INVERSE METHOD FOR INCOMPRESSIBLE VISCOUS FLOW IN STREAM-FUNCTION COORDINATES

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A new, inverse method for viscous 2D, laminar and turbulent duct flows will be presented. The method is based on incompressible Navier-Stokes equations transformed to the stream function coordinate system (von Mises, 1927). For turbulent flows, the Reynolds-averaged Navier-Stokes equations with RANS turbulence model are considered. The flow model and related stream-function is formulated on the chosen 2D manifold with special interest in: blade-to-blade ($r = $ const) and axisymmetric ($d/d\phi = $ const, possibly with swirl) surfaces (see Fig. 1). Next, the von Mises coordinates are obtained from the stream function formulation. The model is transformed to this system.
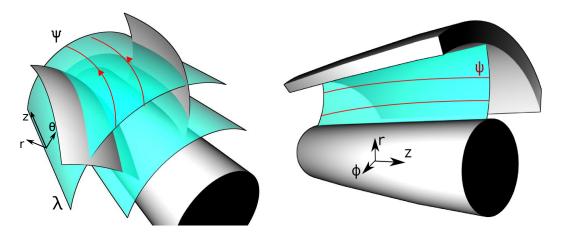
Figure 1: The 2D manifolds λ with streamlines ψ of two different flow problems: blade-to-blade and axisymmetric.

The transformation has unique properties. The coordinate variables become a functional of flow velocity. The relationship between new coordinates and velocity is delegated to a new independent equation. Therefore, the design problem can be formulated easily - that for prescribed pressure distribution one is able to find the shape of the wall. Additionally, the curvilinear flow domain corresponds to a rectangular shape in the new coordinate system. This simplifies the numerical solution of the presented problem.

After the coordinate transformation, the continuity and momentum equations are derived. The model is extended with the additional equation from the transformation. The dimensional analysis of typical flow design problems is presented. Depending on it, simplifications of the model are proposed. The appropriate boundary conditions for viscous flow design are discussed. The no-slip velocity condition in the connection with the Jacobian of the transformation leads to the indeterminate problem at the wall (Dulikravich 1995). A special treatment of those regions to regularise the singularity of discretisation is proposed and its convergence behaviour is analysed.

The design problem is solved numerically in the stream-function coordinates domain. The continuity and momentum equations are coupled with the additional stream function equation. As a result, the pressure and velocity fields as well as shape of streamlines are obtained. The shape of streamline aligned to the wall boundary represents the designed geometry.

The validation of the inverse method for several analytical test cases is presented and analysed, starting with design of straight pipe with the laminar flow. The scheme discretisation errors and general convergence of the flow solver are discussed. Detailed analysis of the singularity influence on the solution is shown.
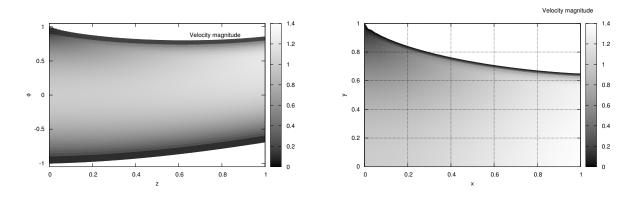

Figure 2: The nozzle and blade-to-blade channel design problem solutions.

Next, nozzle and blade-to-blade flow design problems are solved as an example (see Fig. 2). Potential applications to the design problem of fluid-flow machines are considered. The usage of turbulence modelling in the method is described. The advantages and drawbacks of the method are discussed. The applicability of the method to 3D flow model is prescribed.

References

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