

Challenges in directed network analysis documented on climate reanalysis surface air temperature data

Monster journey

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Data

You are
HERE



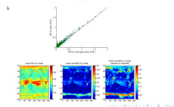
Data description

- 60 years
- 720 time points (monthly averages) of Surface Air Temperature
- (or over 20000 time points in daily averages)
- NCEP-NCAR reanalysis dataset (Kalnay, 2001)
- original data dense resolution (2.5°), i.e. over 10000 time series

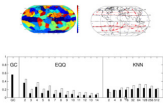
Assessing nongaussianity

⇒ we can quantify the extra dependence (mutual information I) that is not captured by linear correlation ρ :

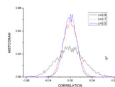
$$I_{\text{total}}(X, Y) = h(X, Y) - \frac{1}{2} \log(1 - \rho^2)$$



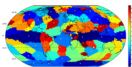
Stability of causality estimators



Correlations



Clustering



Data description

- ▶ ~ 9 minutes, 213 time points of whole brain resting state brain activity
- ▶ 26 (12 males, 19-54 years) healthy volunteers
- ▶ 3T Siemens Magnetom Trio MRI scanner (GE-EPI, TR/TE=2500/30 ms, voxel size=3x3x3mm), a 3D high-resolution T1-weighted image was used for anatomical reference, slice-timing correction, motion correction, spatial normalization to MNI
- ▶ original data ~ 20000 time series, dimensionality reduced to 90 time series by averaging over regions from the Automated Anatomical Labeling atlas
- ▶ orthogonalized wrt motion parameters, white matter and CSF signal

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- ▶ NECP/NCAR reanalysis dataset [Kistler, 2001]
- ▶ original data dense resolution (2.5°), i.e. over 10000 time series
- ▶ only anomalies with respect to average seasonal changes considered

Dependence: how to measure?

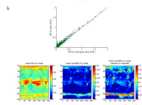
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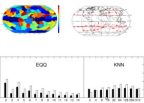
$$I_{\text{total}}(X, Y) = h_{\rho}(X) - \frac{1}{2} \log(1 - \rho^2)$$



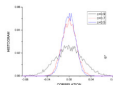
X

Now you are here!

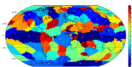
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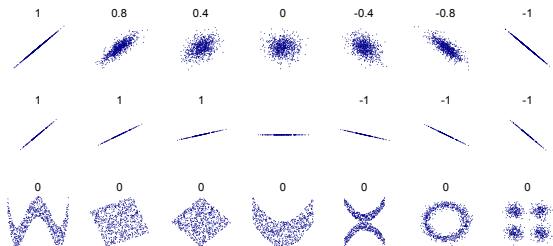
Independence ($X \perp\!\!\!\perp Y$): $p(X, Y) = p(X)p(Y)$



Dependence measures:

Pearson's correlation $\rho_{X,Y} = \frac{\text{cov}(X,Y)}{\sigma_X\sigma_Y} = \frac{E[(X-\mu_X)(Y-\mu_Y)]}{\sigma_X\sigma_Y}$

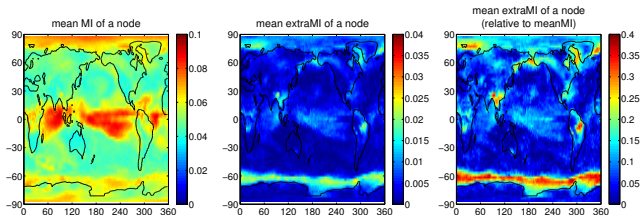
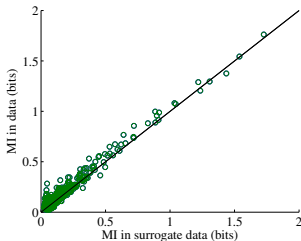
Mutual information: $I(X; Y) = \sum_{y \in Y} \sum_{x \in X} p(x, y) \log \left(\frac{p(x, y)}{p(x)p(y)} \right)$



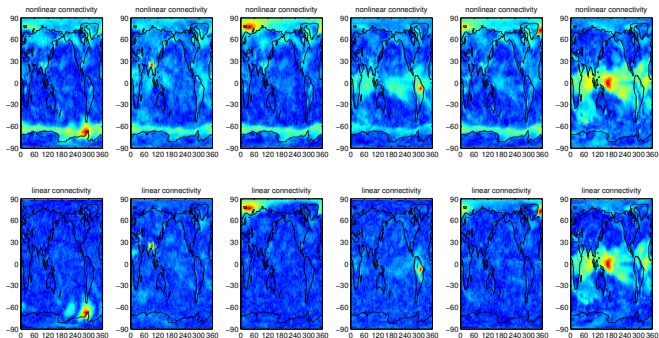
Dependence: Assessing nongaussianity

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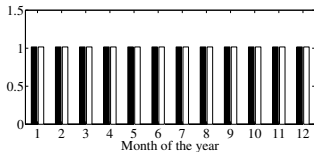
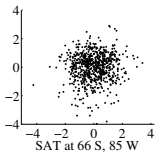
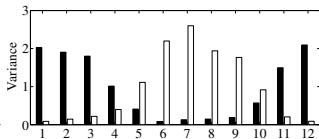
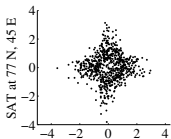
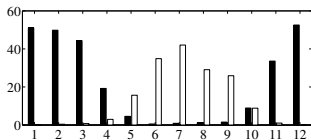
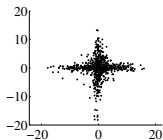
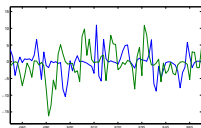
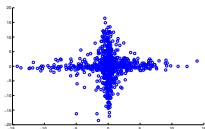
$$I_{\text{extra}}(X, Y) = I_{X,Y} - \frac{1}{2} \log(1 - \rho_{X,Y}^2)$$



Dependence: nonlinearity examples



Dependence: Nongaussian ghost of seasonality



Taking into account autocorrelation

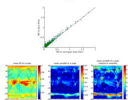
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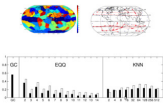
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$$I_{\text{total}}(X, Y) = h(X, Y) - \frac{1}{2}(\log(1 - \rho^2_X) - \rho^2_X)$$

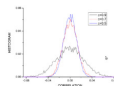


✗ And now here

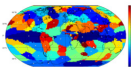
Stability of causality estimators



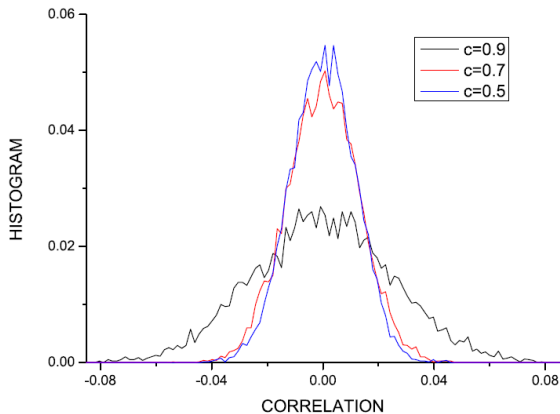
Correlations



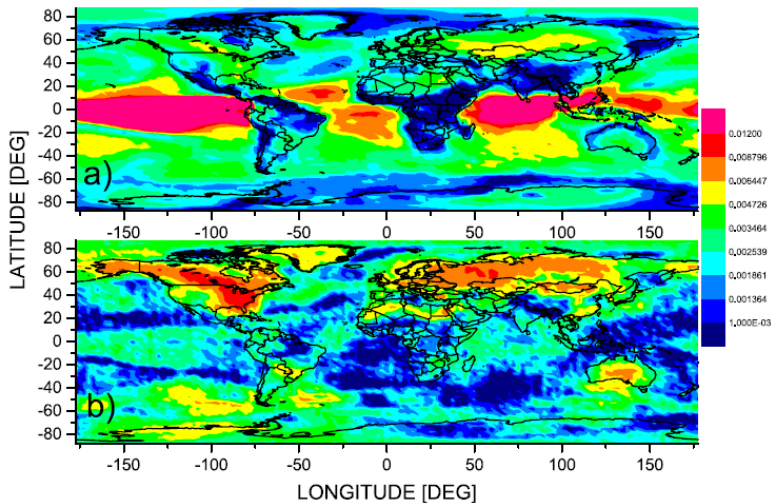
Clustering



Correlation histogram for (autocorrelated) noise



Average weighted correlation with(out) correction



Dimensionality reduction

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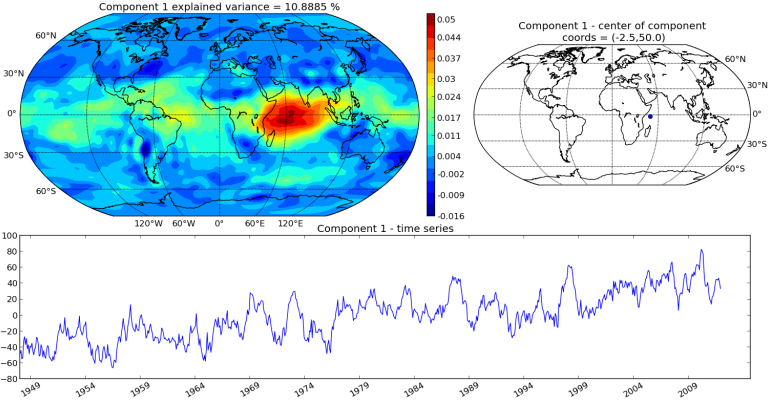

Stability of causality estimators

Correlations

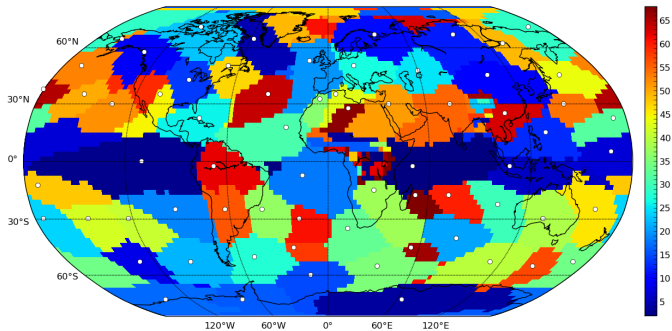
Clustering

X
Half way through!

Decomposition



Clustering



Interaction of subsystems

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→ we can quantify the extra dependence (mutual information I) that is not captured by linear correlation ρ :
 $I_{\text{total}}(X, Y) = h_{\rho}(X) - \frac{1}{2} \log(1 - \rho^2)$



Stability of causality estimators

Correlations

Are we there yet?
Almost!



Clustering



Causality

- ▶ Granger causality: X 'Granger causes' Y iff including the past of Y in a (linear) model of X significantly improves the model fit

$$\mathcal{F}_{Y \rightarrow X|Z} \equiv \ln \left(\frac{|\Sigma(\boldsymbol{\varepsilon}_t)|}{|\Sigma(\boldsymbol{\varepsilon}'_t)|} \right)$$

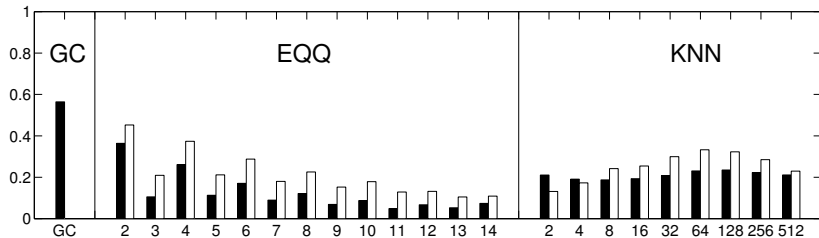
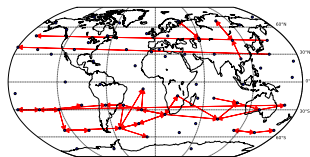
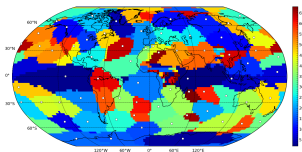
- ▶ Transfer entropy: the difference between entropies of the variable X conditioned (or not) on Y :

$$\mathcal{T}_{Y \rightarrow X|Z} \equiv H(X|X^- \oplus Z^-) - H(X|X^- \oplus Y^- \oplus Z^-),$$

- ▶ for stationary linear Gaussian processes GC and TE **equivalent**

$$\mathcal{F}_{Y \rightarrow X|Z} = 2\mathcal{T}_{Y \rightarrow X|Z}$$

Stability of causality estimators





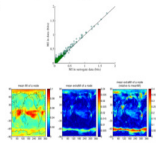
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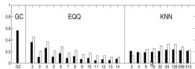
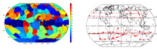
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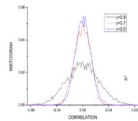
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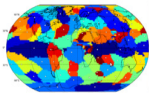
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Thank you for your attention!

Thanks to my colleagues at ICS, Prague: **Milan Palus, Martin Vejmelka, David Hartman, Nikola Jajcay, Lucie Pokorna** and collaborators at IAS, Prague: **Dagmar Novotna** and PIK, Potsdam: **Jakob Runge, Juergen Kurths, Norbert Marwan**

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<http://ndw.cs.cas.cz>, hlinka@cs.cas.cz

References:

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- Palus, M.; Hartman, D.; Hlinka, J. and Vejmelka, M. Discerning connectivity from dynamics in climate networks *Nonlinear Processes in Geophysics*, 2011, 18, 751-763