

# **Povrchové plazmony v integrované fotonice**

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## **Povrchové plazmony v integrované fotonice**

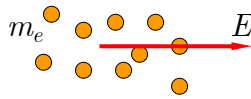
Typické aplikace:

1. vlnovodné polarizátory
2. SPR senzory
3. povrchové plazmony pro přenos informace („plazmonika“)

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## Permitivita kovu (Drudeho model)

„volný“ elektronový plyn v elektromagnetickém poli



Pohybová rovnice:  $-m_e \ddot{x} - m_e \gamma \dot{x} - eE = 0$

Pro harmonické pole  $E = E_0 \exp(-i\omega t)$

získáme ustálené řešení:  $x_0 = \frac{-eE_0}{m_e \omega^2 + im_e \gamma \omega}$

Polarizace:  $P_0 = -n_e e x_0 = \frac{-e^2 n_e}{m_e \omega^2 + im_e \gamma \omega} E_0 = \epsilon_0 \chi E_0$

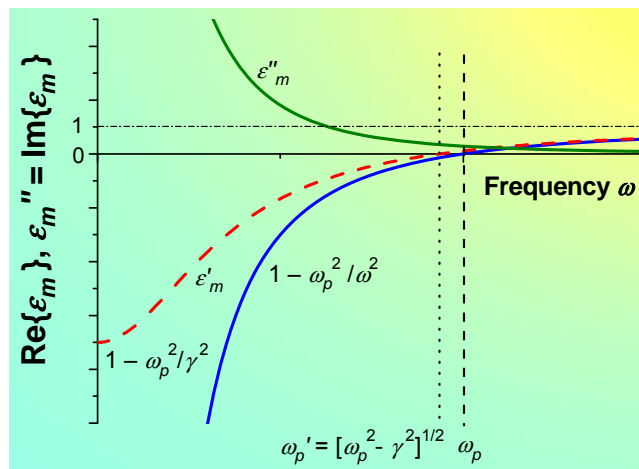
Permitivita:  $\epsilon_m = 1 + \chi = 1 - \frac{e^2 n_e / (m_e \epsilon_0)}{\omega^2 + i\gamma \omega} = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma \omega}$

Plazmová frekvence  $\omega_p = e \sqrt{\frac{n_e}{m_e \epsilon_0}}$

life

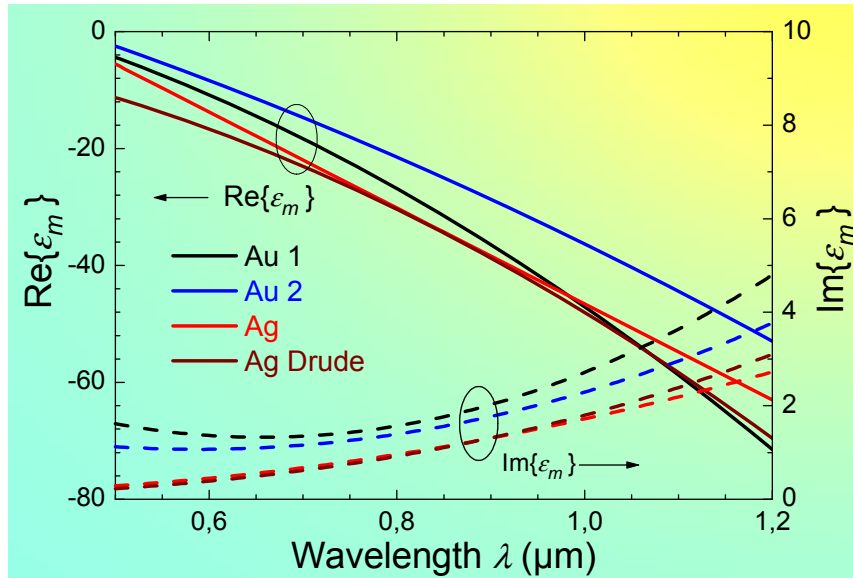
## Disperze kovu (Drudeho model)

$$\epsilon_m = \epsilon'_m + i\epsilon''_m = 1 - \frac{\omega_p^2}{\omega^2 + \gamma^2} + i \frac{\omega_p^2 \gamma}{\omega(\omega^2 + \gamma^2)}$$



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## Disperze kovu (experimentální data)



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### Povrchová plazmová vlna

(povrchový plazmon-polariton, povrchový plazmon)

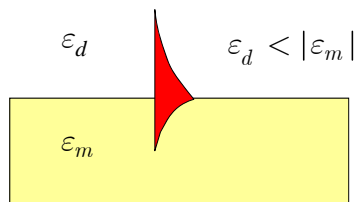
Vzájemně vázaná elektromagnetická a nábojová povrchová vlna  
localizovaná na rozhraní mezi dielektrikem a kovem

Pól  $R(N^2) \Rightarrow N^2$  povrchové vlny

TE:  $\sqrt{\varepsilon_d - N^2} + \sqrt{\varepsilon_m - N^2} = 0$  neexistuje řešení

TM:  $\varepsilon_m \sqrt{\varepsilon_d - N^2} + \varepsilon_d \sqrt{\varepsilon_m - N^2} = 0$  **povrchový plazmon**

$$N_{SP} = \sqrt{\frac{\varepsilon_d \varepsilon_m}{\varepsilon_d + \varepsilon_m}}$$



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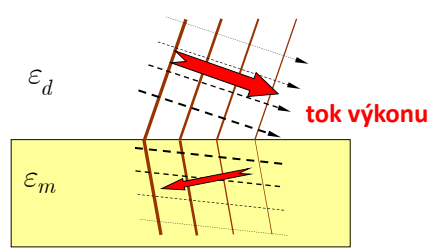
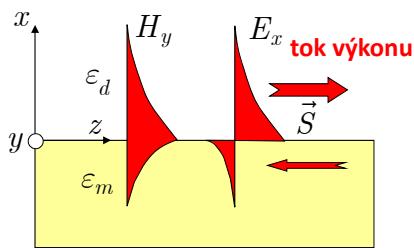
## Rozložení pole povrchového plazmonu

$$H_y(x, z) = H_0 e^{ik_0 N z} \begin{cases} e^{-k_0 \sqrt{N^2 - \epsilon_d} x}, & x > 0 \\ e^{k_0 \sqrt{N^2 - \epsilon_m} x}, & x < 0 \end{cases} \quad \begin{cases} 1/k_0 \sqrt{N^2 - \epsilon_d} = 265 \text{ nm} \\ 1/k_0 \sqrt{N^2 - \epsilon_m} = 26 \text{ nm} \end{cases}$$

$$E_x(x, z) = Z_0 N H_0 e^{ik_0 N z} \begin{cases} \frac{1}{\epsilon_d} e^{-k_0 \sqrt{N^2 - \epsilon_d} x}, & x > 0 \\ \frac{1}{\epsilon_m} e^{k_0 \sqrt{N^2 - \epsilon_m} x}, & x < 0 \end{cases}$$

Pro  $\gamma = 0$ ,  $\text{Im}\{N\} = 0$

Pro  $\gamma > 0$ ,  $\text{Im}\{N\} > 0$



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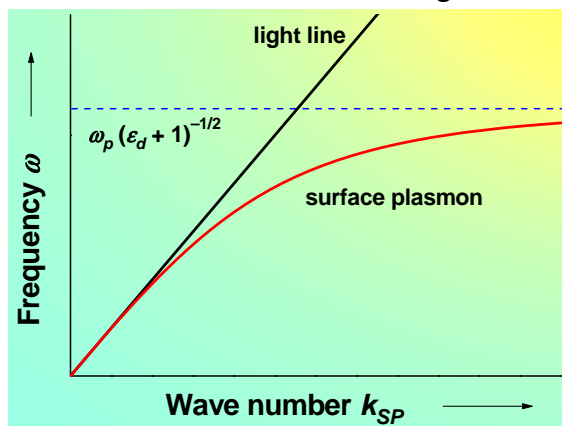
## Disperzní vlastnosti povrchového plazmonu

Pro  $\gamma = 0$ ,  $\omega < \omega_p / \sqrt{\epsilon_d + 1}$

$$k_{SP} = \frac{\omega}{c} N_{SP} = \frac{\omega n_d}{c} \sqrt{\frac{\omega_p^2 - \omega^2}{\omega_p^2 - \omega^2 (\epsilon_d + 1)}}$$

"light line"

faktor  $< 1$



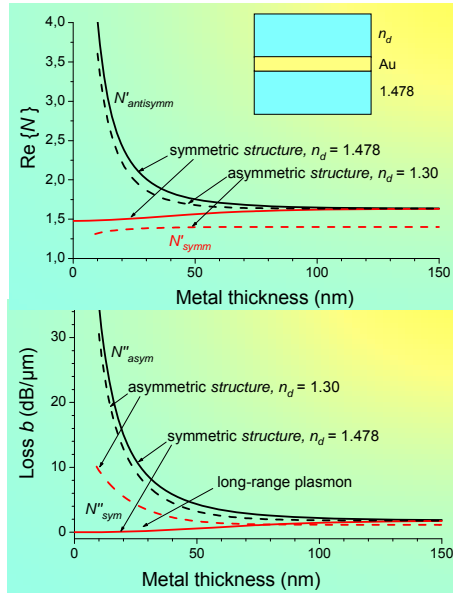
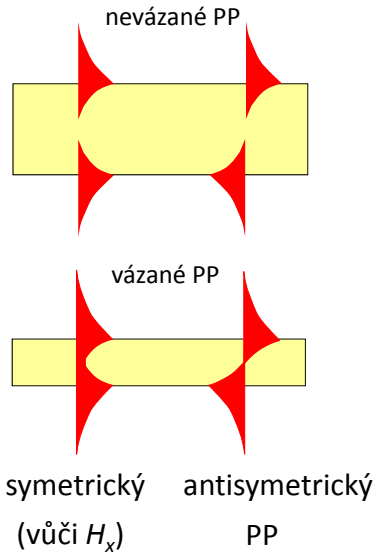
$\text{Re}\{N_{SP}\} > n_d \Rightarrow$

PP je **pomalá vlna**

nemůže být excitována  
zářením z dielektrika

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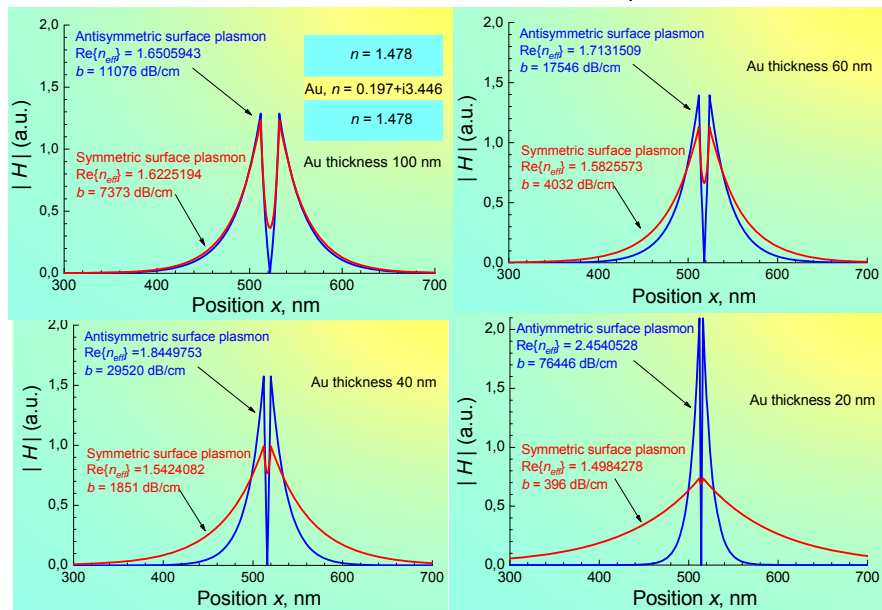
## Povrchové plazmony na kovové vrstvě



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## Rozložení polí PP na kovových vrstvách

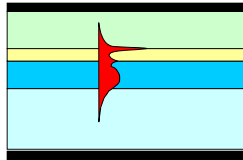
Závislost na tloušťce kovové vrstvy



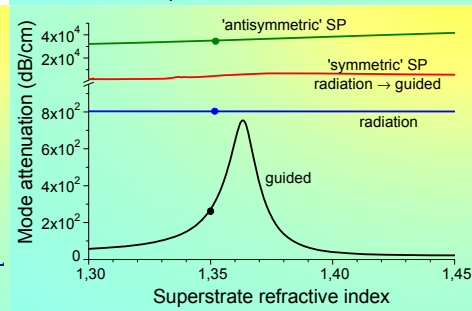
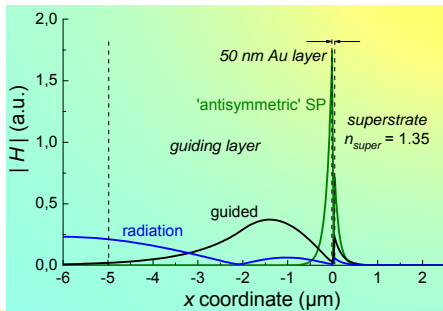
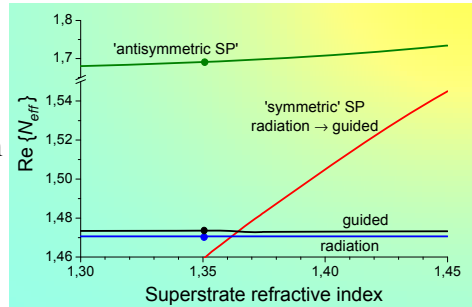
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## Vidy planárních vlnodů s kovovou vrstvou

1.



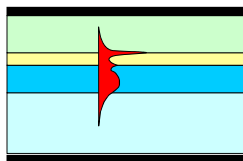
$$d_{Au} = 50 \text{ nm}$$



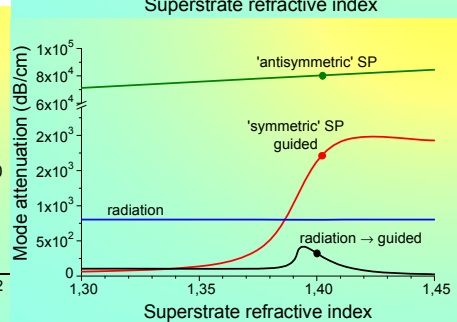
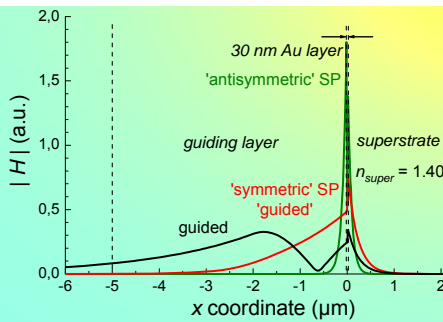
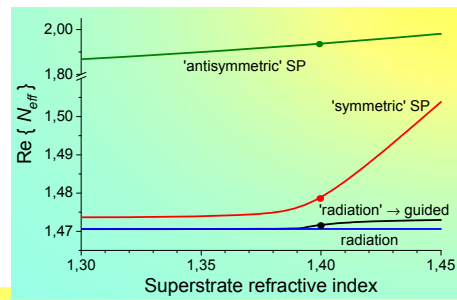
life

## Vidy planárních vlnodů s kovovou vrstvou

2.

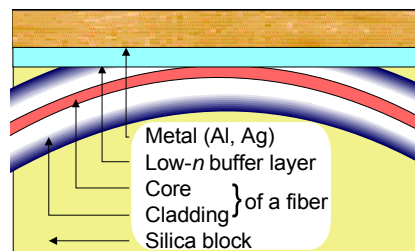
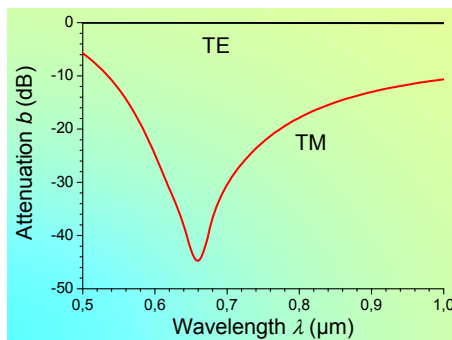
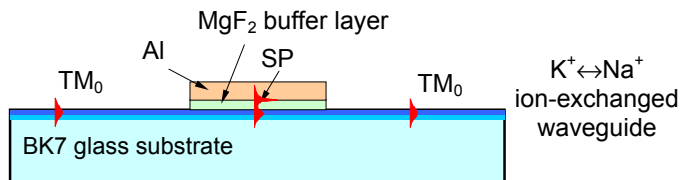


$$d_{Au} = 30 \text{ nm}$$



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## Vlnodný polarizátor založený na rezonanční excitaci PP

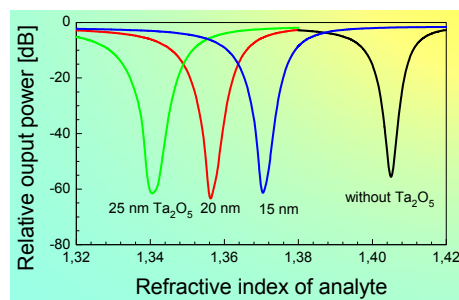
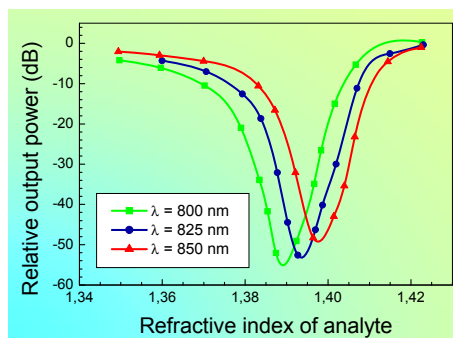


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## Průchod optického záření senzorem s PP

- závislost na indexu lomu analytu (zkoumaného prostředí)

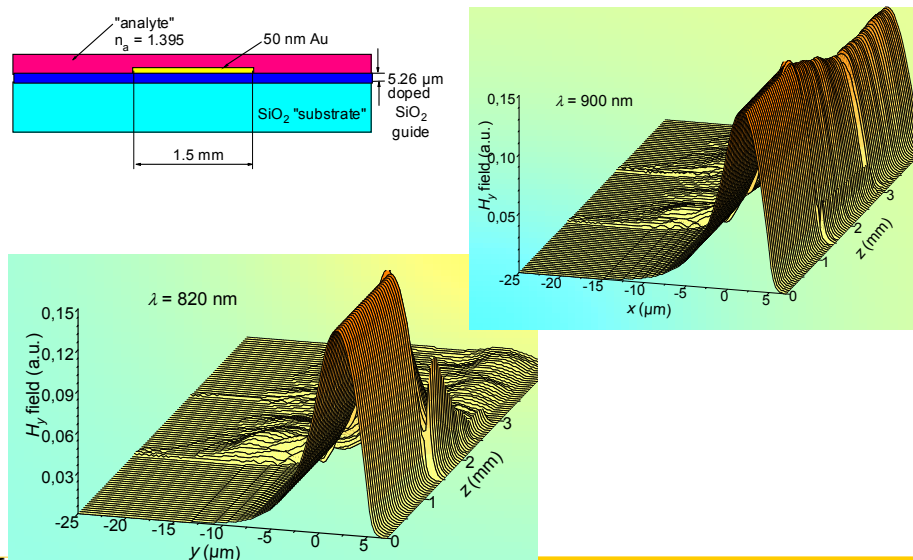
2D (planární) model



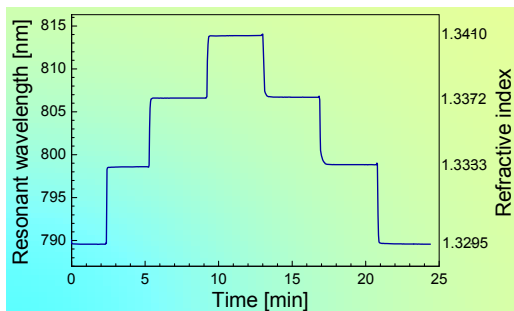
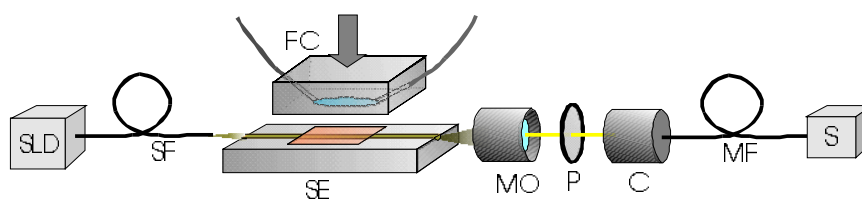
měření laditelným  
Ti:safírovým laserem

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## Rozložení optického záření ve vlnovodu s úsekem, na němž se může šířit PP



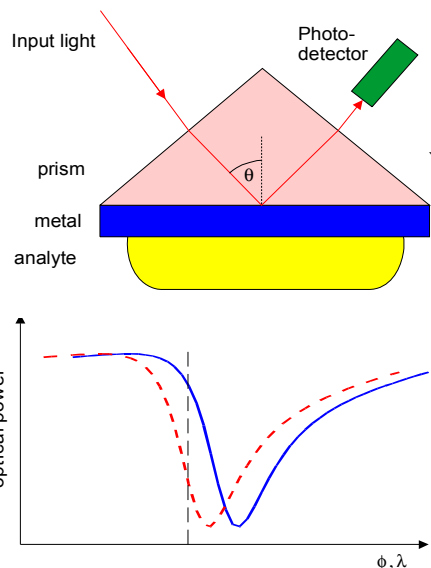
## Experimentální uspořádání integrovaně-optického senzoru s PP



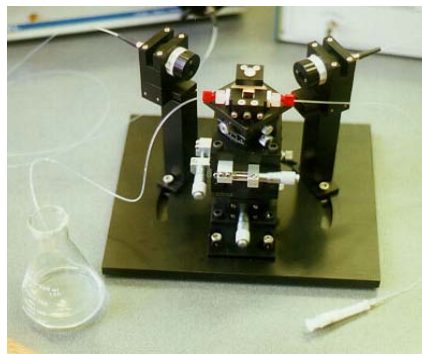
Rozlišení změn indexu lomu menších než  $1.2 \times 10^{-6}$



## Objemové senzory s PP

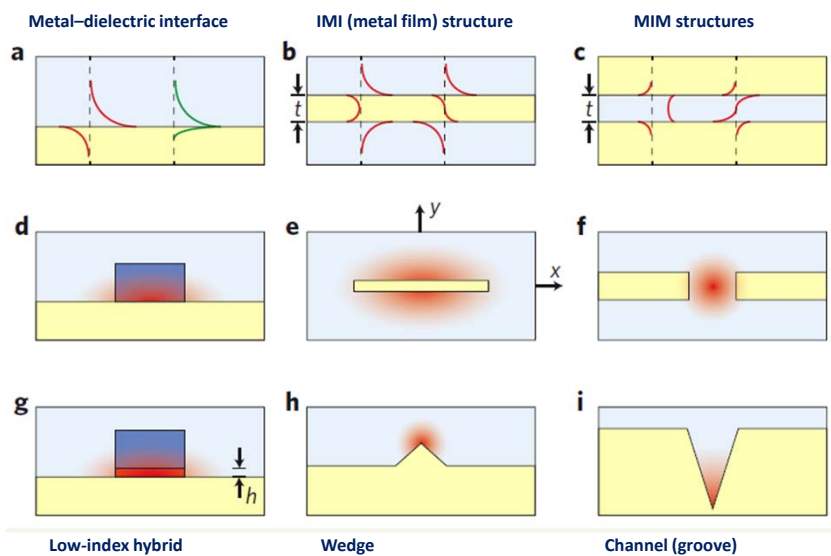


Rozlišení změn indexu lomu  
menších než  $5 \times 10^{-7}$



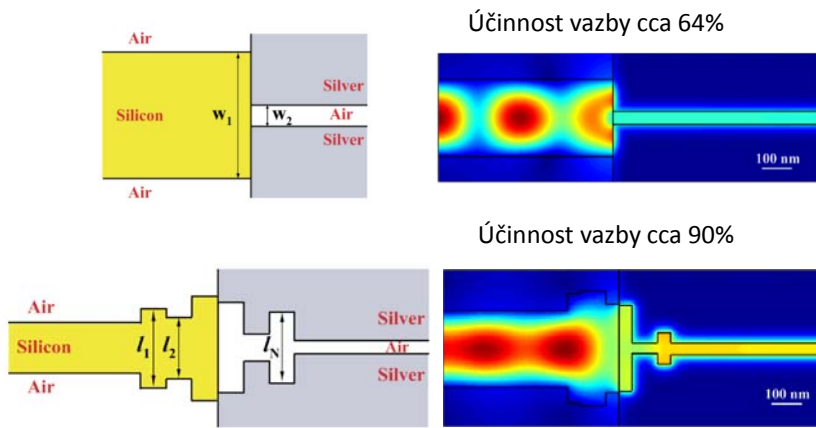
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## PLAZMONICKÉ VLNOVODY

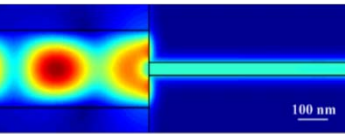


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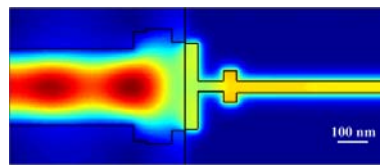
## Přechod mezi vlnovodem SOI a plazmonovým vlnovodem



Účinnost vazby cca 64%



Účinnost vazby cca 90%



G. Veronis, S. Fan, OWTNM 2006, p. 12  
(Stanford university)

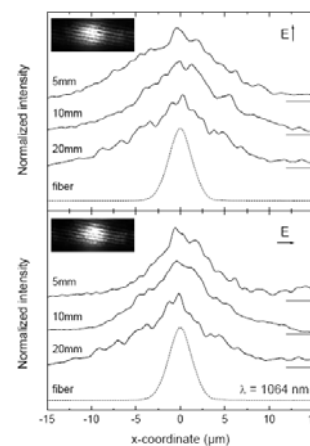
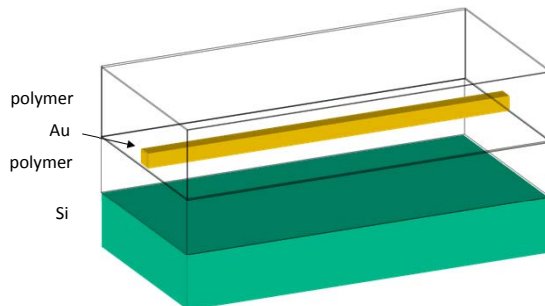
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## „Zlatý nanodrát“ jako vlnovod pro povrchové plazmony

(T. Rosenzweig, ECIO 2007)

Průřez „nanodrátu“  
100×100 nm,

útlum 4.3 dB/cm



Rozložení blízkého pole  
„plazmonů dalekého dosahu“

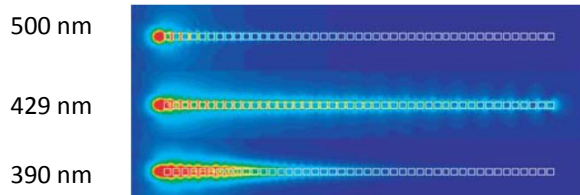
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## „Vlnovod“ tvořený řadou kovových nanočástic – vázané lokalizované plazmony

(S.A.Maier, ECIO 2007)

„Řetízek“ Au krychliček o straně 45 nm vzdálených od sebe 20 nm

Excitační vlnová délka



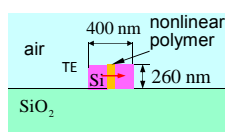
Na vlnové délce 429 je „překlenutelná vzdálenost“  
pro pokles výkonu na  $1/e^2$  celkem 2,2 mm  
(útlum cca 40 dB/cm)

life

2

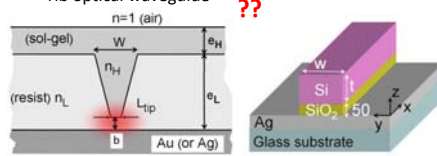
## Nové typy plazmonických vlnovodů

SOI “slot waveguide”



C. Koos & al., *Nat. Photonics* **3**(4), 16–219 (2009)

PIROW – plasmonic inverted  
rib optical waveguide

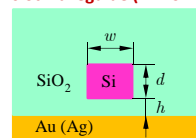


H. Benisty and M. Besbes,  
*J. Appl. Phys.* **108**(6), 063108 (2010).

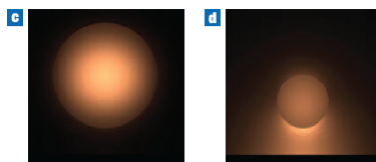
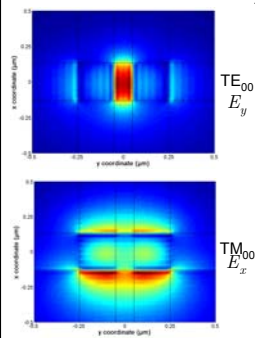
??

H.-S. Chu & al., *J. Opt. Soc. Am. B* **28**(12), 2895 (2011) (others, too)

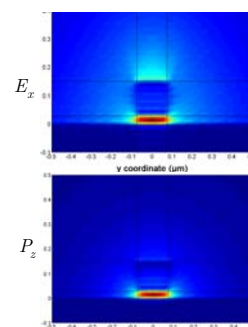
Hybrid dielectric-plasmonic  
slot waveguide (HDPSW)



R. F. Oulton & al., *New J. Phys.* **10**, 105018 (2008)



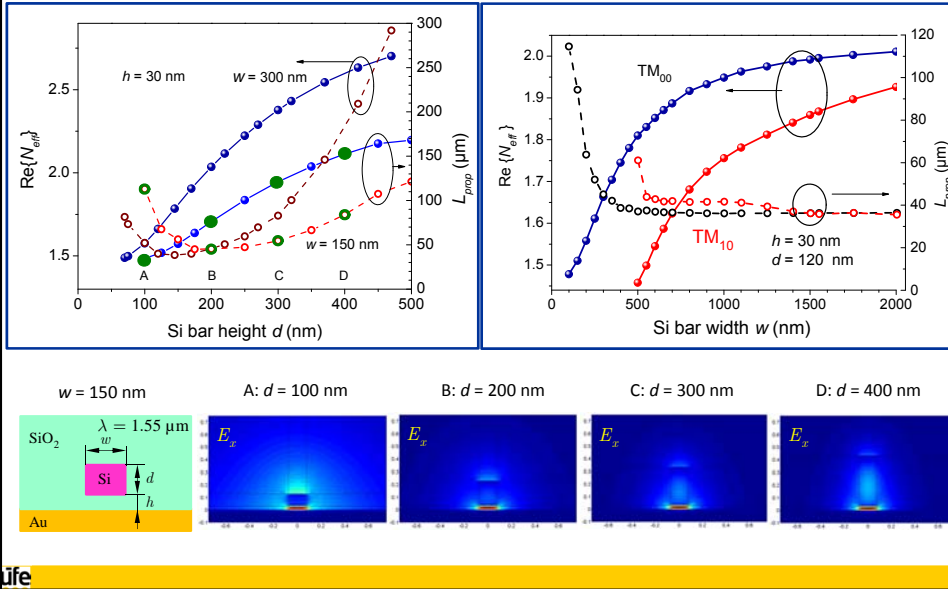
R. F. Oulton & al., *Nat. Photonics* **2**, 496 (2008);



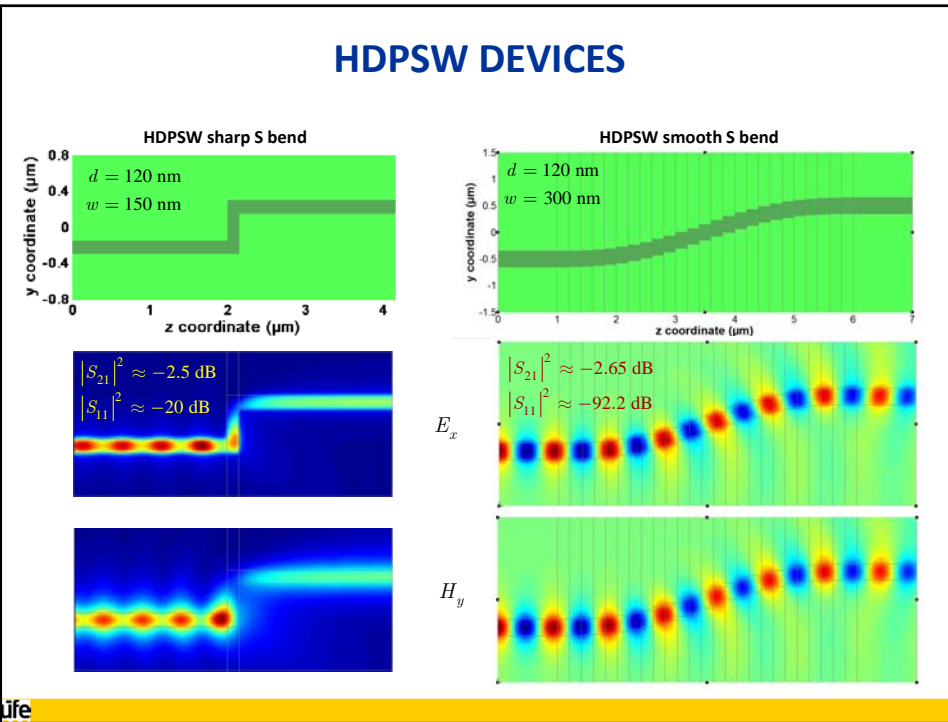
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# Hybrid dielectric-plasmonic slot waveguide

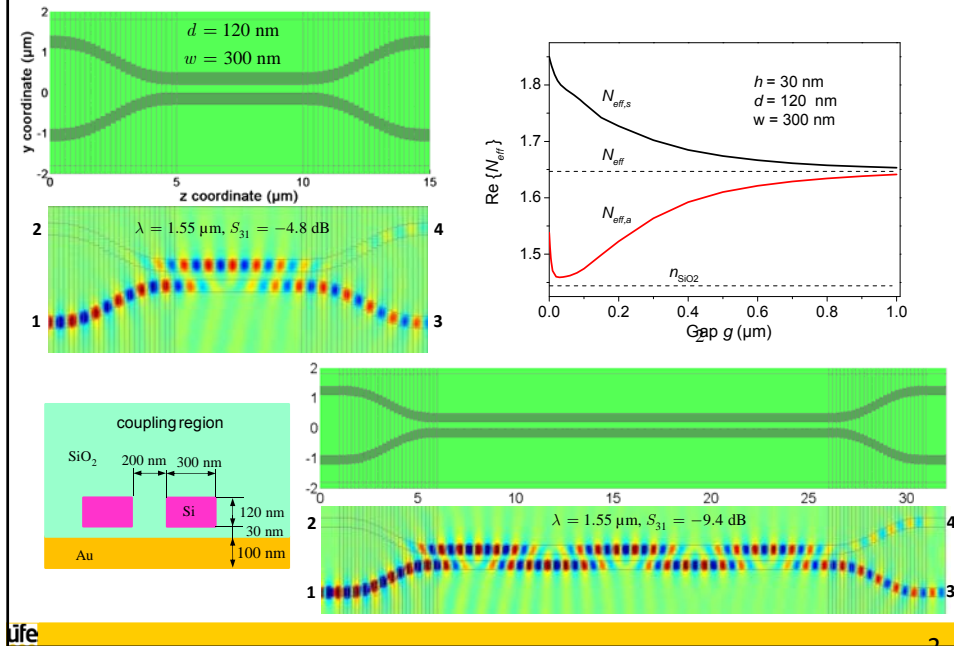
Influence of basic geometric parameters



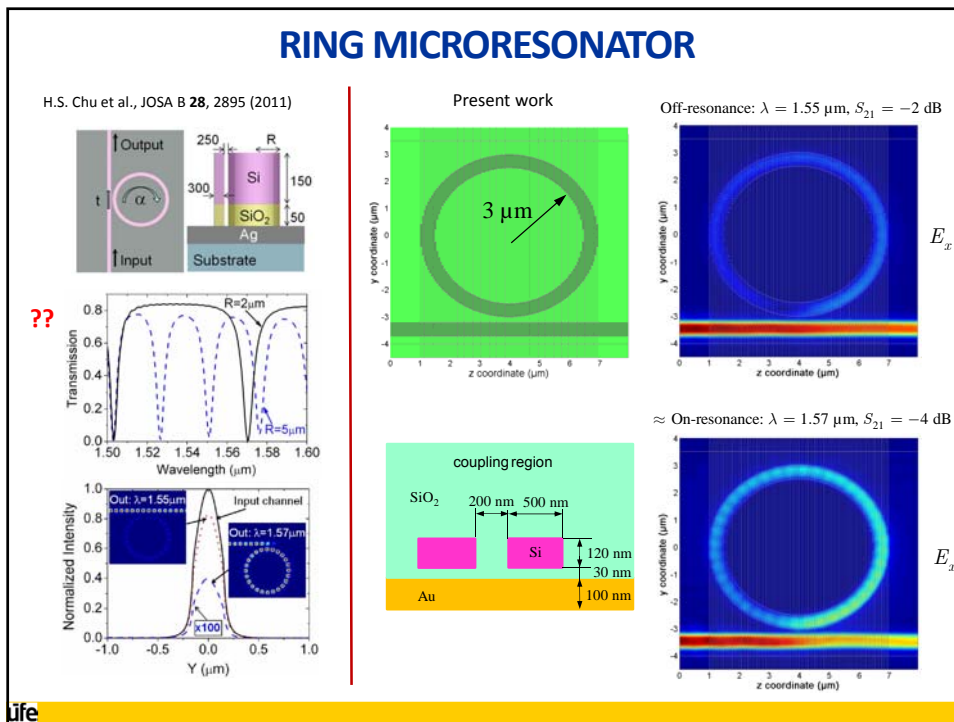
# HDPSW DEVICES



## DIRECTIONAL COUPLER

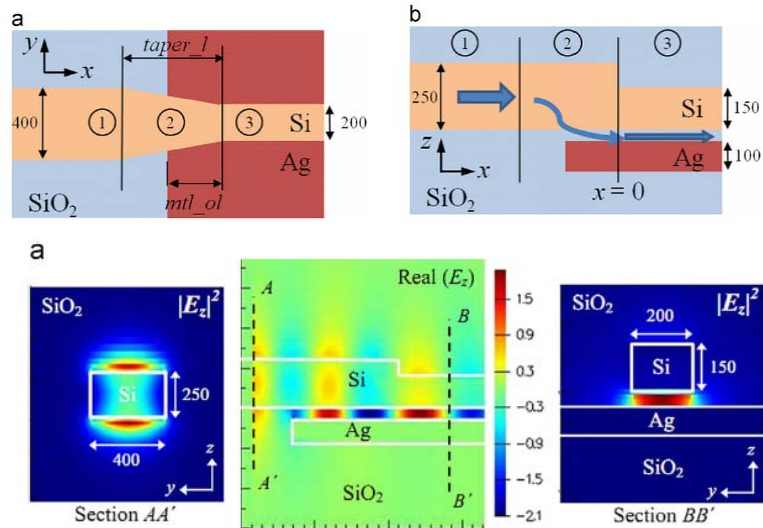


## RING MICRORESONATOR



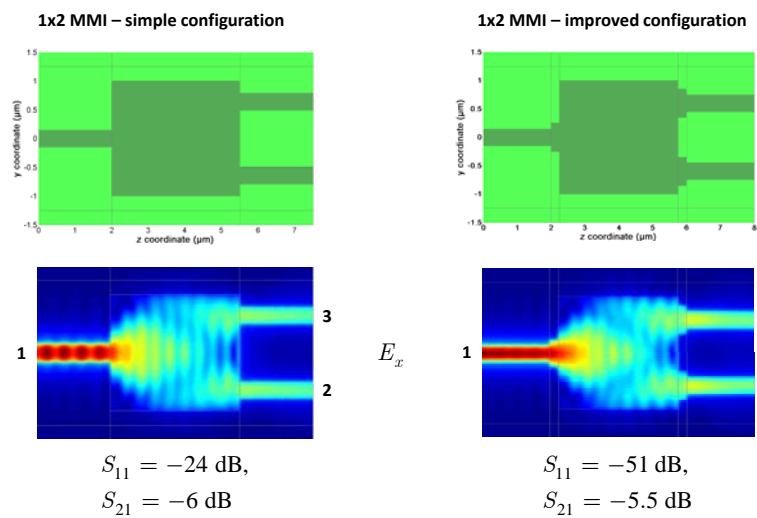
## VAZBA MEZI SOI NANODRÁTEM A HDPSW

R. Mote et al., *Optics Communications* 285 (2012) 3709–3713



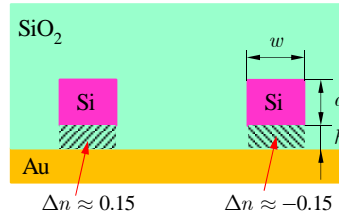
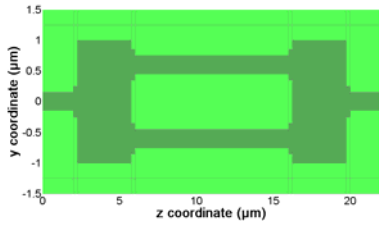
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## MULTIMODE INTERFERENCE COUPLER

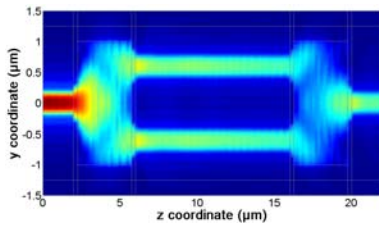


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## MACH-ZEHNDER INTERFEROMETER



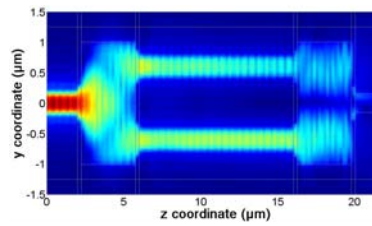
“On” state



$$S_{11} = -37 \text{ dB}$$

$$S_{21} = -6 \text{ dB}$$

“Off” state



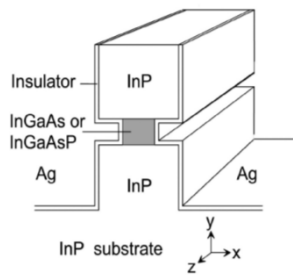
$$S_{11} = -25 \text{ dB}$$

$$S_{21} = -21 \text{ dB}$$

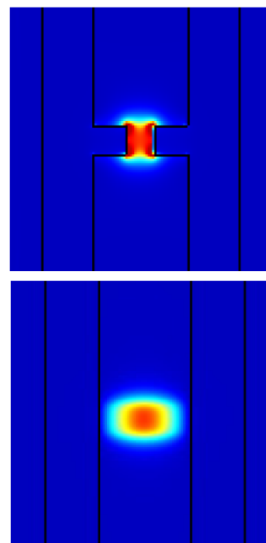
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## Koncept „plazmonového polovodičového laseru“

(M. Hill, ECIO 2007)



Rozměry aktivní oblasti laseru  
26 × 26 × 82 nm



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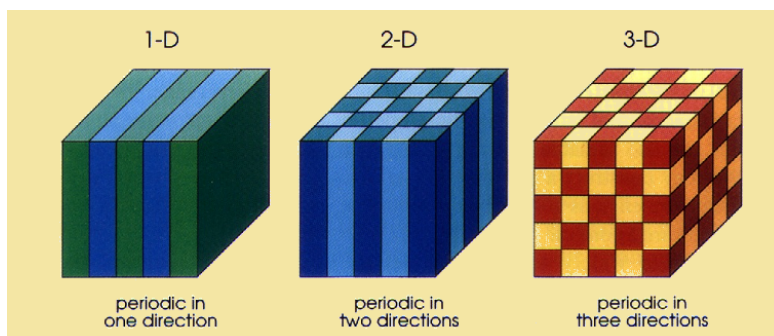
## Fotonické krystaly

### a integrovaná fotonika

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## Fotonické krystaly

1D, 2D nebo 3D periodické struktury s velkým kontrastem permittivity



E. Yablonovitch: „Inhibited spontaneous emission in solid-state physics and electronics“, *Phys. Rev. Lett.*, vol. 58, pp. 2059–2062, 1987

J. D. Joannopoulos *et al.*: *Photonic Crystals: molding the flow of light*, Princeton University Press 1995

S. G. Johnson, J. D. Joannopoulos: *Photonic Crystals, The road from theory to practice*, Kluwer Academic Publishers 2003

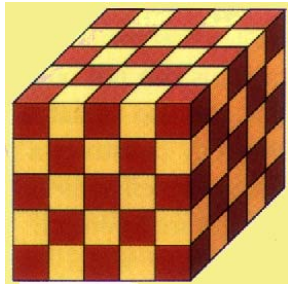
J.-M. Lourtioz *et al.*: *Photonic Crystals : Towards Nanoscale Photonic Devices*, Springer 2005

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**Fotony** se v periodickém dielektrickém prostředí pohybují „podobně“ jako **elektrony** v periodickém potenciálovém poli

Za jistých podmínek existuje **zakázaný pás energií fotonů**. Fotony s energií uvnitř zakázaného pásu se v periodickém prostředí nemohou šířit, záření se tudíž **totálně odráží zpět**



Z pohledu vlnové optiky jde o **braggovský odraz vlny od periodického prostředí**. Totální odraz je možno využít k vytvoření **optických vlnodů ve fotonických krystalech**

Vytvořit trojrozměrné periodické prostředí je však technologicky obtížné.

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### “Pohybové rovnice” pro elektrony a fotony v krystalech

Schrödingerova rovnice pro elektron v periodickém potenciálu:

$$\left[ -\frac{\hbar^2}{2m_e^*} \Delta + V(\mathbf{r}) \right] \psi(\mathbf{r}) = E \psi(\mathbf{r}) \quad V(\mathbf{r} + \mathbf{a}) = V(\mathbf{r}) \quad K = \frac{2\pi}{|\mathbf{a}|}$$

$$\psi(\mathbf{r}) = \sum_m u_m(\mathbf{r}) e^{im\mathbf{K}\cdot\mathbf{r}}$$

periodický potenciál    vlnová funkce    energie fotonu    (Floquetova)-Blochova vlna,

Aproximativní (jednočásticové) přiblížení

“Vlnová rovnice” pro fotony v periodické permitivitě

$$\nabla \times \mathbf{E} = i\omega\mu_0 \mathbf{H}, \quad \nabla \times \mathbf{H} = -i\omega\varepsilon_0 \varepsilon(\mathbf{r}) \mathbf{E},$$

$$\nabla \times \left( \frac{1}{\varepsilon(\mathbf{r})} \nabla \times \mathbf{H}(\mathbf{r}) \right) = \left( \frac{\omega}{c} \right)^2 \mathbf{H}(\mathbf{r})$$

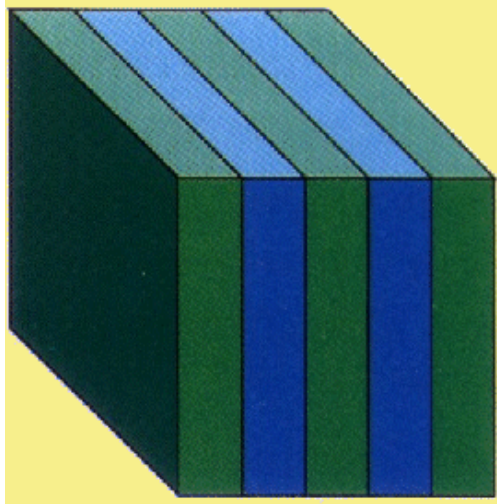
Přesná (“mnohočásticová”) teorie

Rovnice pro vlastní hodnoty energie fotonů a F-B funkce

Tento přístup je jednoduchý a průzračný, ale standardně nebere v úvahu *disperzi permitivity*  $\varepsilon(\mathbf{r}, \omega)$

úfe

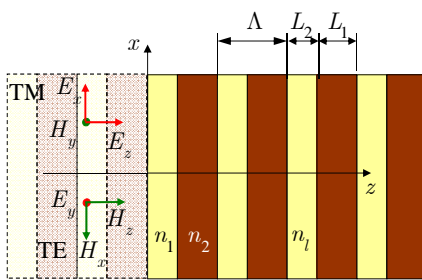
## Periodická vrstevnatá struktura jako jednorozměrný fotonický krystal



úfe

### Jednorozměrný fotonický krystal

Existence zakázaného pásu odvozená metodou přenosové matice  
(fotonická analogie Kronigova - Penneyova modelu krystalu)



Normalizace souřadnic a vln. vektorů

$$\xi = k_0 x, \quad \zeta = k_0 z, \quad k_0 = 2\pi / \lambda$$

$$\mathbf{k}_l = k_0 (\gamma \mathbf{x}^0 + N_l \mathbf{z}^0), \quad l = 1, 2, \dots, L$$

$$\gamma^2 + N_l^2 = \varepsilon_l = \begin{cases} n_1^2 & \gamma - \text{příčná konst.} \\ n_2^2 & \text{šíření } \textit{stejná} \end{cases}$$

konst. šíření

Elektromagnetické pole je popsáno  
komplexními amplitudami  $p_l(\zeta), q_l(\zeta)$

TE

TM

$$\begin{aligned} E_{y,l}(x, z) &= \sqrt{2k_0 Z_0 / N_l} p_l(\zeta) e^{i\gamma \xi}, & H_{y,l}(x, z) &= \sqrt{2k_0 Y_0 \varepsilon_l / N_l} p_l(\zeta) e^{i\gamma \xi}, & Z_0 &= \sqrt{\frac{\mu_0}{\varepsilon_0}}, \\ H_{x,l}(x, z) &= -\sqrt{2k_0 Y_0 N_l} q_l(\zeta) e^{i\gamma \xi}, & E_{x,l}(x, z) &= \sqrt{2k_0 Z_0 N_l / \varepsilon_l} q_l(\zeta) e^{i\gamma \xi}, & & \\ H_{z,l}(x, z) &= \sqrt{2k_0 Y_0 / N_l} \gamma p_l(\zeta) e^{i\gamma \xi}, & E_{z,l}(x, z) &= -\sqrt{2k_0 Y_0 / (\varepsilon_l N_l)} \gamma p_l(\zeta) e^{i\gamma \xi}, & Y_0 &= \sqrt{\frac{\varepsilon_0}{\mu_0}} \end{aligned}$$

úfe

## Elektromagnetické Floquetovy – Blochovy vidy

Průchod  $l$ -tou vrstvou je popsán přenosovou maticí  $\mathbf{A}_l$ , 
$$\begin{pmatrix} p_l(\zeta + \Delta\zeta) \\ q_l(\zeta + \Delta\zeta) \end{pmatrix} = \mathbf{A}_l \cdot \begin{pmatrix} p_l(\zeta) \\ q_l(\zeta) \end{pmatrix}, \quad \mathbf{A}_l = \begin{pmatrix} \cos N_l \Delta\zeta & i \sin N_l \Delta\zeta \\ i \sin N_l \Delta\zeta & \cos N_l \Delta\zeta \end{pmatrix},$$

průchod rozhraním  $l \rightarrow l+1$  a  $l+1 \rightarrow l$  je popsán maticemi

$${}^{l+1,l} \mathbf{A} = \begin{pmatrix} \rho & 0 \\ 0 & 1/\rho \end{pmatrix}, \quad \rho = \sqrt{N_{l+1}/N_l} \quad \text{pro TE polarizaci a}$$

$${}^{l,l+1} \mathbf{A} = \begin{pmatrix} 1/\rho & 0 \\ 0 & \rho \end{pmatrix}, \quad \rho = \sqrt{N_{l+1}\epsilon_l / N_l\epsilon_{l+1}} \quad \text{pro TM polarizaci.}$$

Přenosová matice jedné celé periody je  ${}^\Lambda \mathbf{A} = {}^{12} \mathbf{A} \cdot \mathbf{A}_2 \cdot {}^{21} \mathbf{A} \cdot \mathbf{A}_1$ .

Floquetův-Blochův „vid“ (vlna) je definován pomocí vlastní funkce matice  ${}^\Lambda \mathbf{A}$ ,

$${}^\Lambda \mathbf{A} \cdot \begin{pmatrix} p_1^F \\ q_1^F \end{pmatrix} = s \begin{pmatrix} p_1^F \\ q_1^F \end{pmatrix}, \quad s = \exp(i\varphi^F), \quad \varphi^F = k^F \Lambda, \quad k^F \text{ je konstanta šíření F-B vidu.}$$

$$k^F \text{ je určen až na aditivní konstantu } K = 2\pi/\Lambda: \quad \exp(ik^F \Lambda) = \exp[i(k^F + K)\Lambda]$$

Proto stačí určit  $k^F$  v intervalu  $-K/2 < k^F \leq K/2 \Rightarrow$  první Brillouinova zóna.

úře

## Vlastní hodnoty a fotonický zakázaný pás

Označme  $\Lambda = L_1 + L_2$ ,  $\varphi_1 = k_0 N_1 L_1$ ,  $\varphi_2 = k_0 N_2 L_2$ ,

matice  ${}^\Lambda \mathbf{A}$  má pak vlastní čísla

$$s = \cos \varphi_1 \cos \varphi_2 - \frac{1}{2} \left( \rho^2 + \frac{1}{\rho^2} \right) \sin \varphi_1 \sin \varphi_2 \pm \sqrt{\left[ \cos \varphi_1 \cos \varphi_2 - \frac{1}{2} \left( \rho^2 + \frac{1}{\rho^2} \right) \sin \varphi_1 \sin \varphi_2 \right]^2 - 1}.$$

FB vid se „šíří“, jen pokud  $|s| = 1$ , t.j., pokud

$$\left| \cos \varphi_1 \cos \varphi_2 - \frac{1}{2} \left( \rho^2 + \frac{1}{\rho^2} \right) \sin \varphi_1 \sin \varphi_2 \right| \leq 1.$$

Normovaná konstanta šíření je pak

$$k^{F'} = \frac{k^F}{K/2} = \frac{1}{\pi} \arccos \left[ \cos \left( \frac{\omega}{c} N_1 L_1 \right) \cos \left( \frac{\omega}{c} N_2 L_2 \right) - \frac{1}{2} \left( \rho^2 + \frac{1}{\rho^2} \right) \sin \left( \frac{\omega}{c} N_1 L_1 \right) \sin \left( \frac{\omega}{c} N_2 L_2 \right) \right].$$

$$\text{Pokud } \left| \cos \varphi_1 \cos \varphi_2 - \frac{1}{2} \left( \rho^2 + \frac{1}{\rho^2} \right) \sin \varphi_1 \sin \varphi_2 \right| > 1,$$

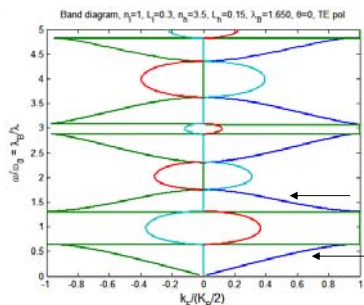
$k^F$  je komplexní, a vlna se nemůže šířit podél nekonečně dlouhého krystalu.

Tak vzniká **fotonický zakázaný pás**.

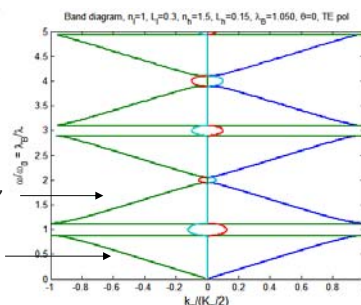
úře

## Pásová struktura jednorozměrného krystalu

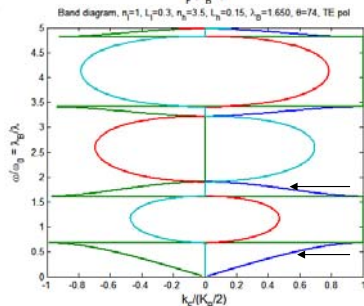
$n_1 = 1$   
 $n_2 = 3.5$   
 $\theta = 0^\circ$



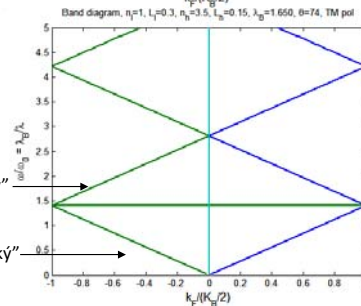
$n_1 = 1$   
 $n_2 = 1.5$   
 $\theta = 0^\circ$



TE  
 $\theta = 74^\circ$



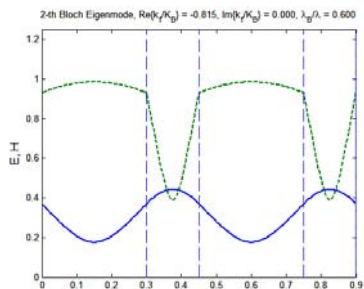
TM  
 $\theta = 74^\circ$



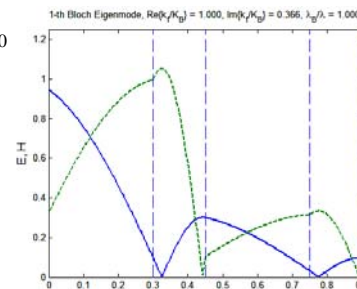
úfe

## Elektromagnetické Floquetovy – Blochovy vlny

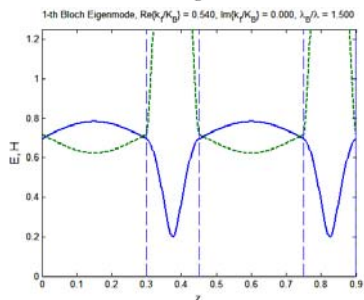
$n_1 = 1$   
 $n_2 = 3.5$   
 $\lambda_B / \lambda = 0.6$



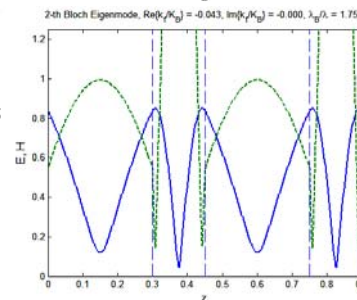
$\lambda_B / \lambda = 1.0$



$\lambda_B / \lambda = 1.5$



$\lambda_B / \lambda = 1.75$

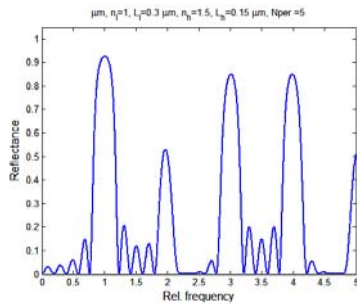


úfe

## Spektrální reflektance

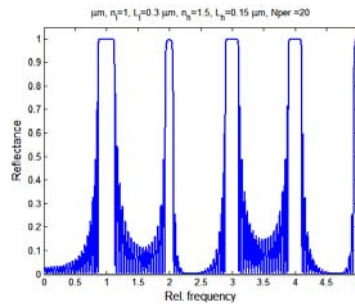
$n_1 = 1$   
 $n_2 = 1.5$   
 $n_{per} = 5$

nízký  
kontrast,  
málo  
period



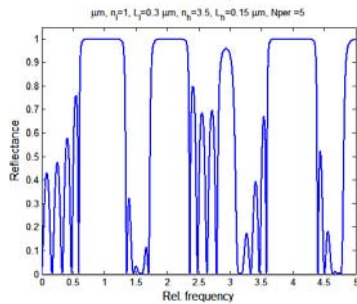
$n_1 = 1$   
 $n_2 = 1.5$   
 $n_{per} = 20$

nízký  
kontrast,  
více  
period



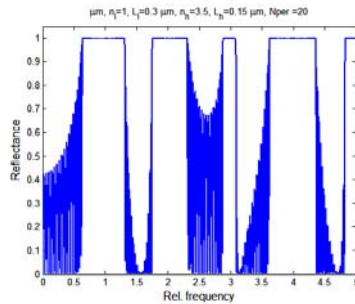
$n_1 = 1$   
 $n_2 = 3.5$   
 $n_{per} = 5$

velký  
kontrast,  
málo  
period



$n_1 = 1$   
 $n_2 = 3.5$   
 $n_{per} = 20$

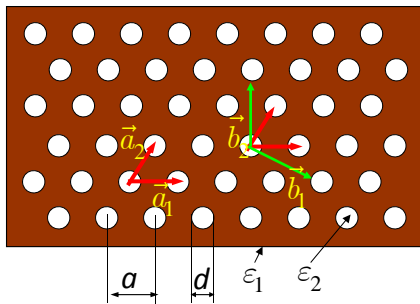
velký  
kontrast,  
více  
period



Fotonické krystaly odpovídají často spíše „nanokrystalům“

úfe

## Dvozměrné „fotonické krystaly“



Elementární vektory prostorové mřížky

$$\vec{a}_1 = (a, 0); \quad \vec{a}_2 = (a/2, \sqrt{3}a/2)$$

Elementární vektory reciproké mřížky

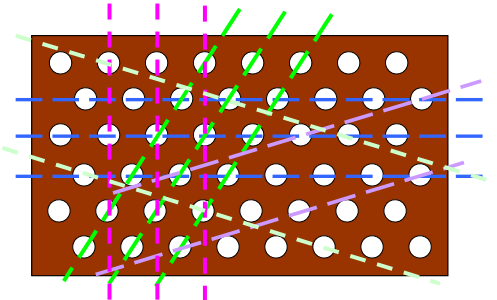
$$\vec{b}_1 = \left(\frac{1}{a}, \frac{1}{a\sqrt{3}}\right), \quad \vec{b}_2 = \left(0, \frac{2}{a\sqrt{3}}\right)$$

Periodické uspořádání otvorů;  
Blochův – Floquetův teorém

$$\left. \begin{matrix} E_z \\ H_z \end{matrix} \right\} = u_{\vec{k}}(\vec{r}_{\parallel}) e^{i\vec{k} \cdot \vec{r}_{\parallel}} e^{i\vec{G} \cdot \vec{r}_{\parallel}}$$

$$u_{\vec{k}}(\vec{r}_{\parallel}) = u_{\vec{k}}(\vec{r}_{\parallel} + \vec{a}_1) = u_{\vec{k}}(\vec{r}_{\parallel} + \vec{a}_2)$$

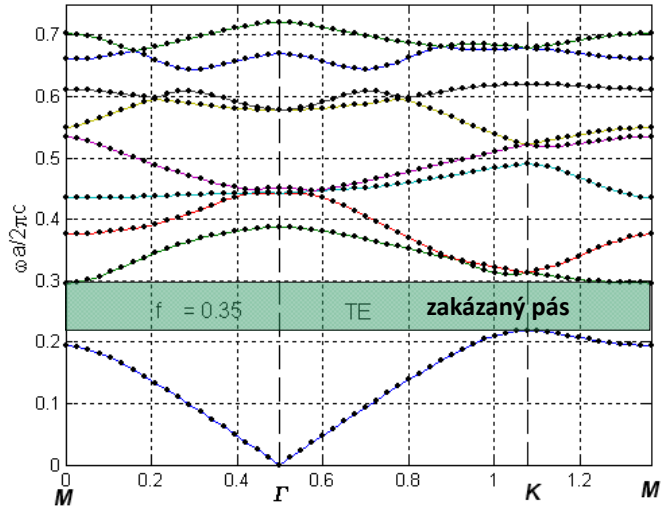
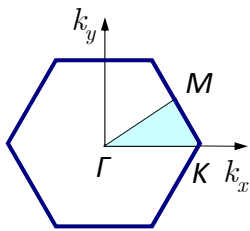
$$\vec{G} = m\vec{b}_1 + n\vec{b}_2; \quad m, n \text{ celé}$$



úfe

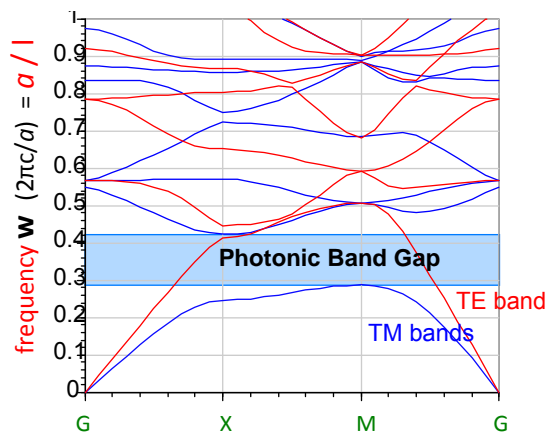
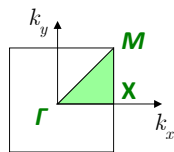
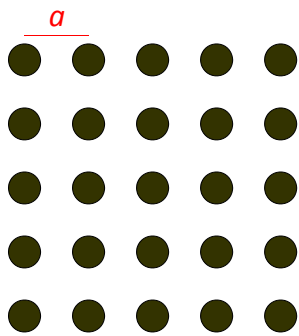
## Pásový diagram energií fotonů 2D krystalu s trojúhelníkovou mřížkou

první  
Brillouinova  
zóna  
prostoru  
vlnových vektorů



úfe

## Pásový diagram energií fotonů 2D krystalu se čtvercovou mřížkou

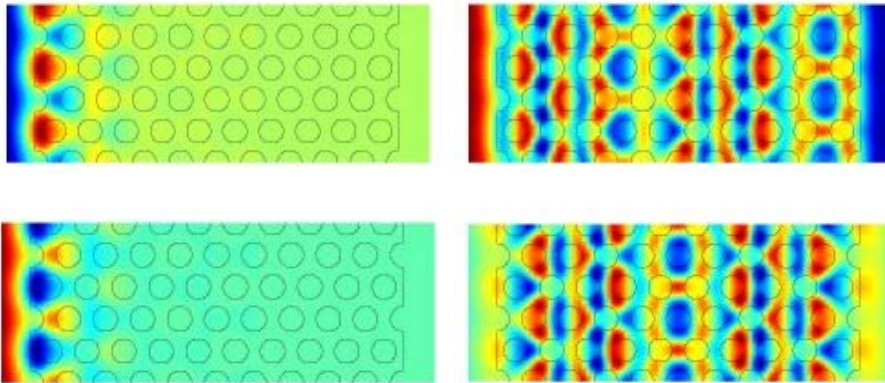


úfe

## Odraz rovinné vlny od 2D fotonického krystalu s trojúhelníkovou mřížkou otvorů v InP

Uvnitř zakázaného pásu

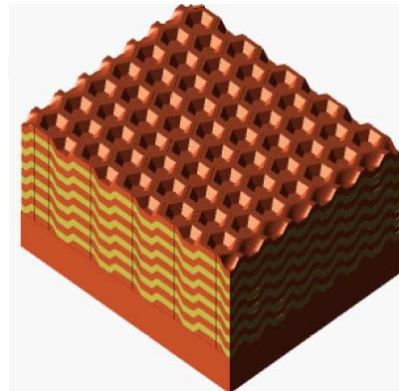
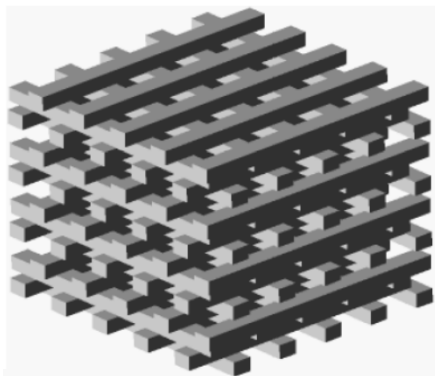
Vně zakázaného pásu



(Ing. Jiří Petráček, Dr., VUT Brno)

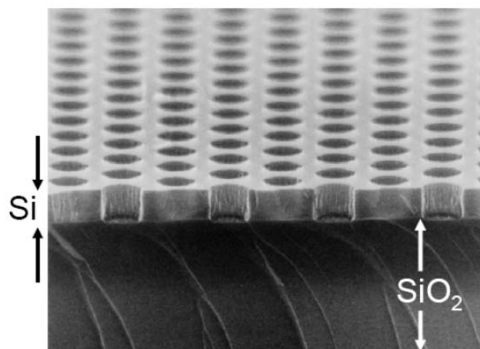
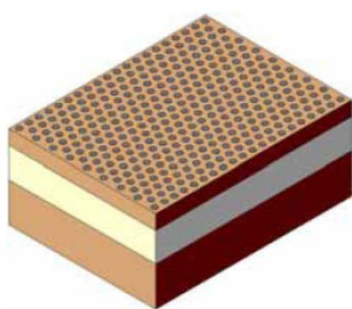
úfe

## Trojrozměrné fotonické krystaly



úfe

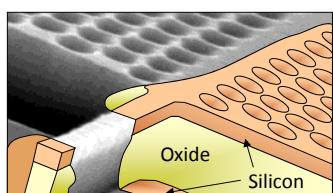
## „2.5-dimenzionální“ fotonické krystaly



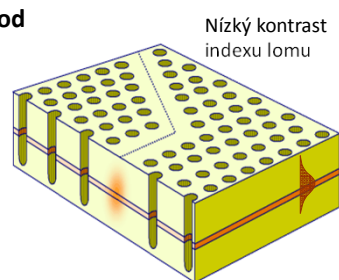
úfe

## Fotonické krystaly a vlnovody

### 1. 2D fotonický krystal + vertikální vlnovod

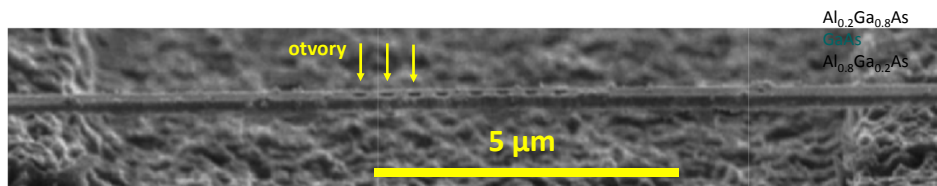


Vysoký  
kontrast  
indexu  
lomu



Nízký kontrast  
indexu lomu

### 2. Čárový 2D dielektrický vlnovod s 1D „fotonickým krystalem“

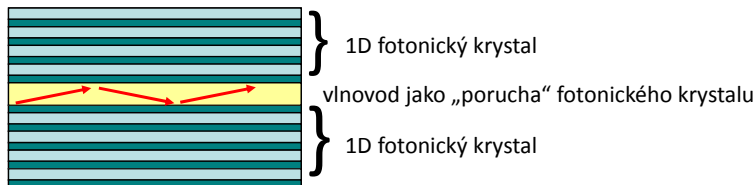


$Al_{0.2}Ga_{0.8}As$   
 $Al_{0.8}Ga_{0.2}As$

úfe



## Vlnovody v 1D fotonickém krystalu



Princip znám od 80. let jako „braggovský vlnovod“  
(antiresonant reflecting optical waveguide, **ARROW**)

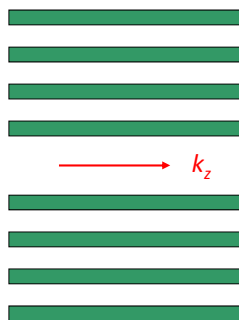
**Rozdíly ARROW vlnovodu vůči konvenčnímu vlnovodu:**

1. pro příušný úhel dopadu vlny **musí** existovat **zakázaný pás**
2. počet period musí být dostatečný, jinak vzniká **útlum vytékáním** („tunelováním“); v krystalu konečných rozměrů existují **pouze vytékající vidy** s komplexní konstantou šíření

úfe

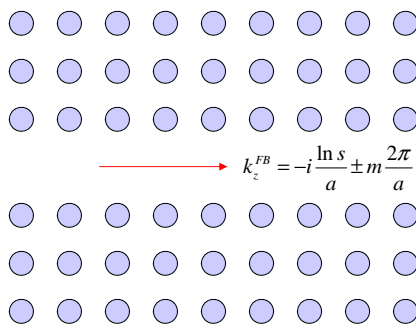
## Vlnovod ve fotonickém krystalu

Braggovský vlnovod  
(ARROW waveguide)



Anti-Reflecting Resonant  
Optical Waveguide

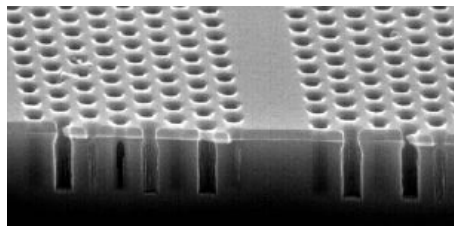
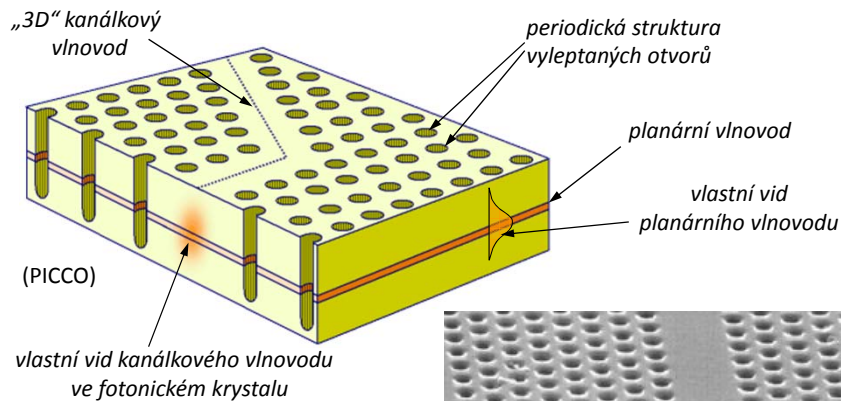
„Čárový defekt“ jako vlnovod  
1D periodičita



$k_z^{FB} a = \varphi \pm 2m\pi$  ... fázový posun při šíření  
o jednu periodu

úfe

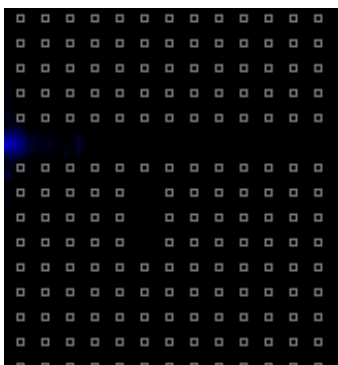
## Realizace 2D fotonických krystalů: 2D krystal v planárním vlnovodu



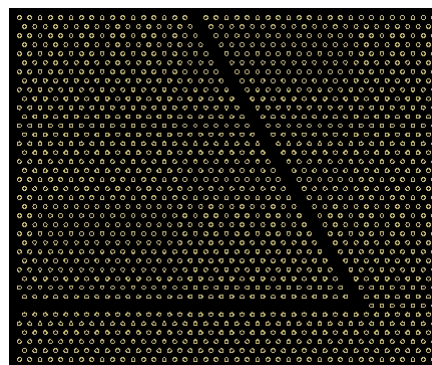
**Zásadní problém:**  
ztráty vyzářováním  
z roviny vlnovodu

úfe

## Numerické modelování šíření vln ve fotonických krystalech



Buzení mikrodutiny  
ve fotonickém krystalu  
femtosekundovým impulsem  
(FDTD, Uni Twente, NL)

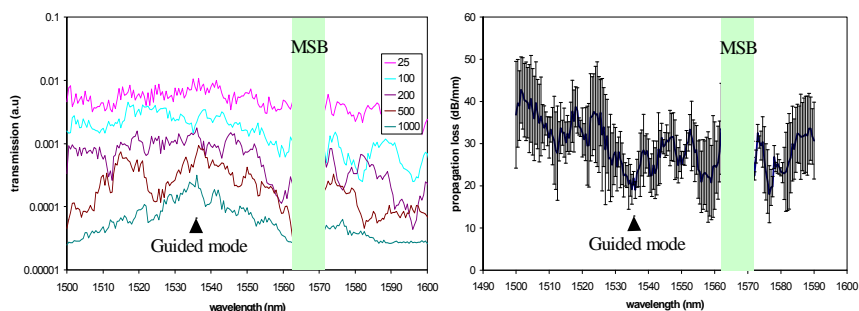
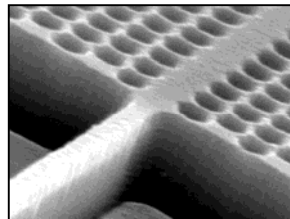


Šíření femtosekundového impulzu  
vlnovodným ohybem ve fotonickém  
krystalu (**F. Lederer et al.**,  
Friedrich-Schiller-Universität Jena, D)

úfe

## Vlnovody ve fotonických krystalech v SOI

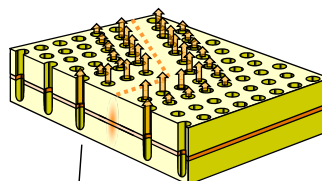
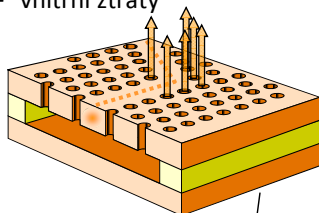
- Příprava:
  - „hluboká“ UV litografie a leptání
- vlnovod W1
  - perioda 500nm,  $\varnothing$  337nm
  - Mini-stop band
  - Nejnižší ztráty:  $20 \pm 3$  dB/mm (lichý vid!)



úfe

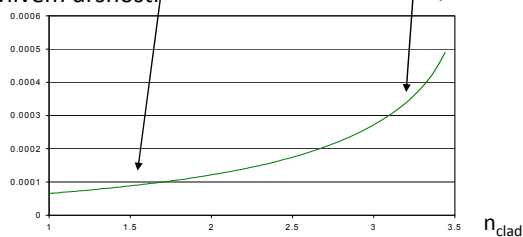
## Ztráty vyzařováním z roviny krystalu

- Vysoký „vertikální“ kontrast indexu lomu
  - vnitřní ztráty
- Malý „vertikální“ kontrast indexu lomu
  - vnitřní ztráty



– ztráty vlivem drsnosti

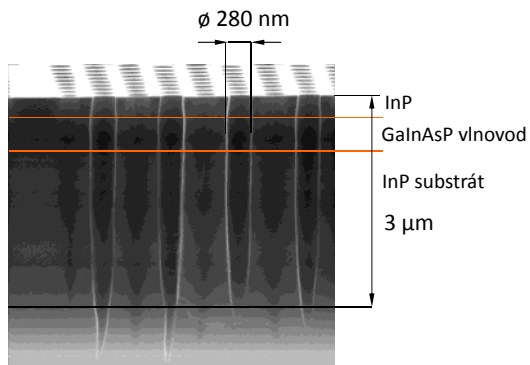
– ztráty vlivem drsnosti



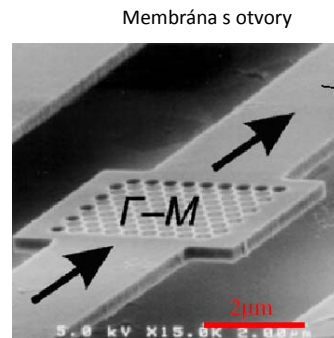
úfe

## Potlačení ztrát vyzářováním do substrátu

1. Leptání hlubokých otvorů: záření „nevnímá“ substrát (kromě vlnovodu ??)



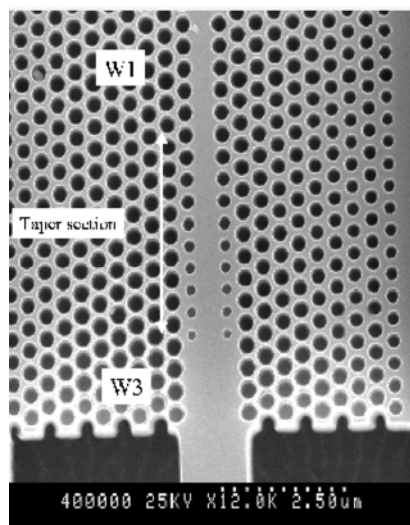
2. Úplné odstranění substrátu (technologicky náročné)



úfe

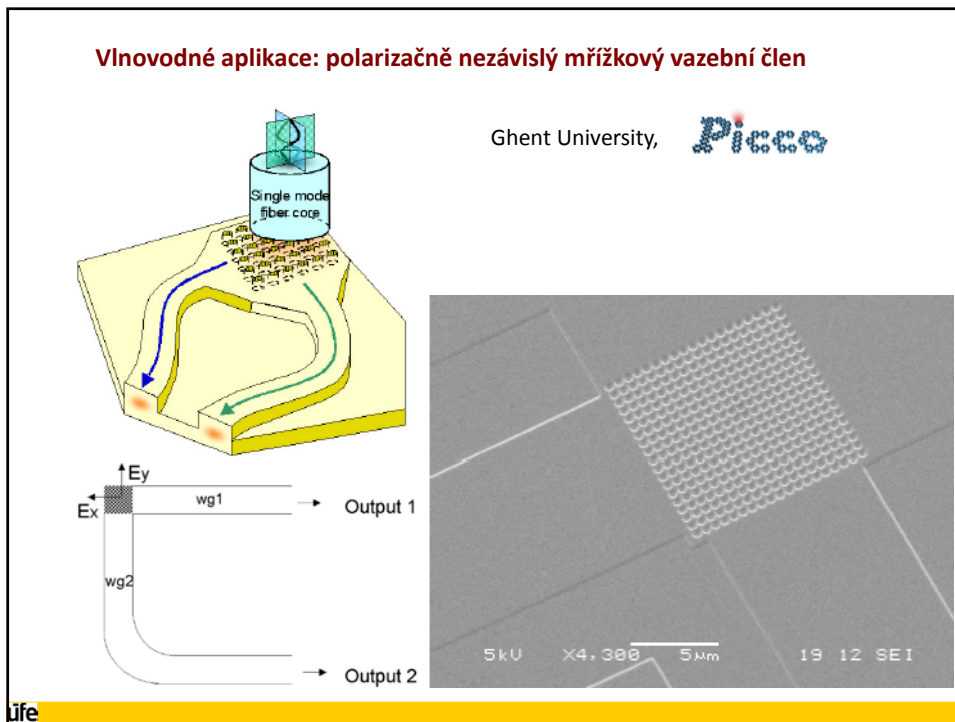
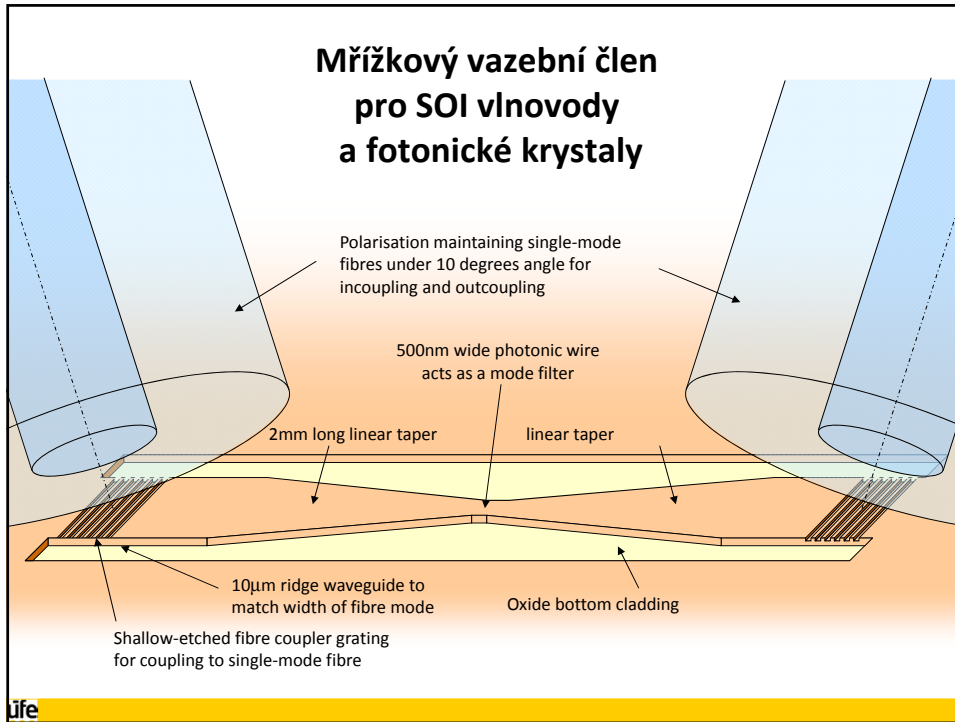
## Vazba s vlnovodem ve fotonickém krystalu

CNRS – LPN, Anne Talneau, Ph. Lalanne



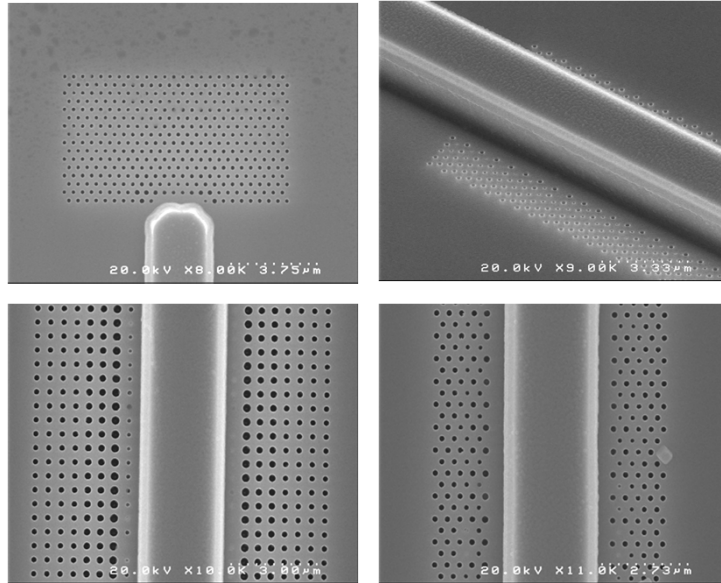
CNRS French patent  
(2001)

úfe



## 2D fotonické krystaly jako zrcadla polovodičových laserů

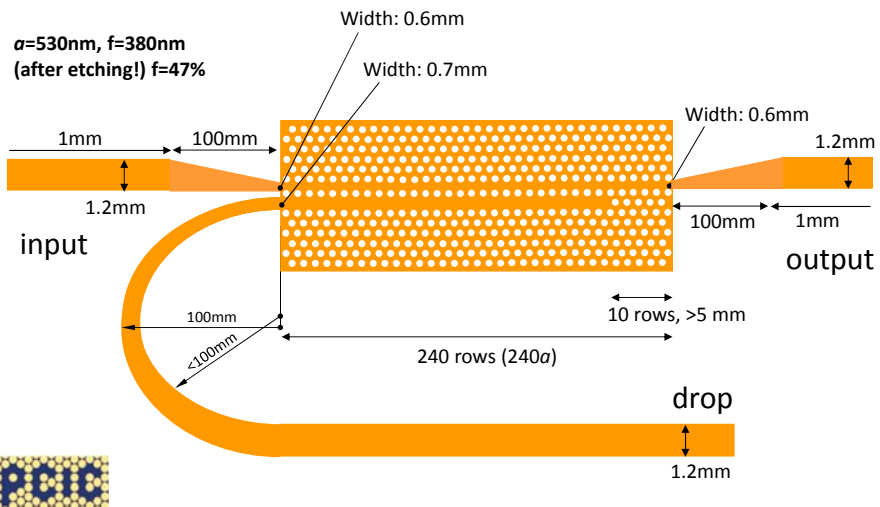
(Alcatel, 2002-3)



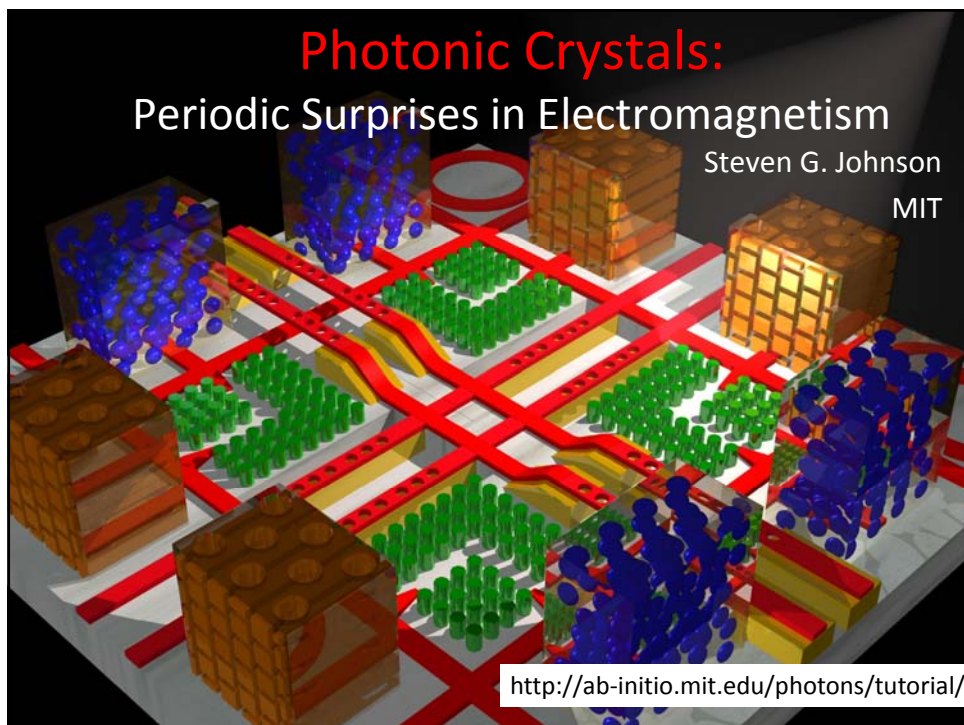
life

## PC Add-Drop Layout

KTH Stockholm, Min Qiu



life



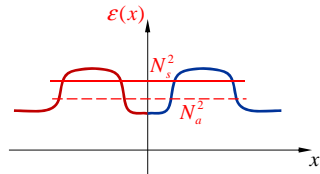
**Vlnodné struktury se ztrátami a ziskem  
jako analogie kvantově-mechanických struktur  
s (porušenou) symetrií parita-čas**

## FORMAL ANALOGY BETWEEN A PHOTONIC WAVEGUIDE AND A POTENTIAL WELL IN QUANTUM MECHANICS

### Eigenmode equation for TE modes of a planar waveguide

$$\frac{1}{k_0^2} \frac{d^2 E(x)}{dx^2} + \epsilon(x) E(x) = N^2 E(x)$$

mode field distribution  $E(x) \Leftrightarrow$   
 wave number  $k_0 \Leftrightarrow$   
 relative permittivity profile  $\epsilon(x) \Leftrightarrow$   
 effective refractive index  $N^2 \Leftrightarrow$

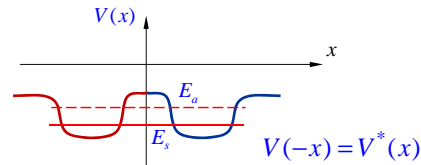


Loss/gain structure:  $\epsilon(-x) = \epsilon^*(x)$

### Schrödinger equation for a particle in a 1D potential well

$$-\frac{\hbar^2}{2m} \frac{d^2 \psi(x)}{dx^2} + V(x) \psi(x) = E \psi(x)$$

wave function  $\psi(x) \Leftrightarrow$   
 mass; Planck constant  $\sqrt{2m} \Leftrightarrow$   
 potential  $-V(x) \Leftrightarrow$   
 particle energy  $-E \Leftrightarrow$

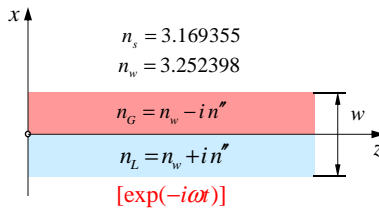


“ $\mathcal{PT}$  symmetry”: **complex potential(!)**,

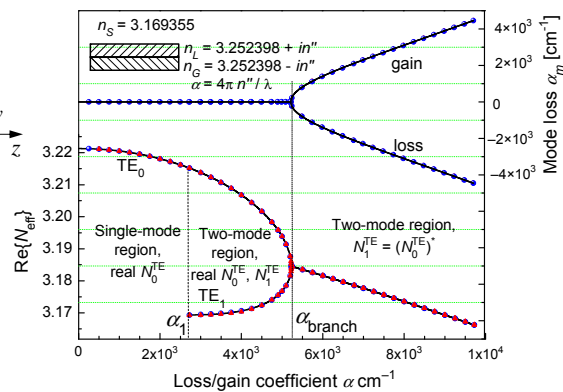
life

## WAVEGUIDE STRUCTURE WITH LOSS/GAIN: HISTORICAL REMARKS

≈ 1995: **COST 240 Action**: Loss/gain waveguide modelling task by **H. P. Nolting (HHI)** (aimed at benchmarking of BPM methods)



$w = 1 \mu\text{m}$ ,  
 $n'' = \frac{\alpha}{2k_0} = \frac{\lambda}{4\pi} \alpha \times 10^{-4} [-; \mu\text{m}, \text{cm}^{-1}]$ ,  
 $\lambda = 1.55 \mu\text{m}$ ,  
 $\alpha \dots$  "loss/gain coefficient"  $[\text{cm}^{-1}]$



1. H.-P. Nolting, G. Sztэфка, J. Čtyroky, "Wave Propagation in a Waveguide with a Balance of Gain and Loss," Integrated Photonics Research '96, Boston, USA, 1996, pp. 76-79.  
 2. G. Guekos, Ed., Photonic Devices for telecommunications: how to model and measure. Berlin: Springer, 1998, pp. 76-78.

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## DISPERSION EQUATION (TE modes)

$$\Phi(N, \alpha) = \gamma_G [\gamma_S \cos(k_0 \gamma_G w) - \gamma_G \sin(k_0 \gamma_G w)] [\gamma_S \sin(k_0 \gamma_L w) + \gamma_L \cos(k_0 \gamma_L w)] + \gamma_L [\gamma_S \cos(k_0 \gamma_L w) - \gamma_L \sin(k_0 \gamma_L w)] [\gamma_S \sin(k_0 \gamma_G w) + \gamma_G \cos(k_0 \gamma_G w)] = 0$$

$$\gamma_S = \sqrt{N^2 - n_s^2}, \quad \gamma_L = \sqrt{n_L^2 - N^2}, \quad \gamma_G = \sqrt{n_G^2 - N^2}, \quad k_0 = \frac{2\pi}{\lambda}$$

"Exceptional" point:  $\frac{dN}{d\alpha} \rightarrow \infty$ .

Since  $\Phi(N, \alpha) \equiv 0$ ,  $\frac{\partial \Phi}{\partial N} \frac{\partial N}{\partial \alpha} + \frac{\partial \Phi}{\partial \alpha} = 0 \Rightarrow \frac{\partial N}{\partial \alpha} = -\frac{\partial \Phi / \partial \alpha}{\partial \Phi / \partial N} \rightarrow \infty \Rightarrow \frac{\partial \Phi}{\partial N} = 0$ .

Exceptional point is given by the simultaneous solution of the following two equations:

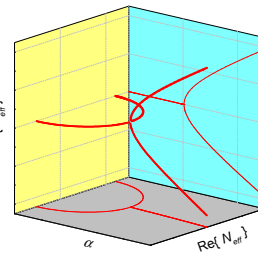
$$\Phi(N_B, \alpha_B) = 0, \quad \Phi'_N = \frac{d\Phi(N_B, \alpha_B)}{dN} = 0$$

Taylor expansion of  $\Phi(N, \alpha)$  in the vicinity of  $N_B, \alpha_B$  sounds

$$\Phi(N, \alpha) \approx \Phi'_\alpha (\alpha - \alpha_B) + \frac{1}{2} \Phi''_N (N - N_B)^2 = 0$$

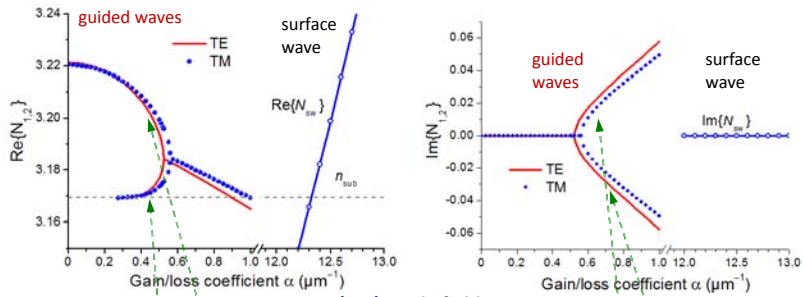
from which it follows

$$N \approx N_B \pm iC \sqrt{(\alpha - \alpha_B)}, \quad C = \sqrt{\frac{2\Phi'_\alpha}{\Phi''_N}}$$

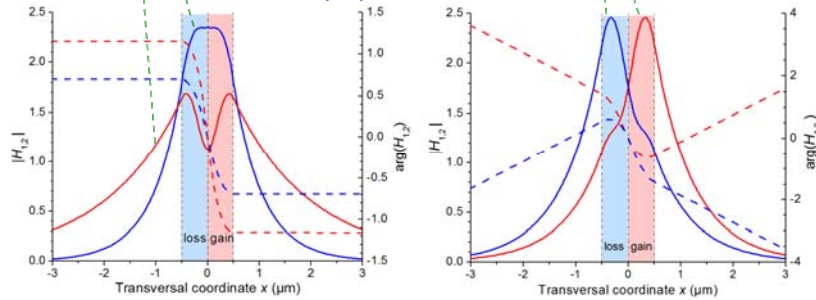


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## 2D ANALYSIS (PLANAR WAVEGUIDES)



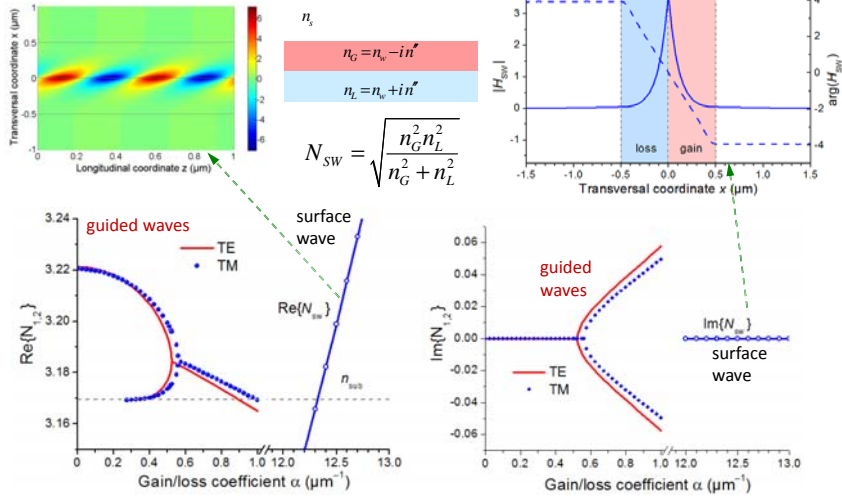
(TM) mode fields



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## WAVEGUIDE STRUCTURE WITH LOSS/GAIN: SURFACE WAVE

Non-attenuated TM surface wave



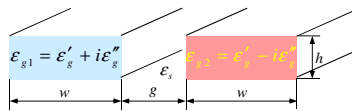
J. Čtyřoký et al., "Waveguide structures with antisymmetric gain/loss profile," *Optics Express*, vol. 18, pp. 21585-21593, 2010.

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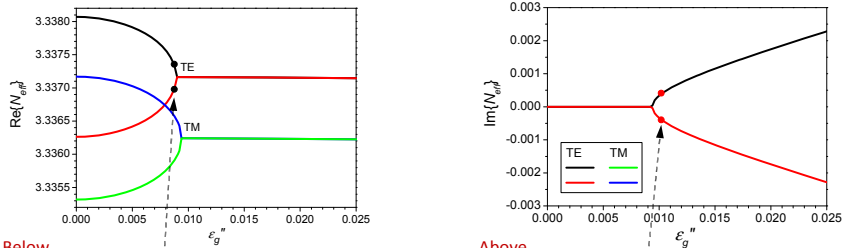
## COUPLED WAVEGUIDES WITH LOSS/GAIN

**Balanced loss/gain switching:**

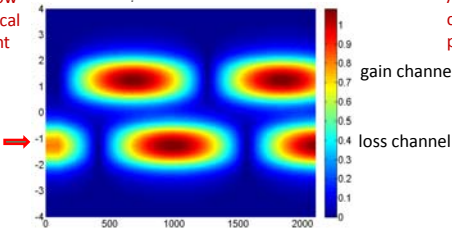
$$\mathcal{E}(-x, y) = \mathcal{E}^*(x, y)$$



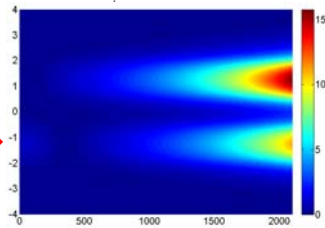
$\epsilon'_s = 10.89, \epsilon''_s = 11.56$   
 $w = 1.5 \mu\text{m}, h = 0.75 \mu\text{m}$   
 $g = 1 \mu\text{m}, \lambda = 1.55 \mu\text{m}$



Below critical point



Above critical point



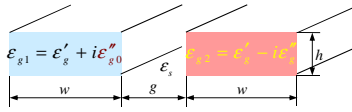
C. E. Rüter et al., "Observation of parity-time symmetry in optics," *Nature Physics*, vol. 6, pp. 192-195, 2010.

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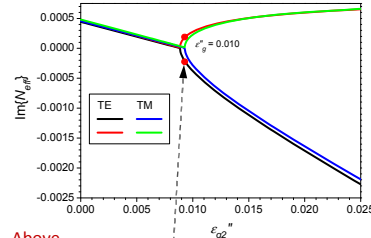
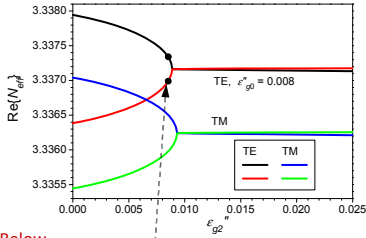
## COUPLED WAVEGUIDES WITH LOSS/GAIN

**Fixed loss/variable gain switching:**

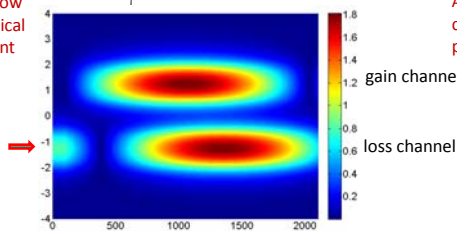
$$\mathcal{E}(-x, y) \neq \mathcal{E}^*(x, y)$$



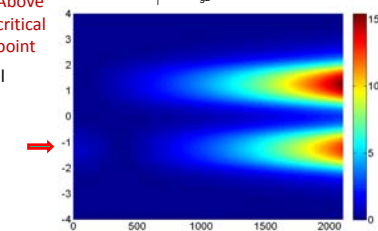
$$\begin{aligned} \epsilon_s &= 10.89, \quad \epsilon_g' = 11.56 \\ w &= 1.5 \mu\text{m}, \quad h = 0.75 \mu\text{m}, \\ g &= 1 \mu\text{m}, \quad \lambda = 1.55 \mu\text{m}. \end{aligned}$$



Below critical point



Above critical point

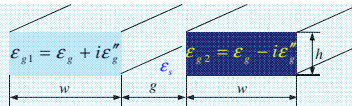


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## COUPLED WAVEGUIDES WITH LOSS/GAIN

**Balanced loss/gain with reduced gain:**

$$\mathbf{E}(x, y, z) = \mathbf{e}(x, y) \exp(ik_0 Nz)$$



$$\begin{aligned} \epsilon_s &= 10.89 + i\epsilon_b'', \quad \epsilon_g = 11.56 + i\epsilon_b'' \\ w &= 1.5 \mu\text{m}, \quad h = 0.75 \mu\text{m}, \\ g &= 1 \mu\text{m}, \quad \lambda = 1.55 \mu\text{m}. \end{aligned}$$

Rigorous equation for transversal field components:  $\Delta_{\perp} \mathbf{e}_{\perp} + \nabla_{\perp} [\nabla_{\perp} (\ln \epsilon) \cdot \mathbf{e}_{\perp}] + k_0^2 (\epsilon - N^2) \mathbf{e}_{\perp} = \mathbf{0}$ ,

Small uniform permittivity modification:  $\epsilon_1(x, y) = \epsilon(x, y) + i\epsilon_b''$ ,  $|\epsilon_b''| \ll |\epsilon|$

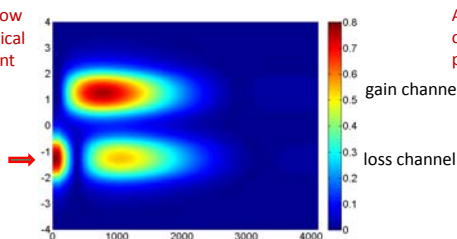
$$\Delta_{\perp} \mathbf{e}_{\perp} + \nabla_{\perp} \left[ \nabla_{\perp} \left( \ln \epsilon + \frac{i\epsilon_b''}{\epsilon} \right) \cdot \mathbf{e}_{\perp} \right] + k_0^2 [\epsilon + i\epsilon_b'' - (N^2 + i\epsilon_b'')] \mathbf{e}_{\perp} = \mathbf{0}; \quad \epsilon \rightarrow \epsilon + i\epsilon_b'' \Rightarrow N^2 \rightarrow N^2 + i\epsilon_b''$$

Uniform background loss can be used to reduce the required gain:

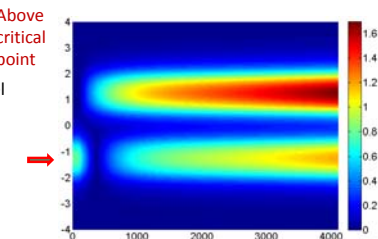
$$\epsilon_{\text{branch}}'' = \pm 0.009$$

$$\epsilon_{g,\text{loss}}'' = 0.0105, \quad \epsilon_{g,\text{gain}}'' = -0.0065, \quad \epsilon_b'' = 0.002, \quad \epsilon_{g,\text{loss}}'' = 0.0115, \quad \epsilon_{g,\text{gain}}'' = -0.0075$$

Below critical point



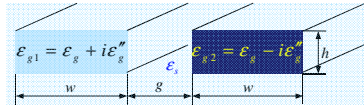
Above critical point



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# COUPLED WAVEGUIDES WITH LOSS/GAIN

Un/balanced loss/gain:



$$\epsilon_s = 10.89 + i\epsilon_b'', \quad \epsilon_g = 11.56 + i\epsilon_b''$$

$$w = 1.5 \mu\text{m}, \quad h = 0.75 \mu\text{m},$$

$$g = 1 \mu\text{m}, \quad \lambda = 1.55 \mu\text{m}.$$

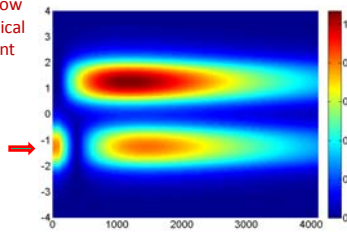
Structure with background loss,  $\epsilon_b'' = 0.002$

Output **power increase** (from both waveguides) by **increasing loss** of the lossy channel:

Lower loss, "subcritical" regime

$$\epsilon_{g,loss}'' = 0.0105, \quad \epsilon_{g,gain}'' = -0.0075, \quad \epsilon_b'' = 0.002,$$

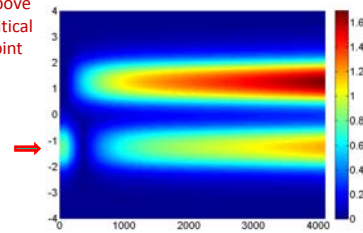
Below critical point



Higher loss, "supercritical" regime

$$\epsilon_{g,loss}'' = 0.0115, \quad \epsilon_{g,gain}'' = -0.0075$$

Above critical point



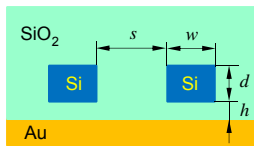
A. Guo et al., "Observation of PT-Symmetry Breaking in Complex Optical Potentials," *Physical Review Letters*, vol. 103, no. 9, pp. 093902-1-4, 2009.

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# PLASMONIC LOSS/GAIN STRUCTURES

A hypothetic "canonic" (balanced) plasmonic loss/gain structure:

Hybrid dielectric-plasmonic slot waveguide directional coupler with "tunable metal" structure:



$$\mathcal{E}(-x, y) = \mathcal{E}^*(x, y)$$

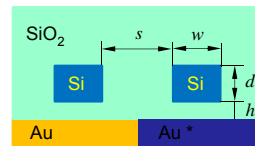
$$w = 300 \text{ nm},$$

$$d = 120 \text{ nm},$$

$$h = 30 \text{ nm},$$

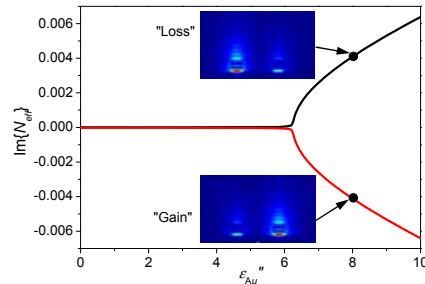
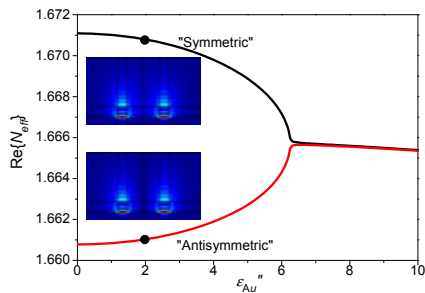
$$s = 1000 \text{ nm},$$

$$\lambda = 1.55 \mu\text{m}$$



Au with tunable loss / Au\* with tunable gain

$$\epsilon_{Au^*} = \epsilon_{Au}^*$$



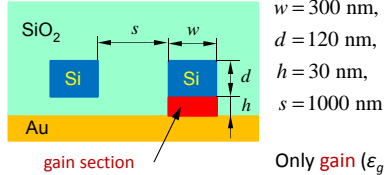
H. Benisty et al., "Implementation of PT symmetric devices using plasmonics: principle and applications," *Optics Express*, vol. 19, pp. 18004-18019, 2011.

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## PLASMONIC LOSS/GAIN STRUCTURES

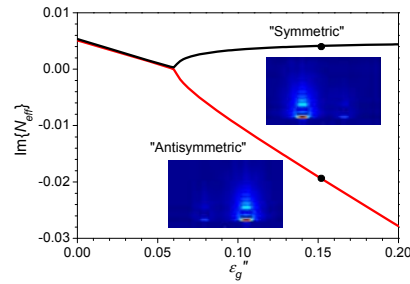
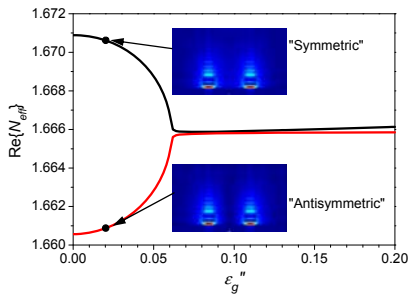
A more realistic model of an **unbalanced** plasmonic loss/gain structure:

Hybrid dielectric-plasmonic slot waveguide directional coupler **with gain section**



$$\mathcal{E}(-x, y) \neq \mathcal{E}^*(x, y)$$

Only gain ( $\epsilon_g''$ ) in the gain section is now tuned:  $\epsilon_{\text{gain}} = \epsilon_{\text{SiO}_2} - i\epsilon_g''$



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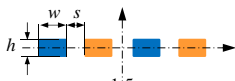
## MORE COMPLEX GAIN-LOSS STRUCTURES

Linear arrays of coupled waveguides with loss and gain

(quasi-TE polarization)

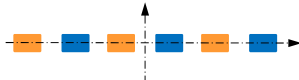
$$\mathcal{E}(-x, y) = \mathcal{E}^*(x, y)$$

4 coupled channel waveguides



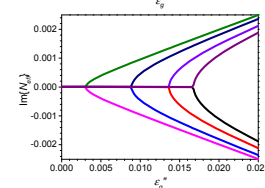
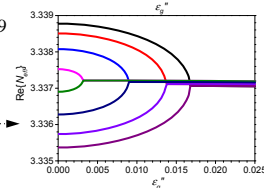
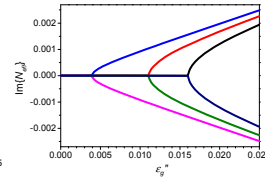
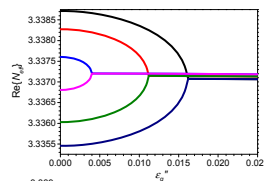
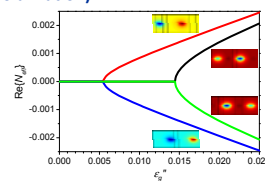
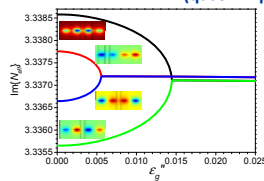
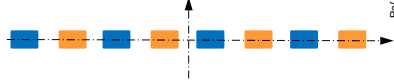
$w = 1.15 \mu\text{m}$ ,  
 $h = 0.75 \mu\text{m}$ ,  
 $s = 1 \mu\text{m}$

6 coupled channel waveguides



$$\epsilon_{g,1} = 11.56 + i\epsilon_g'', \quad \epsilon_{g,2} = 11.56 - i\epsilon_g'', \quad \epsilon_s = 10.89$$

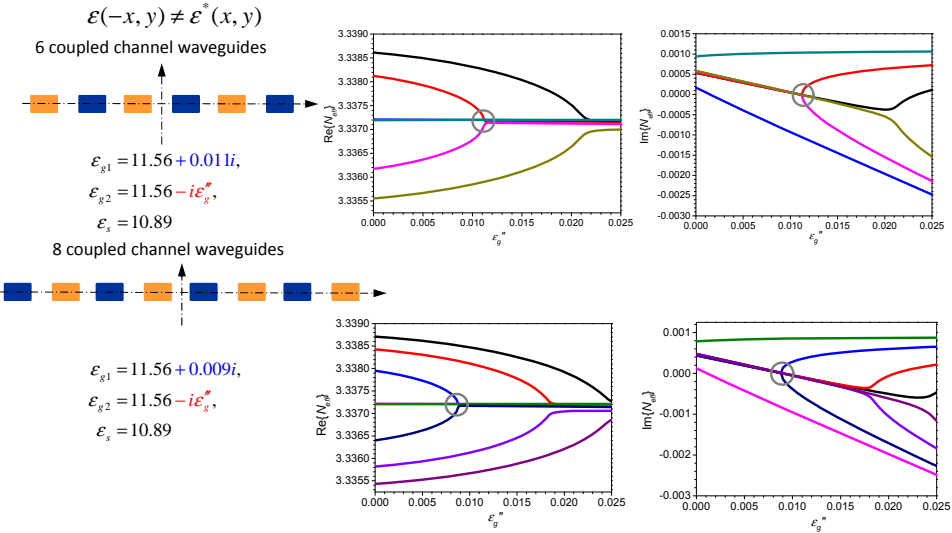
8 coupled channel waveguides



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## LINEAR ARRAY WITH UNBALANCED LOSS/GAIN

Coupled waveguides with *fixed loss* and *variable gain*

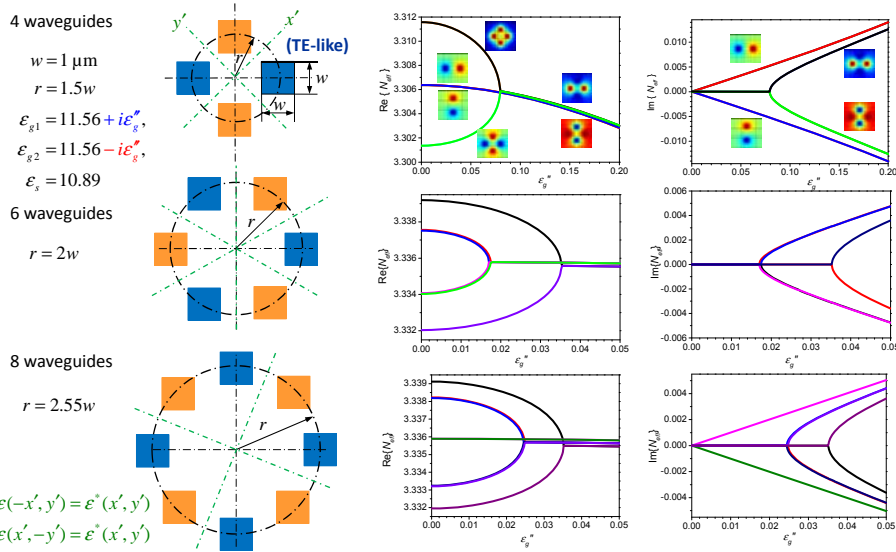


“Switching” by pure gain modulation is feasible also in loss/gain waveguide arrays

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## MORE COMPLEX GAIN-LOSS STRUCTURES

“Circular” arrays of coupled waveguides with loss and gain



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## CIRCULAR ARRAYS WITH UNBALANCED LOSS/GAIN

Coupled waveguides with *fixed loss* and *variable gain*

6 waveguides

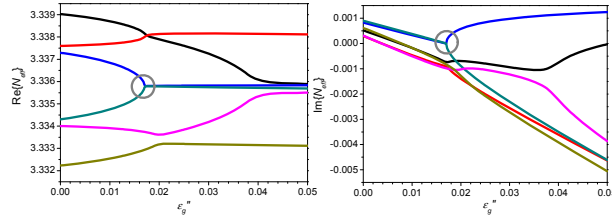
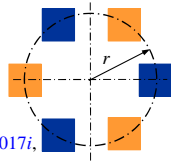
$$w = 1 \mu\text{m}$$

$$r = 2w$$

$$\epsilon_s = 10.89$$

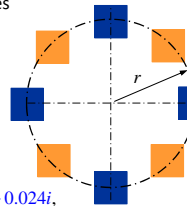
$$\epsilon_{g1} = 11.56 + 0.017i,$$

$$\epsilon_{g2} = 11.56 - i\epsilon_g''$$



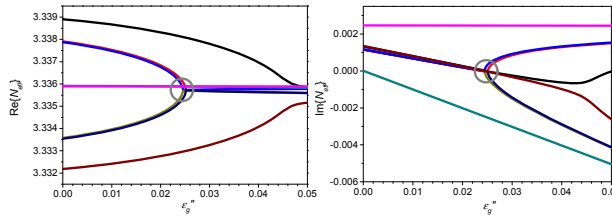
8 waveguides

$$r = 2.55w$$



$$\epsilon_{g1} = 11.56 + 0.024i,$$

$$\epsilon_{g2} = 11.56 - i\epsilon_g''$$



“Switching” by pure gain modulation is feasible also in loss/gain waveguide arrays

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## SOME RELEVANT REFERENCES

1. J. Zenneck, "Über die Fortpflanzung ebener elektromagnetischer Wellen längs einer ebenen Leiterfläche und ihre Beziehung zur drahtlosen Telegraphie," *Annalen der Physik*, vol. 328, pp. 846-866, 1907.
2. H.-P. Nolting, G. Sztefka, M. Grawert, and J. Čtyroký, "Wave Propagation in a Waveguide with a Balance of Gain and Loss," in *Integrated Photonics Research '96*, Boston, USA, 1996, pp. 76-79.
3. G. Guekos, Ed., *Photonic Devices for telecommunications: how to model and measure*. Berlin: Springer, 1998.
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5. K. G. Makris, R. El-Ganainy, and D. N. Christodoulides, "Beam Dynamics in PT Symmetric Optical Lattices," *Physical Review Letters*, vol. 100, pp. 103904(1)-103904(4), 2008.
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7. C. E. Rüter, K. G. Makris, R. E-Ganainy, D. N. Christoulides, M. Segev, and D. Kip, "Observation of parity-time symmetry in optics," *Nature Physics*, vol. 6, pp. 192-195, 2010.
8. H. Benisty, A. Degiron, A. Lupu, A. De Lustrac, S. Chenais, S. Forget, et al., "Implementation of PT symmetric devices using plasmonics: principle and applications," *Optics Express*, vol. 19, pp. 18004-18019, Sep 2011.
9. J. Čtyroký, "3-D Bidirectional Propagation Algorithm Based on Fourier Series," *Journal of Lightwave Technology*, vol. 30, pp. 3699-3708, 2012.
10. A. A. Sukhorukov, S. V. Dmitriev, S. V. suchkov, and Y. S. Kivshar, "Nonlocality in PT-symmetric waveguide arrays with gain and loss," *Optics Letters*, vol. 37, pp. 2148-2150, 2012.
11. ...and many others...
12. ...

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