COMPUTATION OF FLOW AROUND HEATED CYLINDER.

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We present results of computations of flow around heated circular cylinder, with temperature dependent viscosity. The simulation was performed in 2D domain using solver of incompressible Navier-Stokes system with energy equation. Solver is based on spectral element method combined with operator splitting scheme, implemented on the base of Nektar++ library [2]. Momentum and energy equations are coupled through temperature dependent viscosity.

(1a) $\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = -\nabla p + \nabla \cdot (\mu \nabla \vec{u} + \mu (\nabla \vec{u})^T)$

(1b) $\nabla \cdot \vec{u} = 0$

(1c) $\frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T = \nabla \cdot (\lambda \nabla T)$

(1d) $\mu = \mu_0 \big(\tilde{T}(T-1) + 1\big)^{\omega_\mu}$

All variables are dimensionless. $\vec{u}$ denotes velocity vector, p pressure, μ kinematic viscosity with μ_0 and ω_μ fluid parameters (air: ω_μ=0.7774, water: ω_μ=-7), T is temperature, $(\ )^T$ denotes transposition and λ is thermal conductivity, which we take constant.

Our research is focused on the frequency of vortex shedding within range of Reynolds number (Re) 60≤Re≤160 and temperature range $1 \leq \tilde{T} \leq 1{,}8$, where $\tilde{T} = \frac{T_C}{T_\infty}$ with T_∞ the temperature of incoming flow and T_C temperature of cylinder wall. Formula, which describes Strouhal-Reynolds-Prandtl number (Sr-Re-Pr) relationship, based on the experimental data and thermodynamics is given by [1]:

(2) $Sr(\mathrm{Re}_\infty, \tilde{T}) = 0.2665 - \frac{1.0175\frac{(\tilde{T})^{\omega_\mu}}{2}}{\sqrt{\mathrm{Re}_\infty}}\left[1 + \frac{0.227(1-\tilde{T})}{Pr^\gamma(2\tilde{T}-1)}\right]^{\frac{1}{2}}$

We will compare our results with (2), when setting constants: $\gamma = 1/3$, $Pr_{AIR} = 0.72$, $Pr_{WATER} = 8.084$.

Computational scheme brings advantages of spectral accuracy in solving Helmholtz equation (3b), (3c) with smooth data.

(3a) $\frac{\hat{v} - \sum_{q=0}^{J_i-1} \alpha_q v^{n-q}}{\Delta t} = \sum_{q=0}^{J_e-1} \beta_q [(\nabla v)^T \nabla \mu - (v \cdot \nabla) v]^{n-q}$

(3b) $\frac{\hat{\hat{v}} - \hat{v}}{\Delta t} = -\nabla p^{n+1}$

(3c) $\frac{\gamma_0 v^{n+1} - \hat{\hat{v}}}{\Delta t} = \nabla \cdot (\mu^n \nabla v^{n+1}) = \nabla \mu^{n+1} \nabla v^n + \mu^n \nabla^2 v^{n+1}$

(4a) $\frac{\hat{T} - \sum_{q=0}^{J_i-1} \alpha_q T^{n-q}}{\Delta t} = \sum_{q=0}^{J_e-1} \beta_q [-(v \cdot \nabla) T]^{n-q}$

(4b) $\frac{1}{Re\ Pr} \nabla^2 T^{n+1} - \frac{\gamma_0}{\Delta t} T^{n+1} = -\frac{\hat{T}}{\Delta t}$

here α, β, γ are coefficients of multi-step time integration scheme. Solver for incompressible Navier-Stokes equations was upgraded by implementation of variable coefficients to the Helmholtz equation in the third step of splitting scheme. The spectral

accuracy of solution to Helmholtz equation in case of smooth data is preserved also in this case.
Obtaining the Strouhal number needs long computation, because the vortex street was left to develop from zero initial condition. Time step was set to 0,001 and at least 150000 steps were computed in all cases. Our results (Fig. 1) qualitatively agree with the behaviour described by the formula (2). The data also confirm suppressing of the vortex shedding if the heated cylinder is in flow of air and its speed-up in the case of fluid with parameters similar to water.
From the values at $\tilde{T} = 1$ we can conclude, that the above mentioned differences are caused by the Navier-Stokes solver on rough mesh with insufficient resolution on the cylinder boundary and not by the extension to the variable viscosity.
Our results seem to be promissing for further development of the method and for more accurate computations on finer meshes.

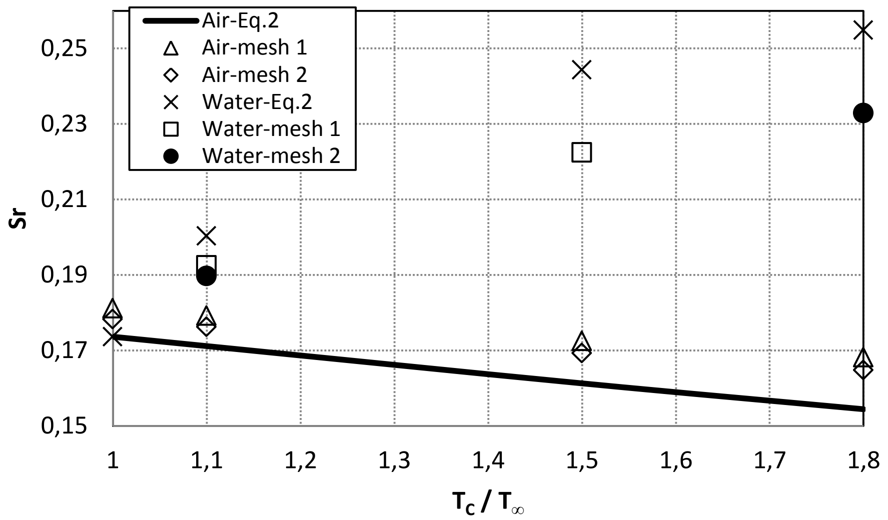

Fig. 1 Dependence of Strouhal number for water and air for Re=120. Mesh1 consists of only 370 elements, mesh2 of 600, both of 6-th order polynomial basis.

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