Thermodynamics of atmosphere-tropical cyclones and their origin and stability

(Taiwan lectures, 2007)

František Maršík
Institute of Thermomechanics CAS v.v.i.
Dolejškova 5, 182 00 Prague 8, E-mail: marsik@it.cas.cz
http://www.it.cas.cz/marsik

Summary

An energy balance of the Earth suggests that most of atmospheric phenomena, such as e.g. climate changes, weather development and consequently global hazards, are primarily determined by the impinging solar radiation energy. The fundamental principles of transformation of solar energy onto movement of atmospheric formations are demonstrated on the origin and development of tropical cyclones and tornado's.

Content

Tropical Cyclone Catarina from the International Space Station on March 26, 2004

A tornado in central Oklahoma.

The tornado itself is the thin tube reaching from the cloud to the ground. The lower part of this tornado is surrounded by a translucent dust cloud, kicked up by the tornado's strong winds at the surface

A waterspout near the Florida Keys (15 miles south of Miami) Tornado over water

Origin of storm terms

In Northwest Pacific: has two possible and equally plausible origins.

The word *typhoon*, is from the Chinese □□ (Cantonese: *daaih fūng*; Mandarin: *dà fēng*) which means "great wind“ (The Chinese term as □□ or □□ *táifēng*, and □□ *taifū* in Japanese, has an independent origin traceable variously to □□, □□ or □□ *hongthai*, going back to Song □ (960-1278) and Yuan □(1260-1341) dynasties.

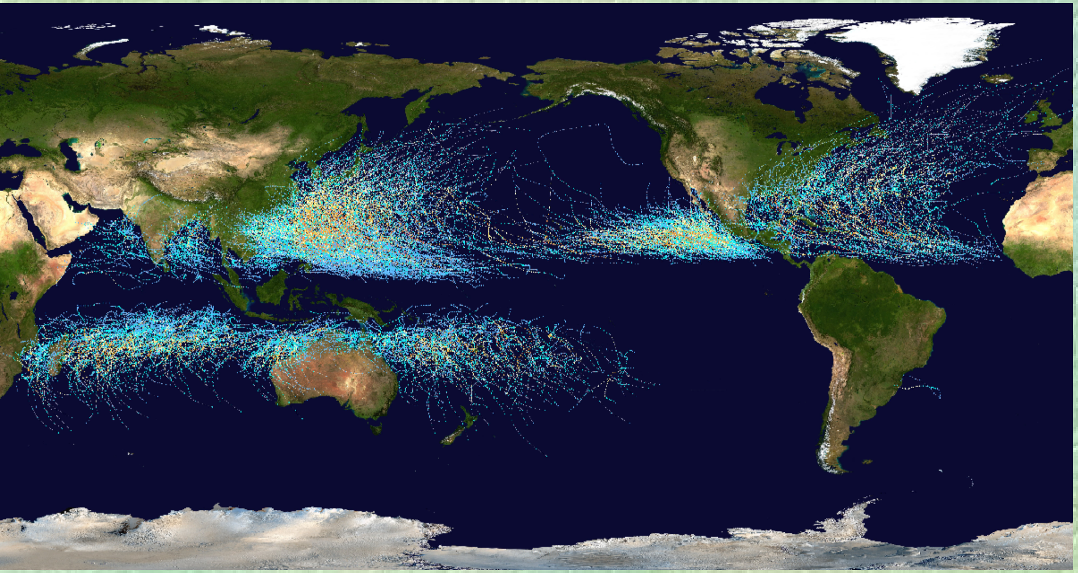
The first record of the character □ appeared in 1685's edition of *Summary of Taiwan* □□□□□). Alternatively, the word may be derived from Persian and Arabic *ţūfān* (طوفان), which in turn originates from Greek typhõn (Τυφών), a monster in Greek mythology responsible for hot winds. The related Portuguese word *tufão*, used in Portuguese for any tropical cyclone, is also derived from Greek *tuphōn*.

In North Atlantic and Northeast Pacific: the word *hurricane*, is derived from the name of a native Caribbean Amerindian God of storm, Huracan, via Spanish, *huracán*.

In Europe : the word *Orcan*- European windstorm, is also derived from huracan.

3. Origination of tropical cyclones and tornadoes

The meaning of “Cyclops” in Greek is “ring-shaped eye”as well.
Three specific characteristics of the tropical cyclone (TC) are:
a) an enormous dimension (often more than 2000 km),
b) one ring-shaped eye (up to 30 km in size),
c) extraordinary rage (the energy reaching up to 10^{18} J).



Map of the cumulative tracks of all TC during the 1985–2005.
In the Pacific Ocean west originates more TC than in any other basin. Almost no activity in the Atlantic Ocean south of the Equator.

Their average number on the Northern Hemisphere counts approximately 40, on the Southern Hemisphere about ten less.

In the Northern Atlantic Ocean, a distinct hurricane season occurs from June 1 to November 30 sharply peaking from late August through September.

In the Northwest Pacific sees tropical cyclones year-round, with a minimum in February and a peak in early September.

Season lengths and seasonal averages

Basin	Season start	Season end	Tropical Storm >63 km/hour	TC >117km/hour	TC >176km/hour
Northwest Pacific	April	January	26.7	16.9	8.5
South Indian	October	May	20.6	10.3	4.3
Northeast Pacific	May	November	16.3	9.0	4.1
North Atlantic	June	November	10.6	5.9	2.0
Australia Southwest Pacific	October	May	10.6	4.8	1.9
North Indian	April	December	5.4	2.2	0.4

COMPLEX DESCRIPTION OF A REAL PHYSICAL SYSTEM *V*

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BALANCE LAWS + ADDITIONAL ASSUMPTIONS (AXIOMS)

Balance laws

- definition of corresponding quantities

$$\phi = \begin{pmatrix} \rho \\ \rho \mathrm{v} \\ x \times \rho \mathrm{v} \\ \dfrac{\rho \mathrm{v}^2}{2} \\ \rho u \\ \rho s \end{pmatrix}$$

- mass

- momentum

- moment of momentum

- mechanical energy

- internal energy

-entropy

Global form $\dot{\Phi} + \mathrm{J}(\phi) = \mathrm{P}(\phi)$

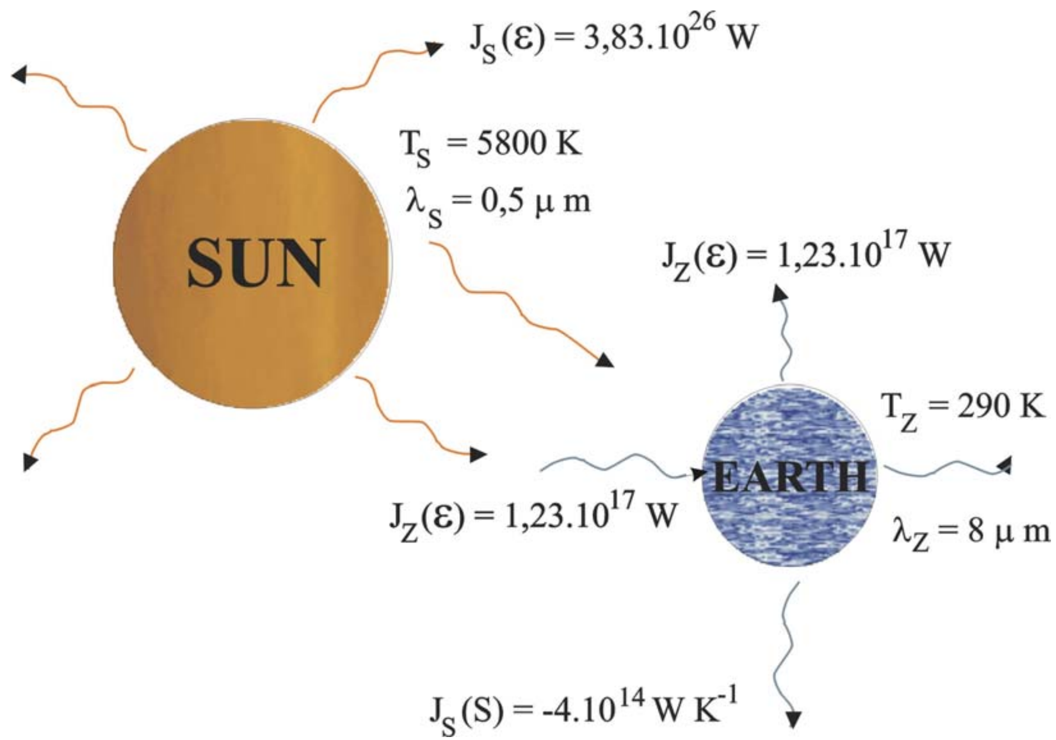
Local form $\dot{\phi} + \nabla j(\phi) = \sigma(\phi)$

1. Energy and entropy balance of the Earth

The starting points are the balance of the energy ε and the balance of the entropy *S* of the Earth. The same amount of energy which reaches the Earth failing down from the Sun has been emitted again.

The negative entropy flow *J*(*S*) has been compensated by the positive entropy production *P*(*S*).

ENTROPY BALANCE OF THE EARTH



ENTROPY FLUX OF THE EARTH

$$J(S) = \frac{4}{3}\left(\frac{J_Z(\varepsilon)}{T_S} - \frac{J_Z(\varepsilon)}{T_Z}\right) = -5{,}37 \cdot 10^{14}\ \mathrm{WK}^{-1}$$

Dynamical equilibrium assumption

$$-J(S)=P(S)=5{,}37\cdot 10^{14}\,[\mathrm{W/K}]$$

$P_{rad}(S)=0{,}94P(S)$ ...**absorption and emission of energy**

$P_{transf}(S)=0{,}06P(S)$...**material changes on earth surface and movement of atmosphere**

Civilization controls by power about 10^{13} W

$$P(S)=3\cdot 10^{10}\,\mathrm{W/K}\approx 0{,}0008P_{transf}(S)$$

Balance law of entropy of a real system

$$\underbrace{\dot{S}}_{\text{time course}} - \underbrace{J(S)}_{\text{interaction with surrounding}} = \underbrace{P(S)}_{\text{thermodynamic non-equilibrium}} \geq 0$$

At time $t = t_o$ the entropy reaches the local maximum

$$S(t_o) = S_o = \max \Leftrightarrow \dot{S}\Big|_{t=t_o} = 0$$

Dynamic equilibrium - steady state

$$S(t) - S_o = \ddot{S}\Big|_{t=t_o} \frac{(t-t_o)^2}{2} + \ldots \leq 0$$

Thermodynamic condition of stability of a real open system (including biological systems)

$$\ddot{S}\Big|_{t=t_o} = \dot{J}(S)\Big|_{t=t_o} + \dot{P}(S)\Big|_{t=t_o} < 0$$

or

$$0 < \dot{P}(S)\Big|_{t=t_o} < -\dot{J}(S)\Big|_{t=t_o}$$

CONDITION OF GROWTH

All material systems are stable when the growth $\dot{J}(S)$ of their interaction (the growth of the flux of negative entropy) is greater than the growth of their dissipation $\dot{P}(S)$ (transformation of external energy for own purpose)

① **Long time constant interaction,** *J(S) = const*, *P(S) = const*, **then**

$$-\dot{J}(S) = 0, \quad \dot{P}(S) = 0$$

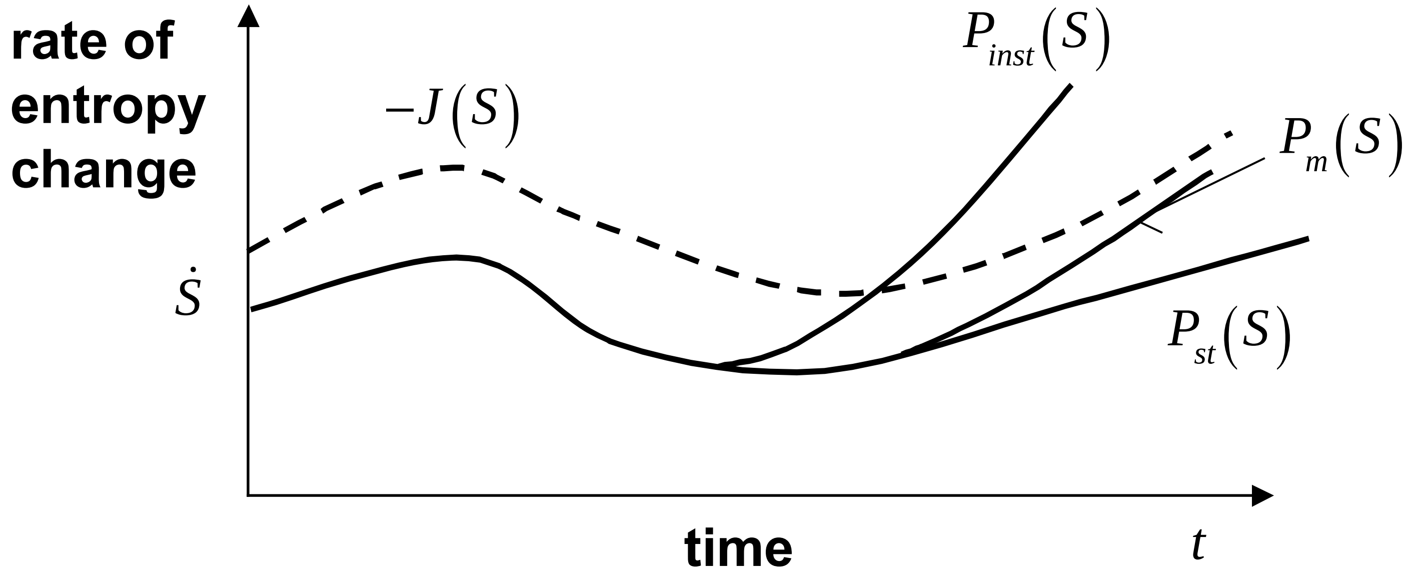
The consequence is marginal stability $\ddot{S} = 0$

② **Closed system** $J(S) = 0,\ P(S) \to 0$

The entropy reaches the global maximum S_{max}, **i.e.**

Thermodynamic equilibrium

Biological point of view - the system is dead



$-J(S)$ ... **flux of negative entropy**

i) $\dot{P}_{st}(S) < -\dot{J}(S)$ ... **stable development**

ii) $\dot{P}_{st}(S) = -\dot{J}(S) \rightarrow P_m(S) = -J(S)$

...**development on the onset of stability (marginal stability)**

$\dot{S} = 0$ ... **steady state**

iii) $\dot{P}_{inst}(S) > -\dot{J}(S)$ ... **instable development**

2. The stability of Earth's atmosphere

The total energy balance $h_c = h + \frac{\mathrm{v}^2}{2} + gz$ written

for 1 kg of the fluid (air) is

$$\frac{\partial h_c}{\partial t} + \mathrm{div}\left(h_c \mathbf{u}\right) = \frac{1}{\rho}\frac{\partial p}{\partial t} - \frac{1}{\rho}\mathrm{div}\left(\mathbf{q}\right) + \mathrm{div}\left(\boldsymbol{\tau} \cdot \mathbf{u}\right)$$

The total enthalpy comprises the sum of the enthalpy h (which contains the internal energy,the pressure energy p/ρ and the energy of phase transition lc_{wl} is included as well), the kinetic energy of the fluid $v^2/2$ and the potential energy gz.

The time change of the total enthalpy of 1 kg of a humid air moving in the upward direction from the Earth (along the axis *z*) is represented as

$$\dot{h}_c = T\dot{s} + P_t = \left[c_p \left(\frac{dT}{dz} \right)_{z=0} + l \left(\frac{dc_w}{dz} \right)_{z=0} + g \right] \mathrm{v}_z + \mathrm{v}_z \dot{\mathrm{v}}_z = P_t .$$

$$P_t = \frac{1}{\rho} \frac{\partial p}{\partial t}$$

When the time change of entropy is zero, i.e. $\dot{s} = 0$, we can derive the relation between the temperature $\frac{dT}{dz}$ and moisture $\frac{dc_w}{dz}$, changes with the height, respectively.

c_p=10^3 $kg^{-1}K^{-1}$ is the specific heat of air at the constant pressure,

c_w is the mass concentration of water steam in 1 kg of the moist air,

l=2460 $kJkg^{-1}$is the evaporation heat of water.

The condition of the air stability is $\dot{\mathrm{v}}_z < 0$

and the condition of the vertical flow existence is $\dot{\mathrm{v}}_z > 0$.

Hence, we have $\dot{\mathrm{v}}_z = \dfrac{P_t - \alpha \mathrm{v}_z}{\mathrm{v}_z} > 0, \quad \alpha = c_p \left(\dfrac{dT}{dz} \right)_o + l \left(\dfrac{dc_w}{dz} \right)_o + g.$

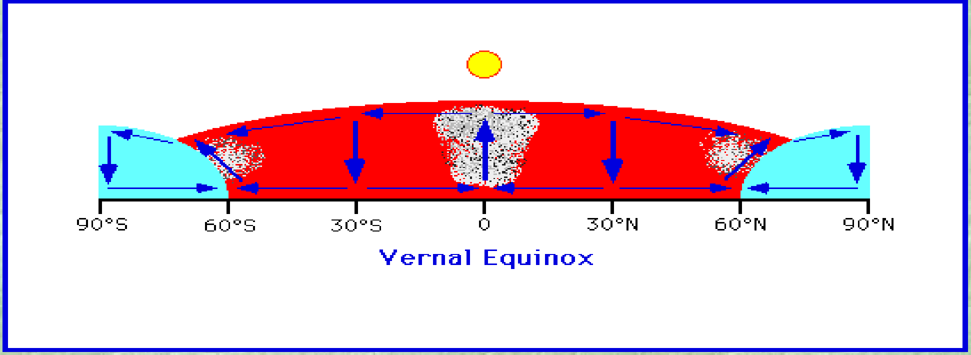
- principal importance for the analysis of the thermal inversion (at $\alpha > 0$, $P_t = 0$ is $\dot{\mathrm{v}}_z \leq 0$), when the air is being depressed to the Earth,

- great importance for the origin of tropical cyclones or tornados as well as, when (at $\alpha < 0$, $P_t = 0$ is $\dot{\mathrm{v}}_z > 0$) the air is being drifted upwards.

The local pressure change (the take up or the retreat of the pressure height or depression) has been expressed by the term

$$P_t = \frac{1}{\rho}\frac{\partial p}{\partial t}$$

where ρ =1.22 kg m^{-3} is the air density.



The limits of the stability at the quiet season and at absolutely dry air (c_w = 0) has been guaranteed by the temperature fall with the gradient higher than

$$\left(\frac{dT}{dz}\right)_o = -\frac{g}{c_p} \approx -10\ \frac{\mathrm{K}}{\mathrm{km}}.$$

For the moist air the gradient is greater

$$\left.\frac{dT}{dz}\right|_o = -6.5\ \frac{\mathrm{K}}{\mathrm{km}}.$$

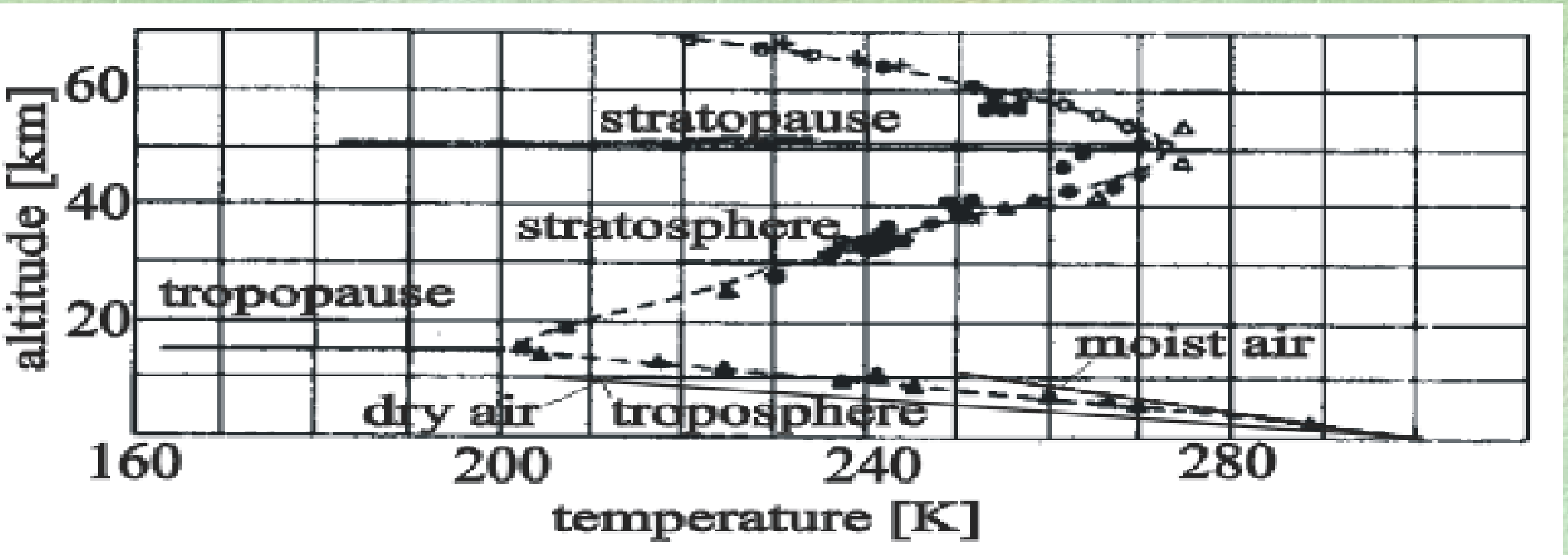
Such decrease of the temperature together with the height we call a standard atmosphere

$T_{z=0}$ is the ground temperature, $T(z) = T_{z=0} - 6.5\times10^{-3} z.$

The instability occurs when the temperature decreases rapidly with the altitude.
The inversion can be dispersed only by a pressure depression of the value

$$P_t < \alpha \mathrm{v}_z < 0.$$

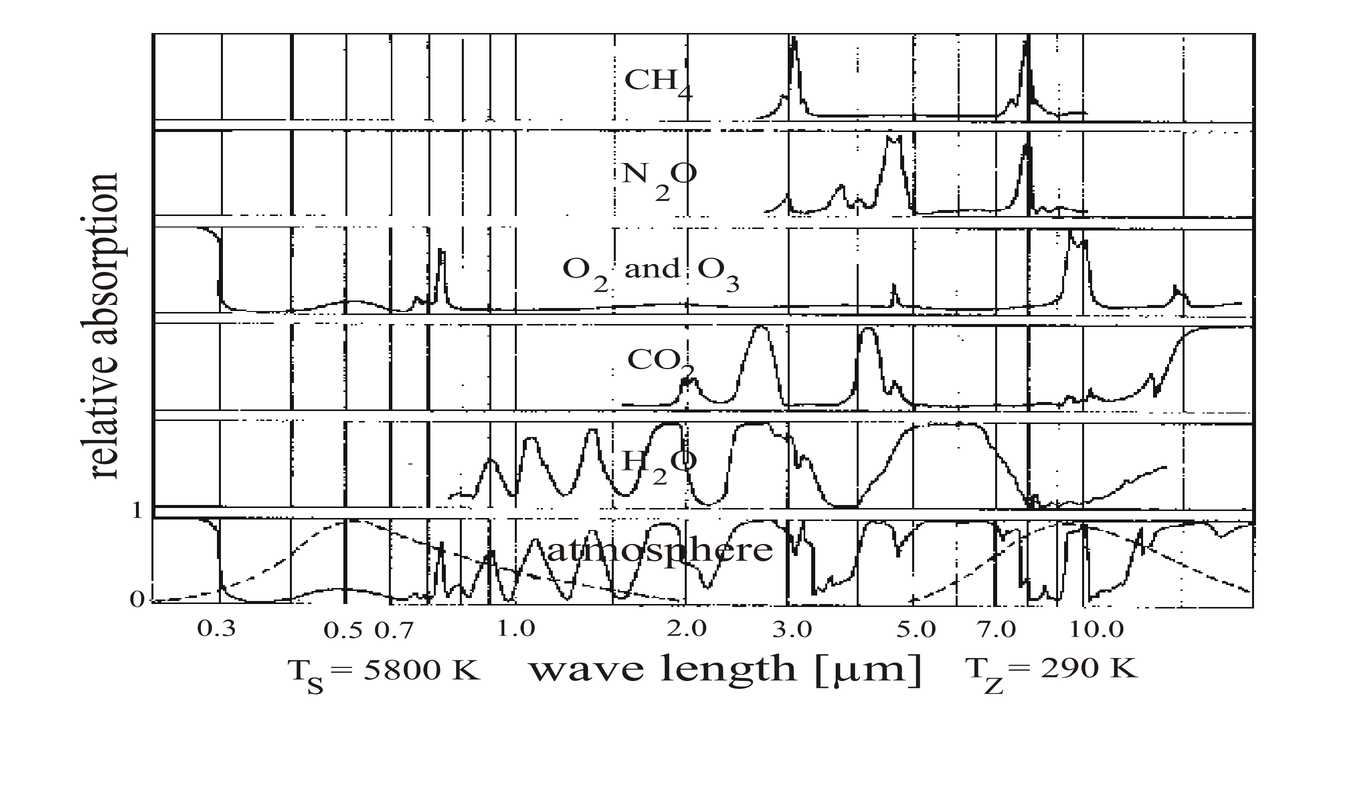
The temperature versus altitude in the Earth's atmosphere. The standard atmosphere with the decrease of 6.5 K over the height of 1 km has been plotted by dashed line and experimental points.



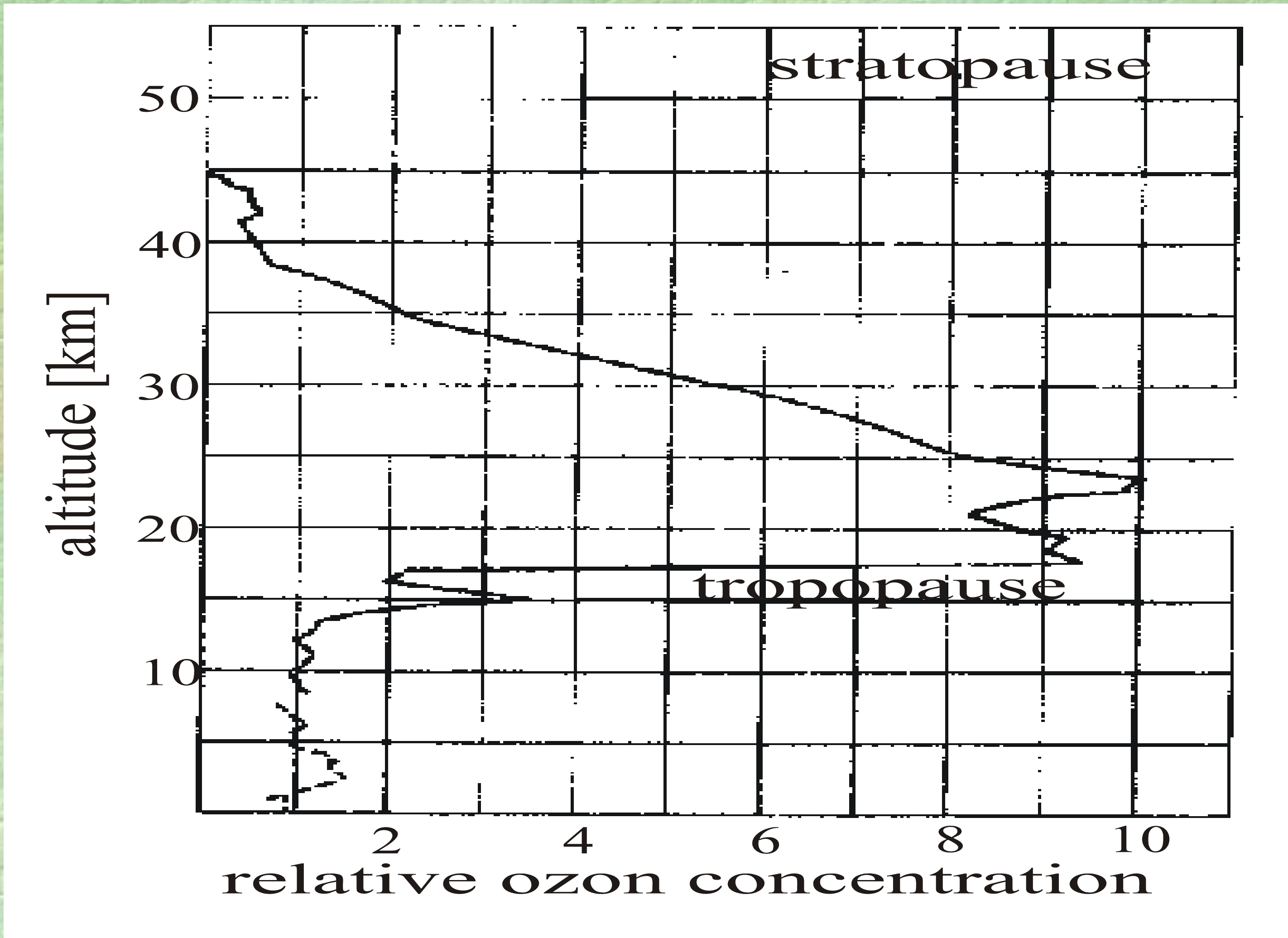
The temperature increase in the stratosphere ensures the stability of the atmosphere (α >0 v_z <0). The stratosphere or ionosphere respectively prevents the escape of the atmosphere of the Earth into the cosmic space.

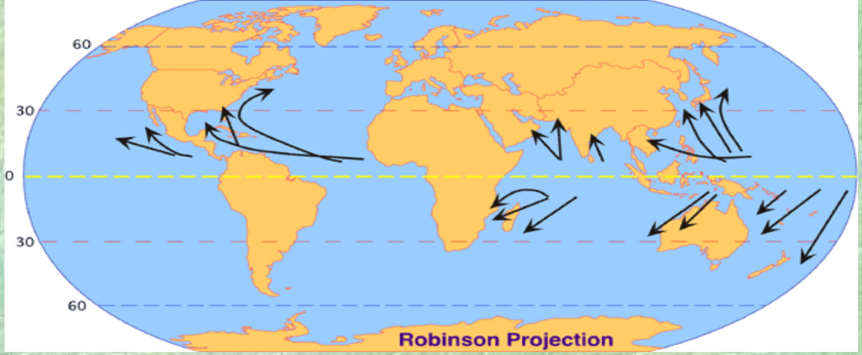
$$\frac{dT}{dz} > 0 \text{ and } \frac{dc_w}{dz} = 0$$

The relative absorption of the Earth's atmosphere and some, so-called greenhouse atmosphere gases CO_2, N_2O, CH_4 and ozone O_3.



The relative ozone concentration in the atmosphere. The ozone has been absorbing up intensively the life threatening short-timed radiation.





In the region of the tropical oceans where there is near the surface a great amount of water steam in the air.

The density of the dry air is greater

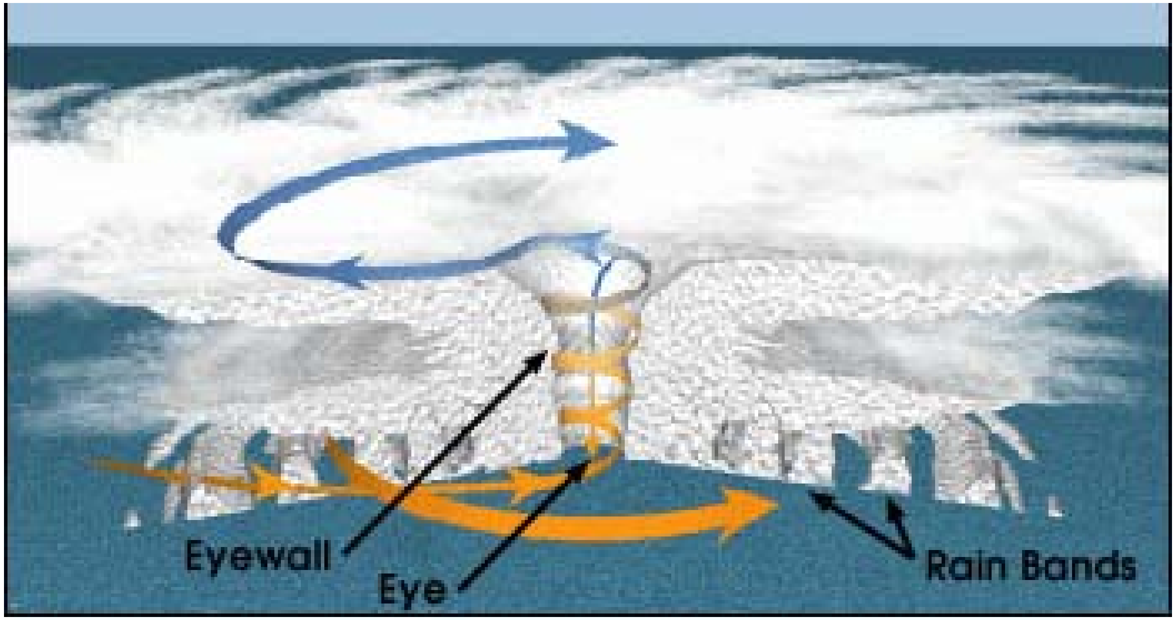
$$\rho_{da} = \rho_{wa}\left[1 + \left(\frac{M_a}{M_w} - 1\right)c_w\right],$$

where ρ_{wa} is the density of moist air,

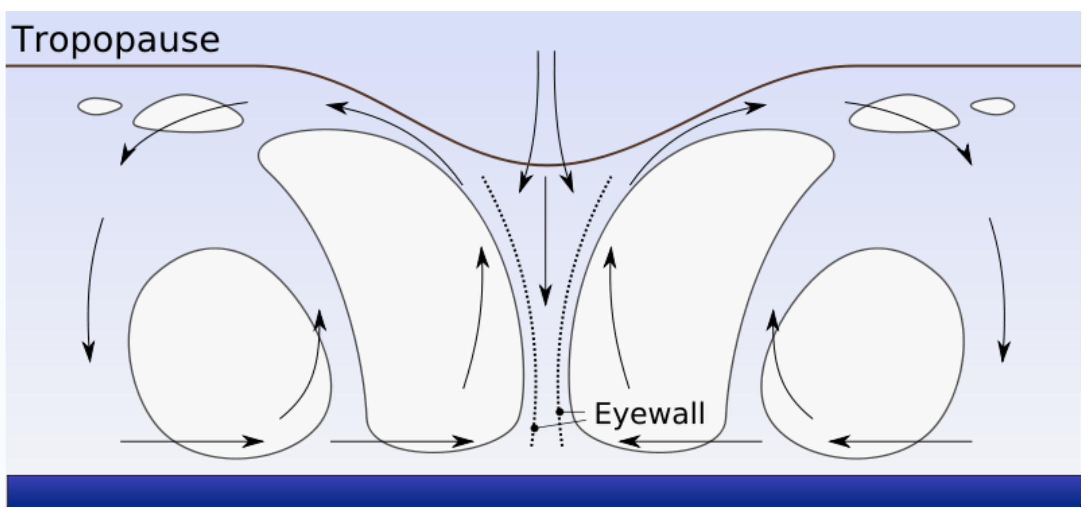
$$c_w = 0.62\frac{p_w(T)}{p}$$

represents the mass concentration of the water steam.
The partial pressure of the water steam $p_w(T)$ growths considerably with the temperature.

At the 30°C the saturation pressure is p_s=4.25 kPa and at 0°C the p_s=0.6 kPa. So that the convection appears already at the temperature decrease approximately at 5 K/km.



Tropical cyclones form when the energy released by the condensation of moisture in rising air causes a positive feedback loop over warm ocean waters.



Balance of momentum – Navier-Stokes equation

Earth rotation $\left(\approx 7{,}3 \cdot 10^{-5}\ \mathrm{s}^{-1}\right)$

$$\dot{\mathbf{v}} = \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\times\nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p + \nu\Delta\mathbf{v} + \boldsymbol{g} - 2\boldsymbol{\Omega}\times\mathbf{v}$$

inertia; pressure gradient; viscous friction; gravitational force; Coriolis force

Balance of energy

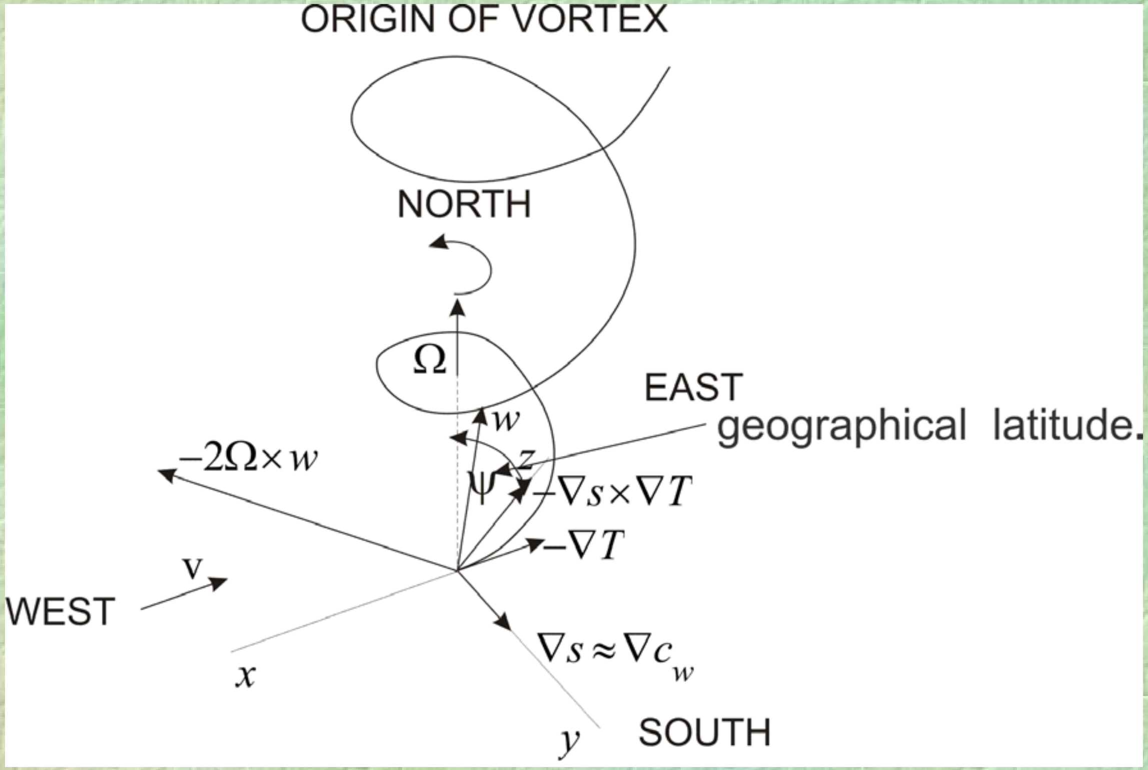
$$T\dot{s} = \dot{h} - \frac{\dot{p}}{\rho}$$

entropy changes including the phase transition; enthalpy changes; change of pressure

$$\left[(1-c_w)\right]c_{p\,\mathrm{air}}\dot{T} + c_w c_{p\,\mathrm{vapor}}\dot{T} + l\,\dot{c}_w$$

c_w is the mass concentration of water steam in 1 kg of the moist air

l evaporation heat



Crocco theorem

For incompressible fluid

div **v**=0

$$\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\times\boldsymbol{w}=\nabla h_c-T\nabla s-\nu\Delta\mathbf{v}\text{-}2\boldsymbol{\Omega}\times\mathbf{v},$$

Total enthalpy

$$h_c=h+\frac{\mathrm{v}^2}{2}+gz$$

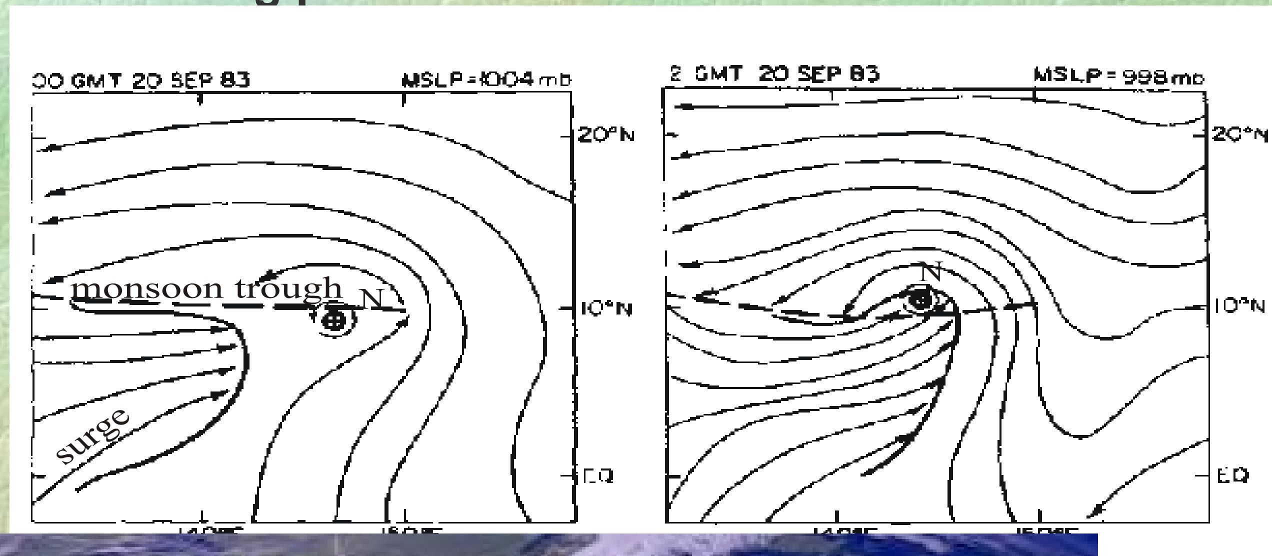
Vorticity

$$\boldsymbol{w}=\mathrm{rot}\ \mathbf{v}$$

Balance of vorticity

$$\frac{\partial \boldsymbol{w}}{\partial t}+\mathrm{rot}(\mathbf{v}\times\boldsymbol{w})=\underbrace{-\nabla T\times\nabla s-\nu\Delta\boldsymbol{w}-2\boldsymbol{\Omega}\times\boldsymbol{w}}_{\text{vorticity source}}$$

Schematic of the possible origin of circulation (due to the Coriolis force, f_{Cor}, which steers the air flow to the right and creates a low pressure domain *N*, which can cause a lasting pressure decrease.



3.1 Conditions for origination and growth of TC

1) The northern or the southern latitude must be at least 5^o (measured from the equator); the reason is that the Coriolis force must be sufficiently large to produce the spiral movement at the water level. The Coriolis force acts on each atmospheric element moving alongside meridians (to the north or to the south) at the velocity *v*. The Coriolis force on the Northern Hemisphere is directed to the right whereas on the Southern Hemisphere is directed to the left relative to the direction of the movement. Its magnitude is $f_{Cor} = 2\rho\Omega v \sin\psi$, where Ω=7.3 10^{-5} s^{-1} is the rotational velocity of the Earth, *v* is the velocity of a fluid element and ψ denotes the geographical latitude.

2) At the temperature gradient of the moist air above the sea surface must be condition athmosphere stability

$$\alpha = c_p \left(\frac{dT}{dz} \right)_o + l \left(\frac{dc_w}{dz} \right)_o + g < 0$$

and the magnitude of the negative temperature gradient lower than 5 K/km. In such case the air elevation will be amplified.

3) The minimum temperature at the surface level of the ocean must be 26^oC. This is to ensure that the steam concentration c_w in the air is sufficiently large and the enthalpy of the air is sufficient to cause a spiral flow of vapor.

4) The initial upward flow of moist air in the core must change its direction to allow the relatively dry and cool air go down through the vortex center and thus start he "Carnot's Thermal Machine".

5) The humidity of air in the middle layers of the troposphere (2-8 km above the sea level) must be elevated just enough to prevent too rapid drying up of the cloud ascending between the double wall of the eye.

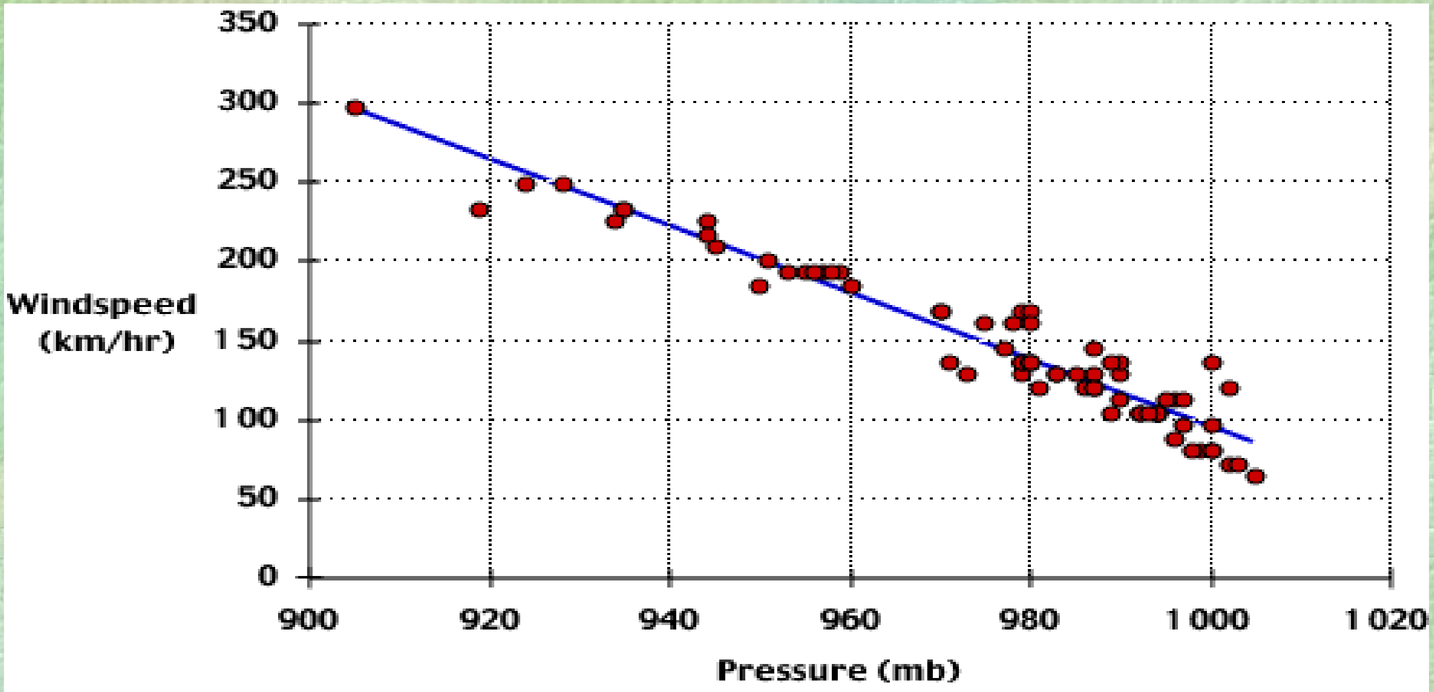
6) The tangential (cyclone) velocity in the lower layers of the atmosphere must be sufficiently large, say greater than 2-3 m/s, to enable the initiation of condition 4) which represents the main source of energy for a TC.

3.2 The balance laws for TC and tornadoes

The relations between the dimensions of a TC, r_o, R_o, H, the thermodynamic parameters such as pressure p, temperature T, or water vapor concentration c_w, and air velocity v_r, v_θ, v_z can be determined only from the balance of mass, momentum and energy.

We suppose that the TC is constituted of the eye and the outer potential vortex. The eye is assumed to rotate as a stiff cylinder with the angular velocity ω_o. The corresponding tangential velocity of rotation of the eye is $v_\theta = \omega_o r$. In the outer potential vortex, the velocity of rotation is

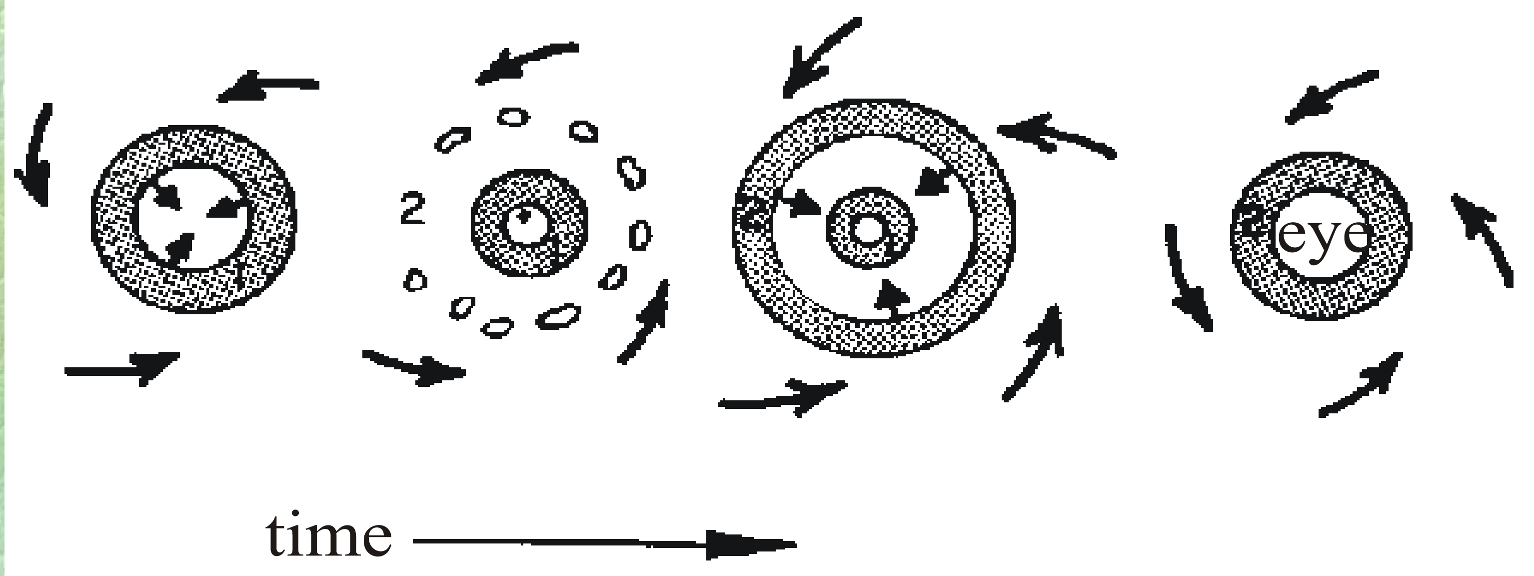
$$v_\theta = \omega_o r_o^2 / r.$$



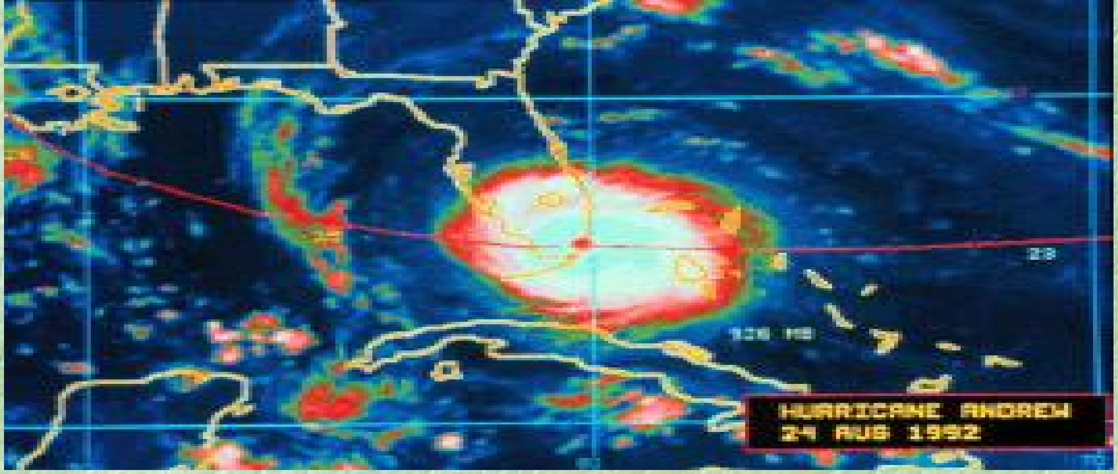
The balance of momentum in the direction of rotation θ for 1 kg of atmosphere (Crocco's theorem) yields

$$2\omega_o \mathrm{v}_\theta = \frac{\partial h_c}{\partial r} - T\frac{\partial s}{\partial r} - a_{dis} \approx l\frac{\partial c_w}{\partial r} \quad \text{or} \quad \Delta c_w \approx \frac{2(\omega_o r_o)^2}{l}.$$

a_{dis} is the damping deceleration caused by the friction of air



One of the possible variants for the cyclone eye development. The rotation of the air and the consequent pressure decrease in the core (the so-called potential vortex) will cause the flow stream directed towards the center. The over-narrowing of the initial loud surrounding the eye (the hatched area) causes its disappearance and the stable eye with the double wall is formed from the succeeding cloud.



Similar to our stability analysis for the atmosphere we assume that the entropy across the hurricane (tornado) eye remains constant (the evaporation heat of water is transformed reversibly into the mechanical energy. Indeed, the acceleration acting on 1 kg of moist air is

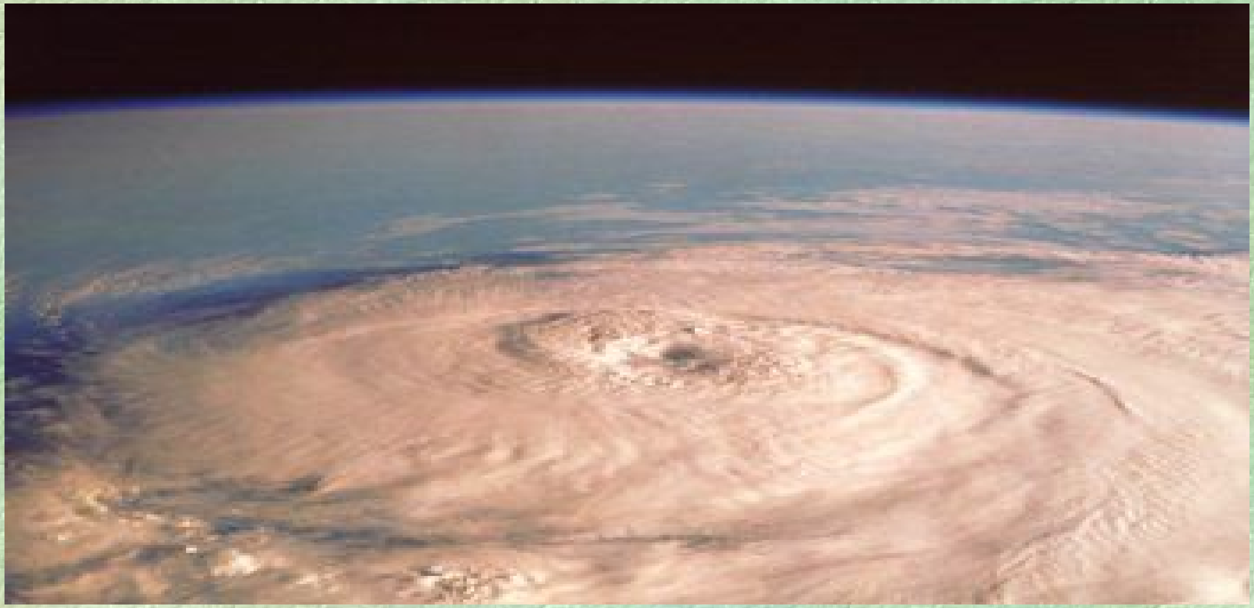
$$l\frac{\partial c_w}{\partial r} = 2.46\cdot10^6 \times 2.6\cdot10^{-2}/1.5\cdot10^4 = 4.2\ \mathrm{m/s}^2$$

and we see that it is comparable with the gravitational acceleration *g* = 9.81m/s².

Considering that the air density inside the eye is about 1 kg/m³ and the air rotates with the average peripheral velocity $v_\theta = \omega_o r_o^2 / r$ =50 m/s, the mechanical power of the whole TC is

$$W = 2\pi\int_0^H\int_{r_0}^{r_{max}} \frac{\rho v_\theta^2}{2} r\,dr\,dz \approx \pi\rho_0 r_0^2 v_0^2 \int_0^H\int_{r_0}^{r_{max}} \frac{dr}{r}dz \approx \pi\rho_0 r_0^2 v_0^2 H \ln\left(\frac{r_{max}}{r_0}\right) \approx 3.14\times1\times\left(1.5\cdot10^4\right)^2\times50^2\times1.4\cdot10^4\times\ln10 = 5.6\cdot10^{16}\ \mathrm{W}$$

In such a configuration, the upper total mechanical power of the TC would be 5.6•10¹⁶ W.



It is assumed that the power of a common TC is one or two orders lower.
Assuming the TC lifetime of about 15 days, I.e. $1.3 \cdot 10^6$ s, the corresponding mechanical work carried out by the TC would reach 10^{20} - 10^{22} J.
For illustration, the electrical power installed on the whole. Earth is about 10^{12} W and the corresponding time for the energy of 10^{20} J to fall on the surface of the Earth is about 1 hour.

The pressure inside the eye of the TC is decreased, however, most damage to the land is caused by the *flooding wave*. The length of the wave is about 250 m and its height can reach up to 32 m. The waves arise due to the interaction of the TC and the ocean on its way to the coast. The velocity of the wave propagation is about 10 m/s.

The global mechanical power and energy performance, Assuming the driving force be again the water vapor gradient and the eye of the tornado does not reach as high as the stratosphere where the water vapor concentration is zero but only as high as the thunderstorm cloud the concentration gradient is

$$\frac{\partial c_w}{\partial r} \approx \frac{\Delta c_w}{r_o} < \frac{2.6 \cdot 10^{-2}}{r_o} \ \mathrm{m}^{-1}.$$

For example,if the gradient is 1.3•10^{-2}/*r_o* the circumferential velocity is

$$v_\theta = \left(\frac{\Delta c_w}{2} \right)^{1/2} \approx 126 \ \mathrm{m/s},$$

i.e. about 450 km/hour.

For a tornado of radius *r_o* =50 m, the acceleration acting on the elements of air is

$$l \frac{\partial c_w}{\partial r} = \frac{2.46 \cdot 10^{6} \times 1.3 \cdot 10^{-2}}{50} = 640 \ \mathrm{ms}^{-2}.$$

Such acceleration can produce forces able to destroy more than just stone walls.

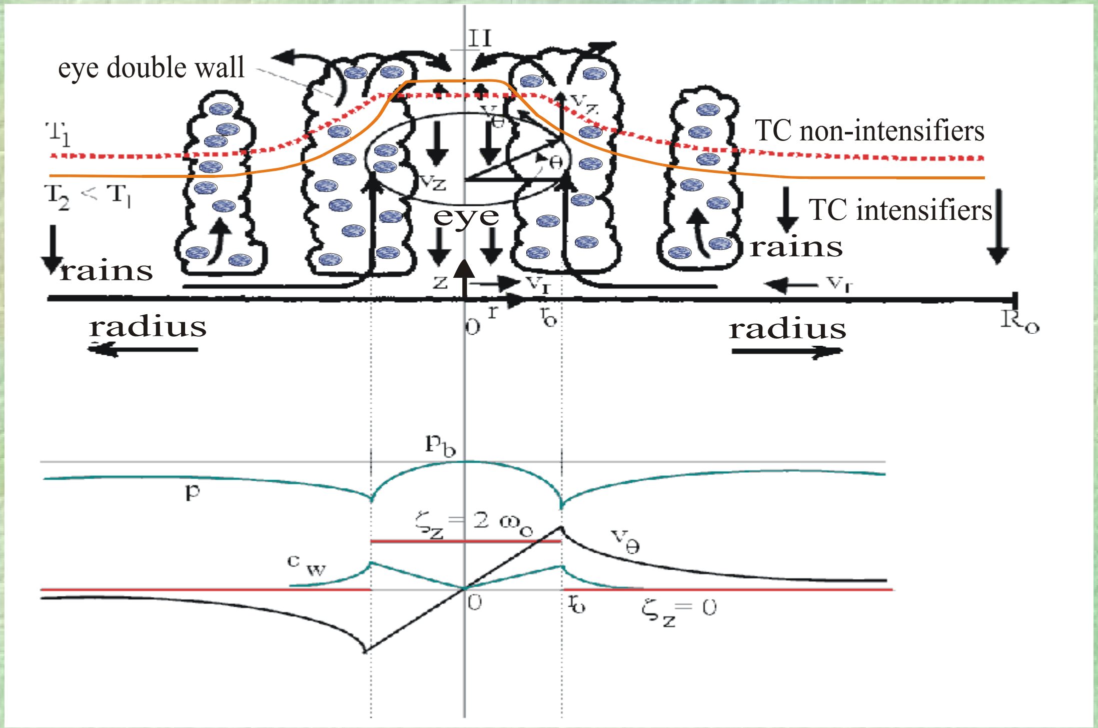
The suction effects of the TC and the tornadoes can be estimated using the following integral of the balance of momentum in its simplified form

$$p_b - p(r) = p_{dis} + \rho \frac{v^2(r)}{2}.$$

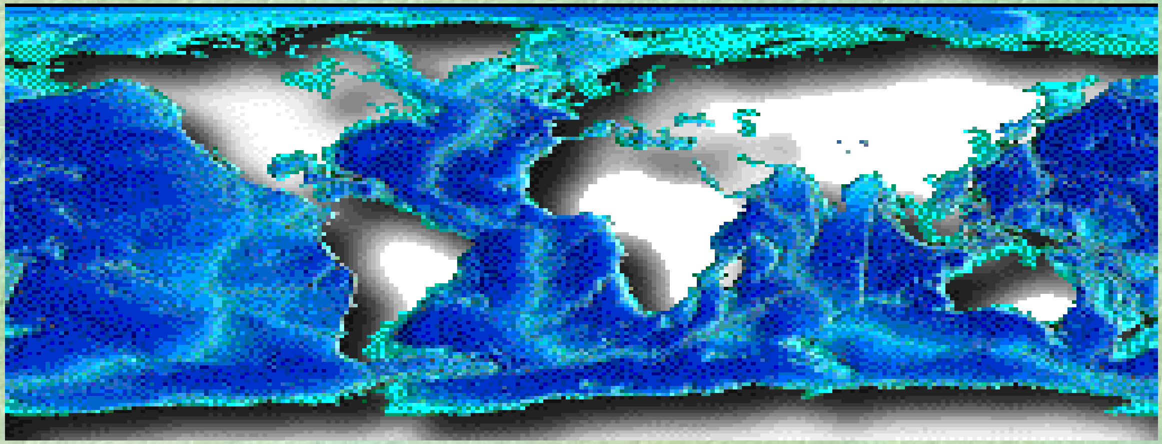
From this equation we can see that the growth of the total velocity of air v increases the underpressure relative to the surrounding barometric pressure p_b. In this equation p_{dis} denotes the pressure loss caused by the friction of air over the Earth's surface.

For example, for v = 100 m/s the *pressure decreases by about 50 mbar*. In tornadoes, the decrease can be twice as large.

• eye is axially symmetric with the radius up to $r_o \approx 15$ km,
• height reaching the stratosphere, $H \approx 14$ km.
• angular velocity is $\omega_o \approx 0.07$ s^{-1},
• maximum pressure difference p_b-p, in the eye wall ranges between 100 and 200 mbar (10-20 kPa).
• water vapor concentration c_w decreased from 20 g/kg (for 30°C at the ocean surface) to 0.4 g/kg (for 0°C somwhere in the upper layers of the troposphere where the temperature is T_1 or T_2).
• outer potential vortex (ζ = 0) is as wide as R = 1000 km.



The eye of the TC works like Carnot's machine between two temperatures T_{ocean} = 30 °C or T_{trop} = -70 °C.



The higher temperature is the temperature of the ocean, for example $T = 300$ K, and the lower temperature is the temperature of tropopause, which is about $T = 200$ K.
The working matter (substance) is the *mixture of air and water vapor*.
The condensed water vapor - rain drops - is expelled by the centrifugal force into the outer (potential) whirl resulting in *heavy rains*.
We can see that the secondary effect of this thermic machine is water pumping.

The thermodynamic efficiency of the cycle (I.e. efficiency of the transformation of the thermal energy to the higher (mechanical) energy is

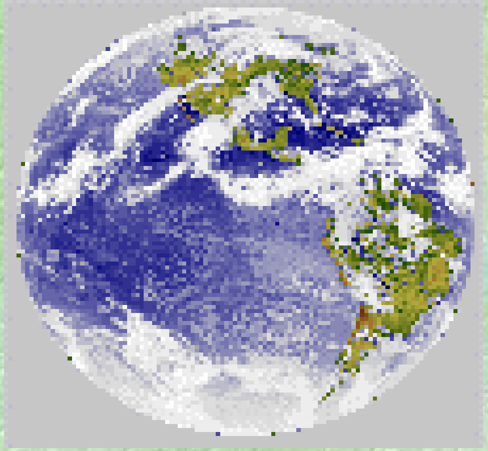
$$\eta_{\mathrm{TC}} = \frac{T_{ocean} - T_{trop}}{T_{ocean}} = \frac{300-200}{300} = 0.33.$$

This equation reveals that the total thermal energy of the TC can be more than three times greater than the estimated mechanical energy (10^{18} J).

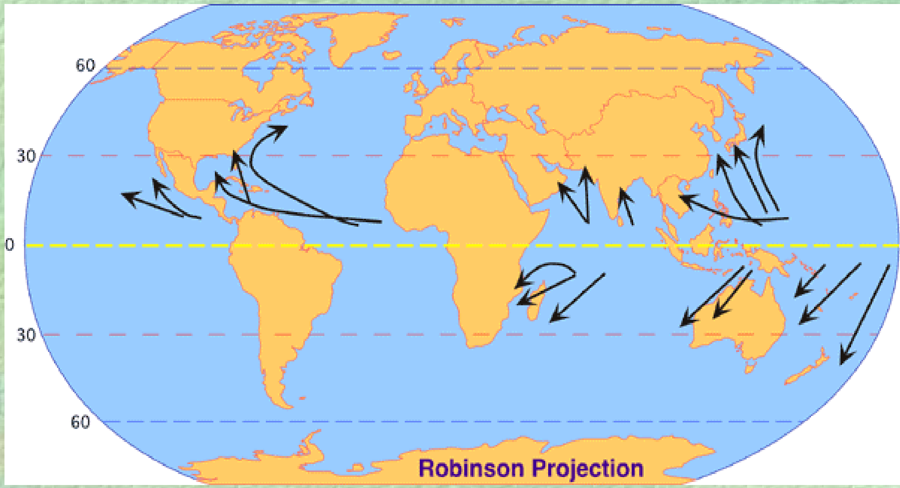
3.3 Forecasting possibilities and origination of TC

Very complex numerical simulations have recently been able to predict the movement of TCs several days in advance, however, their credibility has been querried because of the “peculiar” behavior they sometimes exhibit.

• If the TC has only one eye, its movement can be determined relatively easily.
• However, due to the internal instability of the TC, two eyes can appear very close to each other which can dramatically change the direction of the movement, for example by 90°.
• The onset of this instability cannot be forecasted today and the observation by the meteorological satelites remains the most reliable forecasting tool.
There are attempts to utilize the general theory of deterministic chaos (the method of Ljapunov’s exponents) to estimate the sensitivity of the TC movement to the inaccuracy of the initial conditions.



Conclusion



Further progress can only be made if a complex view of the whole Earth is taken including the activity of mankind whose influence cannot be neglected anymore.

Two major issues are still left to address:

a) The accuracy of the atmospheric model must be improved and robust algorithms must be developed and included in the model.
b) The accuracy of the determination of the initial conditions must be improved.

Since the damages caused by TCs are enormous, the governments of the countries concerned are unanimous in their support for research. As a result, *further progress is not restricted by the lack of money*, which is surprising, *but by our limited knowledge* and the low degree of our understanding of the laws of nature.

Taipei 101 endures a typhoon in 2005

Antenna/Spire 509.2 m (1670.60 ft)

Roof 449.2 m (1473.75 ft)

Top floor 439.2 m (1440.94 ft)

THANK YOU FOR YOUR ATTENTION