APPLICATION OF A CENTER MANIFOLD THEORY
TO A REACTION-DIFFUSION SYSTEM OF COLLECTIVE MOTION
OF CAMPHOR DISKS AND BOATS

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Abstract. Unidirectional motion along an annular water channel can be observed in an experiment even with only one camphor disk or boat. Moreover, the collective motion of camphor disks or boats in the water channel exhibits a homogeneous and an inhomogeneous state, depending on the number of disks or boats, which looks like a kind of bifurcation phenomena. In a theoretical research, the unidirectional motion is represented by a traveling wave solution in a model. Hence it suffices to investigate a linearized eigenvalue problem in order to prove the destabilization of a traveling wave solution. However, the eigenvalue problem is too difficult to analyze even if the number of camphor disks or boats is 2. Hence we need to make a reduction on the model. In the present paper, we apply the center manifold theory and reduce the model to an ordinary differential system.

Keywords: center manifold theory; bifurcation; traveling wave solution; collective motion

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1. Introduction

Self-driven motion of animal and inanimal organisms is observed in several fields, e.g., biology [6], chemistry [1], and nonlinear physics [4], [10], [11]. Organisms move spontaneously to aggregate and form self-organized structures through long-range interactions [5]. Therefore it is important not only to clarify the mechanism of the

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self-sustaining motion of each organism but also to study how organisms behave as a whole system.

Spatiotemporal collective motions in chemical experiments with camphor have been investigated in [7], [8], [9]. A camphor scraping at an air–water surface exhibits several motions, e.g., clockwise/counterclockwise rotation, and translation ([8]). Also, it was shown in [7] and [9] that unidirectional motion can be observed if we put a camphor disk or boat in an annular water channel. In an experimental setup, a camphor boat is composed of a plastic disk and a camphor disk stuck on the edge of the plastic disk with an adhesive. Camphor disks and boats constitute a system for changing the number of particles and with simple interaction. In this system we find two different states depending on the number. It was reported in [9] that when the number of boats is less than 30, camphor boats move with a constant velocity and spatially disperse with the same spacing between the boats, which is called a homogeneous state. On the other hand, when the number is larger than 30, the velocities of the boats change with temporal oscillation, and the shock wave appears in the line of the boats, which is an inhomogeneous state.

Various motions which a camphor disk and boat exhibit have been studied mathematically. In this article we are based on [7] and [9], and introduce the following mathematical model for the self-sustaining motion of a camphor disk and boat:

$$
(1.1) \quad
\begin{cases}
x_c''(t) = -\mu x_c'(t) + \dfrac{\gamma_1}{2\varrho}\left[\dfrac{1}{1 + au(x_c(t) + \varrho, t)} - \dfrac{1}{1 + au(x_c(t) - \varrho, t)}\right], \\[2mm]
\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2} - ku + f(x, x_c(t)),
\end{cases}
$$

where $a, \gamma_1, k, \mu, \varrho$ are positive constants. The first equation is described by the Newtonian equation with the surface tension of water given by $\gamma_1/(1 + au)$ as a function of u. In this model, a camphor scraping is regarded as a particle, and the center of a camphor disk or boat is denoted by $x_c(t)$. The surface concentration, denoted by u, of a camphor molecular layer is supposed to yield to the reaction-diffusion equation with the function $f(x, x_0)$ defined by

$$
f(x, x_0) =
\begin{cases}
1, & 0 < x - x_0 < \varrho, \\
s, & -\varrho < x - x_0 < 0, \\
0, & \text{otherwise},
\end{cases}
$$

which represents that camphor molecules are supplied only from $(x_0 - \varrho, x_0 + \varrho)$ where a camphor disk or boat contacts the water surface. Let $s \in [0, 1]$, which means that a camphor disk considered in this model is an inhomogeneous medium and the amount of the supply on $(x_0 - \varrho, x_0)$ is not larger than on $(x_0, x_0 + \varrho)$.

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The spontaneous motion of a camphor disk and boat can be characterized by a traveling wave solution of

$$(1.2) \quad \begin{cases} z_c'(t) = y_c(t), \quad t > 0, \\ y_c'(t) = -\mu(y_c(t) - c) \\ \qquad + \dfrac{\gamma_1}{2\varrho}\Big[\dfrac{1}{1 + au(z_c(t) + \varrho, t)} - \dfrac{1}{1 + au(z_c(t) - \varrho, t)}\Big], \quad t > 0, \\ \dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial z^2} - c\dfrac{\partial u}{\partial z} - ku + f(z, z_c(t)), \quad -\infty < z < \infty, \quad t > 0, \end{cases}$$

where we set $z_c(t) = x_c(t) + ct$, $y_c(t) = z_c'(t)$, $z = x + ct$ in (1.1). We denote the right-hand side of (1.2) by $F(z_c(t), y_c(t), u(z, t); c)$ for simplicity. A stationary solution of (1.2) is called a traveling wave solution, and the parameter c is called the wave speed, where the function $p(z)$ is supposed to be a C^1-function. It is easy to see that a traveling wave solution $(0, 0, p(z))$ in (1.2) exists with $c \geqslant 0$ for any parameter set.

As shown in [7], there is a critical value such that (1.2) for $s = 1$ has a traveling wave solution with a positive wave speed only in the case that μ is smaller than the critical value. Hence the pair of the critical value and $c = 0$ is obviously a bifurcation point. Actually, there is a bifurcation point with $s < 1$ in (1.2). For example, we set $(a, \gamma_1, k, \varrho) = (0.64, 1.7, 0.011, 0.84)$. Then there are $s \in (0.12, 0.13)$ and a bifurcation point (μ_0, c_0) such that $0.298 < \mu_0 < 0.3$ and $0.271 < c_0 < 0.273$. We will characterize such a bifurcation point in a mathematical sense later.

The existence of the bifurcation point affects the linearized eigenvalue problem

$$(1.3) \quad \lambda\Phi = F'(0, 0, p; c_0)\Phi$$
$$\equiv \begin{pmatrix} Y_\Phi \\ -\mu_0 Y_\Phi - a\dfrac{\gamma_1}{2\varrho}\Big[\dfrac{p'(\varrho)Z_\Phi + \phi(\varrho)}{(1 + ap(\varrho))^2} - \dfrac{p'(-\varrho)Z_\Phi + \phi(-\varrho)}{(1 + ap(-\varrho))^2}\Big] \\ A(\phi, Z_\Phi) \end{pmatrix}$$

for $\Phi = \Phi(z) = {}^t(Z_\Phi, Y_\Phi, \phi)$, where ^{t} denotes the transpose. A linear functional $A(\phi, Z)$ with the domain $H^1(\mathbb{R}) \times \mathbb{R}$ is determined by the trilinear form

$$(A(\phi, Z), \varphi) \equiv \int_{-\infty}^{\infty} (-\phi'\varphi' - c_0\phi'\varphi - k\phi\varphi)\,\mathrm{d}z - Z[s\varphi(-\varrho) - (s-1)\varphi(0) - \varphi(\varrho)]$$

for $\varphi \in H^1(\mathbb{R})$, where $(\cdot, \cdot)$ is the pairing between $(H^1(\mathbb{R}))'$, the dual space of $H^1(\mathbb{R})$, and $H^1(\mathbb{R})$. It is obvious that $\Phi(z) = (-1, 0, \phi(z))$ is a solution of (1.3) for $\lambda = 0$, where $\phi(z) = p'(z)$. Actually, (1.3) has a degeneracy condition at $\mu = \mu_0$, and there is a solution $\Psi(z) = {}^t(Z_\Psi, Y_\Psi, \psi(z))$ of $F'(0, 0, p; c_0)\Psi = -\Phi$. The existence of $\Psi(z)$ generically means that the multiplicity of the zero eigenvalue of $F'(0, 0, p; c_0)$ is equal to 2. More precisely, we suppose that the following conditions hold true at (μ_0, c_0);

(A1) There is a solution $\Psi(z)$ of $F'(0, 0, p; c_0)\Psi = -\Phi$.

(A2) The spectra of $F'(0, 0, p; c_0)$, denoted by Σ_c, consist of $\{0\}$ and Σ_1, where $\Sigma_1 \subset \{\lambda \in \mathbb{C}; \; \mathrm{Re}\,\lambda \leqslant -\kappa\}$ for $\kappa > 0$, and $\mathrm{Re}\,\lambda$ denotes the real part of a complex value λ.

(A3) The generalized eigenspace associated with 0 is spanned by Φ and Ψ.

Now we consider the collective motion of $(N+1)$-camphors on a one-dimensional circuit $(0, L)$. Our system is described by

$$
(1.4) \quad
\begin{cases}
z_i' = y_i, \quad t > 0, \\[2mm]
y_i' = -\mu(y_i - c_0) + \dfrac{\gamma_1}{2\varrho}\left[\dfrac{1}{1 + au(z_i + \varrho, t)} - \dfrac{1}{1 + au(z_i - \varrho, t)}\right], \quad t > 0, \\[4mm]
\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial z^2} - c_0 \dfrac{\partial u}{\partial z} - ku + \displaystyle\sum_{i=0}^{N} f(z, z_i), \quad 0 < z < L, \; t > 0, \\[4mm]
u(0, t) = u(L, t), \quad \dfrac{\partial u}{\partial z}(0, t) = \dfrac{\partial u}{\partial z}(L, t), \quad t > 0
\end{cases}
$$

for $N \geqslant 1$ and $i = 0, \ldots, N$, where $z_i = z_i(t)$, $y_i = y_i(t)$, and $u = u(z, t)$. For simplicity, we denote the operator associated with the right-hand sides of the first, second, and third equation of (1.4) by $\mathcal{L}(U) = {}^t(\mathcal{L}_0^z(U), \mathcal{L}_0^y(U), \ldots, \mathcal{L}_N^z(U), \mathcal{L}_N^y(U), \mathcal{L}^u(U))$, where $U = {}^t(z_0, y_0, \ldots, z_N, y_N, u)$ for $0 \leqslant z_i \leqslant L$, $y_i \in \mathbb{R}$ $(i = 0, \ldots, N)$, $u \in H^1(0, L)$ and $\mathcal{L}_i^z, \mathcal{L}_i^y$ $(i = 0, \ldots, N)$, $\mathcal{L}^u$ are given by

$$
\mathcal{L}_i^z(U) = y_i,
$$
$$
\mathcal{L}_i^y(U) = -\mu(y_i - c_0) + \frac{\gamma_1}{2\varrho}\left[\frac{1}{1 + au(z_i + \varrho, t)} - \frac{1}{1 + au(z_i - \varrho, t)}\right],
$$
$$
\mathcal{L}^u(U) = Tu + \sum_{i=0}^{N} f(z, z_i).
$$

The linear operator $T\colon H^1(0, L) \to (H^1(0, L))'$ is determined by the bilinear form $(Tu, \varphi) = \int_0^L (-u'\varphi' - c_0 u'\varphi - ku\varphi)\,\mathrm{d}z$ for $\varphi \in H^1(0, L)$. In the same way as in (1.2), there exists a traveling wave solution of (1.4) such that $z_i = iL/(N+1)$, which corresponds to the homogeneous state observed in the experiment of [9]. According to the result obtained in [9], the traveling wave solution is expected to be unstable for a large number N. In order to prove the instability of the traveling wave solution, it is necessary to find an unstable eigenvalue. However, the linearized eigenvalue problem is too difficult to analyze theoretically even in the case of $N = 1$. From this point of view, we need to reduce (1.4) and derive a new system. This is our motivation in this article.

Let l be a position of the 0-th particle and denote the distance between the i-th and the $(i+1)$-st particles by h_i for $i = 0, \ldots, N-1$. Since we consider a one-dimensional

circuit, h_N is defined by the distance between the N-th and the 0-th particles. Thus we define a relative position of the i-th particle by $\overline{z}_i = \overline{z}_i(\mathbf{h}) \equiv \sum_{j=0}^{i-1} h_j$, where $\mathbf{h} = {}^t(h_0, \ldots, h_N)$. We put $\overline{z}_0 = 0$. Note that the position of the i-th particle is given by $z_i = \overline{z}_i + l$. Here we assume that one particle is sufficiently separated from any others. In other words, h_i is assumed to satisfy $h_i > h^*$ for any $i = 0, \ldots, N$, where h^* is sufficiently large and fixed.

Due to the interactions described by (1.4), the positions of the particles will be varied. Then the position l and the distance h_i are regarded as functions in time, denoted by $l(t)$ and $h_i(t)$. In the statement of our result, we use the following notation. Put $\mathbf{r} = {}^t(r_0, \ldots, r_N)$, $P(z, l, \mathbf{h}) = \sum_{i=0}^{N}[(\overline{z}_i + l)e_{2i+1} + \overline{p}(z - \overline{z}_i - l)e_{2N+3}]$, $\xi(z, l, \mathbf{h}, \mathbf{r}) = \sum_{i=0}^{N} r_i[Z_\Psi e_{2i+1} + Y_\Psi e_{2i+2} + \overline{\psi}(z - \overline{z}_i - l)e_{2N+3}]$, and $S(z, l, \mathbf{h}, \mathbf{r}) = P(z, l, \mathbf{h}) + \xi(z, l, \mathbf{h}, \mathbf{r})$, where e_i is the unit vector in $\mathbb{R}^{2N+3}$ given by

$$e_i = {}^t(0, \ldots, 0, \overset{\overset{i}{\vee}}{1}, 0, \ldots, 0),$$

and $\overline{\varphi}$ is an L-periodic function for a function φ defined on $(-\infty, \infty)$, which is given by

(1.5) $$\overline{\varphi}(z) = \varphi(z - nL), \quad \left(n - \frac{1}{2}\right)L < z < \left(n + \frac{1}{2}\right)L, \quad n \in \mathbb{Z}.$$

We suppose that $\overline{\varphi}$ is identically equal to 0 in a neighborhood of $z = (n+1/2)L$ for any $n \in \mathbb{Z}$, and if $\varphi \in C^1(\mathbb{R})$, $\overline{\varphi}$ also belongs to $C^1(\mathbb{R})$. Although $\overline{\varphi}$ may not satisfy (1.5) on all $z \in (-\infty, \infty)$, we do not distinguish the extended function from the original one, and we write the extended function as $\overline{\varphi}$. Define $\delta \equiv \delta(\mathbf{h}) = \max_i e^{-\alpha h_i}$ for a constant $\alpha > 0$. Then, if $h_i > h^*$, δ is small. Let $\mathcal{M}(h^*, r^*) = \{S(z, l, \mathbf{h}, \mathbf{r}); \, l \in \mathbb{R}, h_i > h^*, |r_i| < r^*$ for all $i = 0, \ldots, N\}$. Set $\eta = \mu - \mu_0$.

Theorem 1.1. *There exist $h^*, r^*, \eta^*, \alpha, C_0 > 0$ and a neighborhood $\mathcal{N}$ of $\mathcal{M}(h^*, r^*)$ such that for the initial data $U_0 \in \mathcal{N}$ of (1.4) there exist $l(t), \mathbf{h}(t), \mathbf{r}(t)$ such that $\|U(\cdot, t) - S(\cdot, l(t), \mathbf{h}(t), \mathbf{r}(t))\| \leqslant C_0(\delta + |\mathbf{r}|^2 + |\eta|)$ holds true as long as $h_i > h^*$, $|r_i| < r^*$ for any $i = 0, \ldots, N$, where $U(z, t)$ is a solution of (1.4), and $\|\cdot\|$ is a standard norm in $X \equiv \mathbb{R}^{N+1} \times \mathbb{R}^{N+1} \times L^2(0, L)$. In addition, $l(t)$, $h_i(t)$, and $r_i(t)$ $(i = 0, \ldots, N)$ yield to*

(1.6) $$\begin{cases} l' = r_0 + O(\delta + |\mathbf{r}|^2 + \eta), \\ h_i' = r_{i+1} - r_i + O(\delta + |\mathbf{r}|^2 + \eta), \\ r_i' = O(\delta + |\mathbf{r}|^2 + \eta) \end{cases}$$

for $i = 0, \ldots, N$, where $r_{N+1} = r_0$.

This theorem says that the partial differential equation (1.4) can be reduced to the ordinary differential equation (1.6). Our result for $N = 1$ is similar to the one in [3], where the authors considered the interaction between two pulses with very small velocity near a bifurcation point in a reaction-diffusion system. In that article each pulse can be approximated by a stationary solution, which implies that all eigenfunctions $\Phi, \Psi, \Phi^*, \Psi^*$ are expected to be symmetric, that is, odd or even functions. Here Φ^* and Ψ^* will be introduced in the next section. As a result, several calculations in the reduction process become easier than in our case. Actually, we can obtain higher order terms in (1.6) by an argument similar to that in [3] though equations (1.6) have only r_i, which we will show in the forthcoming paper.

In the next section we will describe the outline of the proof of Theorem 1.1. We apply a center manifold theory developed in [2] and [3] for the derivation of (1.6). In order to construct a center manifold, generalized eigenfunctions play an important role. Those functions will be defined in Proposition 2.1. Finally, we do calculations and derive (1.6) from (1.4) in a formal way.

2. Outline of the proof

In this section, we describe the outline of the proof. In the reduction process to show Theorem 1.1, we will project the solution $U(z,t)$ onto a finite dimensional space $E(h)$. This space is constructed by using Φ, Ψ and the generalized eigenfunctions Φ^*, Ψ^* of $(F'(0,0,p;c_0))^*$ which satisfy $(F'(0,0,p;c_0))^*\Phi^* = 0$ and $(F'(0,0,p;c_0))^*\Psi^* = -\Phi^*$, respectively, where $(F'(0,0,p;c_0))^*$ denotes the adjoint operator of $F'(0,0,p;c_0)$. Under (A1)–(A3), Φ^*, Ψ^* do exist thanks to Fredholm's alternative. Moreover, the eigenfunctions Ψ, Φ^*, and Ψ^* are uniquely determined by the orthogonal conditions $\langle \Phi, \Psi^* \rangle = 1$, $\langle \Psi, \Psi^* \rangle = 0$, $\langle \partial\Psi/\partial z, \Phi^* \rangle = 0$ in general, where $\langle \cdot, \cdot \rangle$ is the standard inner product in $\mathbb{R} \times \mathbb{R} \times L^2(\mathbb{R})$, and $\partial\Psi/\partial z = (0,0,\psi'(z))$. Note that $\langle \Psi, \Phi^* \rangle = 1$, $\langle \Phi, \Phi^* \rangle = 0$ automatically hold. Put $\Phi^*(z) = {}^t(Z_\Phi^*, Y_\Phi^*, \phi^*(z))$ and $\Psi^*(z) = {}^t(Z_\Psi^*, Y_\Psi^*, \psi^*(z))$. Note that p', ψ, ϕ^*, ψ^* decay exponentially as $|z| \to \infty$.

To construct $E(h)$, we introduce $\Phi_i(h)$, $\Psi_i(h)$, $\Phi_i^*(h)$, and $\Psi_i^*(h)$ for $i = 0, \ldots, N$.

Proposition 2.1. *There exist $\Phi_i(h)$, $\Psi_i(h)$, $\Phi_i^*(h)$, and $\Psi_i^*(h) \in \mathbb{R}^{N+1} \times \mathbb{R}^{N+1} \times H^1(0,L)$ such that*

$$[\Phi_i(h)](z) = -e_{2i+1} + \overline{\phi}(z - \overline{z}_i)e_{2N+3} + O(\delta),$$
$$[\Psi_i(h)](z) = Z_\Psi e_{2i+1} + Y_\Psi e_{2i+2} + \overline{\psi}(z - \overline{z}_i)e_{2N+3} + O(\delta),$$
$$[\Phi_i^*(h)](z) = Z_\Phi^* e_{2i+1} + Y_\Phi^* e_{2i+2} + \overline{\phi^*}(z - \overline{z}_i)e_{2N+3} + O(\delta),$$

$$[\Psi_i^*(h)](z) = Z_\Psi^* e_{2i+1} + Y_\Psi^* e_{2i+2} + \overline{\psi^*}(z - \overline{z}_i)e_{2N+3} + O(\delta),$$

$$\frac{\partial[\Psi_i(h)]}{\partial z}(z) = \overline{\psi}'(z - \overline{z}_i)e_{2N+3} + O(\delta),$$

and these functions are supposed to satisfy

$$(2.1) \qquad \langle \Phi_i(h), \Psi_i^*(h) \rangle = 1, \quad \langle \Phi_i(h), \Phi_i^*(h) \rangle = 0, \quad \langle \Phi_i(h), \Psi_k^*(h) \rangle = 0,$$
$$\langle \Psi_i(h), \Phi_i^*(h) \rangle = 1, \quad \langle \Psi_i(h), \Psi_i^*(h) \rangle = 0, \quad \langle \Psi_i(h), \Phi_k^*(h) \rangle = 0$$

for any $i = 0, \dots, N$ and $k \neq i$, where $\langle \cdot, \cdot \rangle$ is the standard inner product in $\mathbb{R}^{N+1} \times \mathbb{R}^{N+1} \times L^2(\mathbb{R})$.

This proposition can be proved in the same way as in [2] and [3] so that we omit the details of the proof. Thanks to (2.1), $\{\Phi_i(h), \Psi_i(h)\}_{i=0}^N$ is linearly independent in X. Then we define $E(h)$ to be the linear space spanned by $\Phi_i(h), \Psi_i(h)$ ($i = 0, \dots, N$). Let $Q(h)$ be the projection from X to $E(h)$, where Id is the identity on X. Note that $Q(h)U$ for $U \in X$ is given by $Q(h)U = \sum_{i=0}^N (\langle U, \Psi_i^*(h) \rangle \Phi_i(h) + \langle U, \Phi_i^*(h) \rangle \Psi_i(h))$.

Now we are in a position to derive the reduced equation (1.6) from (1.4) by formal calculations and describe the outline of the proof of Theorem 1.1. We first substitute $S(z, l, \mathbf{h}, \mathbf{r})$ into $U(z, t)$ in (1.4). Note that (1.4) is simply written as $U_t = \mathcal{L}(U)$. Since the function $\xi(z, l, \mathbf{h}, \mathbf{r})$ is represented by $\xi(z, l, \mathbf{h}, \mathbf{r}) = \sum_{i=0}^N r_i[\Psi_i(h)](z - l)$, straightforward calculations give

$$(2.2) \quad -\sum_{i=0}^N (l' + l_i')\left([\Phi_i(h)](z) + r_i\frac{\partial[\Psi_i(h)]}{\partial z}(z)\right) + \sum_{i=0}^N r_i'[\Psi_i(h)](z) = \mathcal{L}(S(z, 0, \mathbf{h}, \mathbf{r}))$$

by putting $z - l \to z$, where $l_i(t) \equiv \overline{z}_i(\mathbf{h}(t))$ ($i = 0, \dots, N$). Then, multiplying by $Q(h)$ both sides of (2.2) and applying Proposition 2.1, we have

$$(2.3) \qquad -(l' + l_i')\left(1 + r_i\left\langle \frac{\partial\Psi_i(h)}{\partial z}, \Psi_i^*(h) \right\rangle\right) = \langle \mathcal{L}(S(z, 0, \mathbf{h}, \mathbf{r})), \Psi_i^*(h) \rangle + O(\delta),$$
$$-r_i' = \langle \mathcal{L}(S(z, 0, \mathbf{h}, \mathbf{r})), \Phi_i^*(h) \rangle + O(\delta).$$

Finally, we obtain the lowest order terms of the two inner products $\langle \mathcal{L}(S(z, 0, \mathbf{h}, \mathbf{r})), \Phi_i^*(h) \rangle$, and $\langle \mathcal{L}(S(z, 0, \mathbf{h}, \mathbf{r})), \Psi_i^*(h) \rangle$. Set $S(z, 0, \mathbf{h}, \mathbf{r}) = {}^t(\hat{z}_0, \hat{y}_0, \dots, \hat{z}_N, \hat{y}_N, u(z))$. Since all particles are separated and $\mathbf{r}$ is small due to the assumptions, we have

$$(2.4) \qquad \mathcal{L}_i^z(S(z, 0, \mathbf{h}, \mathbf{r})) = r_i + O(\delta), \quad \mathcal{L}_i^y(S(z, 0, \mathbf{h}, \mathbf{r})) = O(\delta + |\mathbf{r}|^2 + \eta).$$

By an argument similar to that used to obtain (2.4), it holds true $\mathcal{L}^u(S(z, 0, \mathbf{h}, \mathbf{r})) = \sum_{i=0}^{N} [\overline{f}(z, \hat{z}_i) - \overline{f}(z, \overline{z}_i) + r_i T \overline{\psi}(z - \overline{z}_i)] + O(\delta)$. Since ϕ^*, ψ^* decay exponentially to 0 as $|z| \to \infty$, we have

$$
\int_0^L [\overline{f}(z, \hat{z}_i) - \overline{f}(z, \overline{z}_i) + r_i T \overline{\psi}(z - \overline{z}_i)]\overline{\phi^*}(z - \overline{z}_i)\, \mathrm{d}z
$$
$$
= r_i(A(\psi, Z_\Psi), \phi^*) + O(\delta + |\mathbf{r}|^2) = -r_i \int_{-\infty}^{\infty} p'(z)\phi^*(z)\, \mathrm{d}z + O(\delta + |\mathbf{r}|^2)
$$

owing to $A(\psi, Z_\Psi) = -p'$. Similarly, the inner product of $\mathcal{L}^u(S(z, 0, \mathbf{h}, \mathbf{r}))$ and $\overline{\psi^*}(z - \overline{z}_i)$ is equal to $-r_i \int_{-\infty}^{\infty} p'(z)\psi^*(z)\, \mathrm{d}z + O(\delta + |\mathbf{r}|^2)$. From these calculations, we see that $\langle \mathcal{L}(S(z, 0, \mathbf{h}, \mathbf{r})), \Psi_i^*(h) \rangle = -r_i + O(\delta + |\mathbf{r}|^2 + \eta)$ and $\langle \mathcal{L}(S(z, 0, \mathbf{h}, \mathbf{r})), \Phi_i^*(h) \rangle = O(\delta + |\mathbf{r}|^2 + \eta)$ due to the orthogonal conditions, which concludes Theorem 1.1.

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