Referee's report on PhD Thesis of Ing. Martin Vejmelka entitled "Quantifying Interactions between Complex Oscillatory Systems: A Topic in Time Series Analysis"

Ing. Martin Vejmelka submitted the doctoral thesis dealing with selected aspects of a coupling between oscillatory systems or oscillatory subsystems of complex dynamical systems. The topic of the thesis is highly relevant for all studies on mutually interacting oscillatory or chaotic dynamical systems. Such interactions occur frequently not only in biological systems that are studied in the chapter 7 of the thesis, but also in many technical and technological systems, for example, in complex chemically reacting systems (chemical engineering and technology), in complex mechanical structures (mechanical engineering), in social systems etc. The results of the thesis can be relatively easily adopted in any of these fields. The author makes use of the experimental time series that typically represent the most easily and very frequently the only accessible data from the studied systems and this fact makes the thesis even more important for technical practice. Very important feature of the thesis represent fact that the theoretical considerations are accompanied with a number of numerical experiments. The thesis constitutes significant extension of "classical" nonlinear dynamics.

The thesis consists of 8 chapters, covers 93 pages and includes 30 figures. The list of references includes 91 items. The extent of the thesis corresponds to its significance and amount of new knowledge it presents, especially considering the fact that significant results of the thesis and author's research work have been already published in renowned scientific journals.

The first chapter of the thesis (Introduction) gives precise and sufficiently detailed review of fundamental terms used in the thesis (amplitude, phase, coupling etc.) and discusses in a deep detail problem and methods of determination of the phase on a trajectory orbiting the attractor and definition and characterization of weak coupling and/or synchronization of oscillating systems, where the questions of a coupling strength and a direction of the coupling naturally arise. Model oscillatory systems used in the thesis for examination and validation of evaluation methods and algorithms, i.e., the Lorenz system and the Rössler system, are presented. The author chose well known dynamical systems exhibiting wide variety of dynamical modes easily tunable via their parameters.

The Chapter 2 deals with the problem of quantifying the directionality of coupling between two oscillating systems. Three methods for determination of the directionality are noted and the author prefers usage of the information theory based approach. The indices characterizing strength (intensity) of the directionality are introduced. Numerical examples of the directionality are given in Fig. 2.1 showing also the convergence of the algorithms using various estimators.

The Chapter 3 deals with the problem of quantification of extent of dependence of dependent states of interacting systems. The dependence is expressed, for example, by phase synchronization and numerical experiments with two coupled Rössler oscillators are presented. More discussion or explanation would be desirable on page 33 instead of only simple and brief statement "Some of the indices conform to this ideal characteristic more, some less as can be seen in Fig. 3.3".

The Chapter 4 presents the tests of the significance of evaluated indices (coefficients). This chapter represents very important part of the thesis as a validity of results of each analysis of time series data becomes very often very questionable. The author applies surrogate time series derived from the analysed data for the testing purpose. Brief presentation and comparison of various methods of the surrogate data construction at the end of chapter 4 is highly valuable for anybody aiming to use the author's methods.

The Chapter 5 gives overview of the author's original contributions to the analysis methods presented in previous sections. Despite this chapter is relatively brief it represents the key part of the thesis. The chapter can be brief as the author already published his results in scientific journals.

The Chapter 6 presents results of numerical studies on directionality and coupling strength evaluation using the Rössler system data.

The Chapter 7 deals with the experimental data obtained by measuring several human body signals (EEG, ECG, blood pressure, respiratory effects etc.) and well illustrates applicability of methods presented in the thesis in analyses of real experimental systems. What can be effects of the central nervous system on observed coupling/directionality of the experimental data?

From formal point of view the thesis is written and organized very carefully and neatly. There are no typing errors within the text that is written in clear and concise manner. The only item I miss is a List of Symbols (or Notation) that would improve reader's orientation in the thesis. The symbol $\Delta'\Omega$ used in Fig. 3.1 is not explained.

The questions for a discussion:

1) The author often presents graphs where results obtained with different lengths of time series are shown and compared. Is it possible to formulate some general conclusions or recommendations concerning minimum time series length necessary for data analysis by methods presented in the thesis?
2) Both in technical and biological systems more than two oscillatory subsystems very often interact. What is author's opinion about possibility

of extension of the approach presented by him to systems with more than two interacting subunits?

3) The coupling between oscillating (sub)systems is often accomplished via mass flows. The direction of coupling then seems to be clear, i.e., the same as direction of mass flow. However, in certain cases we do not know the direction of mass flow. Can the information flow take an opposite direction?

4) In past, nonlinear time series analysis has been often denoted as “more art than science”. What is author’s opinion about this statement?

After carefully reading and studying Ing. M. Vejmelka’s thesis I am fully convinced it represents significant and original contribution to the scientific field it deals with. Ing. Vejmelka proved to be skilled scientist. The aims stated in the thesis were reached and its results can be applied in numerous branches of science and technology. In my view the thesis fulfils all requirements posed on theses aimed for obtaining the academic degree of Doctor of Philosophy (PhD) and **I fully recommend the thesis for the defense in front of the respective committee and, after the successful defense, to award Ing. Martin Vejmelka the academic degree of Doctor of Philosophy (PhD).**

Prague, Sept. 21, 2008

Prof. Ing. Pavel Hasal, CSc.