Complex networks of interacting stochastic dynamical systems: Discerning connectivity from dynamics

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Dynamics Days Europe, Oldenburg, September 12, 2011

INTRODUCTION

- Interacting dynamical systems
- Statistical physics
- Graph theory
- COMPLEX NETWORKS
- **Multivariate time series $\longrightarrow$ networks**
  - Nodes: measuring sites
  - Edges: dependence, **"connectivity" measures**
    - weighted graph
    - threshold $\rightarrow$ binary graph

INTRODUCTION

- **Multivariate time series $\longrightarrow$ networks**
  - Edges: dependence, “connectivity” measure
  - linear cross-correlation – the measure of first choice
- correlation – linearity – Gaussianity
- Nonlinearity? hidden connectivity patterns?
- Factors influencing connectivity measures
  - dynamics (serial correlations)
  - temporal and spatial sampling (time lags)
- Factors influencing network structure
  - uniform thresholding or individual statistical testing
  - thresholding Z-score, significance function

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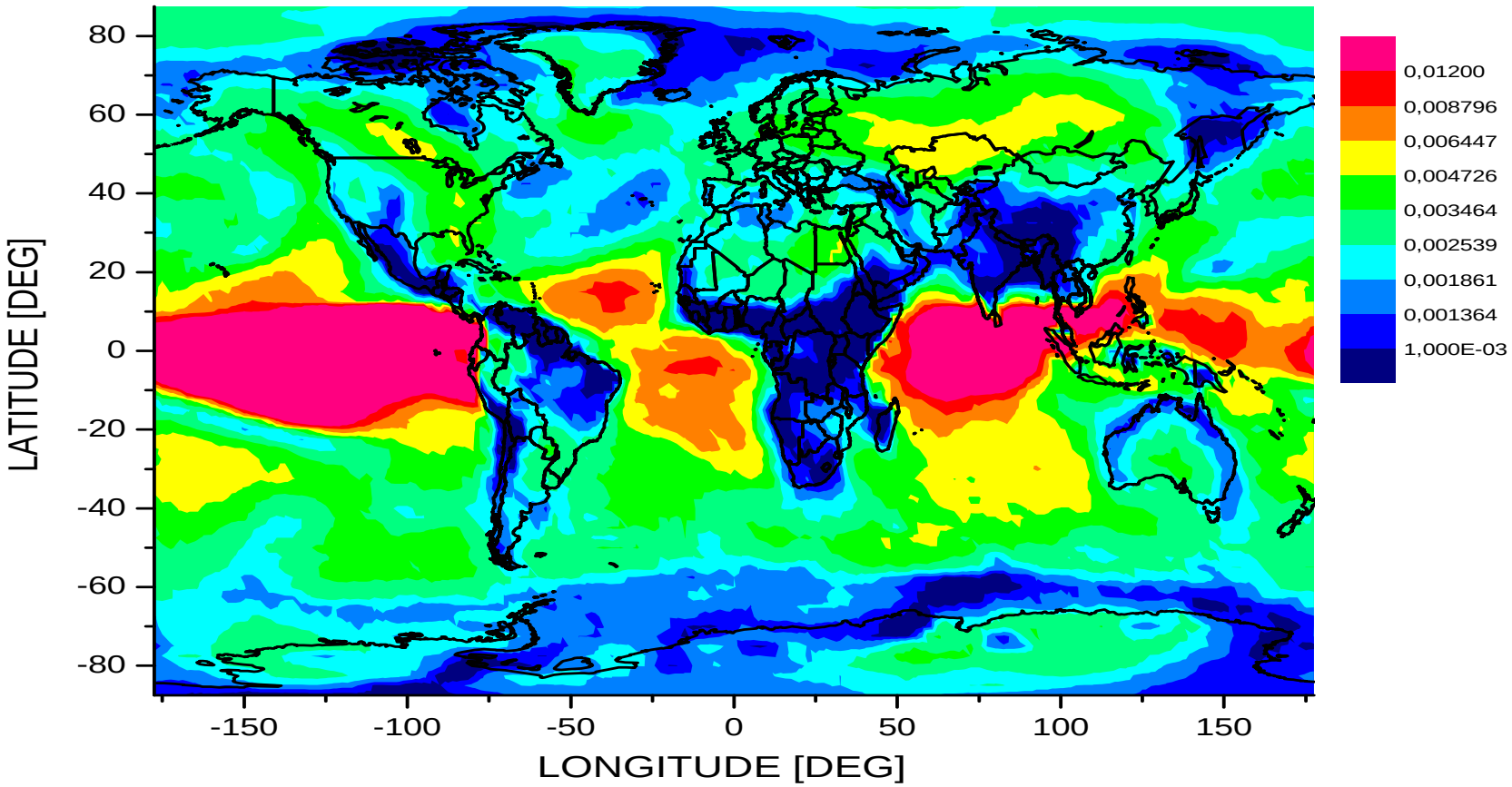
CLIMATE NETWORKS

- **Multivariate time series**: gridded “reanalysis data” of atmospheric variables: air temperature, pressure, humidity, precipitation...
- Here: near-surface air temperature **anomalies**
  subtraction of seasonal means (mean Jan, mean Feb ...)
  removal of the annual cycle
  = **fluctuations** around seasonal means
- grid 2.5° x 2.5° $\longrightarrow$ 10^4 nodes
- Pearson correlation $\longrightarrow$ weighted network
- thresholding $\longrightarrow$ binary network
- $\longrightarrow$ graph-theoretical analysis

Connectivity vs. dynamics

Area Weighted Connectivity $\varrho = 0.005$ for

NCEP/NCAR SAT anomalies – absolute correlations



Connectivity vs. dynamics

- autoregressive process

$$y_t = c \sum_{k=1}^{10} a_k y_{t-k} + \sigma e_t, \tag{1}$$

where $a_{k=1,..,10} = 0, 0, 0, 0, 0, .19, .2, .2, .2, .2$, $\sigma = 0.01$ and e_t are Gaussian deviates with zero mean and unit variance

- **entropy rate**; dynamical entropy

$$h = \lim_{n \to \infty} \frac{1}{n} H(Y(1), \ldots, Y(n)) \tag{2}$$

- dynamical systems: *Kolmogorov-Sinai entropy*
- for a Gaussian process with spectral density function $f(\omega)$

$$h_G = \frac{1}{2\pi} \int_{-\pi}^{\pi} \log f(\omega) d\omega \tag{3}$$

Connectivity vs. dynamics

autoregressive process

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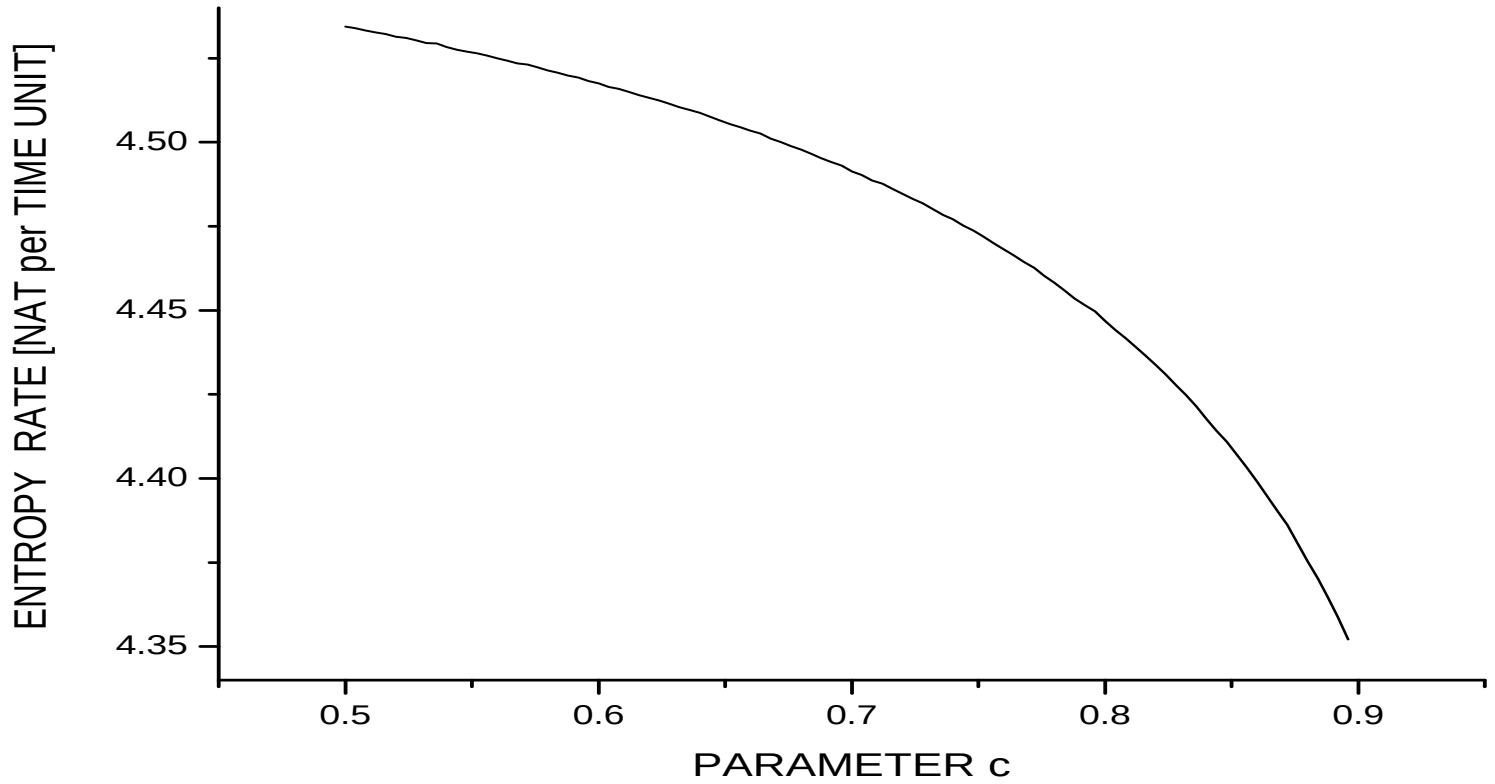
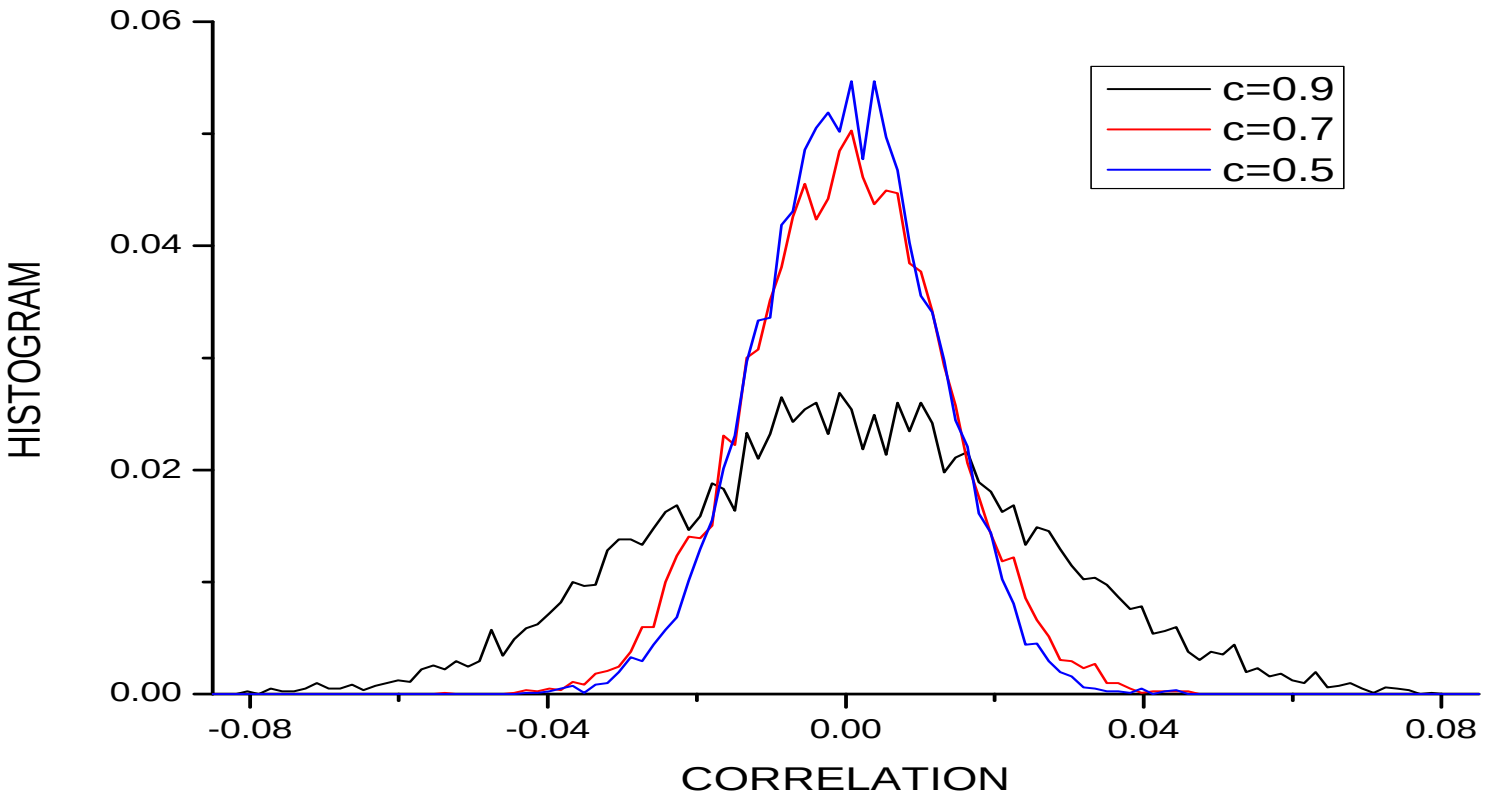


Connectivity vs. dynamics

correlations of INDEPENDENT realizations of

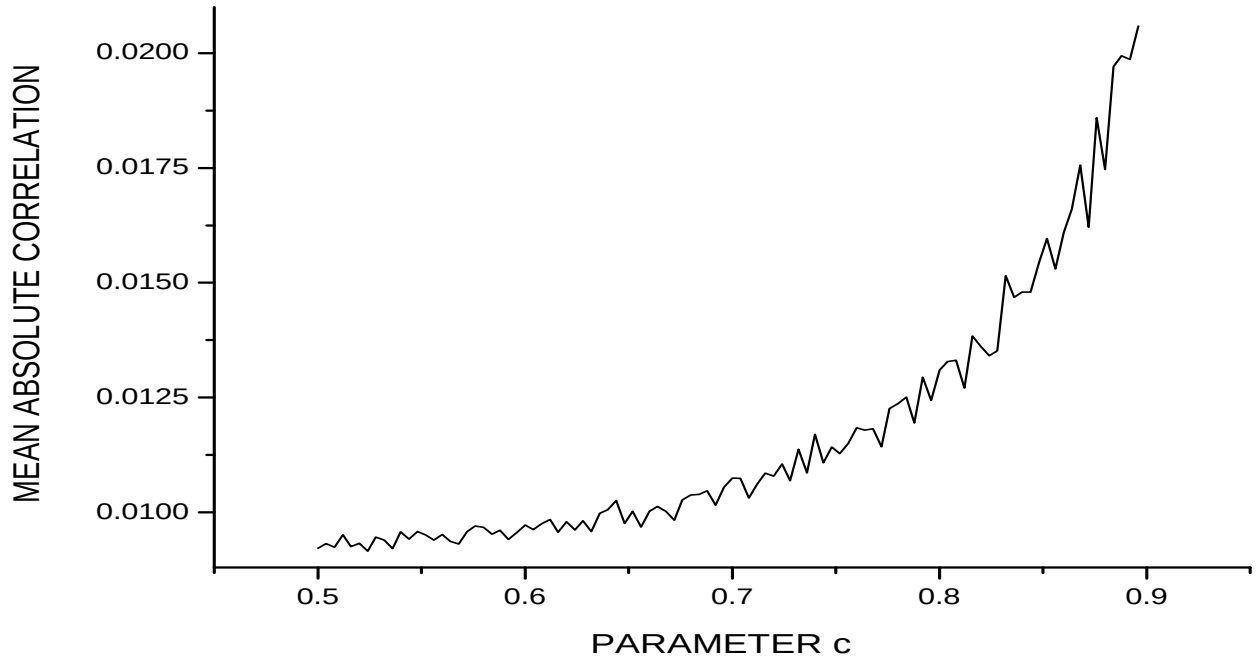
$$y_t = c \sum_{k=1}^{10} a_k y_{t-k} + \sigma e_t$$



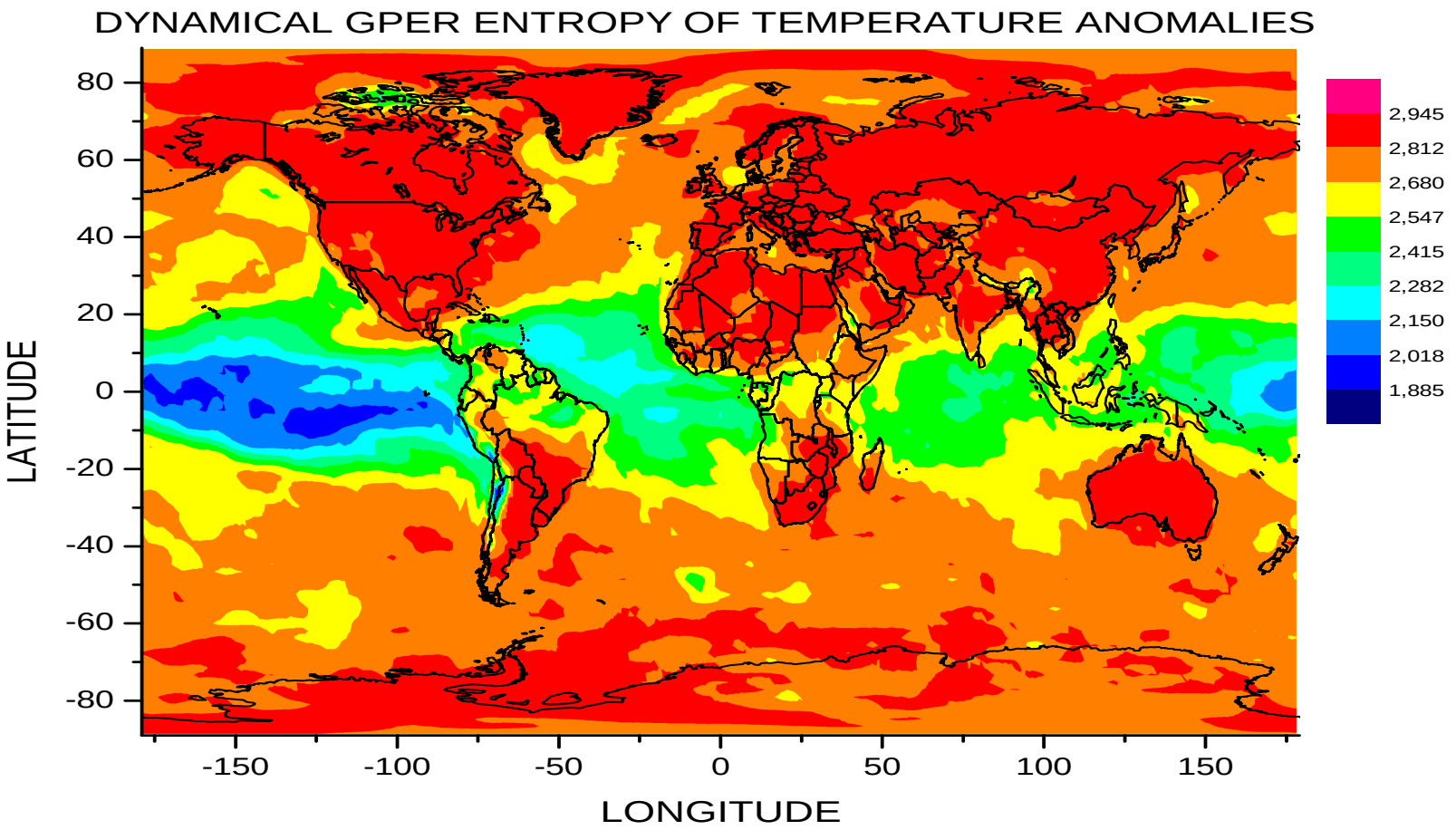
Connectivity vs. dynamics

mean ABSOLUTE correlations of INDEPENDENT realizations of

$$y_t = c \sum_{k=1}^{10} a_k y_{t-k} + \sigma e_t$$



Connectivity vs. dynamics



Dynamical entropy (inverse to regularity) of temperature anomaly time series for each node.

Connectivity vs. dynamics: significance of dependence

SURROGATE DATA / BOOTSTRAP

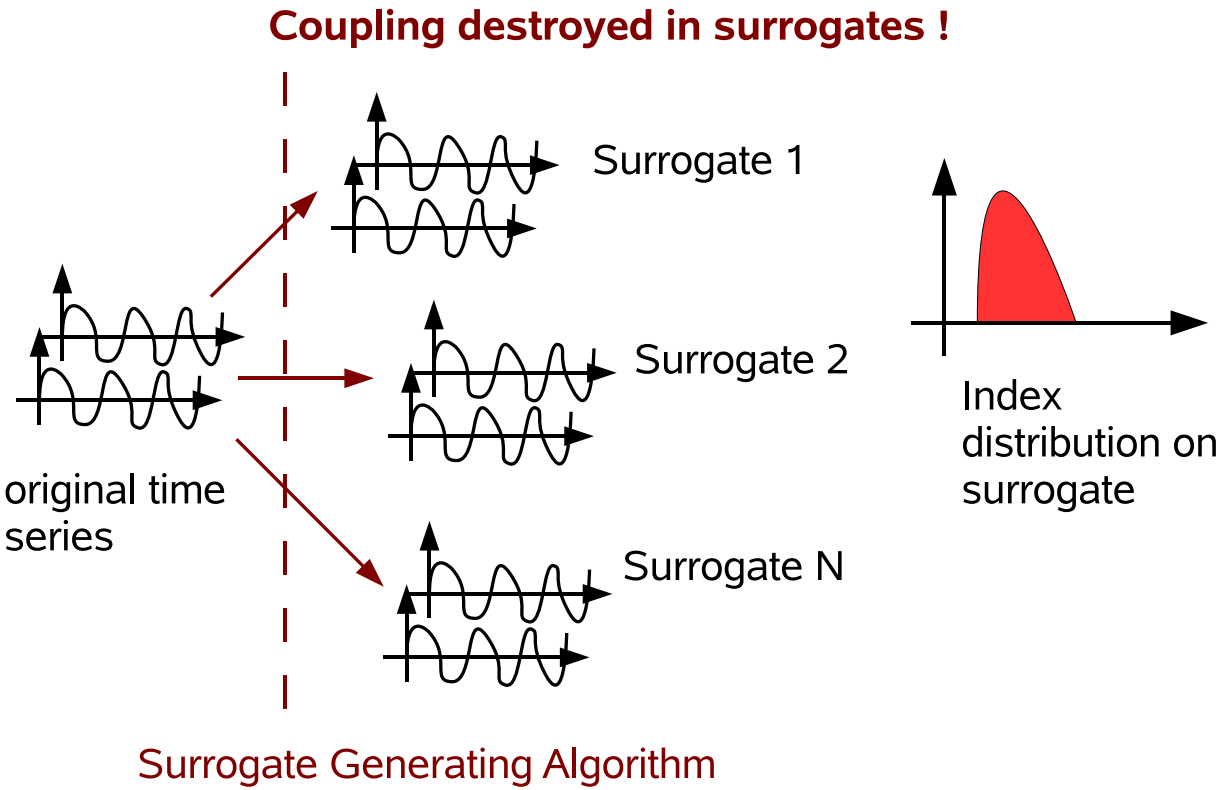
- generated by a model
- obtained by manipulation (randomization) of the original data (surrogate data)

- IID (scrambled) surrogate data
- **FT (AAFT, IAAFT ...) surrogate data**
- wavelet
- recurrence
- constrained randomization ...

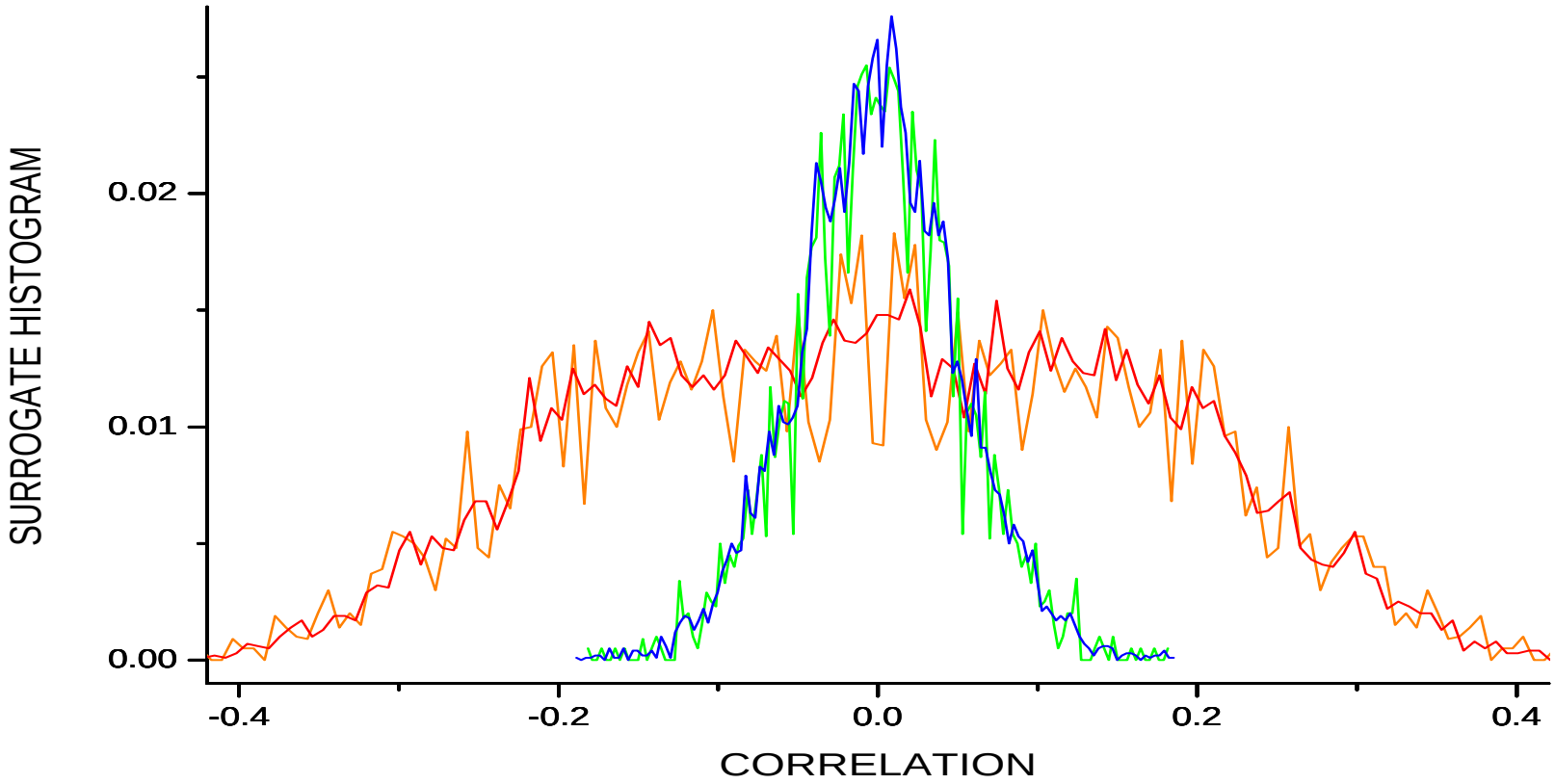
FT surrogates: preserve magnitudes of Fourier coefficients (spectra), randomize Fourier phases

Significance testing using surrogate data

- Use of bootstrap-like strategy (surrogate time series)
- Ideally preserve all properties except tested (coupling)



Connectivity vs. dynamics

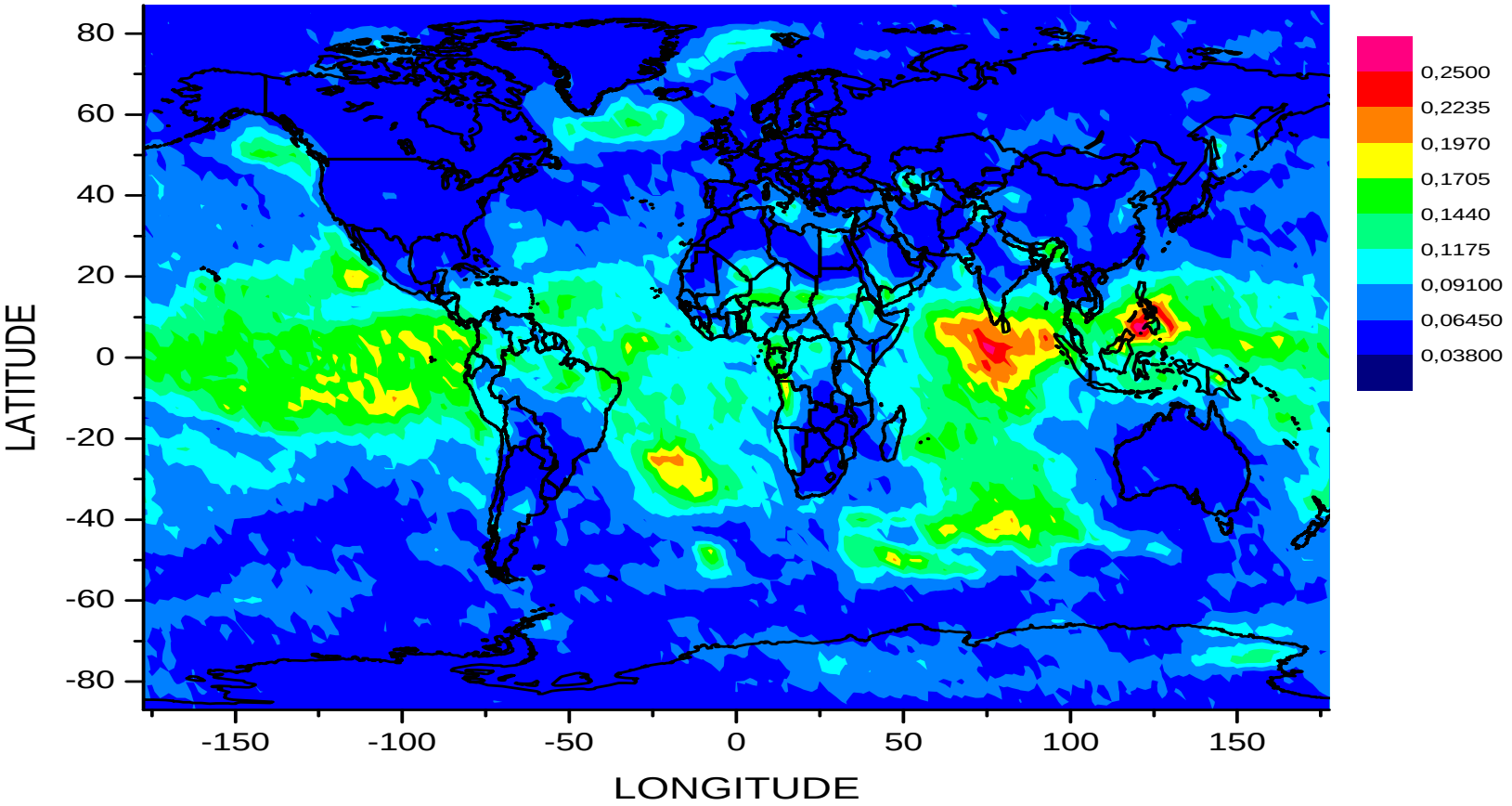


Surrogate cross-correlation for high-ER (green, blue) and low-ER (orange, red) NCEP/NCAR grid-points. FT (green, orange), AAFT (blue, red).

Connectivity vs. dynamics

Mean absolute correlation of NCEP/NCAR SAT anomalies

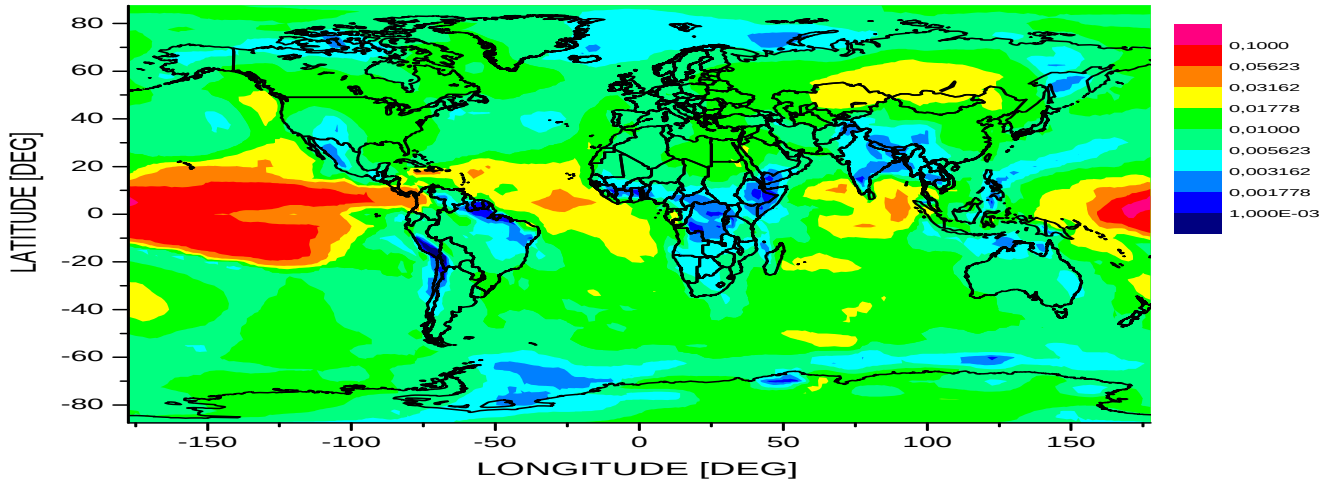
with FT surrogate data



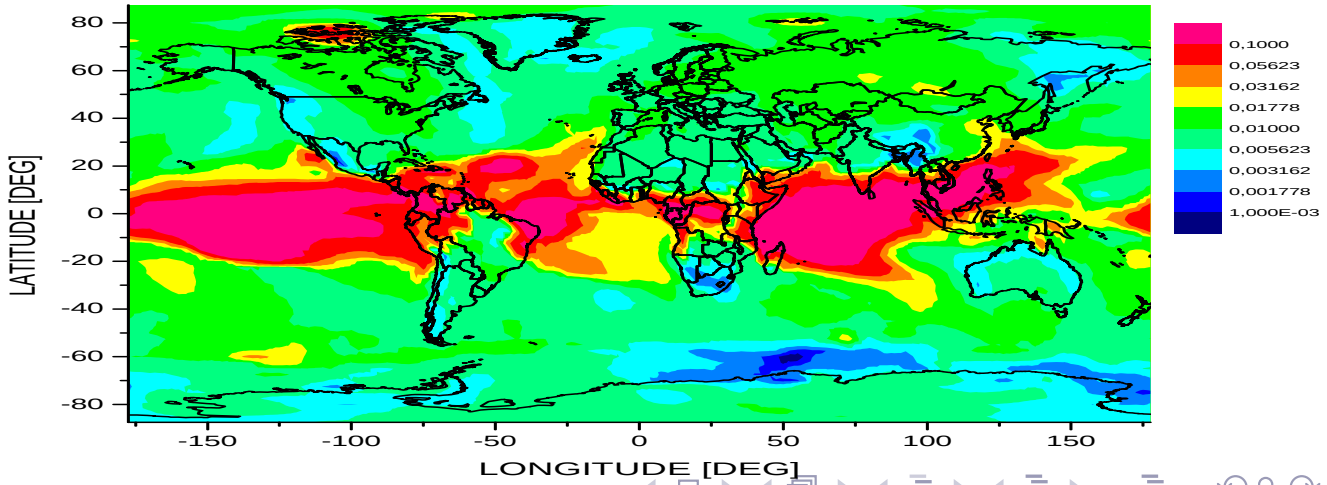
Connectivity vs. dynamics

Area Weighted Connectivity absolute correlations > 0.5
(Tsonis & Swanson, PRL 100, 228502, 2008)

ENSO-

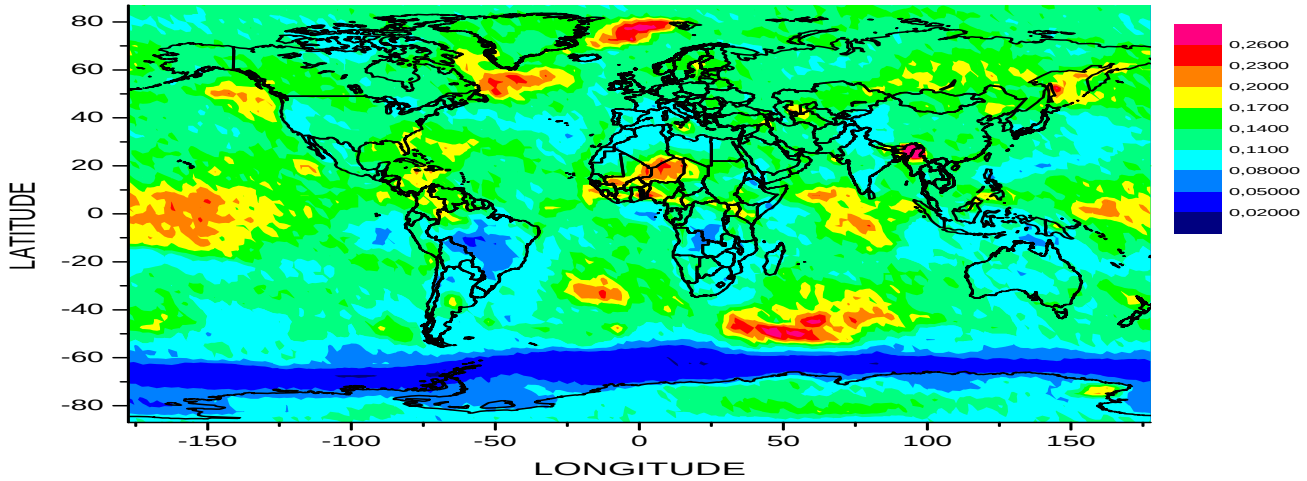


ENSO+



Mean absolute correlations SATA w/ FT surrogates

ENSO-



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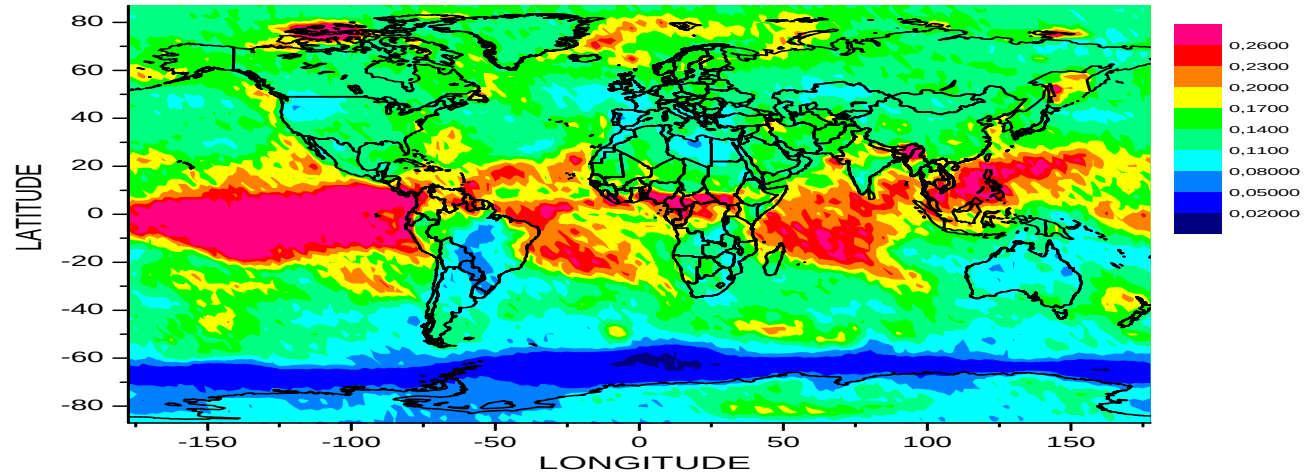


Connectivity vs. dynamics

Correct for dynamics (serial correlations):

For each link a statistical test with FT surrogate data

evaluated by using **Z-score**

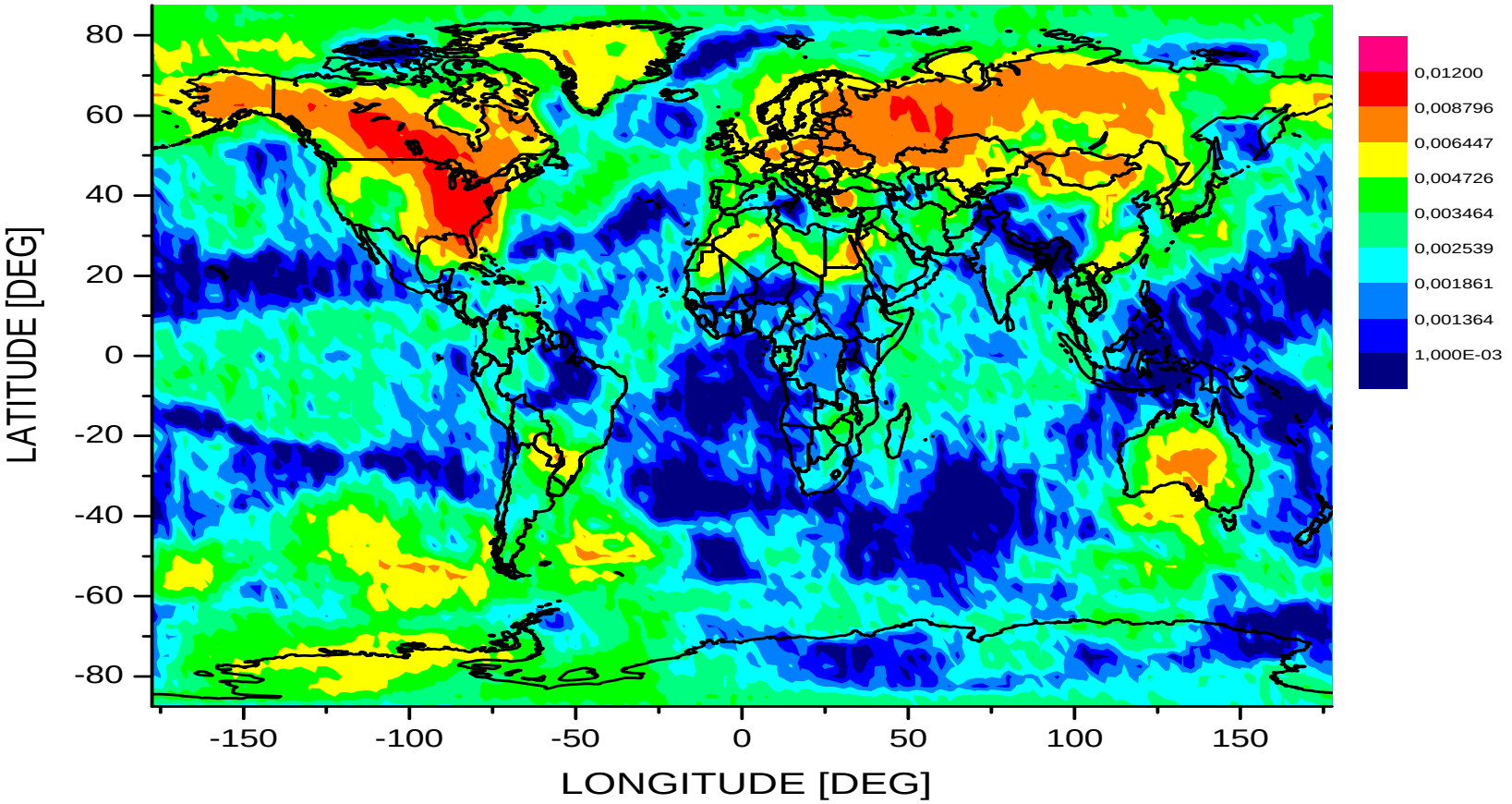
$$Z_{i,j} = \frac{c_{i,j} - mean[c_{i,j}(surr)]}{SD[c_{i,j}(surr)]}$$

Z-score $Z_{i,j}$ used instead of $c_{i,j}$ for the link weights

Connectivity vs. dynamics

Area Weighted Connectivity, NCEP/NCAR SATA, $\varrho = 0.005$

Z-score for absolute correlations + FT surrogate data

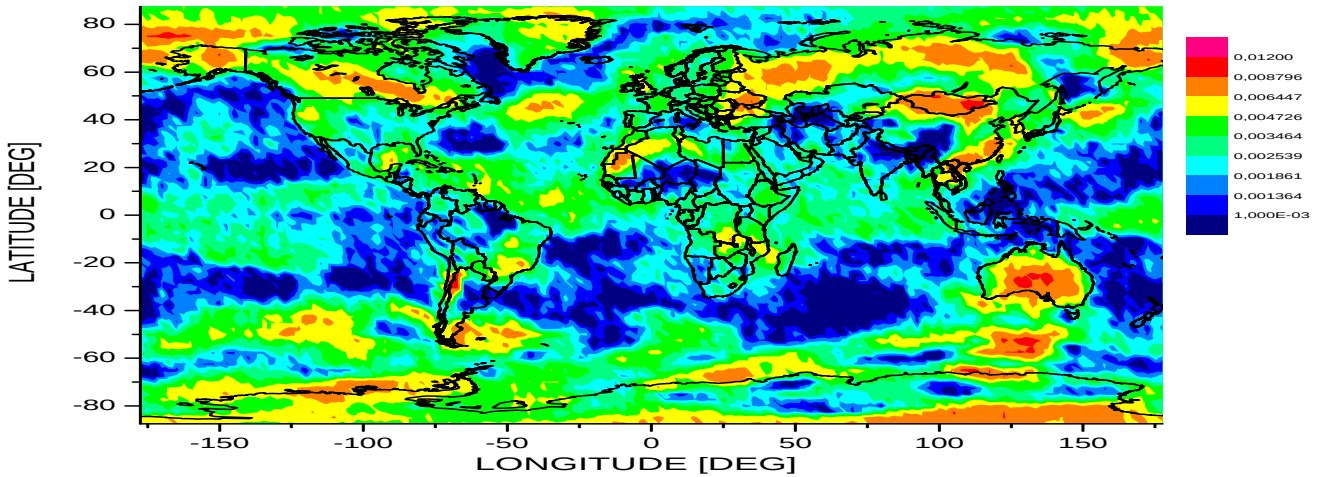


Connectivity vs. dynamics

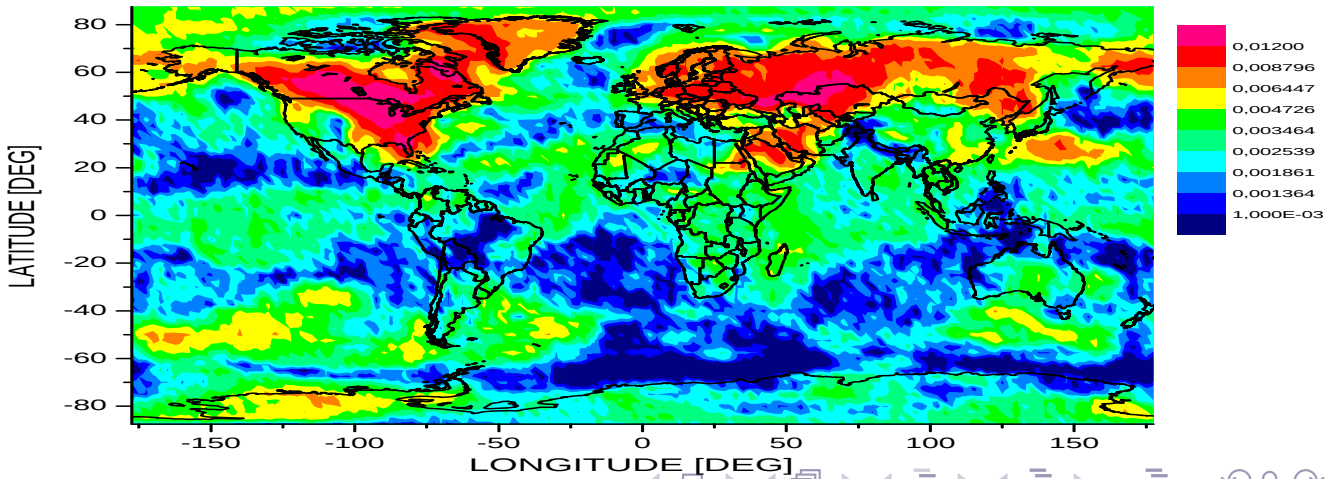
Z-score Area Weighted Connectivity, $\varrho = 0.005$

North Atlantic Oscillation influence

NAO-



NAO+

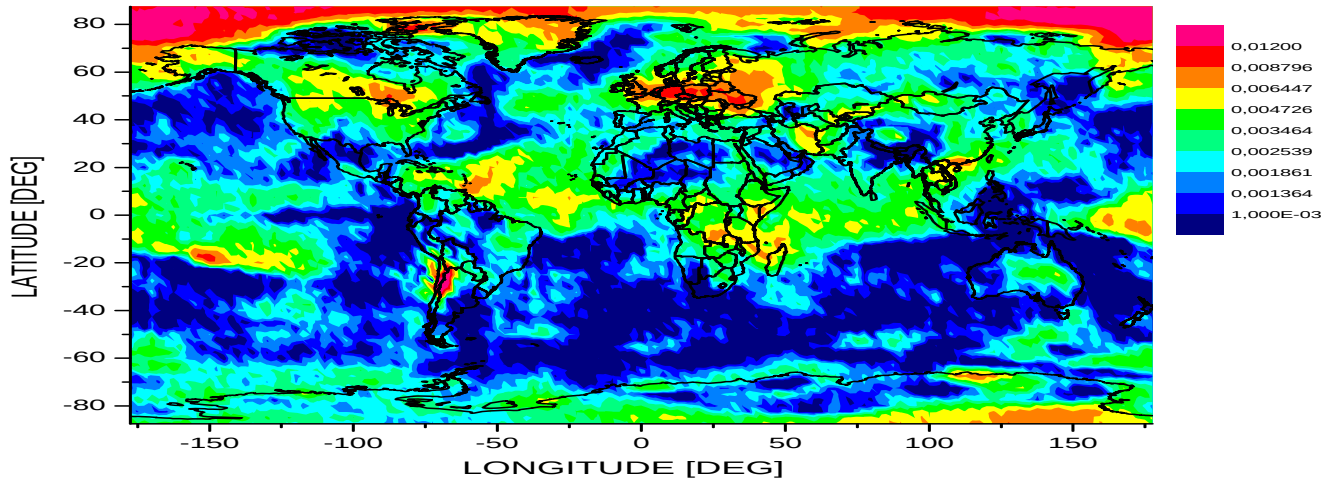


Connectivity vs. dynamics

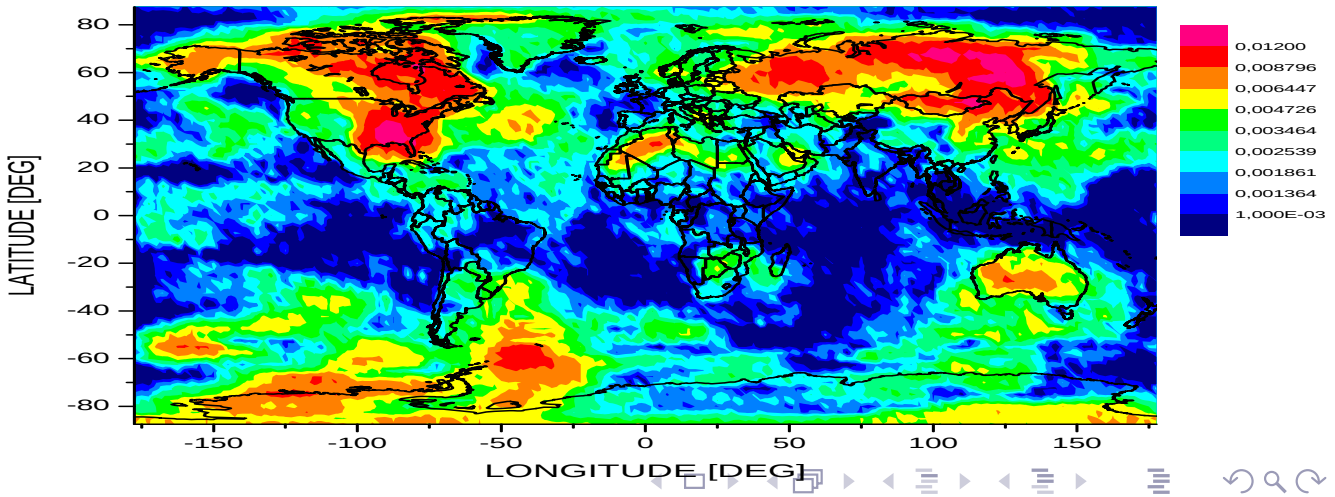
Z-score Area Weighted Connectivity, $\varrho = 0.005$

Solar influence: radio flux at 2800 MHz 10.7 cm

F10.7-



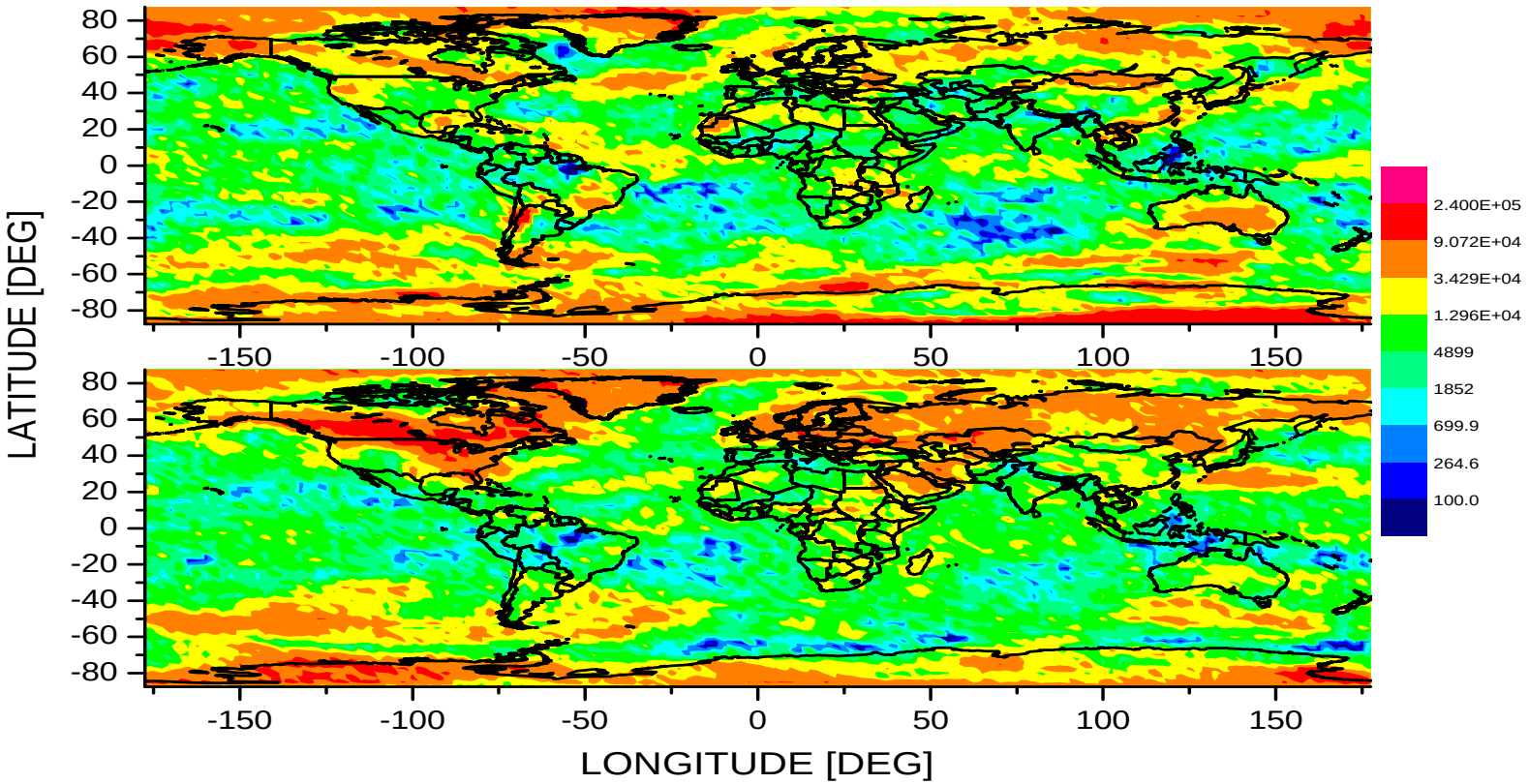
F10.7+



BETWEENNESS CENTRALITY

NCEP/NCAR SATA, $\varrho = 0.005$; NAO- top, NAO+ bottom

Z-score for absolute correlations + FT surrogate data



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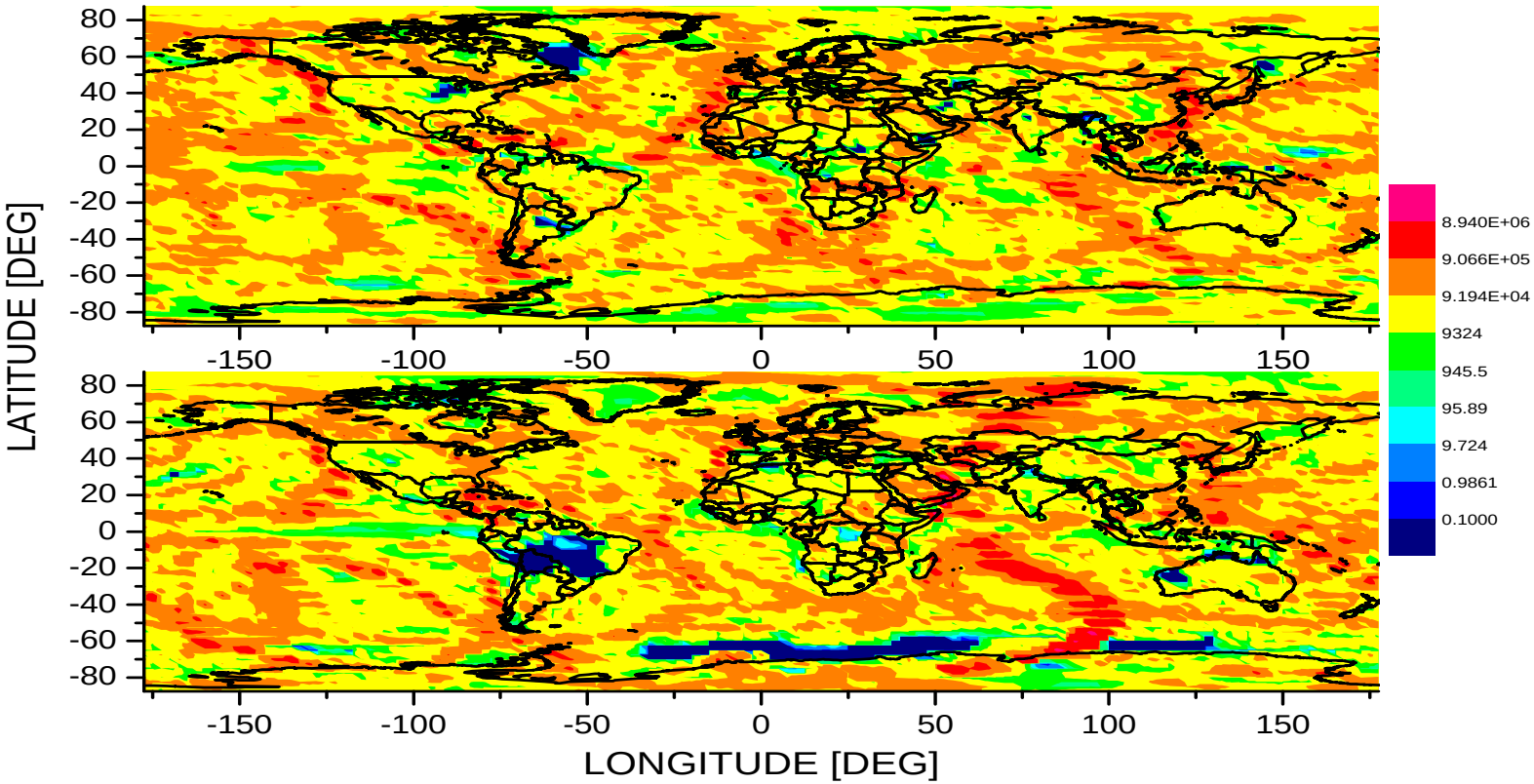
absolute correlations



POSTERS

- Martin **Vejmelka** et al.: Sensitivity of centrality measures to estimation of network structure from multivariate time series
- Jaroslav **Hlinka** et al.: Relation of structure and dynamics in complex systems: consequences for graph-theoretical analysis

CONCLUSION: problems to be solved

- connectivity vs. dynamics
- connectivity vs. spatial/temporal scale
- stability of connectivity, network structure
- significance of changes in time and space
- (climate) network variability vs. external influence

Software package for complex network analysis:

`http://ndw.cs.cas.cz/software/ndw-graph`

CONCLUSION

Thank you for your attention

Preprints:
`http://ndw.cs.cas.cz`
`http://www.cs.cas.cz/mp`

Acknowledgement
This study was supported by the Czech Science Foundation project No. P103/11/J068.