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## **Symmetry, Chaos and Phase Transitions in Collective Dynamics of Atomic Nuclei**

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# 1 Introduction

This dissertation is focused on some general interrelated aspects of collective motions in atomic nuclei and other many-body systems, namely on the aspects connected with the presence of various kinds of dynamical symmetries in collective modes, with the competition between regular and chaotic regimes of collective dynamics, and with transitions between different quantum phases of the collective Hamiltonian.

The dissertation is based on a sample of 25 selected works published in international scientific journals, which are sorted into five topical areas and reprinted here with unifying overview comments and a brief summary. The topical areas are as follows:

1. Classical and quantum chaos in the geometric collective model,
2. Chaos and hidden symmetries in the interacting boson model,
3. Quantum shape phase transitions in the interacting boson model,
4. Thermodynamical analogy for quantum phase transitions,
5. Quantum phase transitions for excited states.

The analyses presented here are mostly performed within two nuclear models (or, more precisely, model families). The first one is the geometric collective model (GCM), formulated already in 1950's by A. Bohr [1–3], which up to now represents a valid phenomenological basis for a quantitative description of nuclear collectivity. This model has been recently extended into several new directions, including in particular various types of explicit analytical solutions of the Bohr Hamiltonian (see, e.g., Ref. [4]). The second model discussed here is the interacting boson model (IBM), elaborated in the 1970's by F. Iachello and A. Arima [5–7], which successfully combines a macroscopic treatment of the nucleus (*à la* the GCM) with some microscopic elements. The IBM can be extended also to other fields of physics, such as molecular physics [8, 9].

The five topics listed above are strongly related. Both GCM and IBM—in spite of their conceptual simplicity—exhibit a rather surprising complexity of behavior. This is reflected by a vast variety of alternative dynamical modes, associated with the same system, and by a tiny interplay between chaotic and regular types of motions [10–14] depending sensitively on the model control parameters and on the selected values of integrals of motion (energy and angular momentum). Such features have been studied in both GCM and IBM.

In Sec. 2 we will focus on a systematic analysis of chaotic properties of the GCM, while the IBM is considered in Sec. 3. It turns out—especially in the IBM—that the regular domains in the space of control parameters, energy and angular momentum can be connected to various specific generalizations of the symmetry concept (see, e.g., Refs. [15, 16]). One of these generalizations, a so-called quasi dynamical symmetry [17], shows that observable signatures of a given dynamical symmetry can persist even in systems in which the actual symmetry is badly broken. This makes a link with an analogous result of classical mechanics (expressed by the so-called Kolmogorov-Arnold-Moser theorem [11]) that the proliferation of irregularity in systems with an increasing strength of a non-integrable perturbation has only a limited, often very moderate extent.

Prevailing chaotic dynamics is typical for those parameter domains where there exists a strong competition between two or more incompatible dynamical symmetries. As shown in Sec. 4, dynamical regimes associated with the different symmetries often have a character of quantum phases, the transitions between these regimes being sharp (non-analytic) in the asymptotic limit of the system's size. This leads to the notion of a quantum phase transition (QPT) [18–24].

A QPT represents an abrupt change of the structure of a given quantum state (or more generally, a set of states) with a varying value of the control parameter, typically represented by an interaction strength in the quantum Hamiltonian. Both GCM and IBM can be considered as specific realizations of the QPT phenomenon, with the phases defined through different geometric forms (shapes) of the nucleus associated with different (quasi) dynamical symmetries of the model [25–27]. In this sense, atomic nuclei provide a fundamental example of quantum many-body systems in which QPT physics is applicable. On the other hand, the above nuclear models can serve as a theoretical tool for studying some important aspects of QPTs, which may be valid in general quantum systems.

Two such aspect will be discussed in Secs. 5 and 6. First, in Sec. 5, we will focus on formal analogies between quantum and thermodynamical phase transitions. It is clear that the Hamiltonian control parameter in a QPT plays a role of the temperature in thermal phase transitions [28]. Exploiting this analogy further on it can be shown that the two types of phase transition bear even much deeper similarities. In particular, it turns out that both phenomena are rooted in the distribution of some anomalies in specific complex extended versions of the model (introducing a complex temperature in case of a thermodynamic phase transition and a complex control parameter in case of a QPT).

Originally, a QPT was defined for the ground state, i.e., attributed to the system at zero temperature [18], but in Sec. 6 we will discuss similar phenomena for individual excited states. A link to the thermodynamical phase transitions then becomes even more obvious, although the notion of excited-state quantum phase transition (ESQPT) is reserved for the microcanonical treatment, in which the excitation of the system is expressed by the excitation energy rather than temperature. The ESQPTs have interesting dynamical consequences and may become important in the processing of quantum information in future quantum technologies [29].

It seems that nuclear collective motions and the models capturing their essential features represent a rich source of interesting general physics and a promising subject for future study.

## 2 Classical and quantum chaos in the geometric collective model

Relevant publications of the author:

Papers in journals:  $\langle 3, 4, 12, 13, 15, 22 \rangle$

Papers in proceedings:  $\{ 2, 4 \}$

Unlike the interacting boson model, chaotic properties of the geometric collective model have not been subject to any analysis before the paper  $\langle 22 \rangle$ . The main message mediated by this paper was a surprise and fascination by the enormous complexity discovered in classical measures of chaos within the GCM. This complexity was compared with some other two-dimensional models, which show just a monotonous increase of chaotic measures with the strength of non-integrable perturbation and with energy. In contrast, the GCM exhibits very complicated dependences in spite of the fact that its Hamiltonian is also represented by a two-dimensional quartic potential (at this stage, we have restricted our analysis to non-rotational states, which are described by a two-dimensional configuration space).

In the subsequent study  $\langle 15 \rangle$ , the first results were extended to other parameter domains and supplied by an evaluation of the dependence on energy and angular momentum. Some

exotic Poincaré sections showing coexisting regular and chaotic features were presented. The most convenient measure of the degree of chaos was found to be the relative fraction of the available phase space (for a given energy) where the trajectories are chaotic (i.e., have positive Lyapunov exponents). This fraction is practically identical with a similar fraction defined on a selected Poincaré section. It was then proved  $\langle 13 \rangle$  that the most pronounced region of regularity inside the non-integrable parameter domain of the GCM is closely connected with the so-called arc of regularity in the symmetry triangle of the IBM [30,31]. This region was shown  $\langle 12 \rangle$  to be stable against modifications of the GCM kinetic energy by adding some position-dependent terms (as it is in the IBM).

The analysis of classical measures of chaos has been subsequently extended to the quantum case  $\langle 3, 4 \rangle$ . Here, according to the so-called Bohigas conjecture [32], the degree of chaos in the classical dynamics should be reflected by some correlation properties of quantum spectra. We have returned to the effectively two-dimensional (non-rotating) regime and performed the quantization of the system in two different ways (as a truly two-dimensional system and as a projection of five-dimensional system if the rotational degrees of freedom are considered). The results described in  $\langle 4 \rangle$  demonstrate a practical equivalence of both quantization schemes from the viewpoint of chaos and a good correspondence of quantum measures of chaos (based on the nearest-neighbor spacing distribution in quantum energy spectra) with the corresponding classical counterparts (chaotic phase-space fractions calculated by an improved method of alignment indices). Such a correspondence has been already observed in many other systems, but the new quality contained in our study is that we have tested the Bohigas conjecture in the situation of a strong variability of chaotic measures with energy and for two different quantizations of the same classical system.

In the paper  $\langle 3 \rangle$  we continued this analysis using a nearly forgotten method of spectral lattices proposed by A. Peres [33]. The method represents an interesting extension of the Bohigas conjecture. In particular, it makes it possible to analyze not only the energy dependence of an averaged degree of chaos (as the method based on the level statistics does) but also enables one (in favorable situations) to attribute regular and chaotic features to individual quantum states. In addition, the Peres method has numerous applications also beyond the field of chaos, as discussed in  $\langle 2, 3, 6 \rangle$  and  $\{2\}$ .

The work on this topic continues by using the GCM as a testing ground for various methods related to classical and quantum chaos. We believe that the enormous complexity of dynamics encoded in simple dynamical equations qualifies this model for such purposes. An ultimate aim would be a deeper understanding of the underlying mechanism that makes the system oscillate between regular and chaotic types of dynamics.

### **3 Chaos and hidden symmetries in the interacting boson model**

Relevant publications of the author:

Papers in journals:  $\langle 2, 6, 11, 13, 21, 32, 37, 38, 39 \rangle$

Papers in proceedings:  $\{2, 5\}$

The analysis of chaotic properties (both quantum and classical) of a class of IBM Hamiltonians mapping the so-called Casten triangle was performed by Alhassid and Whelan [30,31]. A surprising outcome of these works was a discovery of a region with increased regularity,

located inside the triangle, along an arc connecting the SU(3) and U(5) dynamical-symmetry vertices. The increase of regularity in the arc is not uniform in energy but it affects mainly the states just above zero absolute energy (i.e., above the threshold when the deformed shape of the nucleus can change its orientation with respect to the body-fixed frame).

The search of the origin of this effect was motivated by the anticipation that the increase of regularity might be connected with an approximate (hidden) symmetry of the system in the given parameter region. It needs to be stressed that no such a symmetry has been identified yet. Nevertheless, in search of it some interesting results have been obtained.

The papers ⟨39⟩ and ⟨38⟩ attempt to quantify the degree of approximate symmetry of a given transitional (i.e., possessing nominally no dynamical symmetry) IBM Hamiltonian by an entropic measure expressing the overlap of a given transitional eigenbasis with the eigenbasis associated with a fixed dynamical symmetry. It turned out that in a certain region, a combined entropy corresponding to a simultaneous overlap with all dynamical symmetries is reduced. This region precisely coincides with the semiregular arc.

As a by-product of this work we have obtained some expressions ⟨39, 38, 37⟩ for hidden dynamical symmetries  $\overline{\text{SU}(3)}$  and  $\overline{\text{O}(6)}$  derived from the standard IBM symmetries SU(3) and O(6) by gauge transformations of the  $s$ - and  $d$ -boson operators [34, 35]. These hidden symmetries are a consequence of a more general phenomenon, called parameter symmetry [36], which was subject to a general analysis in the paper ⟨32⟩. In the IBM case, the hidden symmetries turned out to have no relevance for the increase of regularity in the Alhassid-Whelan arc, but in generic many-body systems such symmetries should be taken into account.

It was found ⟨21, 13⟩ that the region of increased regularity in the Casten triangle (on the side of the deformed phase) approximately coincides with the locus of degeneracy of the heads of  $\beta$  and  $\gamma$  vibrational bands in the quantum spectrum. Although subsequent work ⟨2⟩ indicated that this coincidence was probably accidental, the finding led to the experimental identification of nuclei which might be situated close to the arc region and further stimulated the interest in an explanation of the arc.

The paper ⟨13⟩ represents a detailed theoretical study focused on the features of quantum spectra and of classical trajectories in the arc region. It shows an interesting connection between the bunching pattern observed in the spectrum of  $0^+$  states along the arc and properties of classical trajectories in the corresponding energy region (just above zero absolute energy), see also ⟨11⟩ and {5}.

The low-energy part of the spectrum along a line crossing the Casten triangle in a perpendicular direction to the arc on the side of the deformed phase has been analyzed in terms of the bosonic mean field in the paper ⟨2⟩. This analysis clearly identified the presence of the quasi dynamical symmetry based on the SU(3) exact symmetry in a wide range of the triangle. The quasi dynamical symmetry is therefore not an explanation of the arc, through it might be relevant as far as the mixing properties of the mean field states are considered. It seems that the behavior of the low-energy part of the spectrum, where the quasi-SU(3) symmetry is present, and the part above the zero absolute energy, where the origin of the increased regularity is located, must be treated separately.

The research along these lines is going on. Note that a useful tool for the quantification of order and disorder in the excited spectrum is represented by Peres lattices, see ⟨6, 2⟩ and {2}.

## 4 Quantum shape phase transitions in the interacting boson model

Relevant publications of the author:

Papers in journals:  $\langle 1, 5, 7, 14, 19, 23, 25, 26, 27, 28, 30, 35, 36, 38 \rangle$

Papers in proceedings:  $\{ 7, 9, 10 \}$

The concept of a quantum phase transition was introduced in the mid 1970's independently in the framework of solid-state physics [18] and nuclear physics [19,20], although in nuclear physics similar ideas can be traced back to the early 1960's. In the context of solid state physics, a QPT usually connects ordered and disordered ground-state configurations and is induced by a varying interaction strength in the system's quantum Hamiltonian. Since the transition happens at zero temperature, the only fluctuations responsible for the onset of disorder are quantum fluctuations.

In nuclear physics, a QPT also affects the ground state and is also driven by the Hamiltonian parameter, but it is not necessarily a transition of the order-disorder type. More commonly it is a transition between two types of order, represented by two incompatible (quasi) dynamical symmetries. This is so, for instance, in the interacting boson model, where the QPT connects spherical and deformed shapes (prolate, oblate, or unstable against the onset of triaxiality) of the nuclear ground state [21]. These shape phases can be associated with individual dynamical symmetries of the IBM.

Whereas the QPT field in solid-state physics has been continuously growing [23,24], in nuclear physics the discovery of the basic IBM phase structure [37,38] was followed by a decade of saturation. The topic was reopened at the beginning of the 1990's [39,40] and sort of exploded with the advent of the new millennium after the introduction of so-called critical-point symmetries in the framework of the geometric model [41,42].

In spite of apparent conceptual similarities of QPTs in solid-state and nuclear physics, there are also some important differences. While the solid-state QPT affect typically systems with an infinite number of degrees of freedom, the configuration space of nuclear (and many-body) models is usually limited. This has an important consequence: In order to make the infinite-size limit of the nuclear system possible, the Hamiltonian must be subject to a convenient scaling (affecting differently interactions of various orders), which ensures that the infinite-size limit of the nuclear system coincides with the classical limit (in contrast with the solid-state systems whose infinite-size limit is represented by the quantum field theory).

This naturally implies a considerable reduction of complexity (in nuclear models, the adequate tool for studying the QPTs is the mean field), but also brings some indispensable advantages. In particular, the association of the QPT phenomena with classical dynamics leads to a deeper insight into the origins of non-analytic behavior and may help to disclose new views on the problem. Examples of such approaches will be discussed in Secs. 5 and 6. Here we focus on the works devoted to the phase structure of the IBM and its various extensions, and to some direct signatures of the QPT behavior.

The origin of our interest in the QPT problem can be seen already in papers  $\langle 38 \rangle$  and  $\langle 36 \rangle$ . In  $\langle 36 \rangle$  we argued against the statement made in the literature that the nucleus  $^{152}\text{Sm}$  exhibits coexisting vibrational and rotational structures due to the existence of two minima of the IBM potential in the first-order QPT region. As a matter of fact, the potential associated with the standard IBM Hamiltonian has only a very small barrier separating

both minima (in a sense, its first-order QPT is not a generic one), so the coexisting spectral structures cannot be interpreted in such a straightforward way.

A more systematic study of the first-order QPT in the IBM was presented in the paper ⟨35⟩ (and briefly in {10}). Here we studied how wave functions of the ground-state and low excited states change in the transitional region and analyzed relations of this evolution to the dynamics caused by avoided crossings of levels with the same symmetry quantum numbers. This paper introduced several problems which we have followed in the forthcoming works. For instance, we noticed an increased density of the low-energy spectrum in the QPT region and investigated its relation to the accumulation of so-called exceptional (branch) points [43–46] of the complex extended Hamiltonian (see Sec. 5). We also studied the level dynamics of the whole spectrum (in an unfolded form) and pointed out some dynamical consequences of “laminar” and “turbulent” flows of levels.

An important step in the description of shape-phase transitions in nuclei was the application of the classical Landau theory of phase transitions [47, 48]. Indeed, this can be done since, as mentioned above, the QPTs in finite many-body models are fully described by the mean-field theory. It is shown in the paper ⟨28⟩ that the phase diagram of nuclear quadrupole deformations with spherical, deformed-prolate, and deformed-oblate equilibrium shapes is the simplest realization of the Landau-type structure, an experimental example of which was unknown at the Landau time. In ⟨28⟩ (which is briefly reflected also in ⟨26⟩ and {7}) a special attention is paid to the triple-point of the phase diagram in between spherical, prolate, and oblate phases. Note that another realization of the triple point in terms of an IBM Hamiltonian with three-body interactions was later presented in the paper ⟨19⟩. A contemporary overview of the work on the Landau theory (along with further extensions of the thermodynamic analogy, as discussed in Sec. 5) can be found in ⟨25⟩. An interpretation of the O(6) dynamical symmetry as a prolate-oblate transitional point is drawn in {9}.

Interesting results were obtained by the application of the Landau theory in the IBM with external rotation, i.e., in the so-called cranking approach. The paper ⟨27⟩, which is a continuation of an earlier analysis presented in ⟨30⟩ (based on the method introduced in Refs. [49]), shows that the phase diagram of the cranked IBM is analogous to the well-known phase diagram of a superconductor in a magnetic field. While the field intensity is equivalent to the cranking rotational frequency (its increase eventually leads to a transition from spherical to deformed shapes, in analogy with the field-induced transition from the superconducting to normal phase in a superconductor), the role of temperature is played by the Hamiltonian control parameter governing the transition between spherical and deformed shapes. It should be mentioned that an earlier application of the Landau theory in hot rotating nuclei [50] led to a different phase diagram because of more restrictive assumptions on the cranking term in the rotating-frame Hamiltonian. Possible experimental consequences of these findings for rotation-driven spherical-deformed transitions were discussed in the paper ⟨23⟩.

An analysis of ground-state phase transition in a family of models of the IBM type was presented in the paper ⟨14⟩. Here, two-level boson models with the scalar  $s$ -boson supplemented by a non-scalar  $b$ -boson with  $k$  components were considered. If the  $b$  boson has a spherical tensorial character with angular momentum  $\lambda$  (the number of components  $2\lambda + 1$  being an odd number), the lowest values of  $\lambda$  yield the Lipkin model ( $\lambda = 0$ ), three-dimensional vibron model of molecular physics ( $\lambda = 1$ ), and the nuclear interacting boson model ( $\lambda = 2$ ). On the other hand, if the number of components of the  $b$ -boson is even (which can be formally expressed by a half-integer value of  $\lambda$ ), one obtains the two-dimensional

molecular vibron model ( $\lambda = \frac{1}{2}$ ) and a four-dimensional bosonic model ( $\lambda = \frac{3}{2}$ ) with no immediate application known at the moment. As shown in ⟨14⟩, the phase structure of all these models can be treated on a unified ground. A simple consequence of this treatment is the finding that the first-order phase transition between “spherical” and “deformed” phases (defined through condensate ground states in pure  $s$  and mixed  $s + d$  bosonic configurations, respectively) can occur only for  $\lambda = 0, 2, 4, \dots$ . Therefore, the Lipkin model (in its parity non-conserving form) and the nuclear IBM are the simplest models where this transition (which is important from the fundamental point of view) is present.

Finally, I would like to mention review articles ⟨7⟩ and ⟨1⟩ devoted to the wide topic of nuclear shape phase transitions (a brief overview also presented in ⟨5⟩).

## 5 Thermodynamical analogy for quantum phase transitions

Relevant publications of the author:

Papers in journals: ⟨1, 7, 10, 20, 25, 34⟩

Papers in proceedings: {6}

The term quantum phase transition reflects an obvious relation to classical, thermal phase transitions. Indeed, in both cases we deal with a certain kind of discontinuity, when an infinitesimal variation of a parameter (temperature or an interaction strength) causes finite changes in the system’s properties. If we formally associate the thermodynamical free energy with the ground-state energy, we obtain the standard Ehrenfest classification of quantum phase transitions, or at least the distinction between first-order and continuous QPTs if the Ehrenfest classification is not applicable. However, how far can the analogy between quantum and classical phase transitions be followed?

The answer to this question is searched in the series of works discussed in this section. My interest in this topic goes back to the paper ⟨34⟩, in which we studied effects of random fluctuations of the IBM control parameter close to the first-order quantum critical point. The fluctuations were considered in a static way—as a certain narrow distribution of the control parameter around its mean value [51]. Instead of wave functions associated with individual eigenstates at a fixed parameter value one deals with density matrices, determined by the properties of the Hamiltonian in the selected parameter range and by the distribution of the parameter in this range. Any of these density matrices can be viewed as a canonical density matrix associated with a fictitious thermal ensemble. This association makes it possible to define an effective temperature, entropy and also the specific heat of the fictitious system. Interestingly, the behavior of these quantities turns out to be very sensitive to the presence of a QPT in the original system. In fact, this procedure can be used to identify an unknown QPT in a general system.

The method introduced in ⟨34⟩ was extended in the paper ⟨25⟩ to the line of a second-order QPT in the IBM. Some alternative definitions of the QPT “specific heat” were considered as well, among them those based on an association of the thermodynamical entropy with a wave-function entropy in an convenient basis.

A more fundamental program has been initiated in the paper ⟨20⟩ (and in the contribution {6}). It is based on a sophisticated approach to thermodynamical phase transitions formulated in a seminal work by Yang and Lee [52], in which the thermodynamical non-



analyticity is seen through zeros of the partition function in a complex-extended variable, e.g., temperature [53]. An infinitesimal approach of these zeros to the real axis in the thermodynamical limit of the system creates a phase transition on the corresponding plane, the type of this transition being dictated by the distribution of zeros in its complex vicinity.

We have anticipated that the role of complex singularities for quantum phase transitions is played by the degeneracies of Hamiltonian eigenvalues in the complex-extended parameter plane (such degeneracies are sometimes called exceptional points [43–46], in contrast to so-called diabolical points which are mostly related to real values of control parameters). Indeed, energy degeneracies of levels with the same symmetry quantum numbers do not typically occur on the real parameter axis (due to so-called no-crossing theorem), but in the complex parameter plane (where the Hamiltonian is not Hermitian) such degeneracies are present. However, there exists a serious obstacle in building a full analogy between complex energy degeneracies and complex zeros of partition function: Complex degeneracies, unlike the zeros, are sorted to individual Riemann sheets associated with the ordering of the levels on the real axis (each degeneracy represents a branch point between a pair of such Riemann sheets). While it is feasible (though difficult) to determine positions of individual complex degeneracies, it is practically impossible (for higher dimensions) to assign them to the corresponding Riemann sheets. But for a ground-state QPT only the degeneracies associated with the ground-state Riemann sheet are relevant.

The way out of this problem was found by creating a certain indirect measure reflecting the density of branch points in a vicinity of the real axis on a selected Riemann sheet. As shown in paper ⟨10⟩, this measure can be constructed from an electrostatic analogy, associating each complex degeneracy with a charge in a plane and evaluating a gradient of the Coulomb force on the real axis. Such a measure neatly reflects the QPT behavior and is capable to distinguish its type. This is so because the measure turns out to represent a direct analog of the specific heat from standard thermodynamics.

The method was tested in the IBM’s second-order QPT ⟨20, 10⟩, where it gives a perfect agreement with a thermal phase transition of the second order. Note that in ⟨10⟩ we have considered not only the ground-state Riemann sheet, but also one associated with an excited state, which exhibits an excited-state QPT, as discussed in Sec. 6. The behavior in this case is consistent with a continuous phase transition with no Ehrenfest order.

Review articles ⟨7⟩ and ⟨1⟩ contain brief résumé of these efforts.

There are still a lot of things to be done (and the work in this field continues), but the above-outlined approach represents a promising way of relating the physics of quantum and classical phase transitions. It may once lead to a more detailed understanding of the underlying mechanisms that drive the systems to develop quantum critical points.

## 6 Quantum phase transitions for excited states

Relevant publications of the author:

Papers in journals: ⟨1, 7, 8, 9, 10, 16, 17, 18⟩

Papers in proceedings: { 1, 3, 4 }

The QPT idea was always considered in connection with systems at zero temperature. Recall that the first word in the term “quantum phase transition” originally referred to quantum zero-point motions playing an important role in the QPT dynamics [18] and that

a name proposed in parallel was “ground-state energy phase transitions” [22]. However, the phenomenon can be easily generalized also to excited states. Clearly, if a ground-state QPT represents a non-analyticity in the evolution of the ground-state energy, an excited-state quantum phase transition (ESQPT) would mean a non-analyticity in the evolution of a single excited state or a certain set of excited states. It turns out that such phenomena indeed exist and play an important role in numerous many-body systems. An interesting question accompanying the above simple extension is how the ESQPT non-analyticity affects the dependence of the spectrum on energy.

ESQPTs are similar and, in fact, very closely related to thermal phase transitions. An important difference is that the former phenomenon is investigated as a function of excitation energy instead of temperature, i.e., in the microcanonical framework instead of the canonical one. In addition, in the systems with a finite number of degrees of freedom a connection with classical dynamics in a finite phase space can be established (the infinite-size limit of the system coincides with the classical limit), which offers a possibility to better understand the origin of the ESQPT behavior. While the ground-state QPT is rooted in a discontinuous change of the system’s equilibrium, static properties, the excited-state QPT implies changes in dynamics—in the type of motions assigned to a given energy.

The first explicit thoughts in this direction came independently in Refs. [54–56], although many closely related ideas had been discussed long before these works (see, e.g., Ref. [57]). We opened this field when working on the papers ⟨18⟩ and ⟨17⟩. It turned out that the transition between  $O(6)$  and  $U(5)$  dynamical symmetries of the IBM is characterized by a rather coherent evolution of states with zero angular momentum and seniority, a quantum number associated with the underlying  $O(5)$  dynamical symmetry. The dynamics of these levels with the control parameter showed a bunching of states resembling a shock wave propagating through a one-dimensional gas of particles ⟨18⟩ (note that the eigenvalue dynamics of linear parameter-dependent Hamiltonians can be treated in terms of a Coulomb gas analogy [58,59]). In ⟨17⟩ (and briefly in {4}) we show that anomalous properties of the spectrum are connected with anomalous properties of classical trajectories. In attempting to explain this effect, we came to the notion of monodromy.

Monodromy is a topological effect present in the phase space of some two-dimensional integrable systems which prevents their full analytic description [60]. It is connected with the existence of a singular class of orbits and turns out to imply also some anomalous properties of the quantum spectrum. The effects of monodromy are known to be present in molecules [61], but in paper ⟨17⟩ we have realized that they are also relevant in the IBM within its integrable regime along the  $O(6)$ - $U(5)$  transition. In particular, monodromy in this regime is linked to the bunching pattern observed in the spectrum. The subsequent paper ⟨16⟩ (and the contribution {3}) show that the non-analyticity associated with monodromy is connected with a non-analytic evolution of individual quantum states across the  $O(6)$ - $U(5)$  transition and introduce the term excited-state QPT.

The paper ⟨9⟩ contains a detailed analysis of a wider class of models showing the same ESQPT effect. Among these models, there are vibron models of molecular physics as well as the Lipkin model and a simple nuclear pairing model. A common factor of all these models is the presence of a second-order critical point at the ground-state level, which extends in excitation energy as a bunching of states propagating through the spectrum. The passage through the bunching implies a singular growth of the level density and, at the same time, a singular evolution of the curvature of each individual level as a function of the control parameter.

In the paper ⟨10⟩ we have shown that the ESQPT mechanism on the level of complex energy degeneracies is the same as for the ground-state QPT (see Sec. 5).

It turns out that the above-described singularity in the excited spectrum represents just one type of the ESQPT, probably the most spectacular one. Other types show up in a less radical way, as discontinuous or singular derivatives of the level density (rather than its infinite growth) accompanied by an anomalous flow within a bunch of levels (rather than non-analytic trajectories of individual levels). Examples of such evolutions have been given in the paper ⟨8⟩ and in the contribution {1}. Of a particular interest was the ESQPT pattern accompanying the first-order ground-state QPT. Indeed, it turned out that close to the first-order critical point very distinctive structures appear in the excited spectrum. These structures are most pronounced in one-dimensional cases (tested by a model based on a quartic, so-called cusp potential) and become less dramatic as the dimension increases (as demonstrated by a two-dimensional collective model).

Apart from presenting specific examples, the paper ⟨8⟩ introduces some general equations which can be used to classify the ESQPT phenomena in other situations. The topic is also put into a broader perspective in the review ⟨7⟩ (and briefly in ⟨1⟩). To identify similar effects in other systems (e.g., in the field of quantum optics) is now of crucial importance since there exist dynamical consequences of the ESQPT behavior which may become relevant in the context of quantum information technologies.

## 7 Résumé

The topics discussed in this dissertation can be compressed into a more economic form under the following two titles: (A) Chaos and the search of approximate symmetries in quantum systems, and (B) Quantum phase transitions in many-body systems. The title (A) covers the topics elaborated in Secs. 2 (classical and quantum chaos in the geometric collective model) and 3 (chaos and hidden symmetries in the interacting boson model), while title (B) includes the topics in Secs. 4 (quantum shape phase transitions in the interacting boson model), 5 (thermodynamical analogy for quantum phase transitions), and 6 (quantum phase transitions for excited states). There is a conceptual link between (A) and (B) since in many-body models ordered dynamics as well as quantum phases of the system are believed to follow from some internal symmetries. The work is in progress to yield a more general theory of these issues. Here is a short summary of our results:

(A) We have studied classical and quantum chaos in the geometric collective model (GCM) and in the interacting boson model (IBM), showing that these models hide enormous complexity of competing regular and chaotic properties. We have used the standard measures of chaos as well as some non-standard ones (Peres lattices) and argued that both models can serve as testing grounds for general studies devoted to chaotic features. We have confirmed the validity of the Bohigas conjecture in these systems, emphasizing a strong dependence of the degree of chaos on energy and control parameters and using different quantization schemes within the GCM. We have probed several types of approximate or exact symmetries associated with the IBM and tested their relation to regular and chaotic features. An ultimate task of these efforts is to establish an unambiguous relation between order (absence of chaos) and symmetry (in any of its incarnations discussed in recent literature, such as hidden dynamical symmetry, partial dynamical symmetry, quasi dynamical symmetry etc.).

(B) We have studied quantum phase transitions (QPTs) in several many-body models

with a finite number of degrees of freedom, in particular in algebraic bosonic models and in some generalized geometric models, which exhibit both first-order and second-order QPTs for the ground state. We have described several types of phase diagrams, satisfying the classical Landau theory of phase transitions, and focused on the search of dynamical effects accompanying the QPT behavior of both types and on their underlying quantum mechanisms. A particularly promising perspective on the QPT problem is associated with the distribution of the Hamiltonian branch points in the complex-extended plane of the control parameter, which is related to the distribution of complex zeros of the partition function in thermal phase transitions. We have identified and described excited-state quantum phase transitions of several types that form an analog of thermal phase transitions translated into the microcanonical language. In finite many-body models, the ESQPTs are linked to anomalies in classical dynamics (like monodromy or a phase-space separatrix) and, in contrast to the ground-state case, require a dynamical definition of phases.

This dissertation is based on 25 of my journal publications (out of the total number 49 journal papers and 20 conference proceeding contributions). In a wider context, 35 of my journal papers and 9 of my proceeding contributions are cited as giving a relevant entry to the topics discussed here.

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