Miniworkshop on Fourier disentangling

Petr Hadrava
email: had@sunstel.asu.cas.cz

Astronomical Institute, Academy of Sciences, Prague

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Plan of the miniworkshop

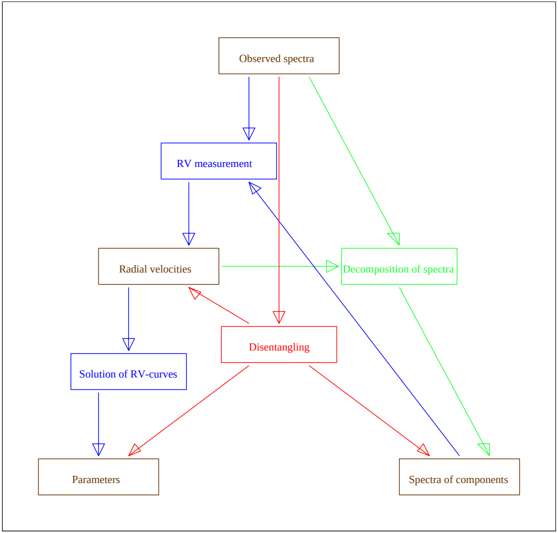
- Morning session: basis of the disentangling
  - Principle of spectra disentangling
  - Method of Fourier disentangling
  - Introduction to KOREL code
- Afternoon session: some advanced topics
  - Disentangling with line-profile variability
    - line-strength photometry
    - limb darkening
    - rotational efect
    - Cepheid pulsations
  - Disentangling with constraints
  - Super-resolution

General preliminaries

arXiv0909.0172: “Disentangling of spectra – theory and practice"

- Disentangling of spectra = a tool for interpretation of observations of spectroscopic variables
- Any interpretation of data is dependent on theoretical model – the answer depends on question we ask
- Exploitation of all available data is preferable
- Disentangling of multiple stars = direct fit of observed spectra yielding simultaneously component spectra and parameters of the system

Scheme of the disentangling



Related methods

- Radial-velocity measurements
  - Classical, photoelectric with mask
  - Cross-correlation, 2-dim
  - Broadening function
  - TIRAVEL
- Decomposition of spectra
  - Subtraction (eclipsing, direct, iterative)
  - Tomographic separation
- Disentangling
  - Wavelength domain (SVD)
  - Fourier wavelength domain

Fourier disentangling

1995 Astron. Astrophys. Supl. 114, 393

$x \equiv c \ln\lambda$

$$I(x,t;p) = \sum_{j=1}^{n} I_j(x) * \delta(x - v_j(t;p))$$

$$\tilde{I}(y,t;p) = \sum_{j=1}^{n} \tilde{I}_j(y) \exp(iyv_j(t;p))$$

LSF by a linear combination

$f(x_l) \simeq f_l$, $f(x) \simeq \sum_j c_j g_j(x)$:

$$0 = \delta \sum_l \left[f_l - \sum_j c_j g_j(x_l) \right]^2$$

$$0 = 2 \sum_{kl} \left[-f_l + \sum_j c_j g_j(x_l) \right] g_k(x_l) \delta c_k$$

$$\sum_j L_{kj} c_j \equiv \sum_j \left[\sum_l g_k(x_l) g_j(x_l) \right] c_j = \sum_l f_l g_k(x_l) \equiv R_k$$

Disentangling with a general broadening function

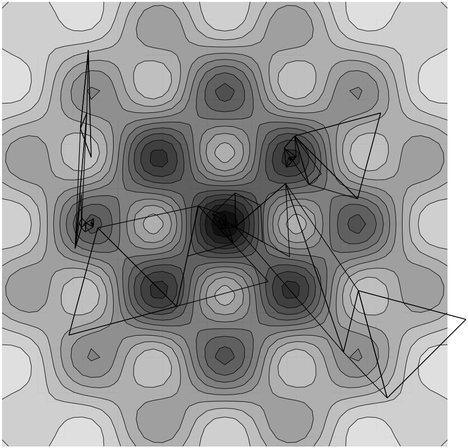
$$0 = \delta \sum_{l=1}^{N} \int |I(x, t_l) - \sum_{j=1}^{n} I_j(x) * \Delta_j(x, t_l, p)|^2 dx$$

$$0 = \delta \sum_{l=1}^{N} \int |\tilde{I}(y, t_l) - \sum_{j=1}^{n} \tilde{I}_j(y) \tilde{\Delta}_j(y, t_l, p)|^2 dy$$

$$\sum_{j} L_{kj}(y, p) \tilde{I}_j(y) = R_k(y, p)$$

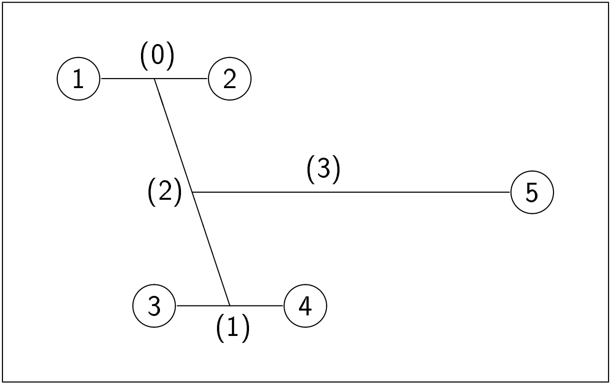
Simplex mehtod

Non-linear optimization in p



KOREL code

http://www.asu.cas.cz/ had/korel.html
https://stelweb.asu.cas.cz/vo-korel/



Input for KOREL (Linux version)

- Input spectra – `korel.dat`
  Rectified, in equidistant sampling of x ($2^n \times 256$ pixels, $n \geq 0$)

```
49158.399 6540. 9. 1. 256
0.98226 0.97913 0.97816 0.9796 0.98242 0.98174 0.97786 0.97278
0.96863 0.95906 0.95437 0.95231 0.95204 0.95837 ...
```

- Controlling codes and starting aproximation of parameters – `korel.par`

```
2 2 0 0 2 1 0 2 2 | key(1,...,5), k= Nr. of spectra>0, filter, plot
o 0 1 0 1 1 12.460326034 0.00001 | 96 Her, sum= 16.0811679
o 0 2 0 1 1 47527.278239438 0.001 = PERIASTRON EPOCH
...
```

- [Template spectra – `korel.tmp`]
- [Others in future]

Relative line photometry

1997 Astron. Astrophys. Supl. 122, 581

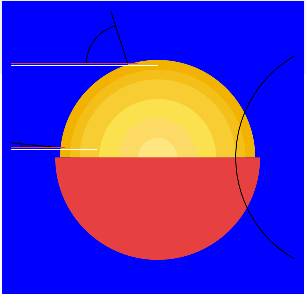
$$\Delta_j(x, t_l, p) = s_{jl}\delta(x - v_j(t_l, p))$$

$$\delta \sum_l \int \left| \tilde{I}(y, t_l; p) - \sum_{j=1}^{n} \tilde{I}_j(y) s_{jl} \exp(iyv_j(t_l; p)) \right|^2 dy = 0$$

Applications:

- Eclipses, elipticity...
- Intrinsic variability
- Telluric lines

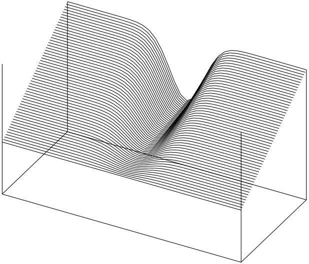
Limb darkening

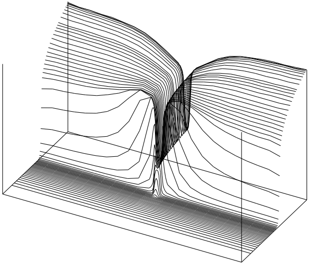


2003 ASP Conf. Ser. 288, 149

$S_\nu(\tau_\nu) = \sum_k S_{\nu,k}\tau_\nu^k/k!$

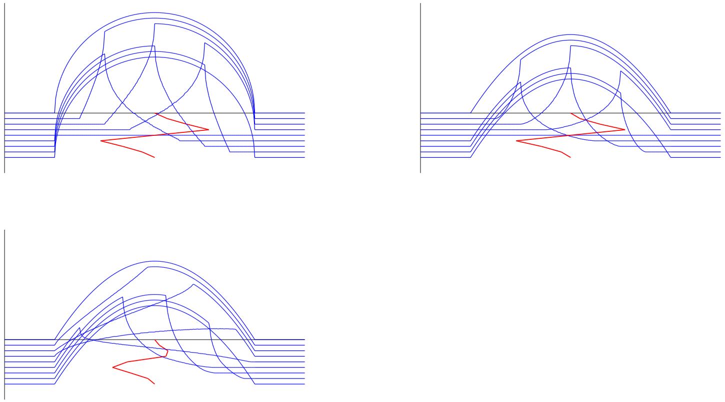
$I_\nu(\tau = 0, \mu) = \int S_\nu(\tau_\nu)\exp(-\tau_\nu/\mu)d\tau_\nu/\mu = \sum_k S_{\nu,k}\mu^k$





Rotational effect

2007 ASP Conf. Ser. 370, 164

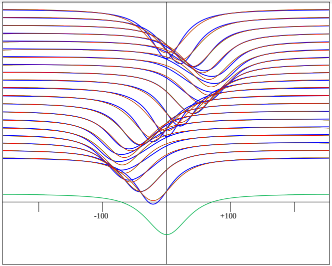


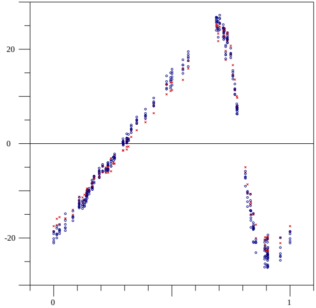
Disentangling of Cepheid pulsations

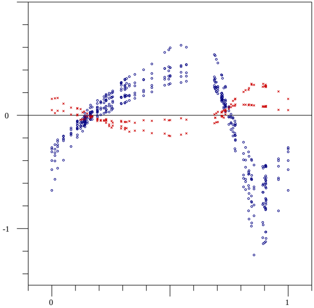
2009 A&A 507, 397

$$\Delta^k(x,t) = \int_s \mu^{k+1}\delta(x - v(s,t))d^2s$$

$$= \frac{2\pi R^2}{v_p^{k+2}} \left[x^{k+1}\right]_0^{v_p}$$







Disentangling with constraints

2006 Astrophys. Sp. Sc. 304, 337
Constrained spectra

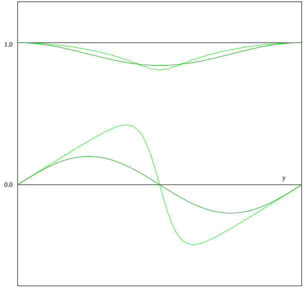
$$\sum_j I_j * \Delta_j(t,p) = I(t) - \sum_j J_j * \Delta_j(t,p)$$

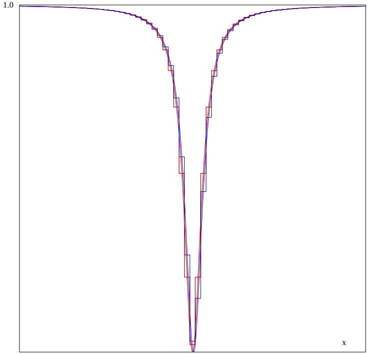
Application in KOREL: `korel.tmp`
Constrained parameters ($F_k(p) = 0$)

$$0 = \delta \left\{ \sum_t \int \left| I(t) - \sum_j I_j * \Delta_j(t,p) \right|^2 dx + \sum_k \lambda_k F_k^2(p) \right\}$$

Super-resolution

2009 A&A 494, 399





$$\delta(x-v) \simeq \delta(x) - \frac{v}{2\Delta_x}(\delta(x+\Delta_x) - \delta(x-\Delta_x)) +$$
$$+\frac{v^2}{2\Delta_x^2}(\delta(x+\Delta_x) - 2\delta(x) + \delta(x-\Delta_x)) + o(v^3)$$

The end