Arrows and Haloes:
Probabilities of Conditionals
and Desire as Belief

Alan Hájek
RSSS, Australian National University

This talk concerns the many parallels between two seemingly disparate debates:

"Stalnaker's Hypothesis", "the Equation":

$P(A \rightarrow B) = P(B \mid A)$ $[P(B \mid A) = P(A \& B)/P(A)$, provided $P(A) > 0)]$

"Desire as Belief"

$V(A) = P(A^{\circ})$ $[V(A) = \Sigma_i V(A \& A_i)P(A_i \mid A)$ for any partition $\{A_i\}$, $P(A) > 0$; V is scaled to the $[0, 1]$ interval.]

- Aerial view:
  - both hypotheses are ill-named
  - Lewis' role in each
  - reductionist ambitions
  - mathematical form
  - quantifiers
  - triviality results
  - fighting back
  - more triviality results
  - but there are still loopholes
  - so we can forecast how the debates will continue …

Now, a view from the trenches…

Probabilities of Conditionals as Conditional Probabilities

- Why care about the Equation?
  - Illuminate the semantics of the conditional:
    - Stalnaker vs Lewis on conditional excluded middle: (p→q) v (p→¬q) is a tautology
    - Adams on 'probabilistic validity'
  - de Finetti: read the equation from right to left
  - Dynamics of credences: the 'Judy Benjamin' problem
- Why believe the Equation?
  - It sounds right; case-by-case evidence; structural similarities
  - Ramsey test: "If two people are arguing 'If *p* will *q*?' and are both in doubt as to *p*, they are hypothetically adding *p* to their stock of knowledge and arguing on that basis about *q*… We can say that they are fixing their degrees of belief in *q*, given *p*".
  - Adams' Thesis: the assertability of the indicative conditional 'if *p* then *q*' is P(*q*|*p*).
- Why disbelieve the Equation? Sources of suspicion
  - It fails for the material conditional
    - But "paradoxes" (?!) of material implication; contra 'pragmatic' accounts
  - Failures of (probabilistic) conditional excluded middle?
    - indeterministic cases
    - indeterminate cases
  - Causal decision theory (with probabilities of counterfactuals) differs from evidential
    - But I have doubts about the lore regarding decision theory
- Four quantified versions
- Lewis' triviality results, in two installments (roughly 10 years apart), refute the *Fixed* → version, and plausibly refute the *Fixed* → *for rational agents* version, but leave the *Indexical* → versions unscathed.
- Fighting back: fallback positions
  - Shrinking the domain of propositions
  - Approximate equality, proportionality, correlation
  - Indexical →
- Perturbation argument; more trouble for *Fixed* → *for rational agents*, and for these fallbacks
- More fighting back
  - Radical indexicality (van Fraassen)
    - Lewis' 'disagreement' argument; retraction, eavesdropping
- Wallflower argument: an example, and overview; trouble for *Indexical* → hypotheses
- A new argument against Adams' Thesis
- Hall showed that if → obeys modus ponens, then P needs to be 'full' (uncountable in a particular way) to sustain PCCP
- But van Fraassen showed that if P *is* full, then it *can* sustain PCCP for a → with a conditional-like logic (which he calls "CE"). And restricting the compounding of sentences with → allows still more logical strength (C2, Stalnaker's preferred logic).
- de Finetti/Stalnaker/Jeffrey: conditionals as *random variables* can sustain a variant of PCCP.

Desire as Belief

- Why care about Desire as Belief?
  - Illuminate the nature of mental states
  - Humeans vs anti-Humeans on motivating rational action
  - Read the equation from right to left (metaethics)
  - Dynamics of desires

- Why believe Desire as Belief?
  - Start with binary desire and binary belief, then generalize

- Why disbelieve Desire as Belief? Sources of suspicion
  - direction of fit
  - 'old lady' example
    - Lewis: *Fine-grained DAB:* $V(A) = \Sigma_i g_i P(A^{\circ i})$ doesn't fare any better

- Four quantified versions

- Lewis' triviality results, in two installments (roughly ten years apart), refute the *Fixed* $^\circ$ version, and cast serious doubt on the *Fixed* $^\circ$ *for rational agents* version, but leave the *Indexical* $^\circ$ versions unscathed.

- Fighting back: fallback positions
  - Shrinking the domain of propositions
  - Approximate equality, proportionality, correlation
  - Indexical $^\circ$

- Perturbation argument; more trouble for *Fixed* $^\circ$ *for rational agents*, and for these fallbacks

- More fighting back
  - Radical indexicality
  - Price: *Desire as Conditional Belief:* (DACB) $V(A) = P(A^\circ \mid A)$

- The future of the debates?
  - The DAB debate guiding the PCCP debate:
    - *Conditional Probabilities of Conditionals as Conditional Probabilities (CPCCP)*
      $P(A \rightarrow B \mid A) = P(B \mid A)$, if $P(A) > 0$.
  - The PCCP debate guiding the DAB debate:
    - Philosophical reply to Indexical $^\circ$ versions: disagreement, retraction, eavesdropping
    - A wallflower argument against indexical $^\circ$ hypotheses?
    - Analogues of Hall's negative and van Fraassen's positive results?
  - *Fine-grained DACB:* $V(A) = \Sigma_i g_i P(A^{\circ i} \mid A)$. Remind you of anything?!

Probabilities of conditionals as conditional probabilities

A '→' function assigns to each pair of propositions <A, B> a proposition A → B. We may interpret it as the 'conditional' operator.

$$P(A \rightarrow B) = P(B|A)$$

(PCCP) $P(A \rightarrow B) = P(B|A)$ for all A, B in the domain of P, with $P(A) > 0$.

Varying the order of quantifiers:

Fixed →: There is some → such that for all P, (PCCP) holds.

Indexical →: For each P there is some → such that (PCCP) holds.

Varying the domains:

Fixed → for rational agents: There is some → such that for all P that could represent a rational agent's credences, (PCCP) holds.

Indexical → for rational agents: For each P that could represent a rational agent's credences, there is some → such that (PCCP) holds.

Lewis's triviality results and a perturbation argument refute *Fixed →* and cast serious doubt on *Fixed → for rational agents*.

A cardinality argument refutes *Indexical →* and casts serious doubt on *Indexical → for rational agents*.

Desire as Belief

A '°' function assigns to each proposition A a proposition A°. We may interpret it as the 'is good' operator.

$$V(A) = P(A^{\circ})$$

(DAB) $V(A) = P(A^{\circ})$ for all A in the domain of P and of V, with $P(A) > 0$.

Varying the order of quantifiers:

Fixed °: There is some ° such that for all <V, P>, (DAB) holds.

Indexical °: For each <V, P> there is some ° such that (DAB) holds.

Varying the domains:

Fixed ° for rational agents: There is some ° such that for all <V, P> that could represent a rational agent's desires/credences, (DAB) holds.

Indexical ° for rational agents: For each <V, P> that could represent a rational agent's desires/credences, there is some ° such that (DAB) holds.

Lewis's triviality results and a perturbation argument refute *Fixed* ^{o} and cast serious doubt on *Fixed* ^{o} *for rational agents*. I'm not aware of any results against *Indexical* ^{o} or *Indexical* ^{o} *for rational agents*.