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Handling Memory Properties of Smart Materials: a Review on Modeling, Compensation and Control

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Abstract— **The interest in smart alloys as active materials to design new devices is constantly increasing. This brings the necessity to develop robust and accurate models for the underlying complex physical phenomena in order to improve the device performance. Special attention has to be paid to understanding the coupling of mechanical and electromagnetic characteristics in dynamical processes. In particular, rate independent memory effects are commonly seen as a serious drawback in the design of the controller. In the manuscript we aim to present a clear and unified view of the basic points to be faced with, when the analysis and design of control algorithms for smart devices are concerned. So, the focus is pointed out on the basic issues of hysteresis modeling, based on the Preisach operator, and its ability to describe a dissipation process, which opens the way to an affordable and reliable design of control algorithms for smart devices.**

I. INTRODUCTION

In nature it is quite common to observe phenomena where the response lags behind an input field, independently of its rate of change. The most known is the magnetization process of any ferromagnetic material, where Magnetization (**M**) lags behind the applied magnetic field (**H**). The description of magnetization phenomena, at a macroscopic scale of observation (i.e. phenomenological modeling), starts with the seminal papers, [1]-[2] while, in 1935, F. Preisach in [3] proposed the model bearing his name. Only in the '70s this model has been reformulated by Krasnosel'skii, [4], who is considered the founder of the mathematical theory of hysteresis that experienced a great growth in later years, [5]-[7].

More recent contributions focused on Piezoelectric or Magnetostrictive phenomena, [8], [9] refined the available tools for handling these phenomena in the field of control systems and technology and, in particular, the idea of inverse hysteresis operator, [10]-[15]. When observing the behavior of ferromagnetic material, a lag of the measured magnetization M with respect to the applied magnetic field H is evident. The basic mechanism relating input and output fields in hysteresis processes was originally described by Madelung, with a set of rules, bearing its name and listed as follows [1]:

- The shape of a monotone magnetization curve does not depend on the rate of change.
- The local shape of a curve starting from a turning point does not depend on the previous history.
- After second turn the curve returns back to its starting point.
- As soon as the minor loop is closed, the process continues as if no turn had taken place.

These properties allows one to identify the *fingerprints* of hysteresis. This helps to select or define those operators with proper memory properties and therefore able to describe hysteresis phenomena.

The first property, in other words, tells us that this *delay*, or *lag* is in reality completely independent on the *rate* of input variation. Formally, let us define the admissible time transformation function

$$\varphi : t \in [0,\hat{t}] \to \varphi(t) \in [0,\hat{t}], \tag{1}$$

with: $\varphi(0) = 0$ and $\varphi(\hat{t}) = \hat{t}$. The operator $\mathscr{W}$ is defined as *rate independent* if and only if the following condition holds, [6]:

$$\mathscr{W}[x \circ \varphi] = \mathscr{W}[x] \circ \varphi, \tag{2}$$

for any admissible time transformation φ. Here $\circ$ denotes the composition operator. Rate independence is therefore an *intrinsic* property of any operator with hysteresis which implies the impossibility to describe the process by the aid of a trivial dynamical system, where the lag is, conversely, strictly dependent on the input rate. The remaining properties define which local input extrema are stored.

A specific and well known hysteresis operator fulfilling Madelung's rules is the Preisach operator. It shows *rate independence* and fulfills *wiping out* and *congruency* properties. While the former is quite general, the latter, conversely, represents a too restrictive constraint, limiting the applicability of the Preisach operator in hysteresis process modeling. For this reason, an important attempt to relax such a property, has been carried out in [6], where a wider class of operators based on Preisach memory updating rules was proposed.

The tools of hysteresis modeling spread out from the field of magnetics and mechanics and were rapidly adopted in several other fields of physics and engineering. In particular, the availability of new materials, universally referred to as smart, opened the chance to develop new and promising devices. So, new systems exploiting, among others, piezoelectric or magnetostrictive materials were proposed, even if the presence of hysteresis limited their performances. For this reason, the need to model and *compensate* memory effects was a mandatory task for the design of a reliable and effective

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control of such *smart devices*.

When a smart material is of concern, in order to provide an adequate modeling of its behavior, the following preliminary assumptions can be made:

1) The magnetization/polarization, z, drives the deformation (ε).
2) The field x drives z through a hysteresis process which evidences the dissipative phenomena (ferroelectric or ferromagnetic) taking place in the material;
3) Deformation at a first approximation is linked to z with no memory, i.e.
$$\varepsilon = f(z);$$
4) The link strain/field shows therefore hysteresis:
$$\varepsilon = \mathscr{W}(x).$$

Such a relationship allowed to employ classical hysteresis models to describe the behavior of these materials. These models were able to naturally describe dissipation and guaranteed a 'thermodynamic' compatibility. Such a modeling strategy perfectly fitted the needs of smart actuators control in *quasi-static* working conditions, where the stress experienced by the material could be considered as constant.

For faster dynamics, i.e. high input rates, the stress could not be considered anymore as a constant and required to be considered as a further independent variable. In this case, therefore, a new structure of the model should be considered,

$$\begin{aligned} y_1 &= \mathscr{F}_1(x_1, x_2), \\ y_2 &= \mathscr{F}_2(x_1, x_2), \end{aligned} \tag{3}$$

and the *thermodynamic* compatibility cannot be trivially assumed. In other words a specific thermodynamic constraint is needed. To better illustrate this specific point, let us choose x_1 and x_2 as *state variables*, while y_1 and y_2 are *state functions*. It is implicit that their physical meaning is related to the specific physical process. So, when a magnetostrictive process is of concern, x_1 and x_2 can be interpreted as the *magnetic field* and applied *stress*, while y_1 and y_2 represent *magnetization* and *strain*, respectively. Similar interpretation could be provided, for example, when a piezo-electric material should be modeled. When hysteresis is taken into account, the *memory state* will constitute an additional state variable. Consider a process $(x_1(t), x_2(t))$ in a time interval $t \in [0,T]$. The work $W(t_1,t_2)$ done by the magnetic and mechanical forces in an arbitrary interval $[t_1,t_2] \subset [0,T]$ is given by the integral

$$W(t_1,t_2) = \int_{t_1}^{t_2} \sum_k \dot{y}_k(t) x_k(t)\, \mathrm{d}t, \tag{4}$$

where the dot denotes time derivative. The process is said to be *reversible*, if there exists another state function $F = F(x_1,x_2)$ (the *free energy*) such that the work $W(t_1,t_2)$ is completely transformed into the energy increase with no energy losses, that is

$$W(t_1,t_2) = F(x_1(t_2),x_2(t_2)) - F(x_1(t_1),x_2(t_1)). \tag{5}$$

If losses are present, part of the work is dissipated into heat. In isothermal processes, the dissipation

$$D(t_1,t_2) = W(t_1,t_2) - (F(x_1(t_2),x_2(t_2)) - F(x_1(t_1),x_2(t_1))) \tag{6}$$

is nonnegative in agreement with the second principle of thermodynamics. Losses make the process *irreversible*. In differential form, the energy balance relations for irreversible processes reads

$$\sum_k \left(\frac{d}{dt} \mathscr{F}_k(x_1,x_2)(t) x_k(t) \right) \geq \frac{\mathrm{d}}{\mathrm{d}t} F(x_1(t),x_2(t)). \tag{7}$$

A constitutive law, such as (3), is said to be *thermodynamically admissible*, if (7) holds for every process $(x_1(t),x_2(t))$. In what follows a discussion of some operators, able to fulfill the above fundamental constraint will be provided.

II. MODELING OF RATE-INDEPENDENT MEMORY EFFECTS

A. Preisach operator

The original form of the Preisach operator, [3] can be represented as a *linear superposition* of *non-ideal relays* $R_{r,v}$ with parameters $v \in \mathbb{R}$ (interaction field) and $r > 0$ (critical field of coercivity), sketched in Fig. 1. More formally, the relay operator $R_{r,v}$ associates to each continuous input function $u : [0,T] \to \mathbb{R}$ and each initial state $R^0_{r,v} = \pm 1$ the output $R_{r,v}[u](t) : [0,T] \to \{-1,1\}$ according to the following rule: For $t \in [0,T]$, let $A_\pm(t)$ be the sets

$$\begin{aligned} A_+(t) &= \{\tau \in [0,t] : u(\tau) \geq v + r\}, \\ A_-(t) &= \{\tau \in [0,t] : u(\tau) \leq v - r\}. \end{aligned} \tag{8}$$

We then define

$$R_{r,v}[u](t) = \begin{cases} R^0_{r,v} & \text{if } A_+(t) = A_-(t) = \emptyset, \\ 1 & \text{if } \max A_+(t) > \max A_-(t), \\ -1 & \text{if } \max A_+(t) < \max A_-(t), \end{cases} \tag{9}$$

with the convention $\max \emptyset = -\infty$. The half-plane with coordinates (r,v), $r > 0$, $v \in \mathbb{R}$, is called the Preisach half-plane. At each time t, the Preisach half-plane is divided into two

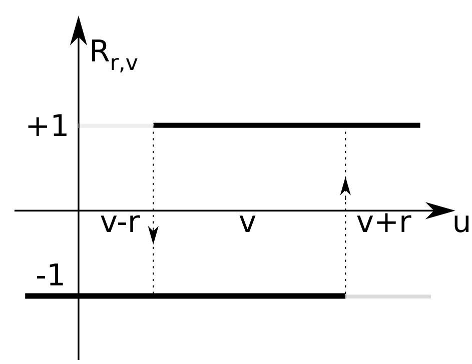

Fig. 1. A diagram of the relay with thresholds $v+r$, $v-r$.

regions corresponding to the values ± 1 of the relays at time t. The Preisach operator is then defined as follows:

$$\mathscr{P}[u](t) = \int_0^\infty \int_{-\infty}^\infty R_{r,v}[u](t)\, \psi(r,v)\, \mathrm{d}v\, \mathrm{d}r, \tag{10}$$

where $\psi(r,v) \geq 0$ is the density of relay distribution in the Preisach half-plane. The special class of *Prandtl-Ishlinskii operators* is obtained if $\psi = \psi(r)$ is independent of v.

Formula (10) can be re-arranged by introducing a one parameter family of operators known as *play operator*, formally defined as follows. Given a function $u : [0,T] \to \mathbb{R}$ which is monotone in each interval $[t_{j-1}, t_j]$ of a partition $0 = t_0 < t_1 < \cdots < t_m = T$, the play operator with parameter $r > 0$ is recursively defined by the formula

$$\mathfrak{p}_r[x_r^0,u](t) = \max\{u(t) - r, \min\{u(t) + r, \mathfrak{p}_r[x_r^0,u](t_{j-1})\}\} \tag{11}$$

for $t \in (t_{j-1}, t_j]$, $j = 1, \ldots, m$, and with initial condition $\mathfrak{p}_r[x_r^0,u](t_0) = u(0) - x_r^0$, where $x_r^0 \in [r,r]$ is given. The definition can be extended to the space $C[0,T]$ of continuous functions $u : [0,T] \to \mathbb{R}$, see [4]. We restrict the initial

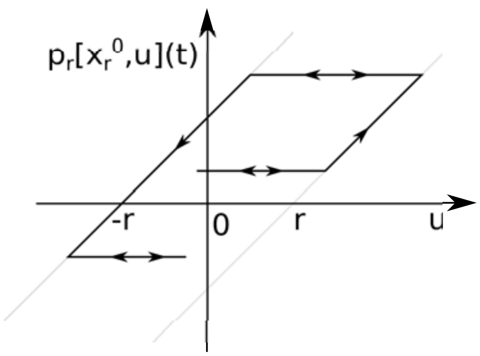

Fig. 2. A diagram of the play operator $\xi_r = \mathfrak{p}_r[x_r^0,u](t)$ for a fixed threshold r.

conditions for the play and for the relay by choosing the unperturbed initial state

$$x_r^0 = \max\{-r, \min\{r, u(0)\}\}\,,\; R_{r,v}^0 = \begin{cases} 1 & \text{if} \quad v \leq u(0) - x_r^0, \\ -1 & \text{if} \quad v > u(0) - x_r^0. \end{cases} \tag{12}$$

In [16], it was shown that for every t, the relay (9) is related to the play by the following formula:

$$R_{r,v}[u](t) = \begin{cases} 1 & \text{if} \quad v < \mathfrak{p}_r[x_r^0,u](t), \\ -1 & \text{if} \quad v > \mathfrak{p}_r[x_r^0,u](t). \end{cases} \tag{13}$$

Formula (10) then can be written as

$$\mathscr{P}[u](t) = \int_0^\infty \left(\int_{-\infty}^{\mathfrak{p}_r[x_r^0,u](t)} - \int_{\mathfrak{p}_r[x_r^0,u](t)}^{\infty} \right) \psi(r,v)\,\mathrm{d}v\,\mathrm{d}r, \tag{14}$$

or, alternatively, putting

$$g(r,w) = \left(\int_{-\infty}^{w} - \int_{w}^{\infty} \right) \psi(r,v)\,\mathrm{d}v\,, \tag{15}$$

as a single integral over the memory variable r,

$$\mathscr{P}[u](t) = \int_0^\infty g(r, \mathfrak{p}_r[x_r^0,u](t))\,\mathrm{d}r\,. \tag{16}$$

The advantage of this representation is that the Preisach energy balance can be stated in explicit form by using the energy balance for each individual play

$$u(t)\frac{\mathrm{d}}{\mathrm{d}t}\mathfrak{p}_r[x_r^0,u](t) = \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(\mathfrak{p}_r[x_r^0,u](t))^2 + r\left| v\frac{\mathrm{d}}{\mathrm{d}t}\mathfrak{p}_r[x_r^0,u](t)\right|. \tag{17}$$

The thermodynamic interpretation of (17) is clear: the left hand side is the energy supplied to the system, which is partly used for the potential increase (left term on the right hand side), and the rest is dissipated. Formula (17) suggests to choose the Preisach energy potential as

$$\mathscr{V}[u](t) = \int_0^\infty G(r, \mathfrak{p}_r[x_r^0,u](t))\,\mathrm{d}r\,, \tag{18}$$

with

$$G(r,w) = \left(\int_{-\infty}^{w} - \int_{w}^{\infty} \right) v\psi(r,v)\,\mathrm{d}v\,, \tag{19}$$

and the dissipation operator as

$$\mathscr{V}[u](t) = \int_0^\infty r g(r, \mathfrak{p}_r[x_r^0,u](t))\,\mathrm{d}r\,. \tag{20}$$

The energy balance for the Preisach operator that can be written in the form

$$u(t)\frac{\mathrm{d}}{\mathrm{d}t}\mathscr{P}[u](t) = \frac{\mathrm{d}}{\mathrm{d}t}\mathscr{V}[u](t) + \left| v\frac{\mathrm{d}}{\mathrm{d}t}\mathscr{D}[u](t)\right|, \tag{21}$$

see [16]. In the terminology of [6], $\mathscr{P}[u](t)$ is a *counter-clockwise* operator.

B. Prandtl-Ishlinskii operator

Prandtl-Ishlinskii operators represent an important class of Preisach operators that can be characterized in terms of the Preisach density $\psi(r,v)$ by the condition that $\psi = \psi(r)$ is a function of r only and does not depend on v. Its popularity in the control community is due to the fact that the Prandtl-Ishlinskii operators are based on the superposition of plays or *backlash* operators, widely applied in this field.

C. Example: magnetoelastic coupling

The operator $\mathscr{P}[u](t)$ is able to 'mimic' irreversible and dissipative phenomena such as the typical magnetization process. In that case, as well-known, the operator describes in the *M-H* plane hysteresis loops, where the area represents the total loss. This important interpretation can be drawn whenever x_k and y_k can be assumed as *conjugate variables*, i.e. their product is the *work* done in the process involving those variables. The idea of passivity for the Preisach operator has been exploited for stability analysis in [17], [18]. However, when a model that describes the full coupling between *all* involved variables is of concern, a completely new approach is necessary to guarantee a thermodynamic consistency of the operator, as sketched in the previous section. In order to describe a specific case of physical interest a full coupled magneto elastic process is discussed. To this aim, the constitutive relations of a magnetostrictive alloy:

$$\begin{aligned} \varepsilon &= \varepsilon(\sigma, H), \\ B &= B(\sigma, H) \end{aligned} \tag{22}$$

are considered between the state variables σ (stress) and H (magnetic field) on the one hand, and ε (strain) and B

(magnetic induction) on the other hand. It is well known (and easily be checked by the chain rule) that if no hysteresis is present, the constitutive law (22) is thermodynamically well posed if the *potentiality condition*

$$\frac{\partial \varepsilon}{\partial H} = \frac{\partial B}{\partial \sigma} \tag{23}$$

holds. This is equivalent to the existence of an energy potential. If hysteresis is taken into account, the chain rule is not available any more. Instead, we assume that all hysteresis phenomena in the system (22) are due to a single Preisach operator acting on one aggregated quantity $u = u(\sigma, H)$. A more detailed discussion on the subject will be carried out in the forthcoming paper [19]. Here, we only mention that a particularly good agreement with experiments is obtained if the hysteresis counterpart of (22) is considered in the form

$$\begin{aligned} \varepsilon &= \tfrac{1}{E}\sigma - f'(\sigma)\mathscr{V}[u], \\ B &= \mu H + \mathscr{P}[u], \\ u &= H/f(\sigma), \end{aligned} \tag{24}$$

where E is the elasticity modulus, μ is the permeability, $\mathscr{P}$ is a Preisach operator, $\mathscr{V}$ is its potential, and f is a positive self-similarity function. If we choose the Helmholtz free energy F as

$$F = F(\sigma, H) = \frac{1}{2E}\sigma^2 + \frac{\mu}{2}H^2 + (f(\sigma) - \sigma f'(\sigma))\mathscr{V}[u],$$

we easily check, using (21), that the energy dissipation

$$D = \dot{\varepsilon}\sigma + \dot{B}H - \dot{F} = f(\sigma)(u\dot{\mathscr{P}}[u] - \dot{\mathscr{V}}[u])$$

is nonnegative in agreement with the second principle of thermodynamics.

III. COMPENSATION OF RATE-INDEPENDENT MEMORY EFFECTS

The inversion of a hysteresis operator requires some care due to the existence of an internal state. To this aim the inverse can be defined as follows:

Definition: A hysteresis operator $\mathscr{P}$ with initial state p, is called a *compensator* (or inverse) of the operator $\mathscr{Q}$, with initial state q if, for any state p, there is a state q, such that $\mathscr{Q}_q \circ \mathscr{P}_p x(t) = \mathscr{P}_p \circ \mathscr{Q}_q x(t) = x(t)$, for every input function x(t).

The availability of the inverse is strictly linked to the mathematical properties of the operator and should be specified case by case. Referring to the Preisach operator, Brokate in [20] specified its invertibility conditions, while in [21] the invertibility conditions of a Prandtl-Ishilinskii operator were drawn. In the latter case, assuming the Prandtl-Ishilinskii operator:

$$\pi[u](t) = \int_0^\infty \mu(r)\mathfrak{p}_r[x_r^0, u](t))\,\mathrm{d}r, \tag{25}$$

with $\mu(r) > 0$, the inverse, can be put in the form

$$\pi^{-1}[u](t) = \int_0^\infty \nu(r)\left(u - \mathfrak{p}_r[x_r^0, u]\right)(t)\,\mathrm{d}r, \tag{26}$$

with a suitable choice of μ and ν functions, [21]. The latter represents a further reason of the popularity of these operators, where the availability of compensator in closed form is advisable. However, the above operator is unable to describe saturation and this limits its applicability to Piezo- actuators or magnetostrictives for low driving fields. This pushed to define a restricted class of operators, with Preisach memory updating rules, [6] admitting the inverse and showing saturation phenomena, [10], [22]:

$$y = G \circ \pi \circ f(x), \tag{27}$$

with suitable functions G and f. General Preisach operators admit inverse operators, too, but no decomposition formula is known, and the inverse has to be constructed numerically. A fast algorithm, updating the Preisach state by the sequence of output field has been proposed in [23] where it is also shown that its computational weight is equivalent to the direct Preisach algorithm.

The general approach to smart materials, as mentioned in section II, is taking into account state variables x_1 and x_2 and state functions y_1 and y_2 simultaneously. This implies a modification of compensation algorithms described so far. Specifically, the approach is based on the introduction of a modified 'direct' operator where x_1 and x_2 are assumed as independent variables, [25]:

$$y = \mathscr{W} \circ \zeta(x_1, x_2) + q(x_2), \tag{28}$$

where $y = \varepsilon$, is the measured strain, $x_2 = \sigma$, is the applied stress and x_1 is electric or magnetic field depending on weather a Piezo-electric or magnetostrictive material is considered. Finally, $q(x_2)$ is a pure elastic response to be identified with almost trivial mechanical measurements at zero field. The latter assumption is not strictly necessary, but convenient during the identification procedure. There, by exploiting the invertibility of the Preisach operator $\mathscr{W}$, the model can be re-arranged as:

$$z \equiv \mathscr{W}^{-1}[y - q(x_2)] = \zeta(x_1, x_2). \tag{29}$$

Finally, by exploiting the monotonicity of magnetostrictive or piezo-electric characteristics,which reflects on the properties of the function ζ, it can be shown that the mapping

$$\Phi : x_1 \in X \subseteq \mathbb{R} \longrightarrow (x_2, z) \in S, \tag{30}$$

admits the inverse and then the input field x_1 can be determined according to the following equation:

$$x_1 = \Phi^{-1}(\mathscr{W}^{-1}[y - q(x_2)], x_2). \tag{31}$$

The issue is discussed in detail in [26], [27].

IV. MODEL-BASED CONTROL STRATEGIES IN SMART DEVICES

A. *Controller design through a compensator algorithm - one variable case*

If a compensator algorithm is employed within a control system, either in open or closed loop, then, within the identification limit, the chain of the compensator and the smart actuator acts as a linear system. Therefore, even a classical Proportional-Integral (PI) controller can be employed

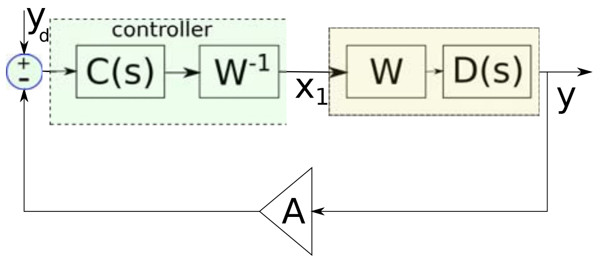

Fig. 3. Control scheme with a compensator.

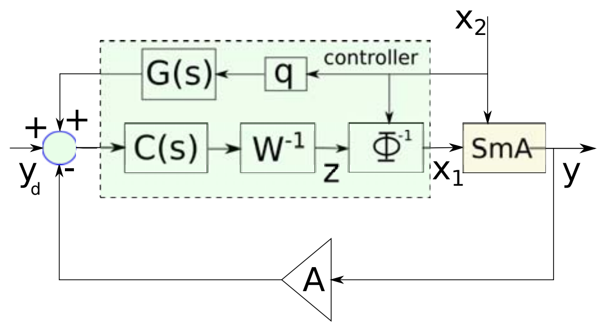

Fig. 4. Control scheme with a two-variable compensator.

in a closed loop. This approach has been widely used in actuation tasks where smart materials like Piezo, SMAs or magnetostrictives were employed, [11], [12], [13], [24], [28].

The Fig.3 shows one example of application of the compensator algorithm in a control scheme. There, the blocks W and W^{-1} represent, respectively, the rate-independent hysteresis of the smart actuator and its compensator, while $D(s)$ represents the actuator dynamic behavior that can be taken into account by properly designing the controller $C(s)$. This control scheme is used in quasi-static micro-positioning applications where the mechanical stress over the smart actuator is much lower than its internal pre-stress. Applications and results of this technique can be found in [27].

B. Controller design through a compensator algorithm - two variable case

There are applications where the mechanical stress cannot be considered constant anymore. For example, if a plant must be positioned with a settling time comparable with the internal time constants of the actuator. In this case, it can happen that the stress variations over the smart material are not negligible anymore and have to be considered into the modeling and the control strategy.

The control scheme in Fig.4 shows a first attempt to consider such a case. The x_2 variable is the mechanical stress acting on the smart actuator and it is measured by a load cell sensor (represented by the block A). The variable x_1 is the magnetic field supplying the device which returns a displacement represented by y. The block q implements the non-linear function of eq. (28). The compensator is composed by the series of two blocks, implementing the formula (31). The block $G(s)$ can take into account, at first, the transfer function of the stress sensor but, it should include another controller because the stress represents another feedback loop within the control scheme. This controller can be simply an integrator, limiting the bandwidth of the stress variable in the feedback.

The control schemes presented above can be used in real-time applications and implemented in micro-controllers. In Fig. 5 shows the performances of the feedback system, sketched in Fig. 4, in tracking a triangular waveform, with period of $5\,s$, and with variable stress. Here the relative error lies below 3% and so evidencing the effectiveness of the approach.

V. Conclusions and Perspectives

Smart materials employed in actuation or sensing devices need further effort in their modeling in order to exploit all the possibilities and performances related to their ‘smartness’. The development of more accurate compensation and control techniques, such as the 2DoF control, [29], also in connection to the new modeling approach proposed so far, is still an open problem and requires further effort. The evolution of devices with embedded control, able to exploit all potentialities of smart materials (most of them of recent invention), is strongly linked not only to the materials themselves or to the computational power of the employed hardware, but chiefly to the ability to design new control systems able to fully take into account the actual characteristic of the material. Such task cannot be successful without a multidisciplinary effort involving researchers of different areas.

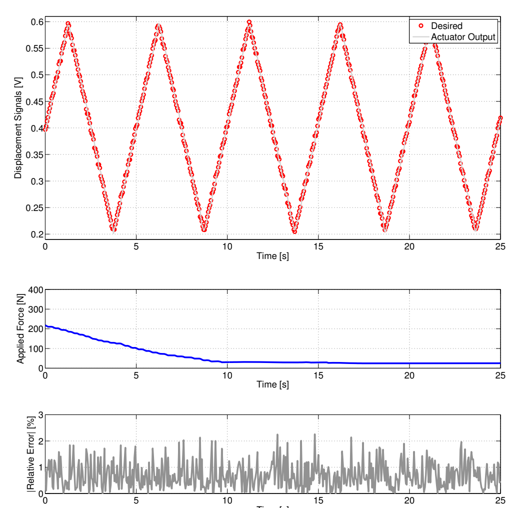

Fig. 5. Tracking performances of a PI control system with 2 variables compensation. The desired output has a period $T = 5$s

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