}$$

http://www.iram.fr/IRAMFR/IS/IS2010/presentations/gueth-mmarray-101004.pdf

Extended source



A(s) Antenna power pattern

I(s) Sky brightness distribution

$$dr = A(\boldsymbol{s})I(\boldsymbol{s})d\Omega d\nu \cos(2\pi\nu\tau_g)$$

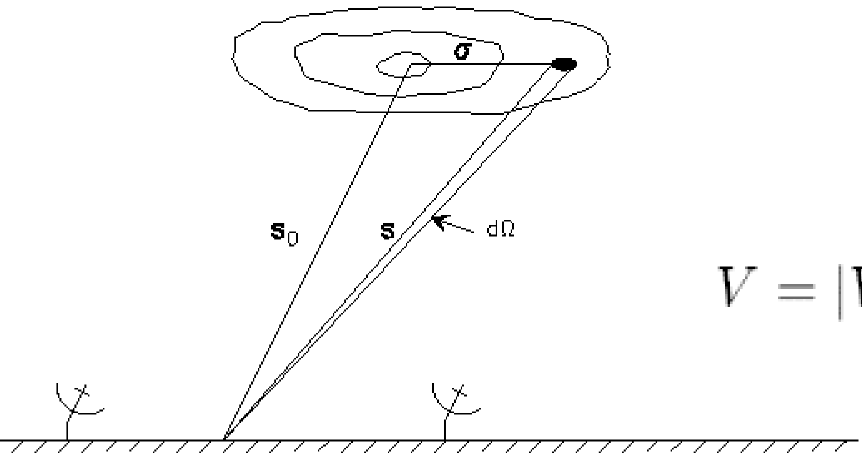
$$r = d\nu \int_{Sky} A(\boldsymbol{s})I(\boldsymbol{s}) \cos(2\pi\nu \boldsymbol{b}.\boldsymbol{s}/c)d\Omega$$

For a source in direction $\boldsymbol{s}_o$, with $\boldsymbol{s}=\boldsymbol{s}_o + \boldsymbol{\sigma}$

$$\begin{aligned} r &= d\nu \cos(2\pi\nu \boldsymbol{b}.\boldsymbol{s}_o/c) \int_{Sky} A(\boldsymbol{\sigma})I(\boldsymbol{\sigma}) \cos(2\pi\nu \boldsymbol{b}.\boldsymbol{\sigma}/c)d\Omega \\ &- d\nu \sin(2\pi\nu \boldsymbol{b}.\boldsymbol{s}_o/c) \int_{Sky} A(\boldsymbol{\sigma})I(\boldsymbol{\sigma}) \sin(2\pi\nu \boldsymbol{b}.\boldsymbol{\sigma}/c)d\Omega \end{aligned}$$

http://www.iram.fr/IRAMFR/IS/IS2010/presentations/gueth-mmarray-101004.pdf

Extended source



We define the Complex Visibility *V*

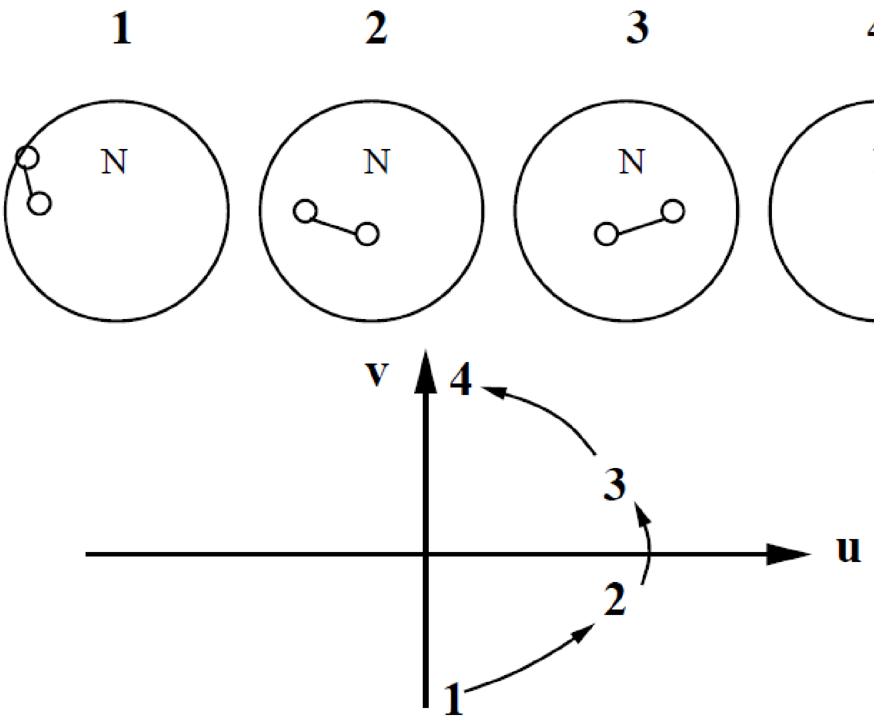
$$V = |V|e^{i\Phi_V} = \int_{Sky} A(\boldsymbol{\sigma})I(\boldsymbol{\sigma})e^{(-2i\pi\nu \boldsymbol{b}.\boldsymbol{\sigma}/c)}d\Omega$$

$$\begin{aligned} r &= d\nu \left(\cos(2\pi\nu \boldsymbol{b}.\boldsymbol{s}_o/c)|V|\cos(\Phi_V) - \sin(2\pi\nu \boldsymbol{b}.\boldsymbol{s}_o/c)|V|\sin(\Phi_V)\right) \\ &= d\nu|V|\cos(2\pi\nu\tau_G - \Phi_V) \end{aligned}$$

Correlator output is proportional to amplitude of visibility, (also contains a phase relation with it)

http://www.iram.fr/IRAMFR/IS/IS2010/presentations/gueth-mmarray-101004.pdf

U-V plane



Components of the projected baseline vector ***b***, in units of ***λ***

http://www.jb.man.ac.uk/~njj/lofardata.pdf

U-V plane

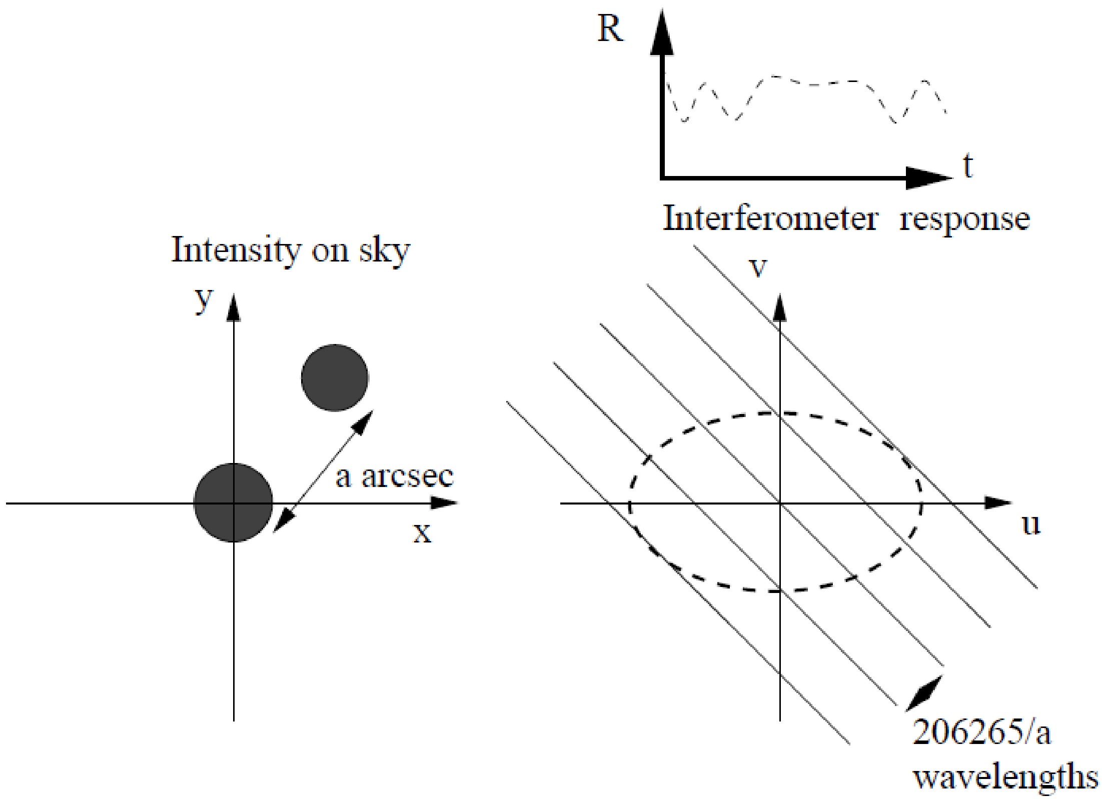


Fig. 5. Diagram showing the interferometer response as a function of u and v for a double source on the sky.

http://www.jb.man.ac.uk/~njj/lofardata.pdf

Visibility function

- Fourier transformation of the brightness distribution of source

$$V(u,v,w)e^{-i2\pi\omega} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(x,y)I(x,y)e^{-i2\pi(ux+uy)}\,dx\,dy.$$

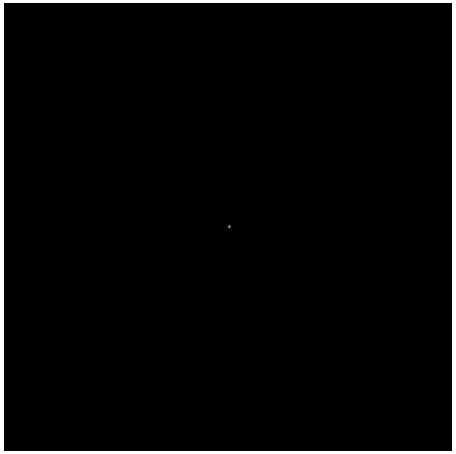
$$I'(x,y) = A(x,y)I(x,y) = \int_{-\infty}^{\infty} V(u,v,0)e^{-i2\pi(ux+uy)}\,du\,dv.$$

- Each observation of the source with a particular baseline and orientation is one point in the UV plane

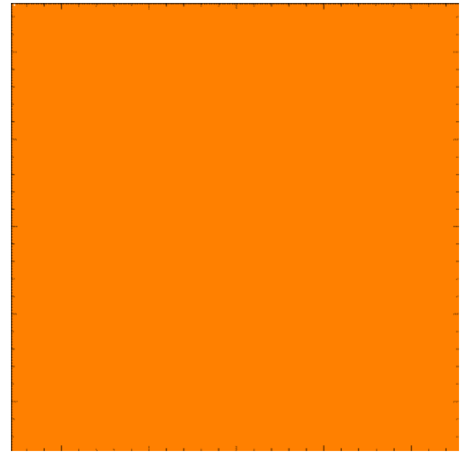
Some 2D Fourier Transform Pairs

T(x,y) Amp{V(u,v)}

δ Function

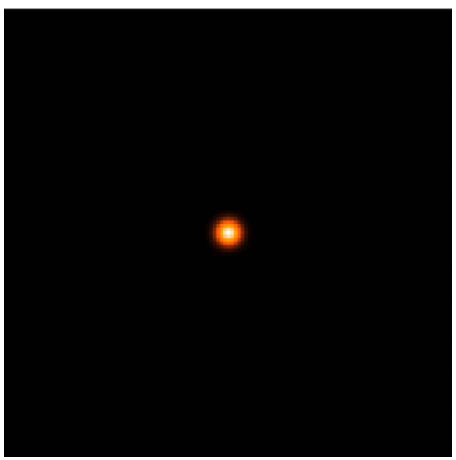


$\rightleftharpoons$

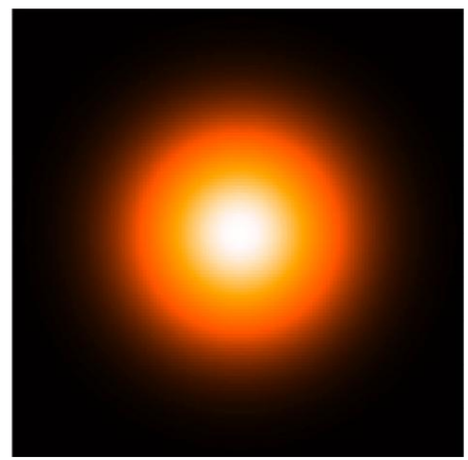


Constant

Gaussian



$\rightleftharpoons$



Gaussian

narrow features transform to wide features (and vice-versa)

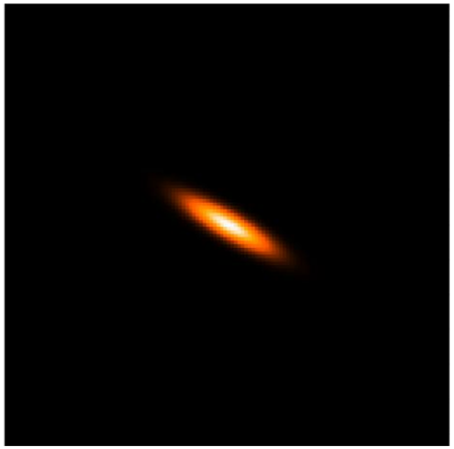
http://www.aoc.nrao.edu/events/synthesis/2010/lectures/wilner_synthesis10.pdf

More 2D Fourier Transform Pairs

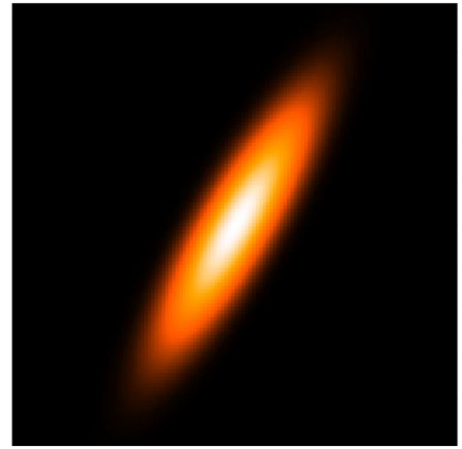
T(x,y)

Amp{V(u,v)}

elliptical Gaussian



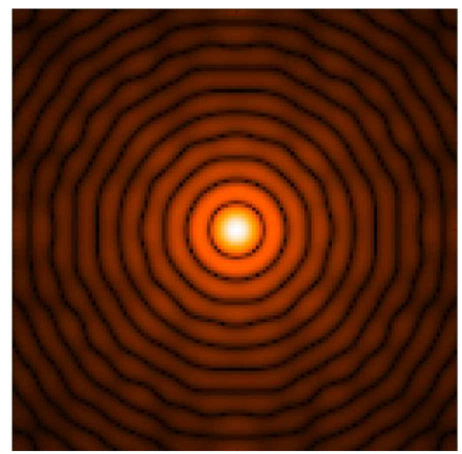
$\rightleftharpoons$



elliptical Gaussian

Disk

$\rightleftharpoons$



Bessel

sharp edges result in many high spatial frequencies

http://www.aoc.nrao.edu/events/synthesis/2010/lectures/wilner_synthesis10.pdf

Imaging basics

- Dirty Image – Image from FT of visibility, incomplete sampling of UV plane
- Dirty beam – Response of the interferometer to a point source, or a PSF (point spread function)

Deconvolution

Correct for sampling effects, to retrieve the Clean Image from the Dirty Image

$$I_D(x,y) = \iint I(u,v)S(u,v)e^{2\pi i(ux+vy)}dudv$$

where ***S(u,v)*** is the sampling function

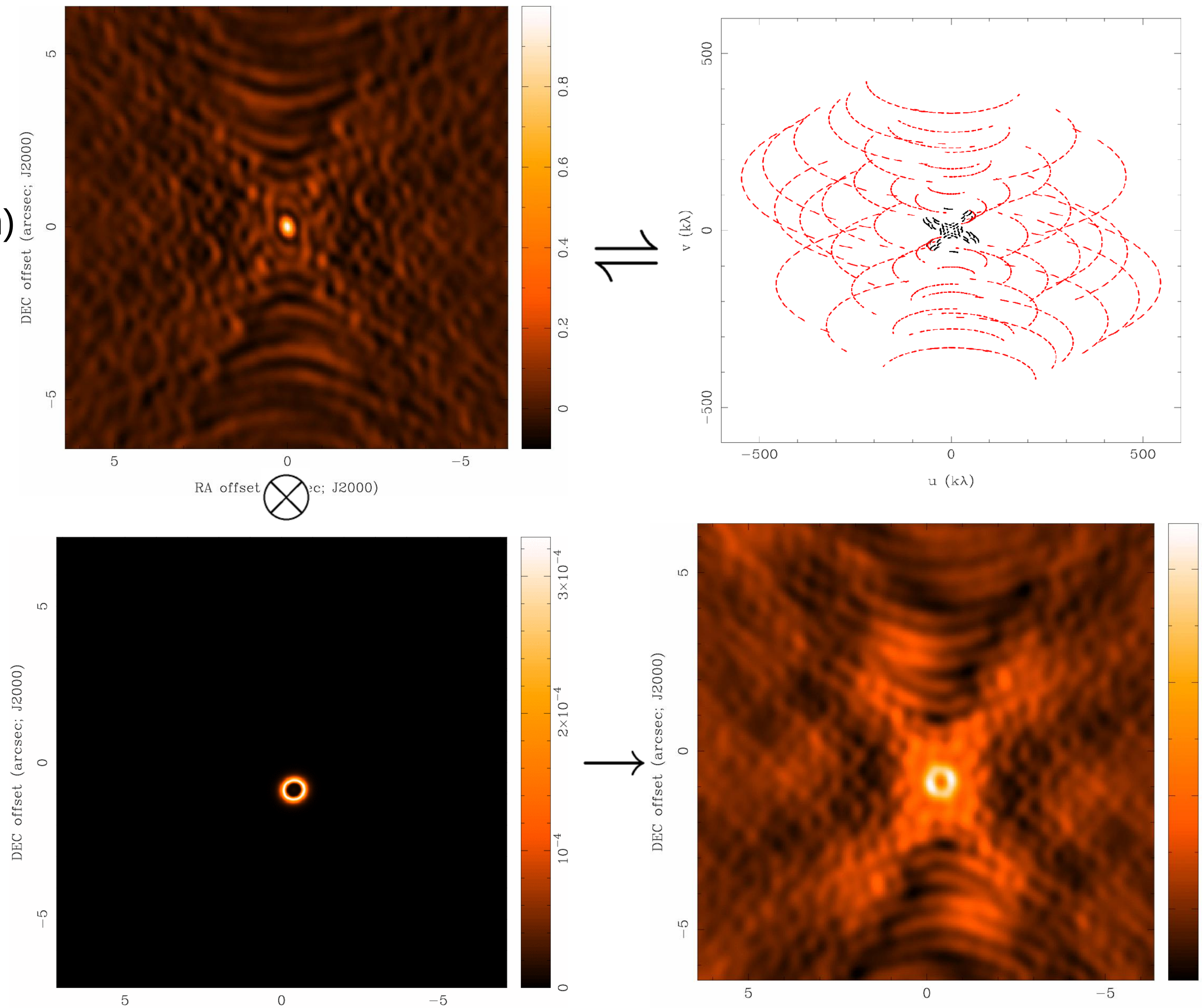
$$B(x,y) = \iint S(u,v)e^{2\pi i(ux+vy)}dudv,$$

where ***B(x,y)*** is the dirty beam

Dirty Beam and Dirty Image

b(x,y)
(dirty beam)

B(u,v)



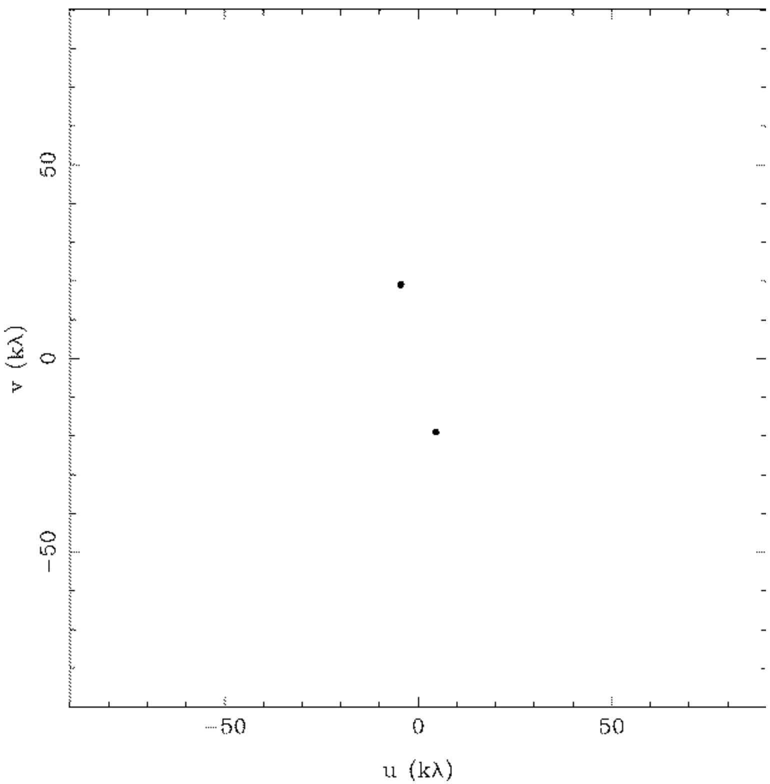
T(x,y)

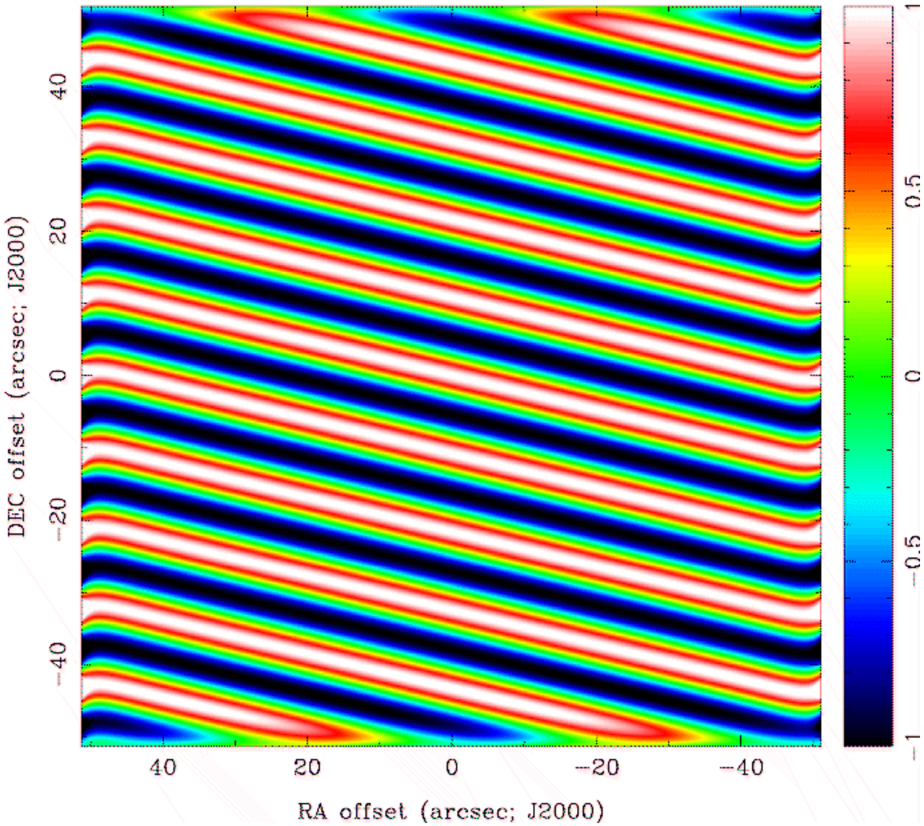
$T^D(x,y)$
(dirty image)

http://www.aoc.nrao.edu/events/synthesis/2010/lectures/wilner_synthesis10.pdf

Dirty Beam Shape and N Antennas

2 Antennas

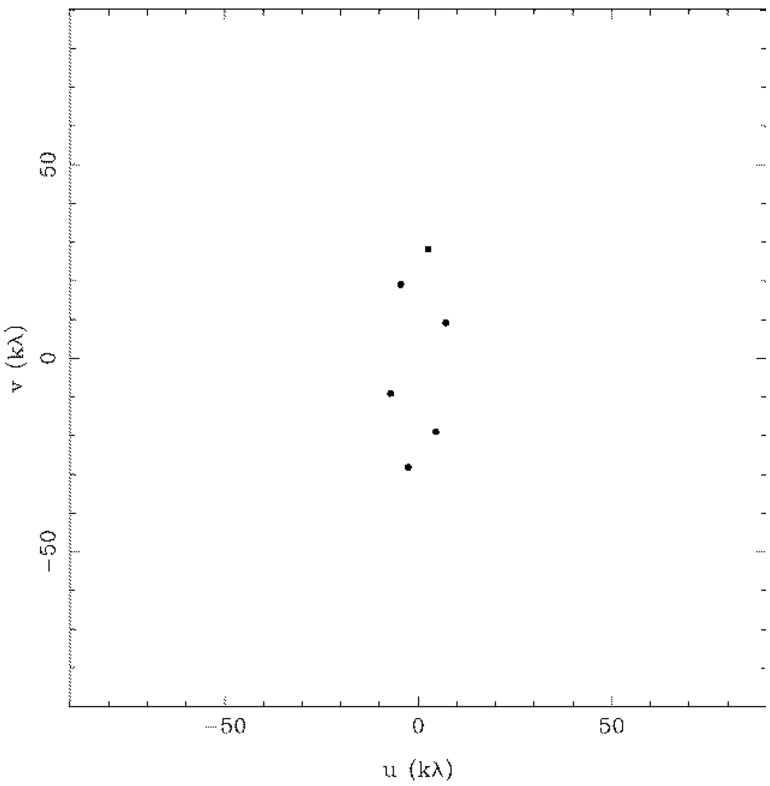


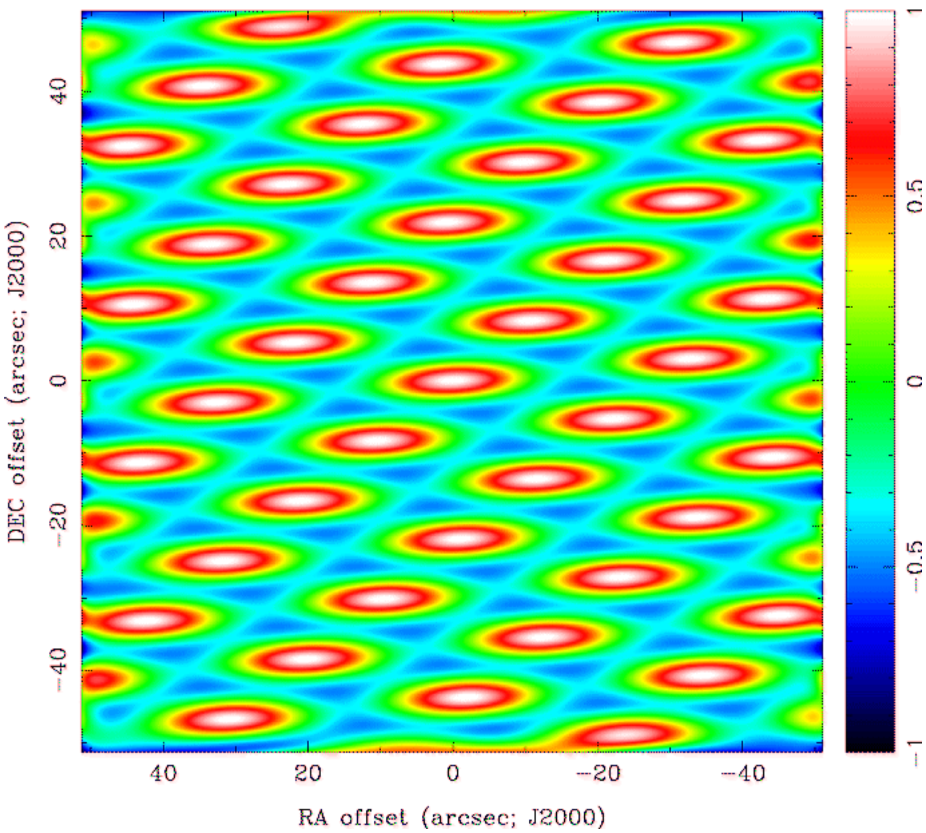


http://www.aoc.nrao.edu/events/synthesis/2008/lectures/wilner_synthesis08.pdf

Dirty Beam Shape and N Antennas

3 Antennas

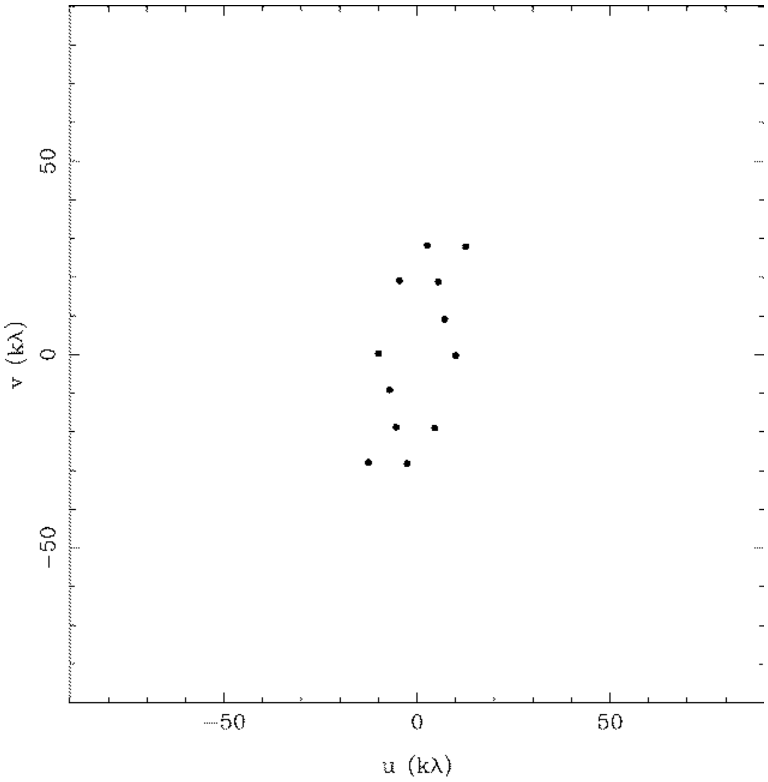


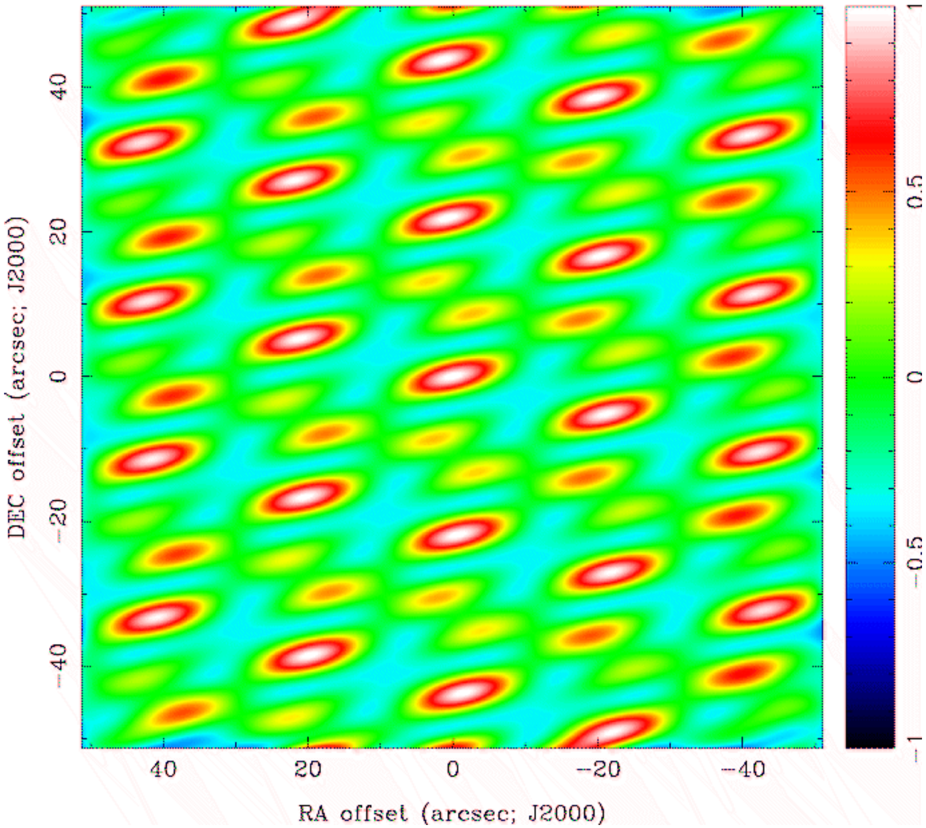


http://www.aoc.nrao.edu/events/synthesis/2008/lectures/wilner_synthesis08.pdf

Dirty Beam Shape and N Antennas

4 Antennas

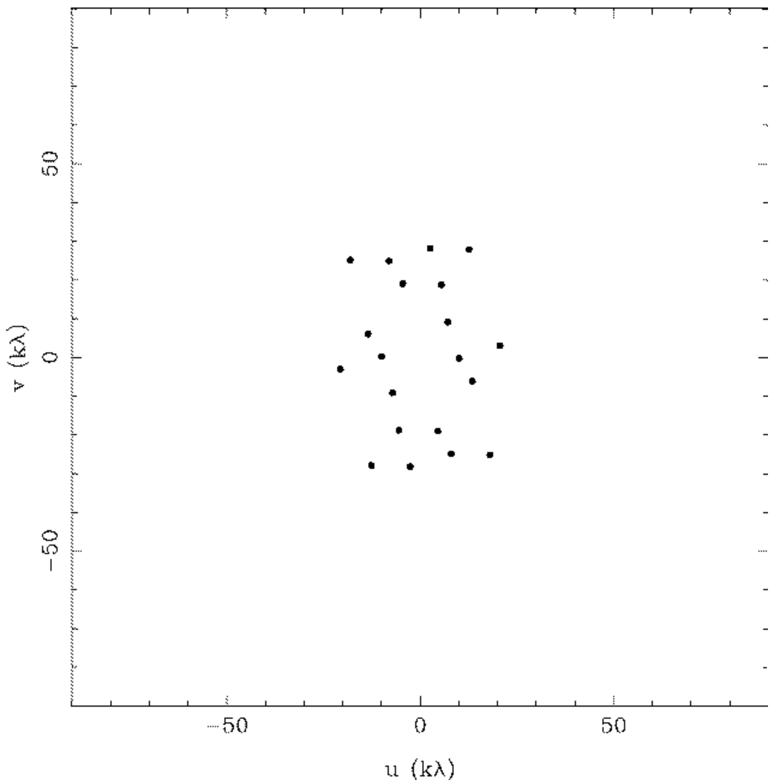


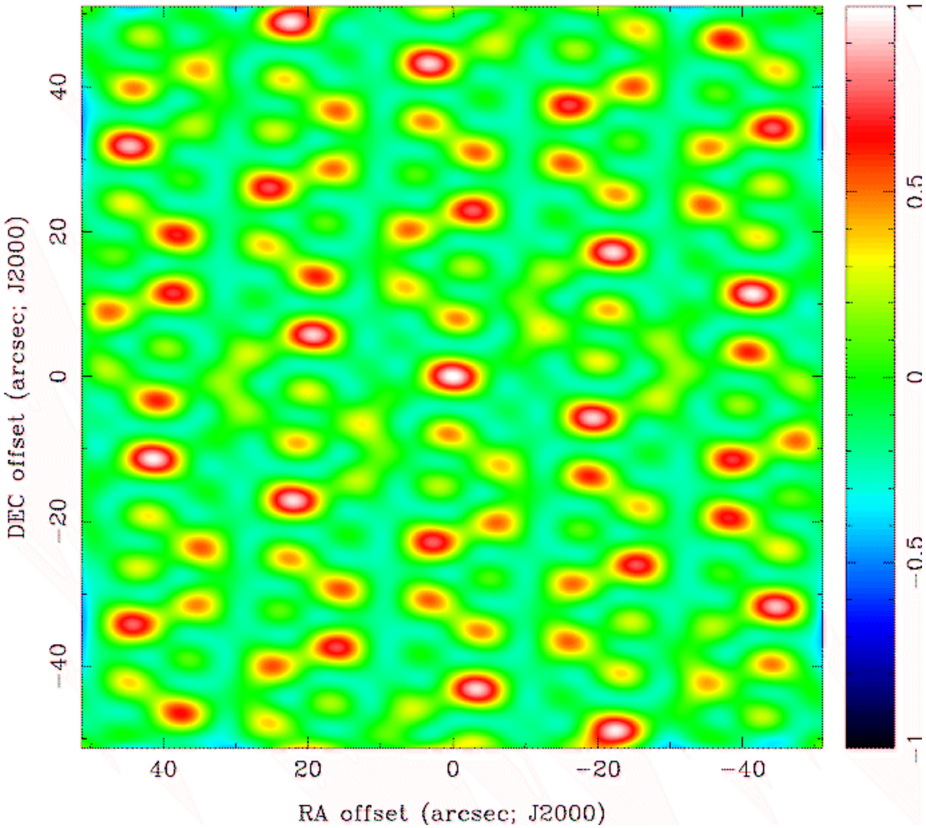


http://www.aoc.nrao.edu/events/synthesis/2008/lectures/wilner_synthesis08.pdf

Dirty Beam Shape and N Antennas

5 Antennas

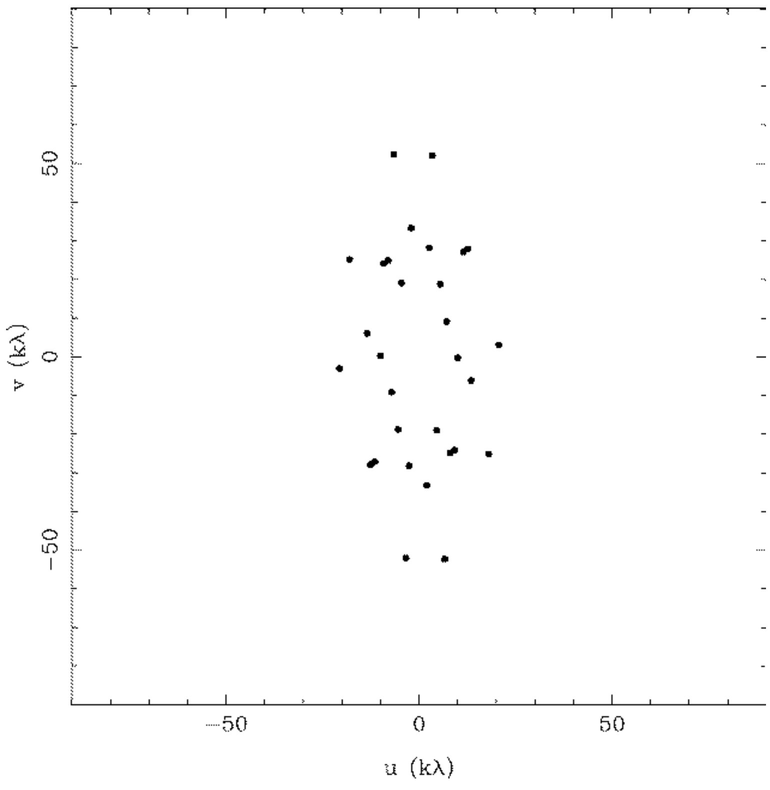


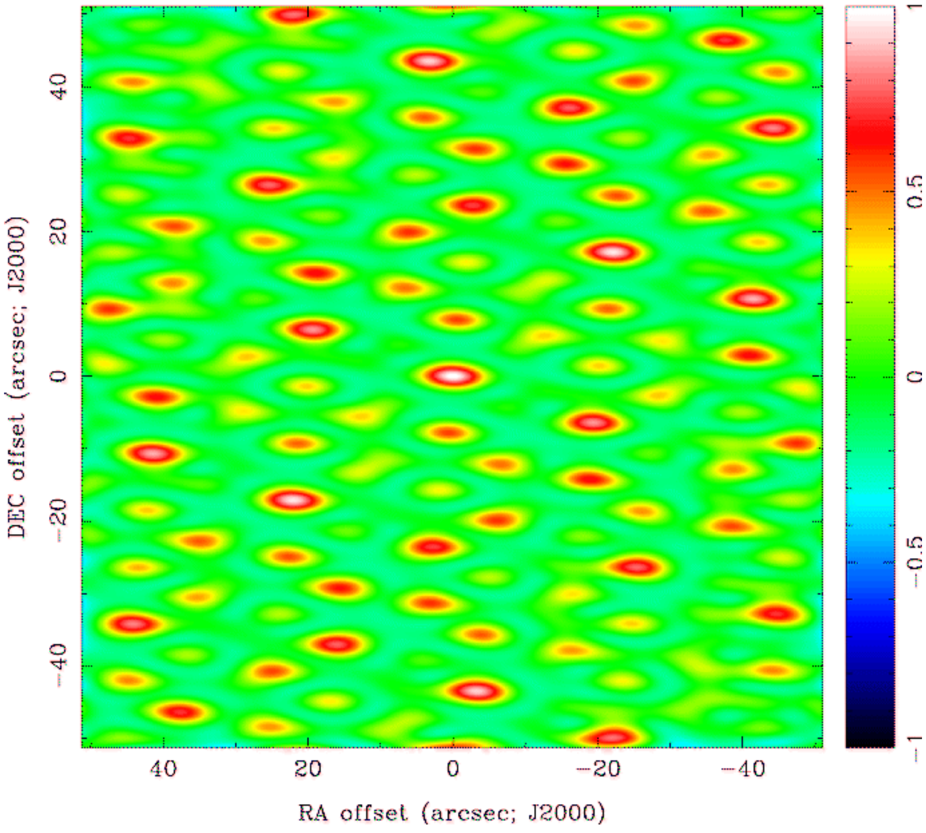


http://www.aoc.nrao.edu/events/synthesis/2008/lectures/wilner_synthesis08.pdf

Dirty Beam Shape and N Antennas

6 Antennas

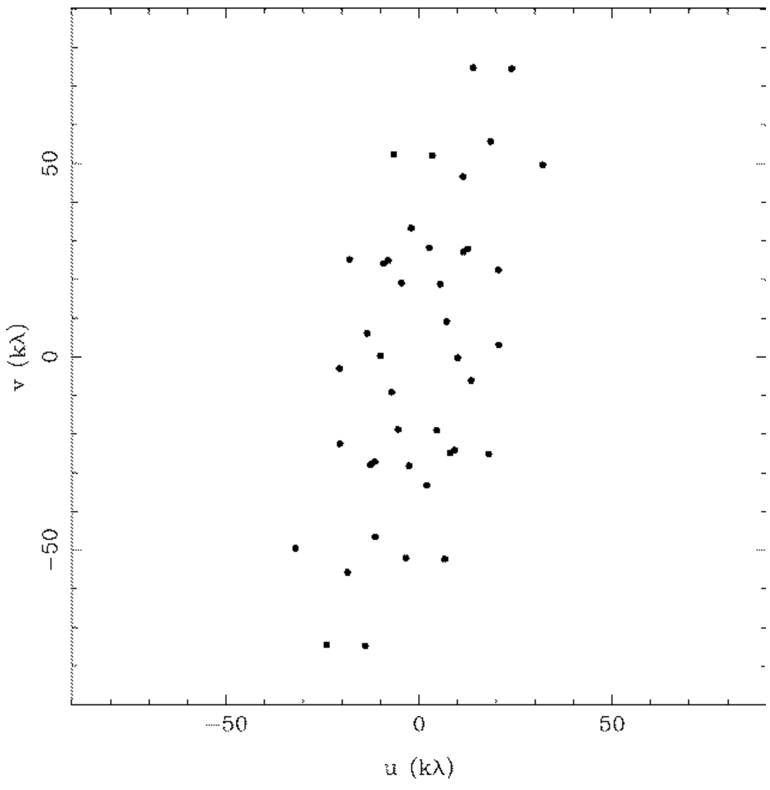


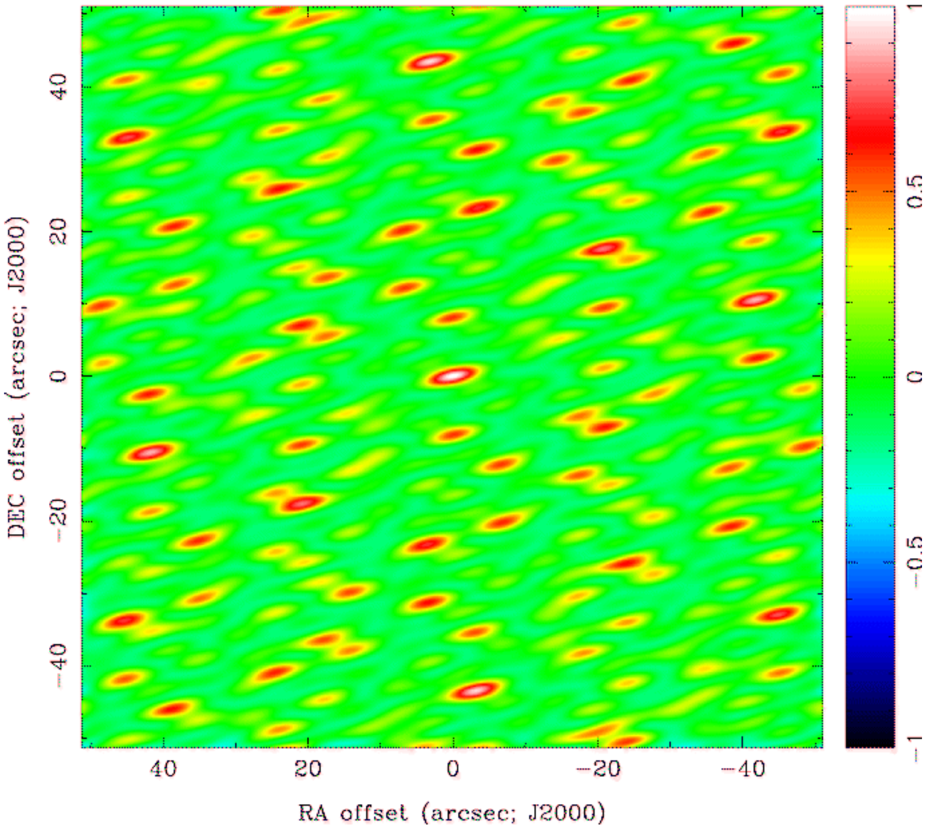


http://www.aoc.nrao.edu/events/synthesis/2008/lectures/wilner_synthesis08.pdf

Dirty Beam Shape and N Antennas

7 Antennas

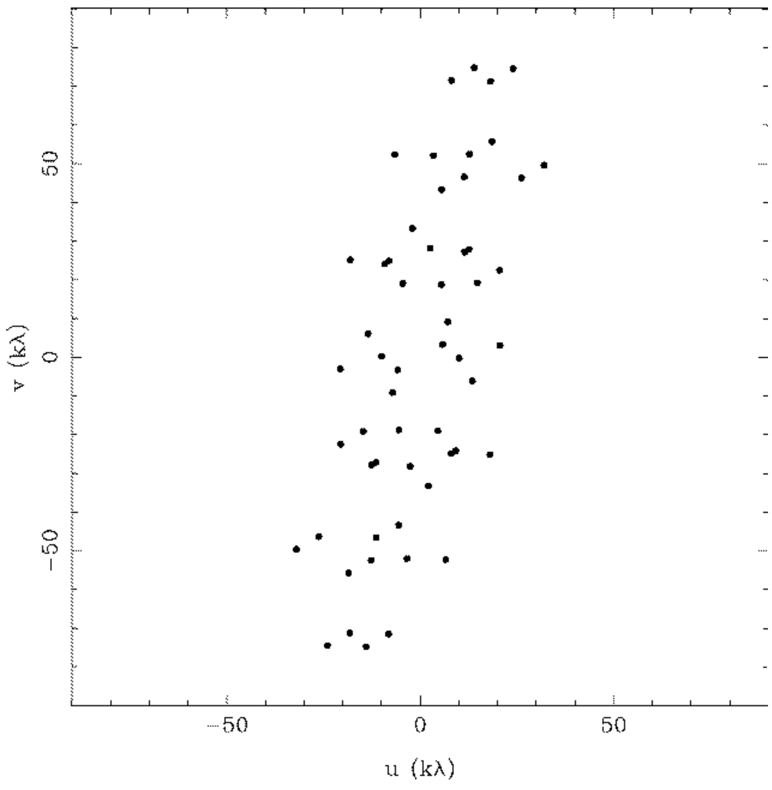


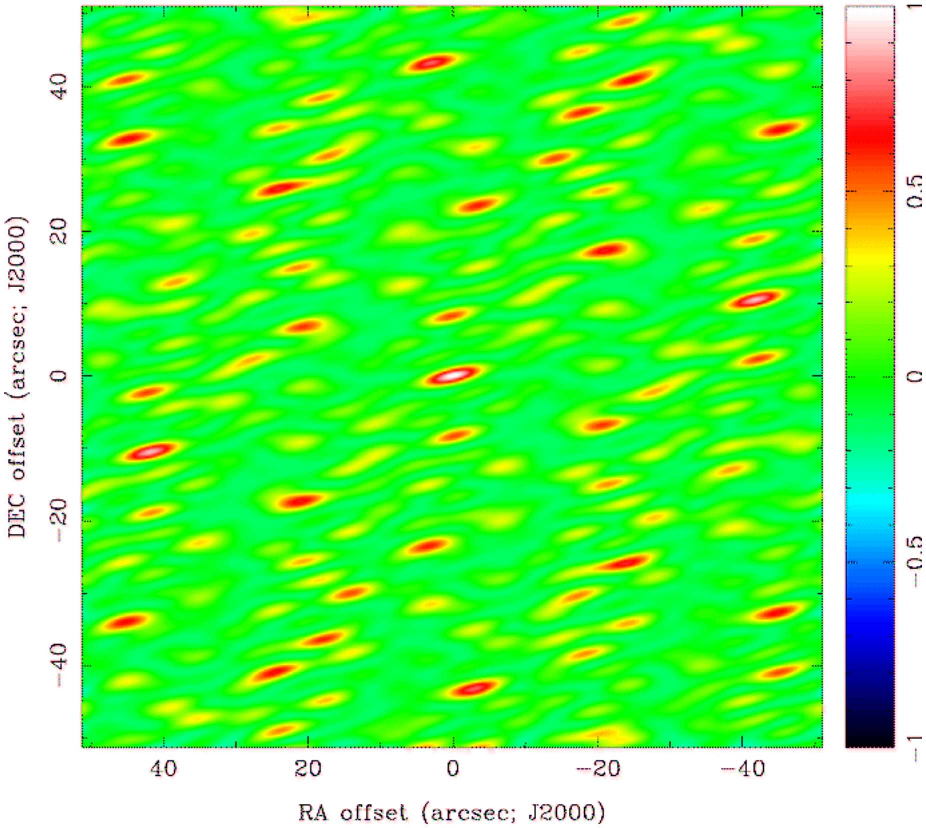


http://www.aoc.nrao.edu/events/synthesis/2008/lectures/wilner_synthesis08.pdf

Dirty Beam Shape and N Antennas

8 Antennas

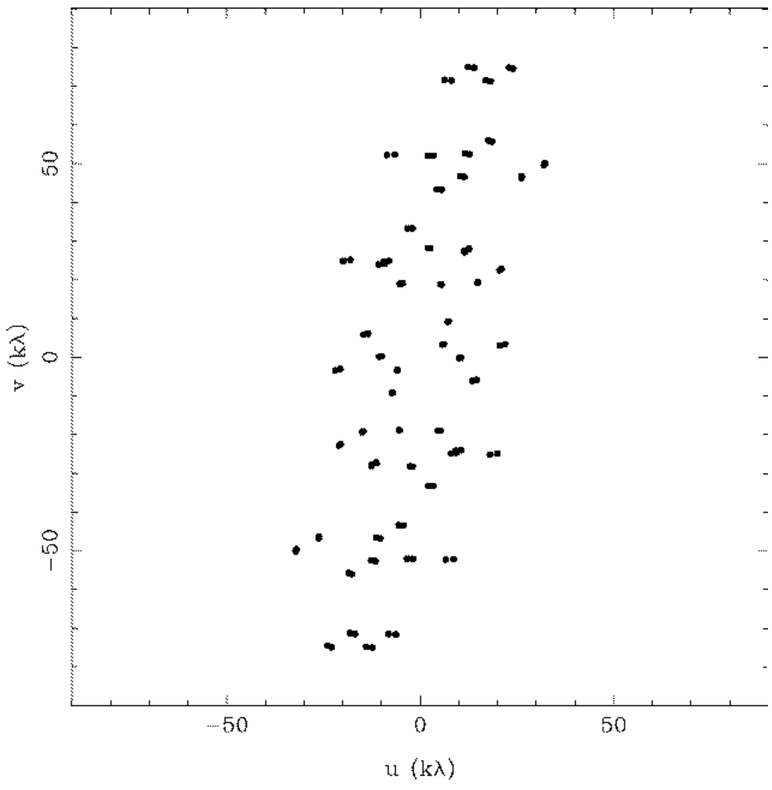


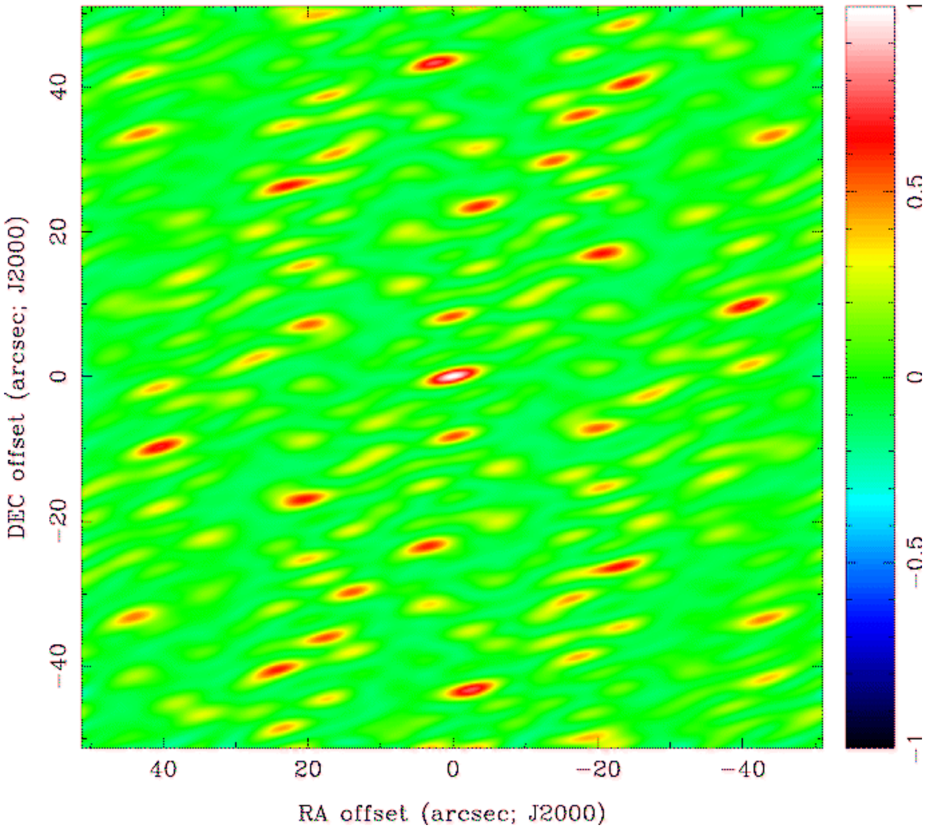


http://www.aoc.nrao.edu/events/synthesis/2008/lectures/wilner_synthesis08.pdf

Dirty Beam Shape and N Antennas

8 Antennas x 2 samples

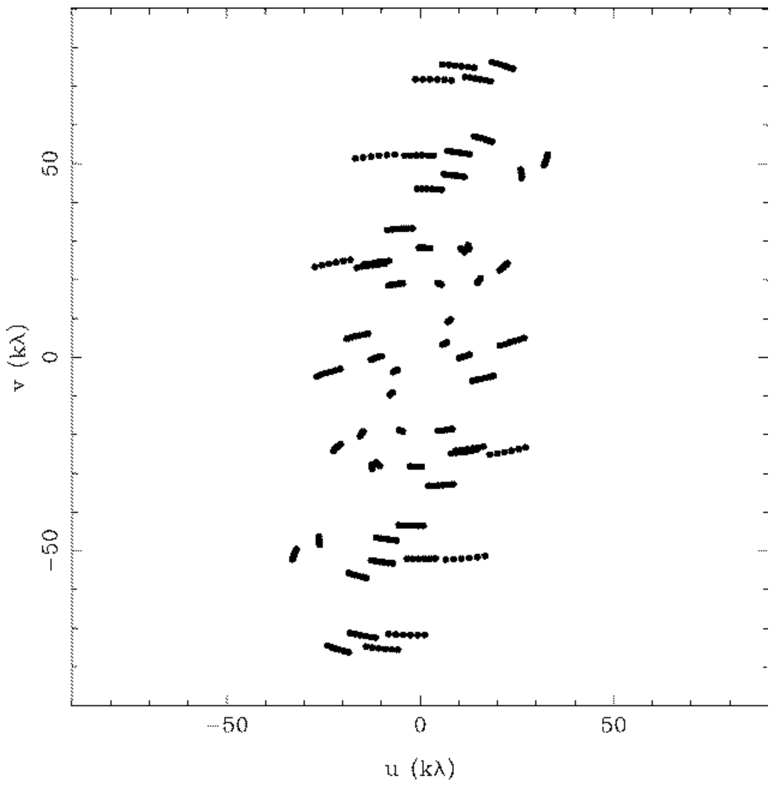


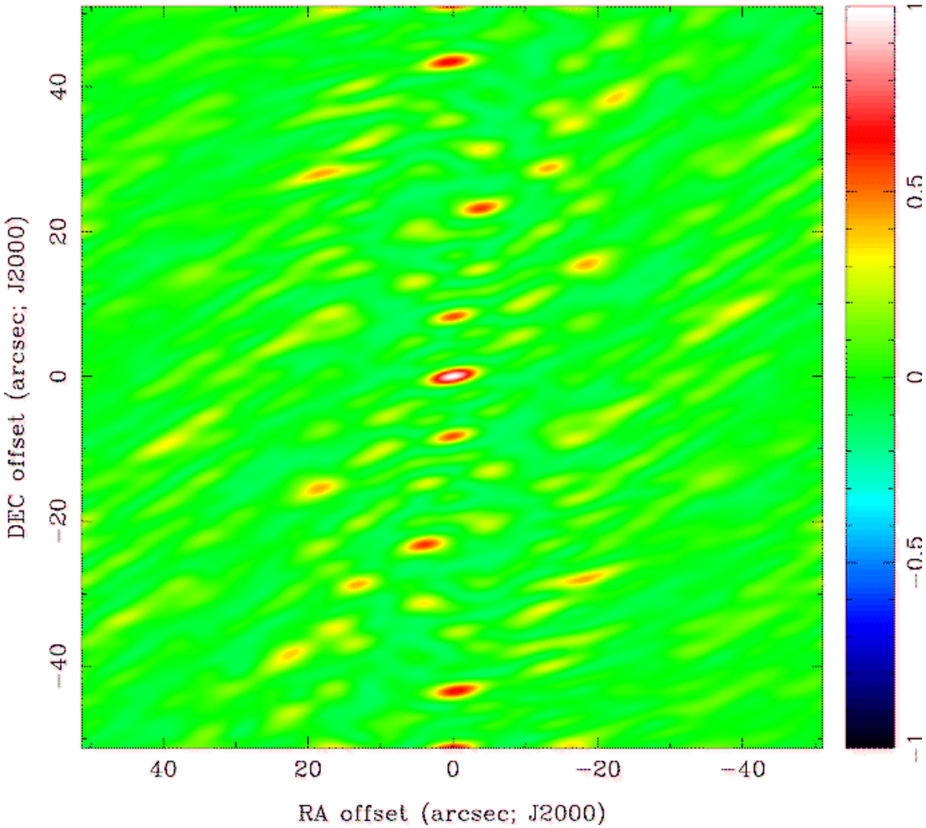


http://www.aoc.nrao.edu/events/synthesis/2008/lectures/wilner_synthesis08.pdf

Dirty Beam Shape and N Antennas

8 Antennas x 6 samples

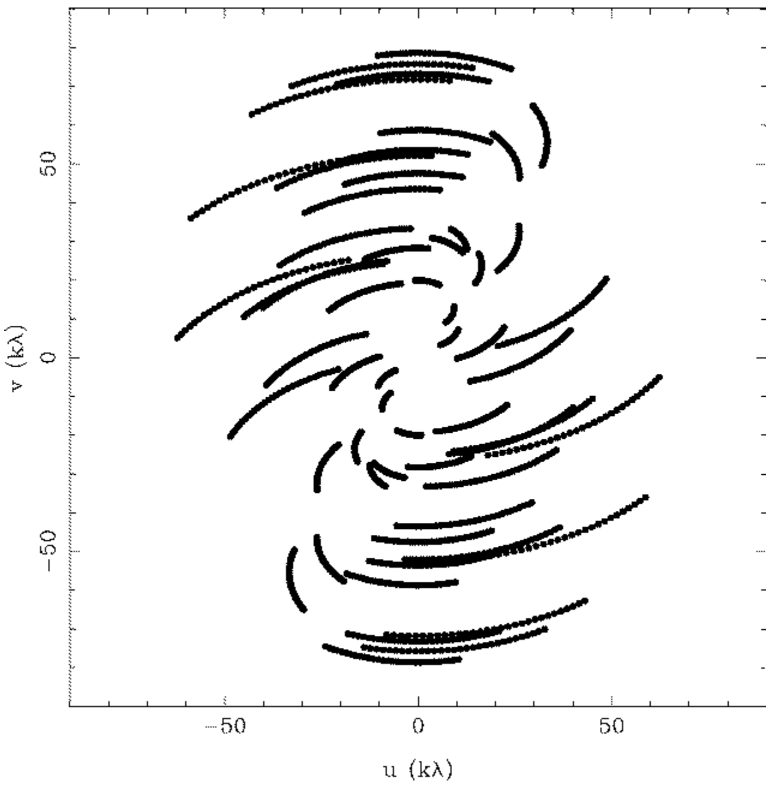


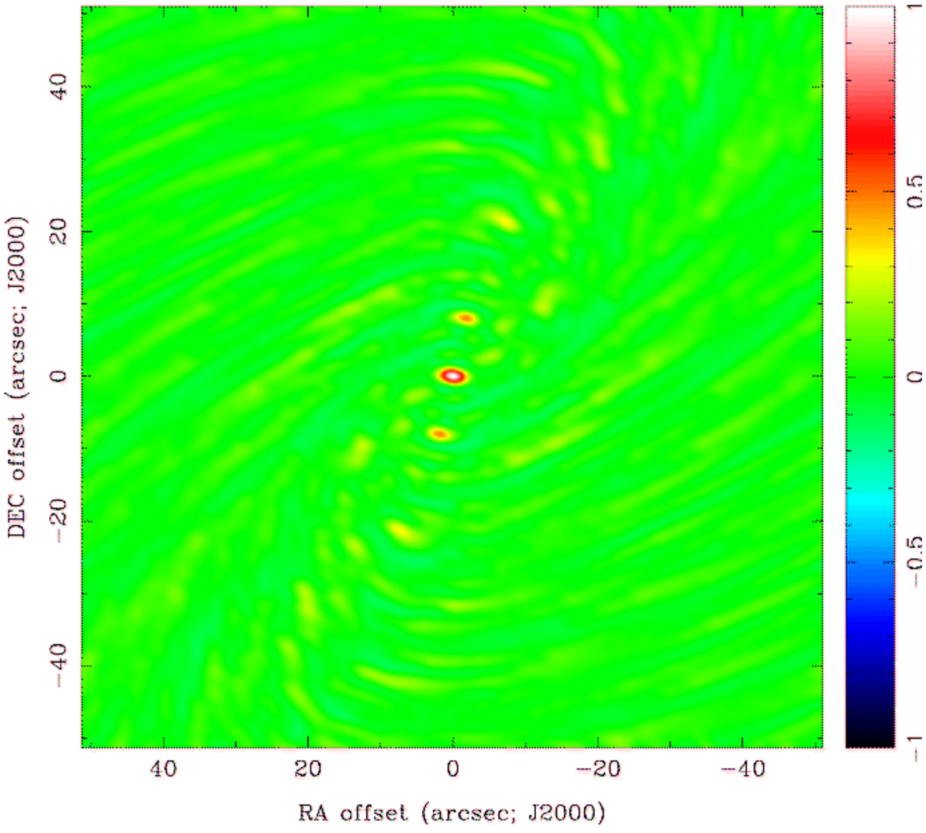


http://www.aoc.nrao.edu/events/synthesis/2008/lectures/wilner_synthesis08.pdf

Dirty Beam Shape and N Antennas

8 Antennas x 30 samples

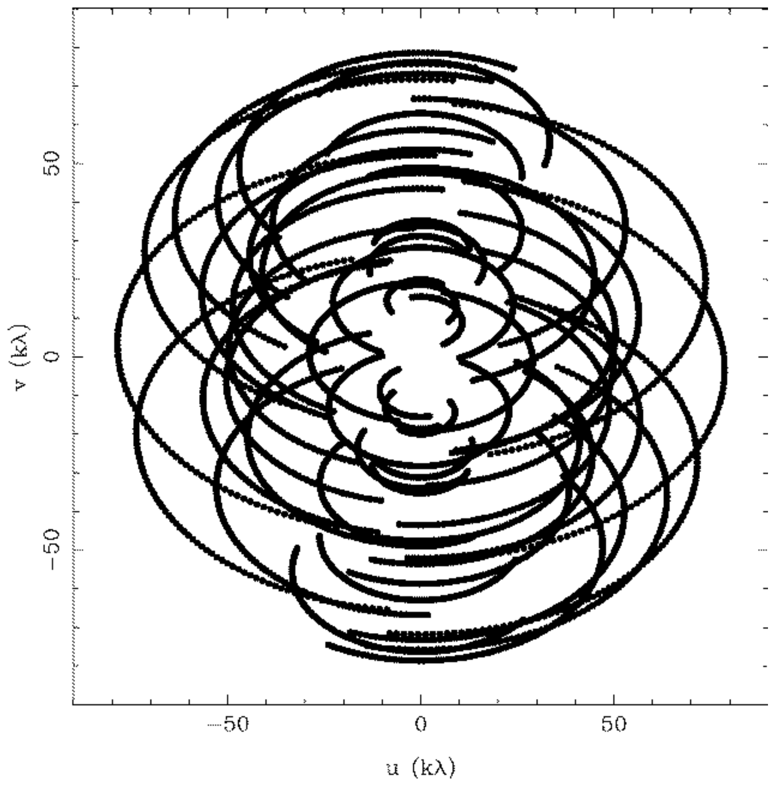


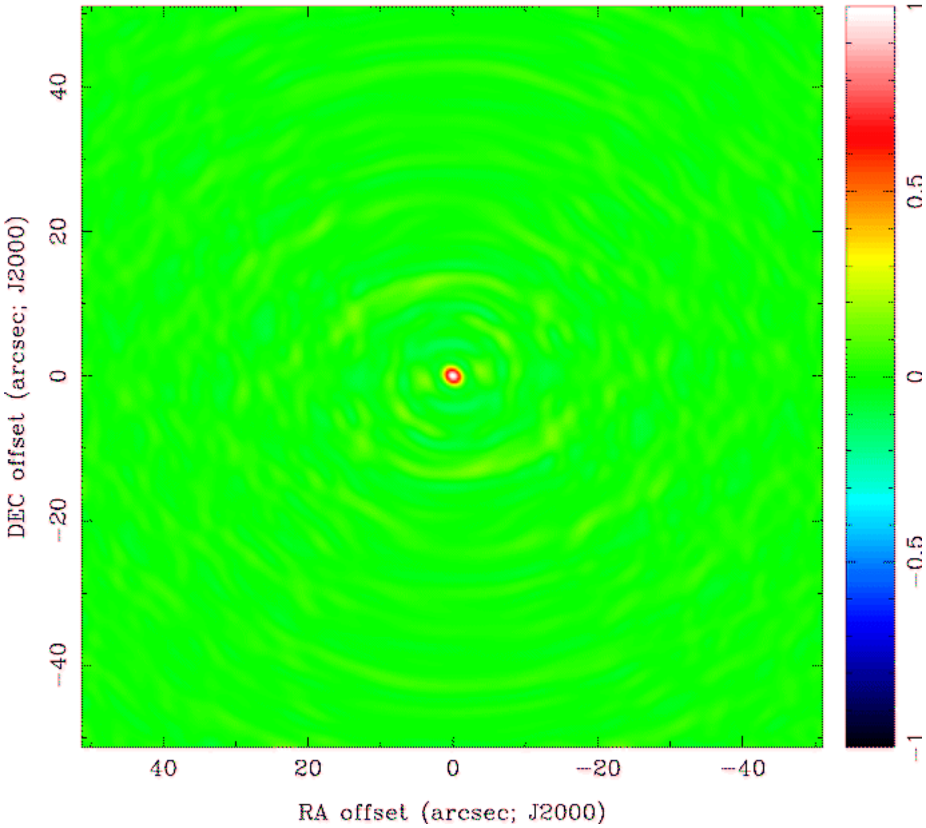


http://www.aoc.nrao.edu/events/synthesis/2008/lectures/wilner_synthesis08.pdf

Dirty Beam Shape and N Antennas

8 Antennas x 107 samples





http://www.aoc.nrao.edu/events/synthesis/2008/lectures/wilner_synthesis08.pdf

Calibration

- Correct errors due to instrumental and atmospheric effects
- Observing calibrator sources – ideally point sources close to target
- Bandpass, phase and amplitude calibration

Sensitivity (r.m.s noise)

$$S = \frac{\sqrt{2} k_B T_{\rm sys}}{A \eta \sqrt{n_b \Delta\nu t_{\rm int}}}$$

where T_{sys} is the system temperature,

A is the area of each antenna

η is the aperture efficiency

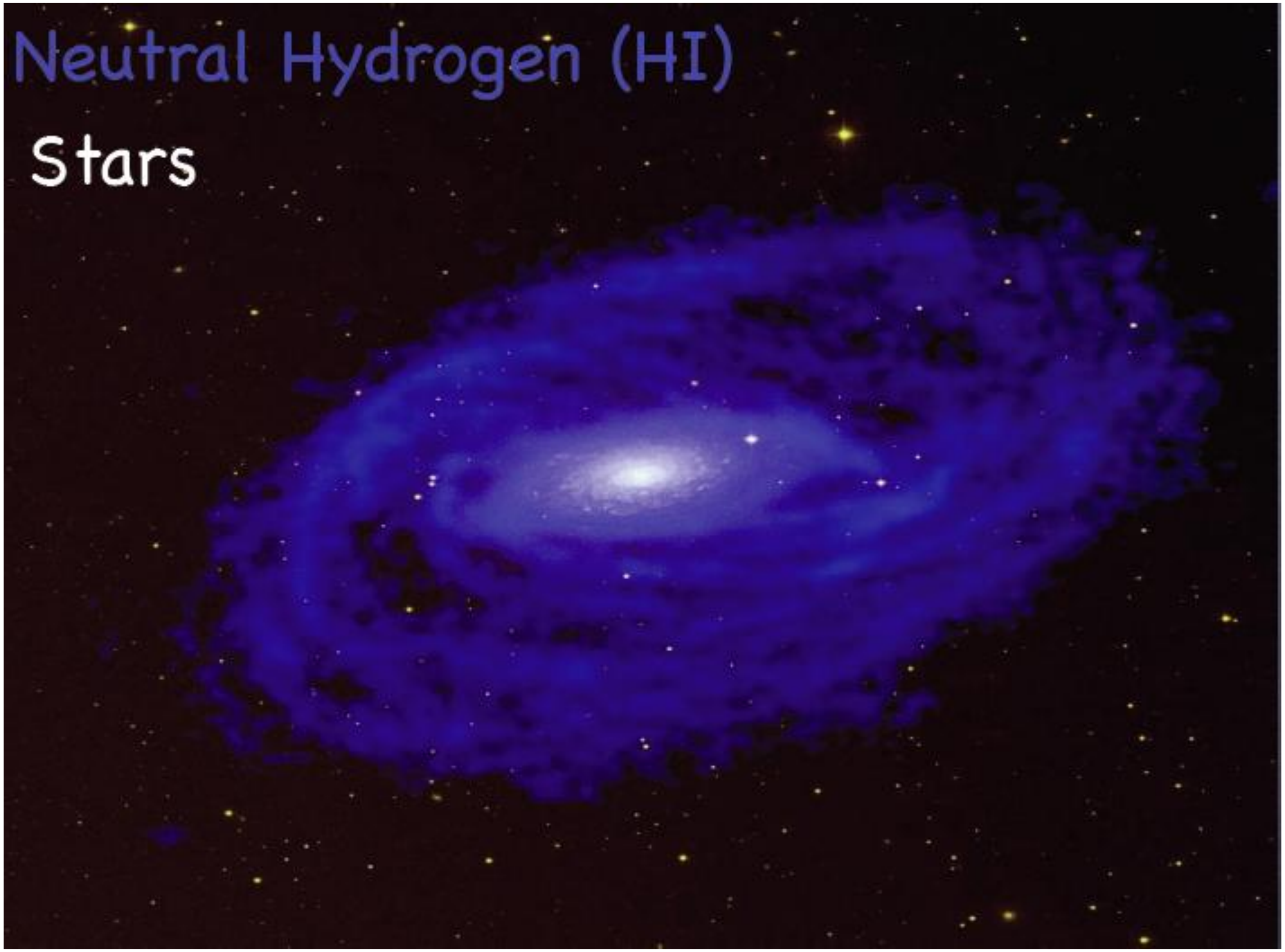
n_b is the number of baselines

Δν is the observing bandwidth

t_{int} is the integration time

Science with radio interferometers

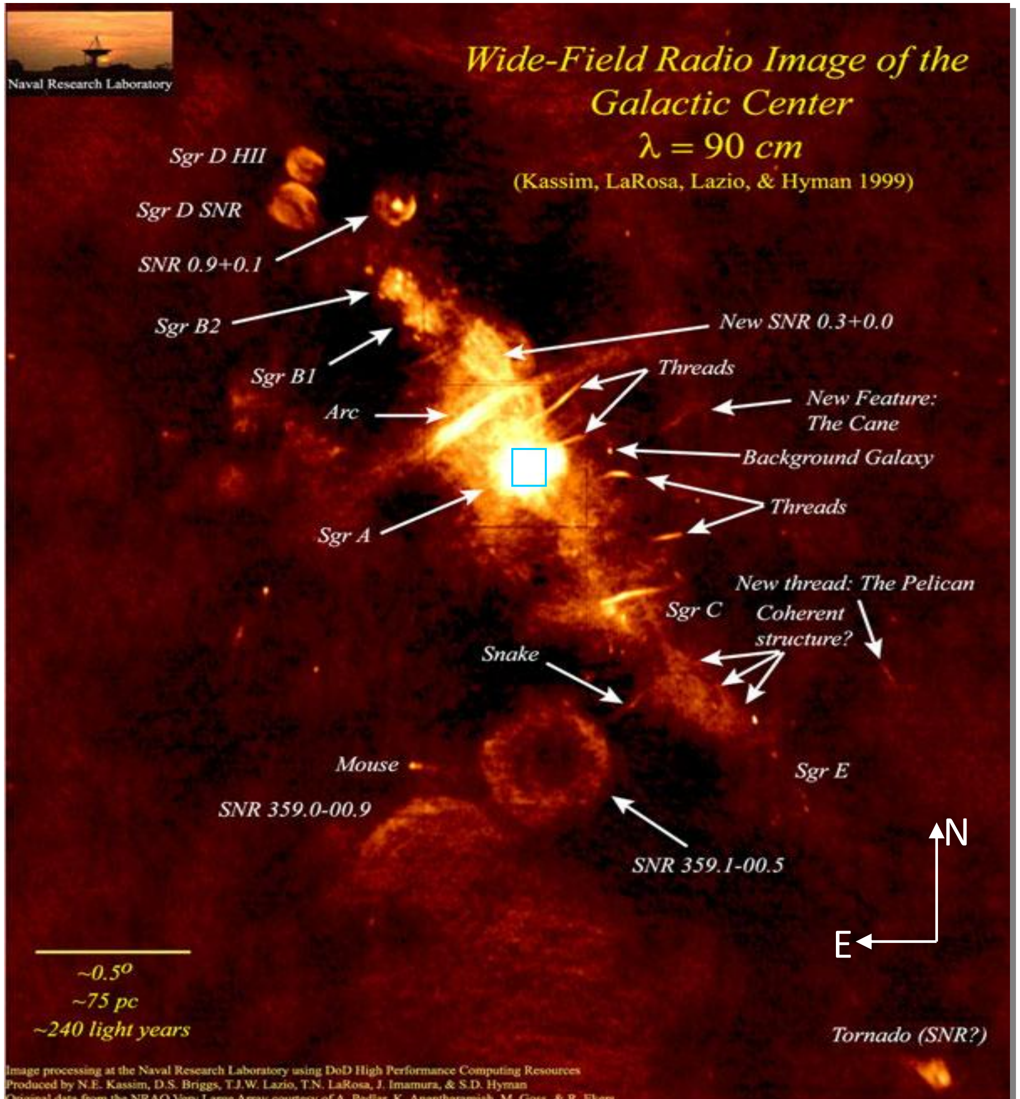
Spectral line emission

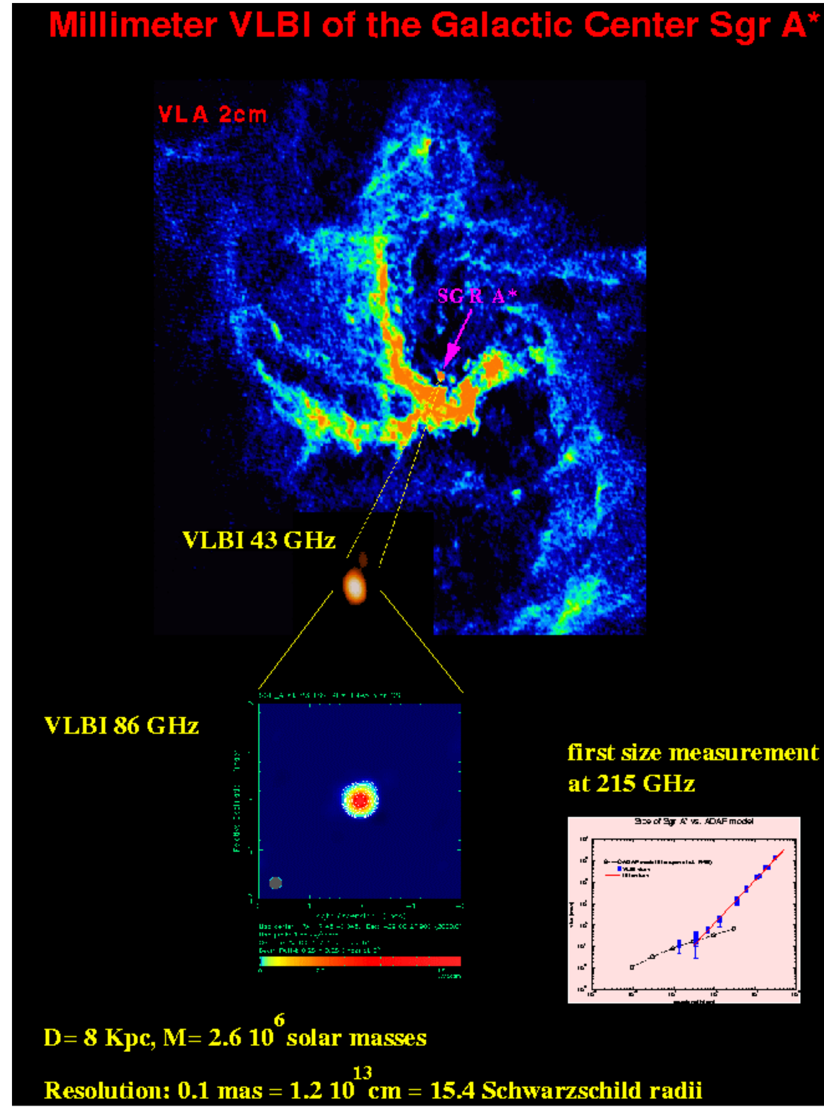


http://www.mpifr-bonn.mpg.de/div/eris/talks/Garrett_RadioAstronomy_red.pdf

Science with radio interferometers

Continuum emission – Thermal and non-thermal



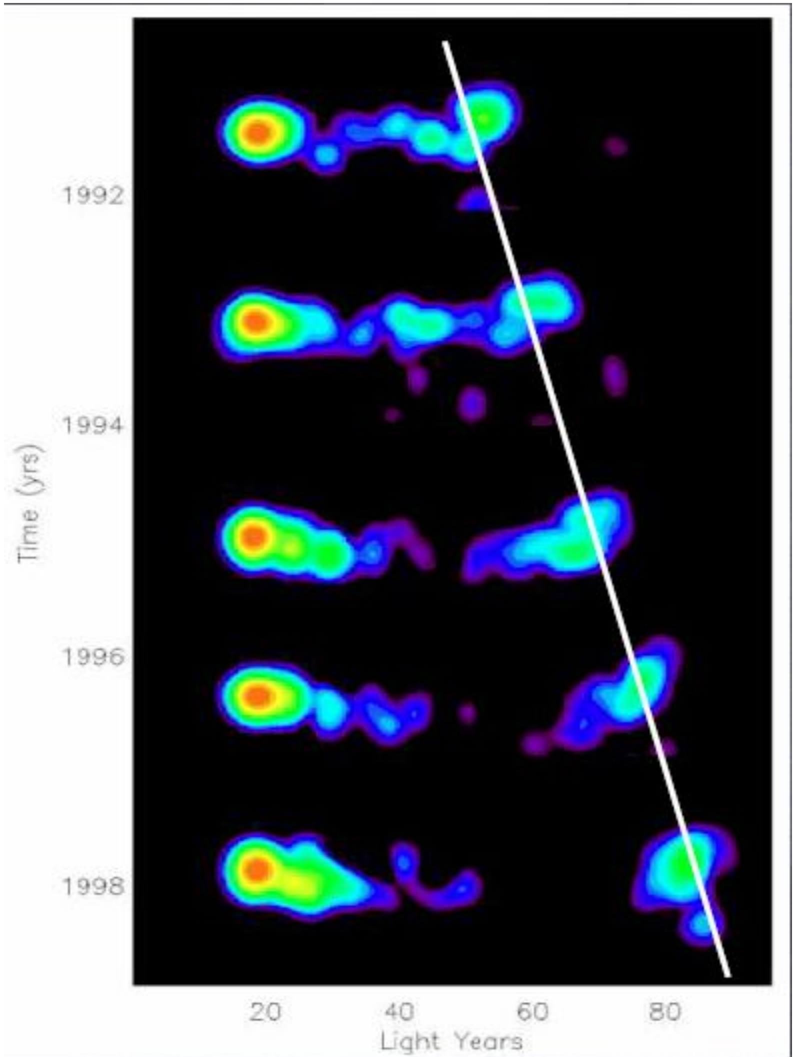


Kassim et al. 1990

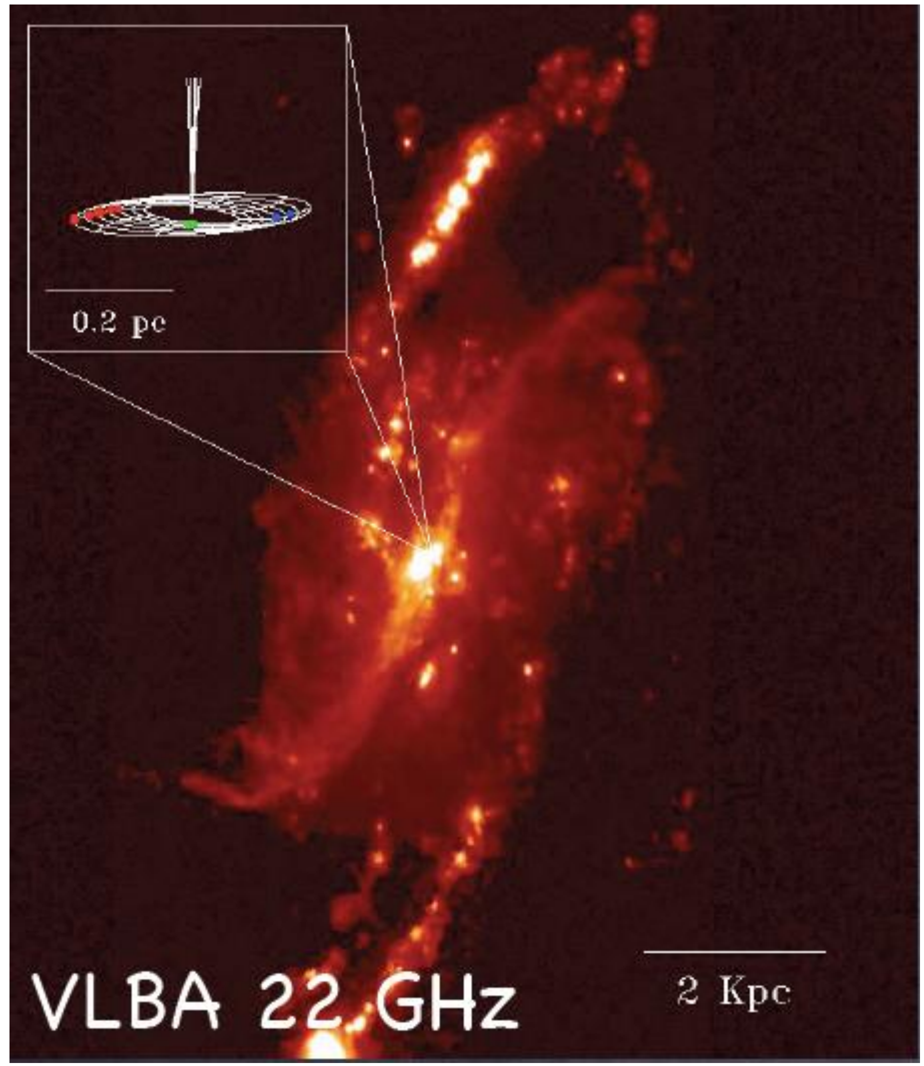
Krichbaum 1998

Science with radio interferometers

Superluminal motion

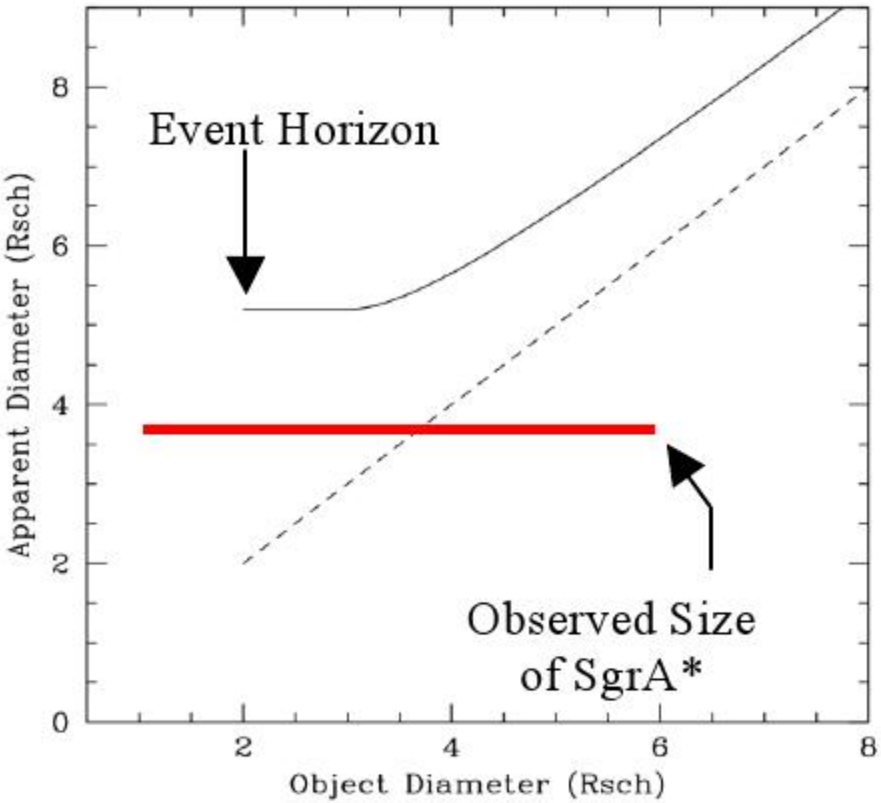


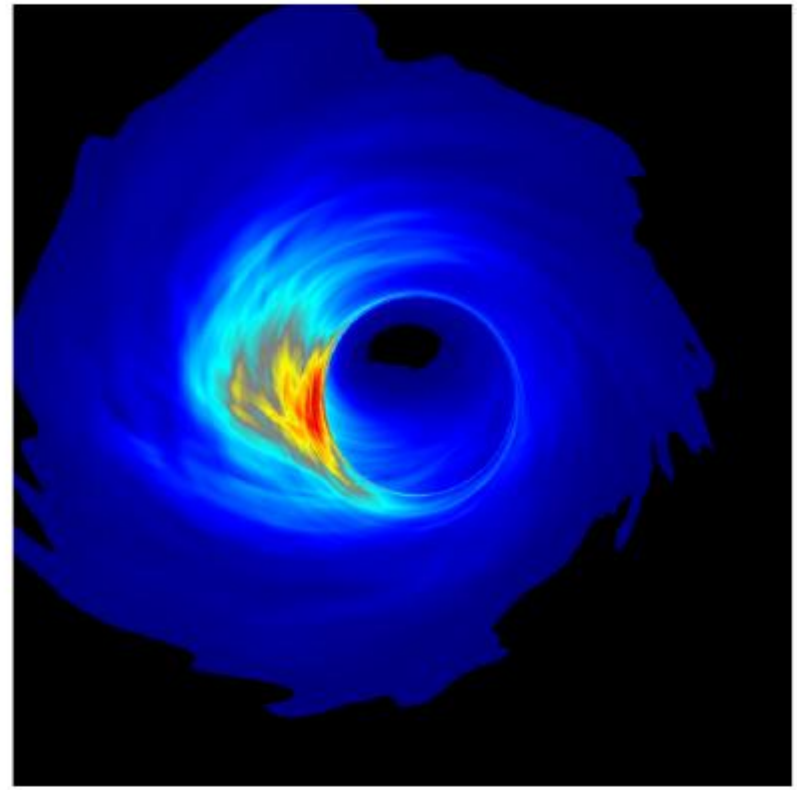
Water maser measurement NGC4258 (Miyoshi et al.1995)



http://www.mpifr-bonn.mpg.de/div/eris/talks/Garrett_RadioAstronomy_red.pdf

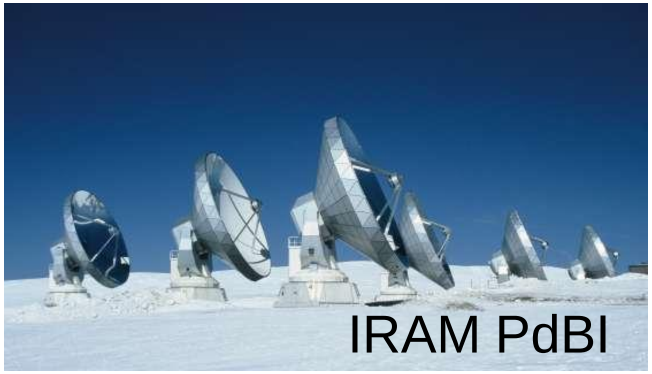
Future mm-VLBI measurements

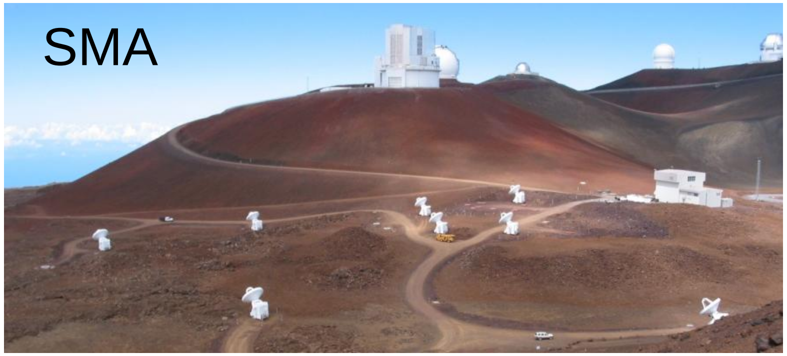


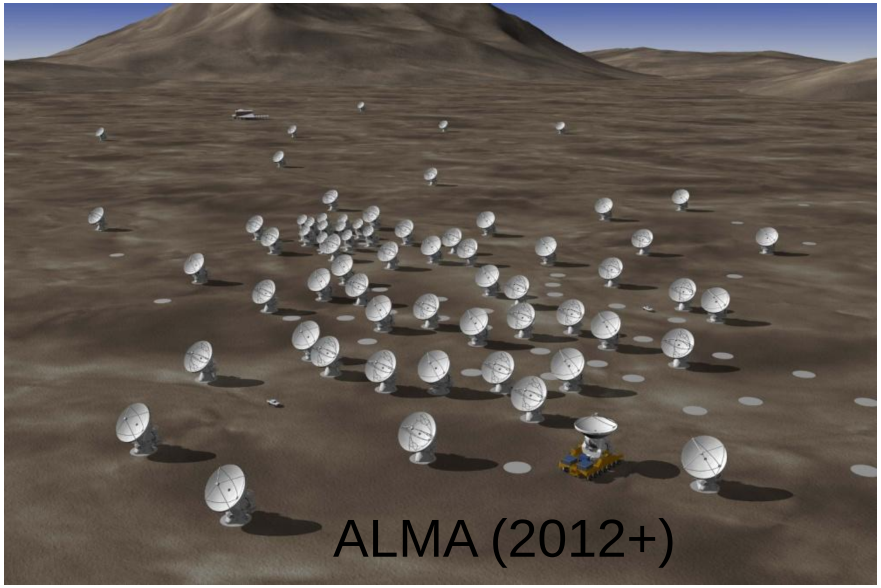


Doeleman et al. 2009

Examples of Millimeter Aperture Synthesis Telescopes







http://www.aoc.nrao.edu/events/synthesis/2010/lectures/wilner_synthesis10.pdf

Thank you!